Introduction to Accelerator Physics

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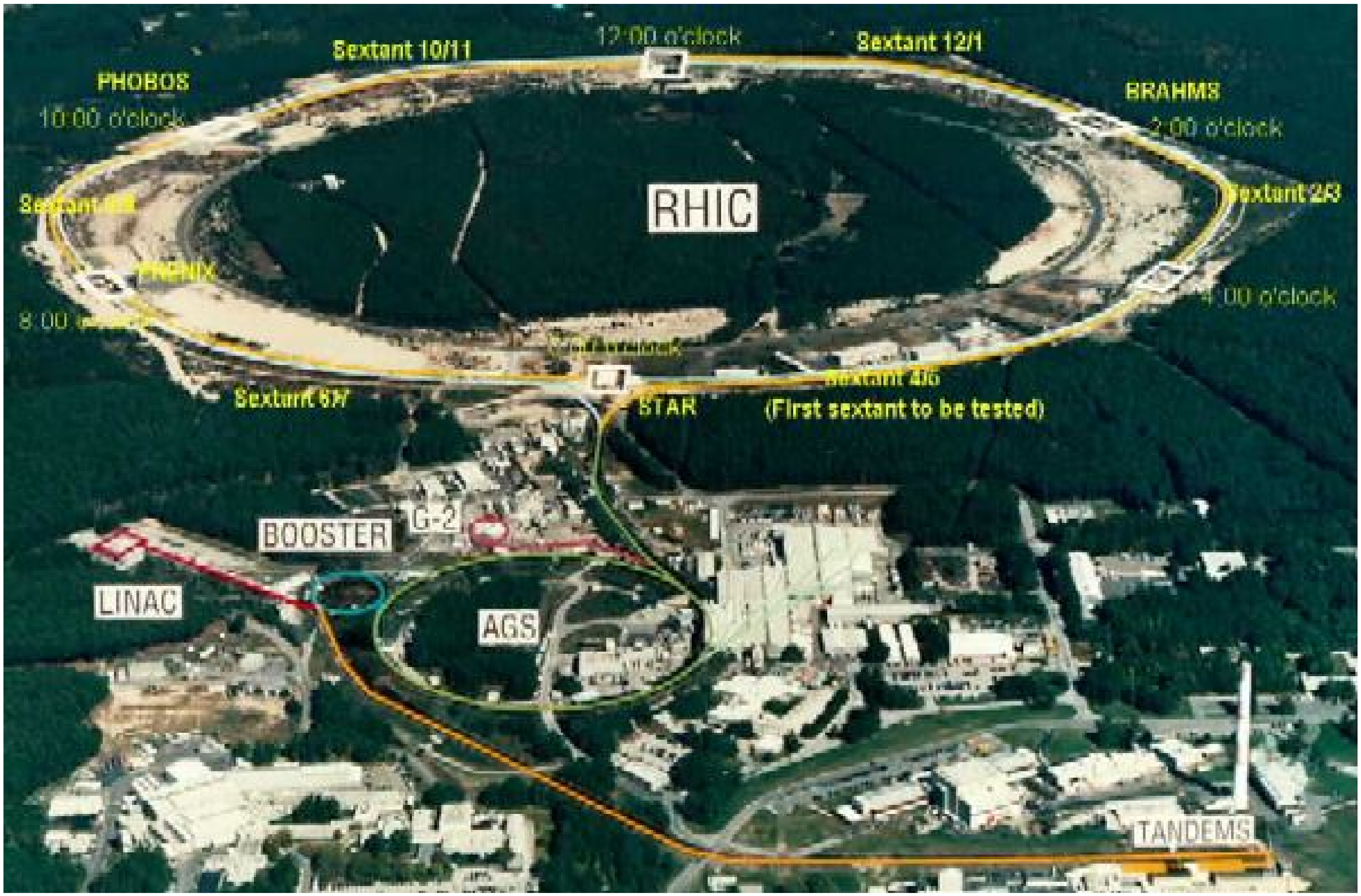
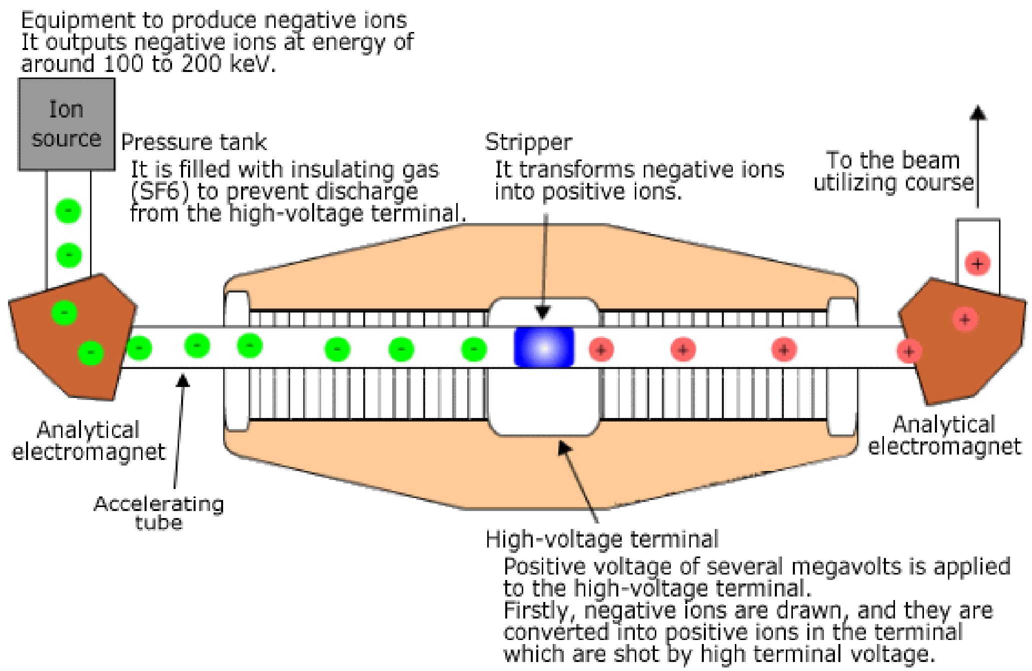
The RHIC complex



Tandem Van-de-Graaf



Electrostatic pre-accelerator up to a few MeV

Synchrotrons and storage rings

Synchrotrons (Booster, AGS):

- Circular machines used to rapidly accelerate particles to higher energies

- Typical cycle time: 1 sec

Storage rings (RHIC):

- Circular machines used to store beams over many hours

- May be used to slowly accelerate beams from injection to top energy in minutes

Storage ring

The Lorentz force

Charged particles are guided by magnetic fields, using the Lorentz force:

$$\vec{F} = q \cdot (\vec{v} \times \vec{B})$$

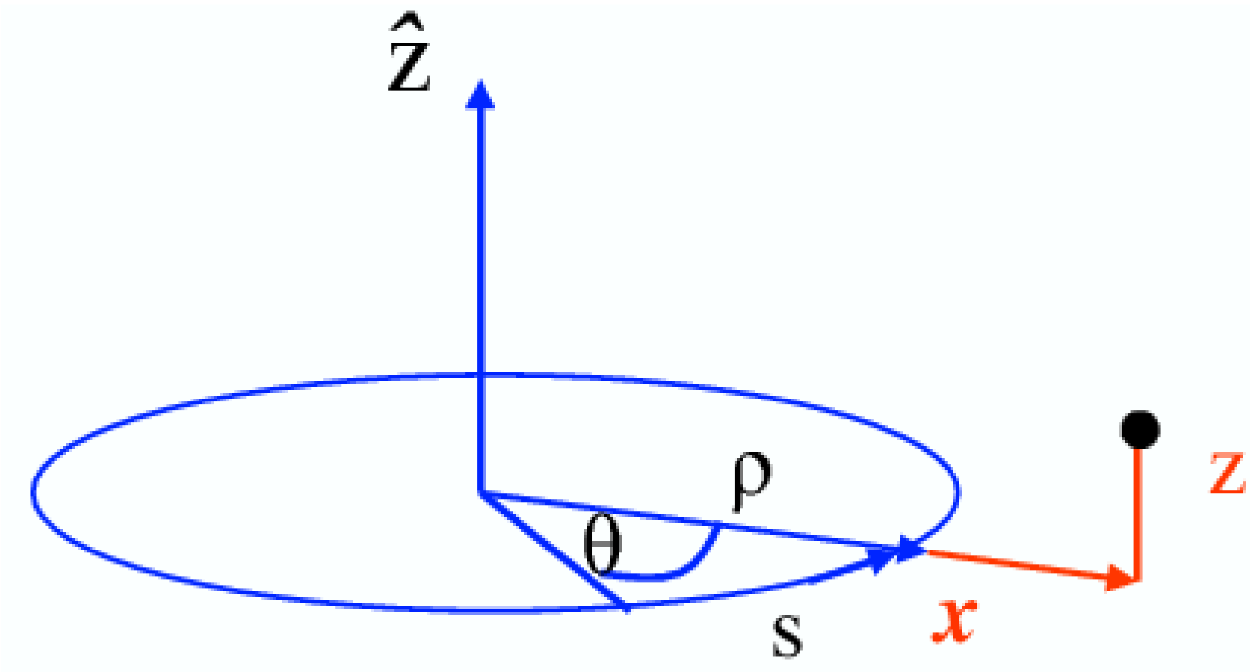
$\vec{F}$: force
q : electric charge of the particle
$\vec{v}$: particle velocity
$\vec{B}$: magnetic field

Vector equation - $\vec{F}$ is perpendicular to $\vec{v}$ and $\vec{B}$

Important consequence:
Magnetic fields can only deflect particles, but cannot change their velocity (or energy)

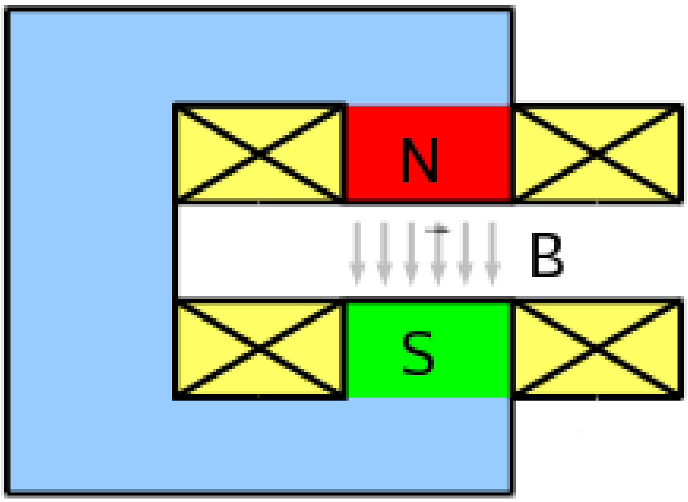
Coordinate system

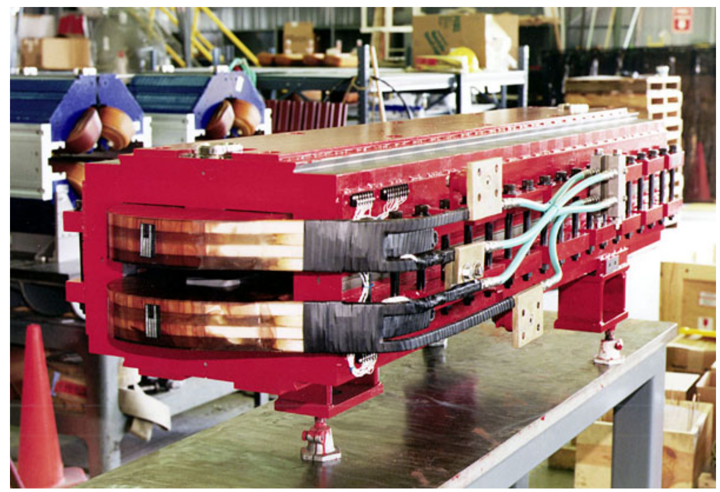
Particle motion is described in a co-moving reference frame:



Transverse coordinates x, z w.r.t. ideal (nominal) particle on the *closed orbit*

Beam deflection

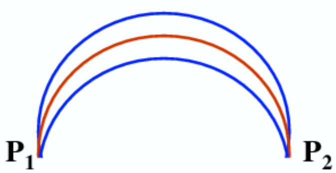




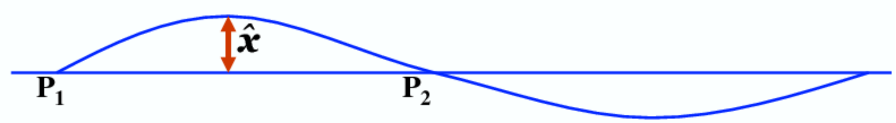
Dipole magnets bend the beam on a curved trajectory with constant radius $\rho = p/(q \cdot B)$

Weak focusing

In a homogeneous dipole field, all particles travel on circles with slightly different centers depending on initial particle direction:



„Geometric focusing" in a homogeneous field: consider three particles, starting at the same point with different angles



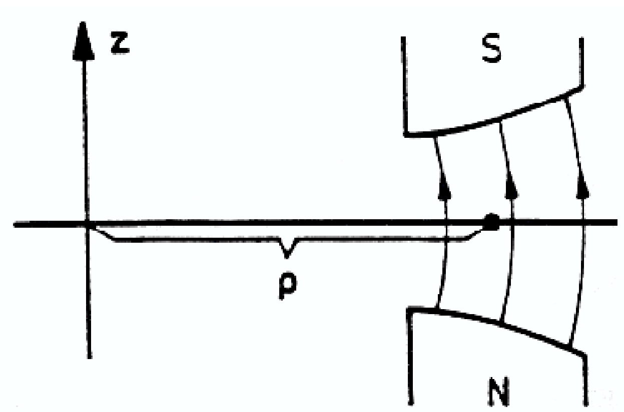
Problem: amplitude of betatron oscillation in this case $\hat{x} \approx \alpha * \rho$

$$\left.\begin{array}{l} \alpha \approx 1\ mrad \text{ for a particle beam} \\ \rho \approx \text{several } 100m \end{array}\right\} \quad \hat{x} \approx 1\ m$$

Geometric focusing in the horizontal plane

The vertical plane

Without vertical focusing, particles inevitably spiral out of the horizontal plane
Solution: Provide a restoring force $F_z \propto z$



Modified pole faces provide horizontal field component $B_x(z) = z \cdot \mathrm{d}B/\mathrm{d}z = z \cdot \mathrm{const.}$

$\Rightarrow$ Restoring force $F_z = q \cdot v \cdot z \cdot \mathrm{d}B/\mathrm{d}z$ (harmonic oscillator)

Summary of weak focusing

- Simultaneous bending and focusing by combined-function magnets (dipoles with modified pole face shape)

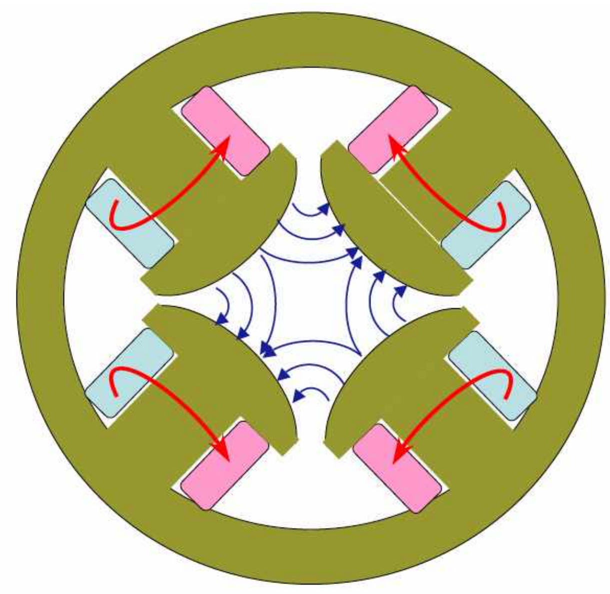
- Typical beam size: 1 m

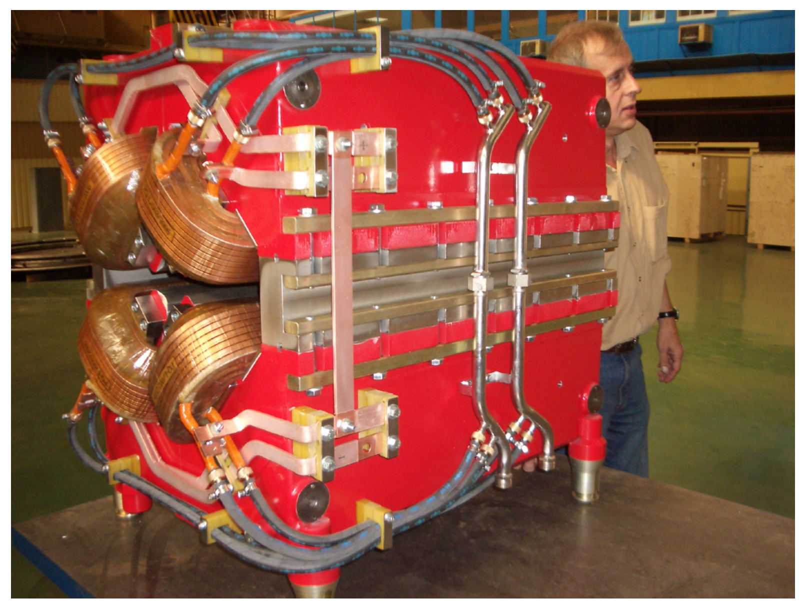
- Requires large vacuum chambers (beam pipes) that become more and more impractical in larger machines

Remedy:
Separate bending and focusing functions (= "strong focusing")

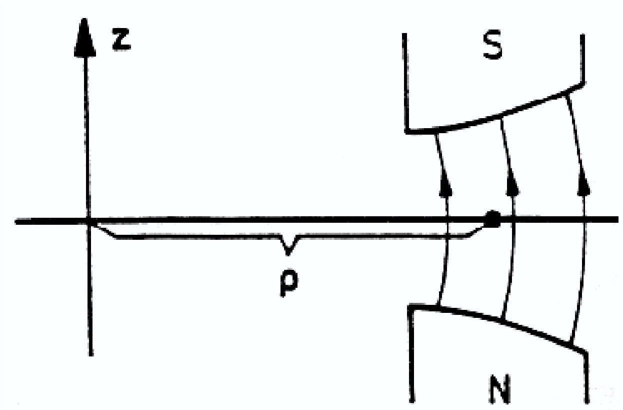
Strong focusing

Quadrupole magnets focus the beam in one plane, and de-focus in the other





Equations of motion



Restoring force F for a particle at position $\rho + x$ is the difference between the centripetal force F_c and the Lorentz force F_L at this location:

$$\begin{aligned} F = m\ddot{x} &= F_c - F_L \\ &= \frac{mv^2}{\rho + x} - q \cdot v \cdot B_z(\rho + x) \end{aligned}$$

Approximating (Taylor expansion)

$$B_z(\rho + x) = B_z(\rho) + \left.\frac{\partial B_z}{\partial x}\right|_{\rho} \cdot x + \ldots$$

and

$$\frac{1}{1+\frac{x}{\rho}} \approx 1 - \frac{x}{\rho},$$

we get

$$\begin{aligned} m\ddot{x} &= \frac{mv^2}{\rho\left(1+\frac{x}{\rho}\right)} - qv\left(B_z\Big|_{\rho} + \left.\frac{\partial B_z}{\partial x}\right|_{\rho} \cdot x + \ldots\right) \\ &= -mv^2\frac{x}{\rho^2} - qv\left.\frac{\partial B_z}{\partial x}\right|_{\rho} \cdot x \end{aligned}$$

Substituting the time t by the longitudinal coordinate $s = v \cdot t$ and defining $\frac{\mathrm{d}^2 x}{\mathrm{d}s^2} = x''$, and $\frac{q}{m \cdot v} \frac{\partial B_z}{\partial x} = k$:

$$\boxed{x'' + \left(\frac{1}{\rho^2} + k\right) \cdot x(s) = 0}$$

In the vertical plane, $1/\rho = 0$ (no vertical bends):

$$\boxed{z'' - k \cdot z = 0}$$

The opposite sign in front of the quadrupole strength k reflects the opposite focusing in the two planes (focusing vs. de-focusing)

Solving the equations of motion inside a quadrupole

Example:

Horizontally defocusing quadrupole,

$$
\begin{aligned}
k &> 0 \\
\frac{1}{\rho} &= 0
\end{aligned}
$$

Educated guess (horizontal plane only):

$$
\begin{aligned}
x(s) &= A\cosh(\sqrt{k}s) + B\sinh(\sqrt{k}s) \\
x'(s) &= \sqrt{k}A\sinh(\sqrt{k}s) + \sqrt{k}B\cosh(\sqrt{k}s)
\end{aligned}
$$

Using the initial conditions $x(0) = x_0$, $x'(0) = x'_0$:

$$\begin{aligned} x(s) &= x_0 \cosh(\sqrt{k}s) + \frac{x'_0}{\sqrt{k}} \sinh(\sqrt{k}s) \\ x'(s) &= x_0 \sqrt{k} \sinh(\sqrt{k}s) + x'_0 \cosh(\sqrt{k}s) \end{aligned}$$

In matrix notation:

$$\begin{aligned} \begin{pmatrix} x(s) \\ x'(s) \end{pmatrix} &= \begin{pmatrix} \cosh(\sqrt{k}s) & \frac{1}{\sqrt{k}} \sinh(\sqrt{k}s) \\ \sqrt{k} \sinh(\sqrt{k}s) & \cosh(\sqrt{k}s) \end{pmatrix} \cdot \begin{pmatrix} x_0 \\ x'_0 \end{pmatrix} \\ &= M_{\mathrm{QD}} \cdot \begin{pmatrix} x_0 \\ x'_0 \end{pmatrix} \end{aligned}$$

Matrices for the vertical plane (remember the opposite sign!), and for other elements (field free drifts, dipoles, focusing quadrupoles), are derived in a similar fashion

To describe an accelerator beamline, we multiply the matrices of its constituing elements in the correct order:

$$M_{\text{beamline}} = M_1 \cdot M_2 \cdot M_3 \cdot \ldots,$$

with

$$\begin{aligned} M_i &= M_{\text{QD}}, M_{\text{QF}}, M_{\text{drift}}, M_{\text{dipole}}, \ldots, \\ i &= 1, 2, 3, \ldots \end{aligned}$$

Stability condition in a circular accelerator:

$$|\text{Tr}(M)| < 2$$

Twiss parameters

So far, we have described a behavior of a single particle, by deriving and solving its equation of motion.
Now, we want to describe the collective behavior of a beam of many particles.

Define generalized focusing strength

$$K(s) = \begin{cases} \frac{1}{\rho(s)^2} + k(s) & \text{(horizontal)} \\ -k(s) & \text{(vertical)} \end{cases}$$

Using the generalized focusing strength, the equations of motion can be re-written as

$$y''(s) + K_y(s)y(s) = 0, \; y = x, z$$

Ansatz:

$$y(s) = A_y \cdot u_y(s) \cos[\Psi_y(s) + \Phi_y]$$

Inserting in differential equation yields two expressions that must be fulfilled simultaneously:

$$\begin{aligned} u_y'' - u_y \Psi_y'^2 + K_y(s) u_y &= 0 \\ 2u_y' \Psi_y' + u_y \Psi_y'' &= 0 \end{aligned}$$

The second of these equations can be integrated directly:

$$\Psi_y(s) = \int_0^s \frac{\mathrm{d}\sigma}{u_y^2(\sigma)}$$

Inserting the expression for $\Psi(s)$ in the first equation yields

$$u_y'' - \frac{1}{u_y^3} + K_y(s) u_y = 0,$$

and with the substitutions

$$\begin{aligned} \beta_y(s) &= u_y^2(s) \\ A_y &= \sqrt{\epsilon_y} \end{aligned}$$

finally

$$y(s) = \sqrt{\epsilon_y \cdot \beta_y(s)} \cos[\Psi_y(s) + \Phi_y]$$

The amplitude of this oscillation (the beam size) is therefore

$$\hat{y}(s) = \sqrt{\epsilon_y \cdot \beta_y(s)}$$

"Emittance" ϵ_y is a beam parameter, while $\beta_y(s)$ describes the accelerator optics.

Summary of strong focusing

- Separate bending and focusing magnets

- Alternate focusing and defocusing quadrupoles (Alternating Gradient Synchrotron - AGS)

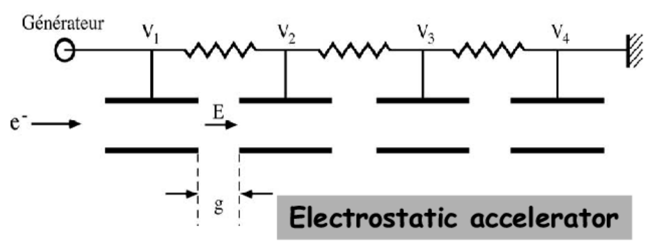
- Typical beam size: 1 mm to 1 cm

- Beam optics described by matrix multiplication

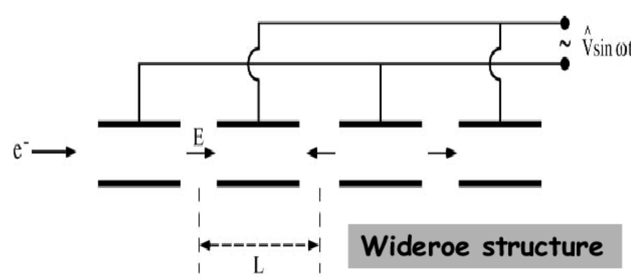
Longitudinal dynamics

How does an accelerator accelerate the beam?

Magnetic fields only deflect the beam, but electric fields can change the beam energy



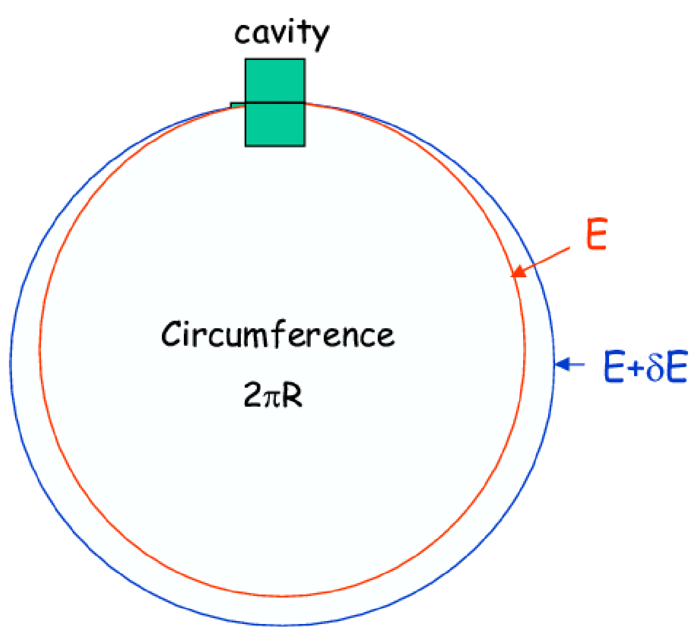
Electrostatic accelerator



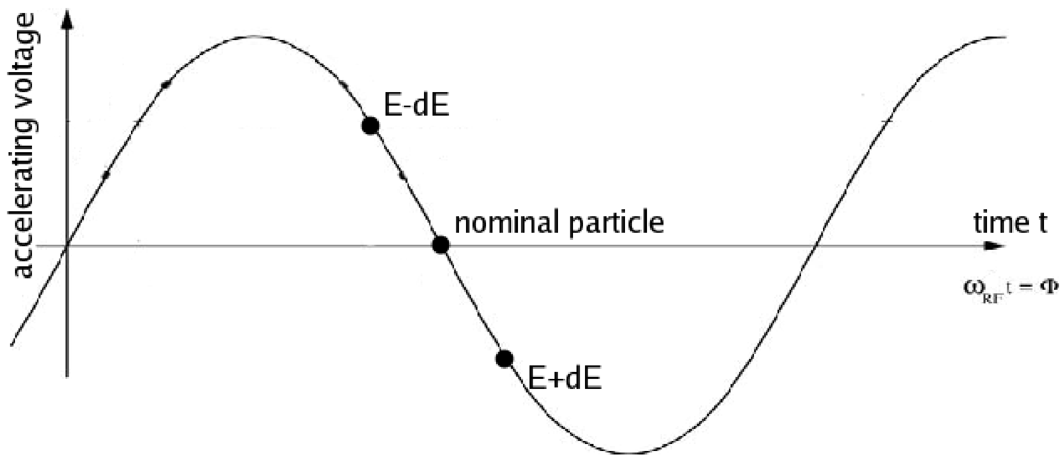
Wideroe structure

While the particle is inside the field-free drift tube, the polarity changes

A highly relativistic particle ($v \approx c$) with an energy $E + \delta E$ is heavier ($E + \delta E = (m + \delta m)c^2$) then the nominal particle at energy E, and therefore travels at a larger radius $R + \delta R$.



Since the pathlength (circumference) $2\pi(R + \delta R)$ at this larger radius is larger while the velocity v is practically unchanged, the particle arrives late at the accelerating section (cavity).



At fixed energy (no acceleration):

• the nominal particle receives no longitudinal kick, so its energy E remains unchanged
• a particle with higher energy $E + \delta E$ arrives at a later time, receives a negative kick, and gets decelerated
• a particle with lower energy $E - \delta E$ arrives early, receives a positive kick, and gets accelerated

To accelerate the entire beam, gently increase the dipole field to reduce the path length, so all particles arrive early and get accelerated

Luminosity

Experimenters want to observe as many interesting "events" per unit time as possible. What does that mean?

Probability to throw a "6" with a single die: $P(6) = 1/6$

Total number of "sixes" after one hour:
$N(6) = P(6) \cdot [\text{number of dice}] \cdot [\text{number of throws per hour}]$

In archery, the number of hits per hour on the target depends on:

- the number of arrows shot during that hour
- the archer's ability to aim
- the size of the target
- the number of targets

In a collider, the “event rate” (=number of “events” per second) is

$$\begin{aligned}\dot{N} &= \frac{1}{4\pi}\frac{f \cdot N_1 \cdot N_2}{\sigma^2} \cdot \sigma_{\mathrm{event}} \\ &= L \cdot \sigma_{\mathrm{event}}\end{aligned}$$

The higher the luminosity L, the more interesting events per second

Reminder: $\sigma^2 = \epsilon \cdot \beta$

Note:

In nuclear physics, cross sections σ_{event} are measured in barns, with

$$1\,\mathrm{barn} = 10^{-24}\,\mathrm{cm}^2$$

Thank you!

Brain teaser

Like all matter, the beam particles are subject to gravity.

Why doesn’t the beam simply drop to the bottom of the beam pipe?