Studying 3D structure of proton with neural networks

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[JHEP07(2011)073,arXiv:1106.2808[hep-ph]]

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Outline

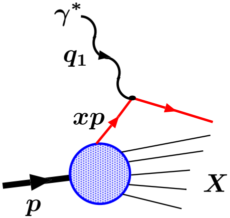
Introduction to Generalized Parton Distributions (GPDs) and Deeply Virtual Compton Scattering (DVCS)

Model-dependent global analysis of unpolarized target DVCS data

Neural networks approach

Parton distribution functions

- Deeply inelastic scattering, $-q_1^2 \to \infty$, $x_{BJ} \equiv \frac{-q_1^2}{2p \cdot q_1} \to \mathrm{const}$



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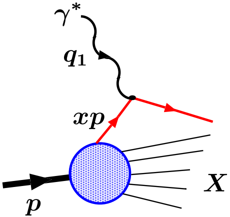
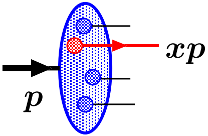
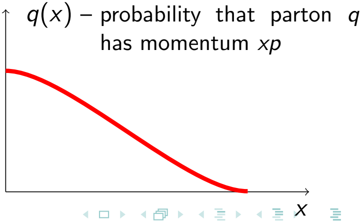
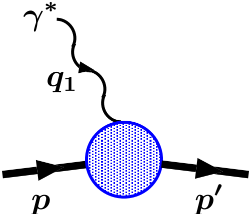






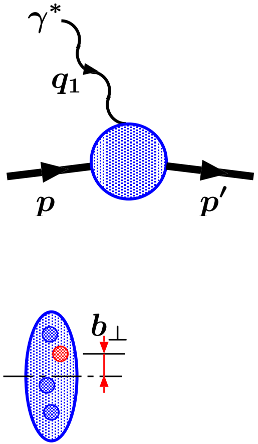
Electromagnetic form factors



- Dirac and Pauli form factors:

$$F_{1,2}(t = q_1^2)$$

Electromagnetic form factors



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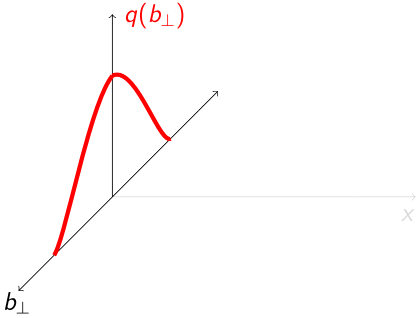
$$q(b_\perp) \sim \int \mathrm{d}q_1 \, e^{-iq_1 \cdot b_\perp} F_1(t = q_1^2)$$



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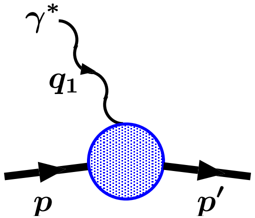

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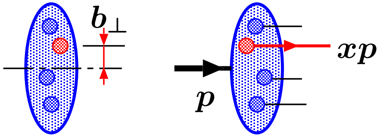

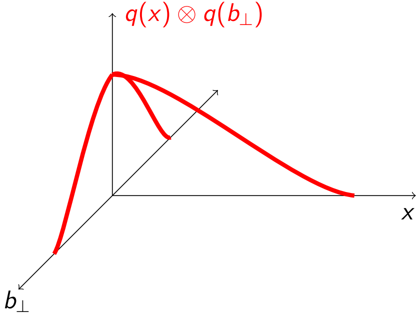

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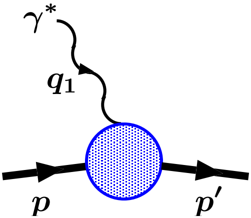

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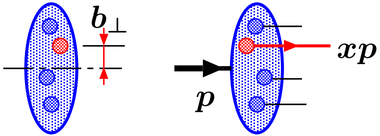

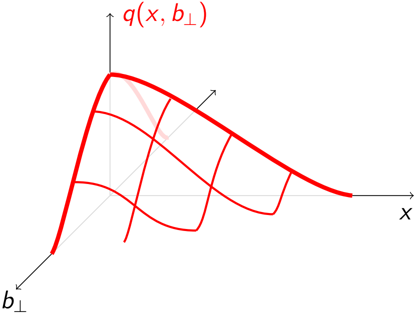

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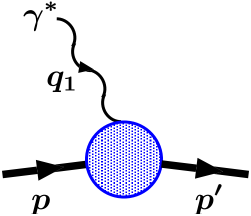

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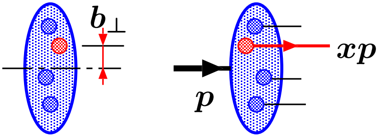
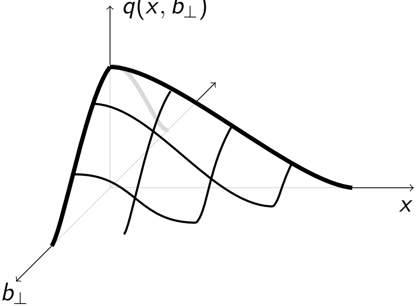
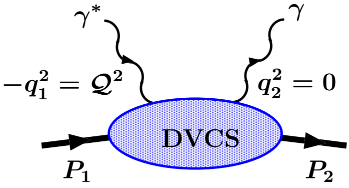




- "skewless" GPD: $H^q(x, 0, t = \Delta^2) = \int \mathrm{d}b_\perp\, e^{i\Delta\cdot b_\perp} q(x, b_\perp)$

Probing the proton with two photons

- Deeply virtual Compton scattering (DVCS) [Müller '92, et al. '94]



$$P = P_1 + P_2\ , \qquad t = (P_2 - P_1)^2$$

$$q = (q_1 + q_2)/2$$

Generalized Bjorken limit:

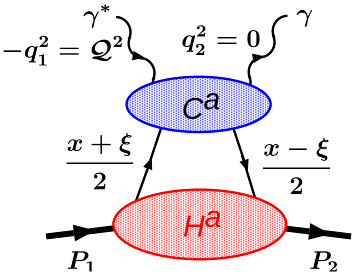
$$-q^2 \simeq \mathcal{Q}^2/2 \to \infty$$

$$\xi = \frac{-q^2}{2P\cdot q} \to \text{const}$$

- To leading twist-two accuracy cross-section can be expressed in terms of Compton form factors (CFFs)

$$\mathcal{H}(\xi, t, \mathcal{Q}^2), \mathcal{E}(\xi, t, \mathcal{Q}^2), \tilde{\mathcal{H}}(\xi, t, \mathcal{Q}^2), \tilde{\mathcal{E}}(\xi, t, \mathcal{Q}^2), \ldots$$

Factorization of DVCS $\longrightarrow$ GPDs



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- Compton form factor is a convolution:

$$^a\mathcal{H}(\xi, t, \mathcal{Q}^2) = \int \mathrm{d}x \; C^a(x, \xi, \mathcal{Q}^2/\mathcal{Q}_0^2)\; H^a(x, \eta = \xi, t, \mathcal{Q}_0^2)$$

a=NS,S(Σ,G)

- $H^a(x, \eta, t, \mathcal{Q}_0^2)$ — Generalized parton distribution (GPD)

Dispersion-relation access to GPDs at LO

[Teryaev '05; K.K., Müller and Passek-K. '07, '08; Diehl and Ivanov '07]

- LO perturbative prediction is "handbag" amplitude

$$\mathcal{H}(\xi, t, \mathcal{Q}^2) \stackrel{\mathrm{LO}}{=} \int_{-1}^{1} dx \, \left(\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon} \right) H(x, \xi, t, \mathcal{Q}^2)$$

- giving access to GPD on the "cross-over" line $\eta = x$

$$\frac{1}{\pi} \Im\mathrm{m}\mathcal{H}(\xi = x, t, \mathcal{Q}^2) \stackrel{\mathrm{LO}}{=} H(x, x, t, \mathcal{Q}^2) - H(-x, x, t, \mathcal{Q}^2)$$

- while dispersion relation connects it to $\Re\mathrm{e}\mathcal{H}$ and at the most one subtraction constant $\mathcal{C}_{\mathcal{H}} = -\mathcal{C}_{\mathcal{E}}$; $\mathcal{C}_{\tilde{\mathcal{H}}} = \mathcal{C}_{\tilde{\mathcal{E}}} = 0$

$$\Re\mathrm{e}\mathcal{H}(\xi, t, \mathcal{Q}^2) =$$
$$\frac{1}{\pi} \mathrm{PV} \int_{0}^{1} d\xi' \left(\frac{1}{\xi - \xi'} - \frac{1}{\xi + \xi'} \right) \Im\mathrm{m}\mathcal{H}(\xi', t, \mathcal{Q}^2) + \mathcal{C}_{\mathcal{H}}(t, \mathcal{Q}^2)$$

Model-dependent extraction of GPDs

- Revealing GPD H from DVCS on *unpolarized* proton target at LO [K.K. and D. Müller '09]
- Valence quarks model (ignoring $\mathcal{Q}^2$ evolution):

$$\Im\mathrm{m}\mathcal{H}(\xi,t) = \pi\left[\frac{4}{9}H^{u_{\rm val}}(\xi,\xi,t) + \frac{1}{9}H^{d_{\rm val}}(\xi,\xi,t) + \frac{2}{9}H^{\rm sea}(\xi,\xi,t)\right]$$

$$H(x,x,t) = n\, r\, 2^{\alpha}\left(\frac{2x}{1+x}\right)^{-\alpha(t)}\left(\frac{1-x}{1+x}\right)^{b}\frac{1}{\left(1-\frac{1-x}{1+x}\frac{t}{M^2}\right)^{p}}\,.$$

- Fixed: n (from PDFs), $\alpha(t)$ (eff. Regge), p (counting rules)

$$\alpha^{\rm val}(t) = 0.43 + 0.85\, t/\mathrm{GeV}^2 \quad (\rho,\ \omega)$$

- Sea partons modelled in conformal moment space + partial wave expansion + $\mathcal{Q}^2$ evolution

- $\Re\mathrm{e}\mathcal{H}$ determined by dispersion relations

$$\Re\mathrm{e}\mathcal{H}(\xi, t, \mathcal{Q}^2) = \frac{1}{\pi}\mathrm{PV}\int_0^1 d\xi' \left(\frac{1}{\xi-\xi'} - \frac{1}{\xi+\xi'}\right) \Im\mathrm{m}\mathcal{H}(\xi', t, \mathcal{Q}^2) - \frac{C}{\left(1-\frac{t}{{M_C}^2}\right)^2}$$

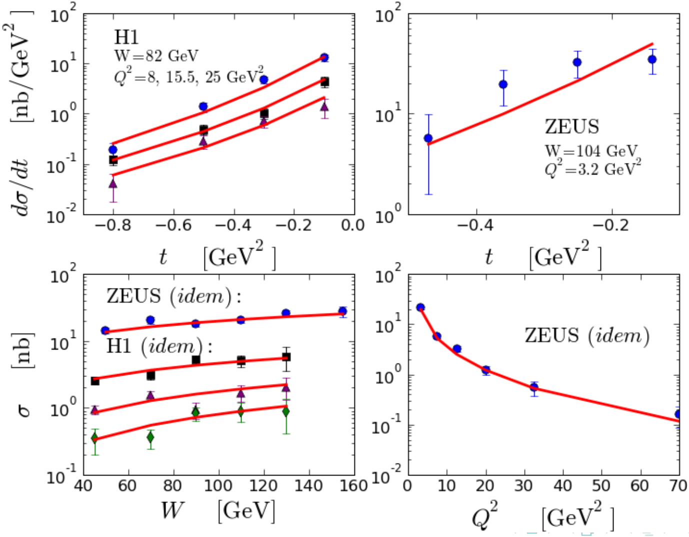
- Typical set of free parameters:

M_0^{sea}, s_{sea}, s_G	sea* quarks and gluons H
r^{val}, M^{val}, b^{val}	valence H
C, M_C	subtraction constant (H, E)
($\tilde{r}^{\mathrm{val}}$, $\tilde{M}^{\mathrm{val}}$, $\tilde{b}^{\mathrm{val}}$)	valence $\widetilde{H}$ (if needed)

- Global fit to 150–200 data points is fine: $\chi^2/d.o.f. \approx 1$

$^*s_{\mathrm{sea,G}}$ = strengths of subleading partial wave. LO evolution is included.

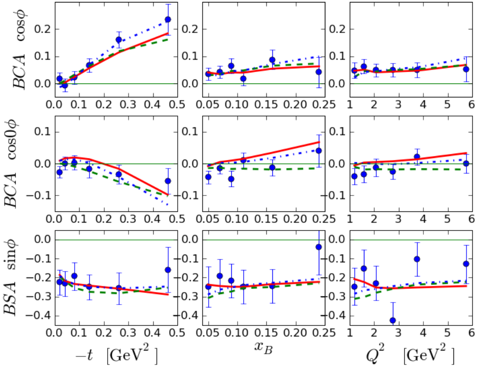
H1 (2007), ZEUS (2008)



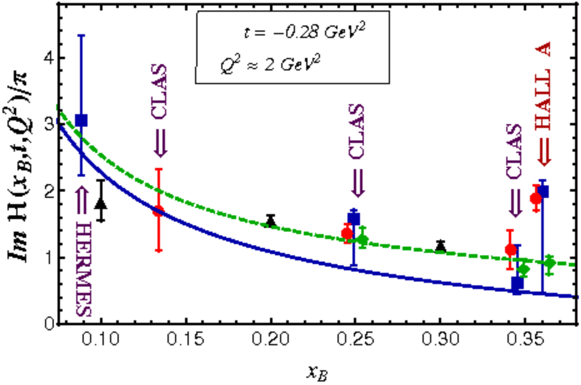
HERMES (2008)

$$BCA \equiv \frac{d\sigma_{e^+} - d\sigma_{e^-}}{d\sigma_{e^+} + d\sigma_{e^-}} \sim A_C^{\cos 0\phi} + A_C^{\cos 1\phi}\cos\phi \sim \Re\mathrm{e}\mathcal{H}$$

$$BSA \equiv \frac{d\sigma_{e\uparrow} - d\sigma_{e\downarrow}}{d\sigma_{e\uparrow} + d\sigma_{e\downarrow}} \sim A_{LU}^{\sin 1\phi}\sin\phi \sim \Im\mathrm{m}\mathcal{H}$$



Result and comparison to others



[Guidal '08, Guidal and Moutarde '09], seven CFF fit (blue squares), [Guidal '10] $\mathcal{H}$, $\tilde{\mathcal{H}}$ CFF fit (green diamonds), [Moutarde '09] H GPD fit (red circles)

Models are available at WWW

- http://calculon.phy.hr/gpd/

```
% xs.exe

  xs.exe  ModelID  Charge  Polarization  Ee  Ep  xB  Q2  t  phi

returns cross section (in nb) for scattering of lepton of energy Ee
on unpolarized proton of energy Ep. Charge=-1 is for electron.

ModelID is one of
   0 debug, always returns 42,
   1 KM09a - arXiv:0904.0458 fit without Hall A,
   2 KM09b - arXiv:0904.0458 fit with Hall A,
   3 KM10  - preliminary hybrid fit with LO sea evolution, from Trento presentation,
   4 KM10a - preliminary hybrid fit with LO sea evolution, without Hall A data
   5 KM10b - preliminary hybrid fit with LO sea evolution, with Hall A data

xB Q2 t phi  -- usual kinematics (phi is in Trento convention)

% xs.exe 1 -1 1 27.6 0.938  0.111 3. -0.3 0

  0.18584386497251
```

Curse of dimensionality

- *It is relatively easy to find a coin lying somewhere on* 100 *meter string. It is very difficult to find it on a football field.*

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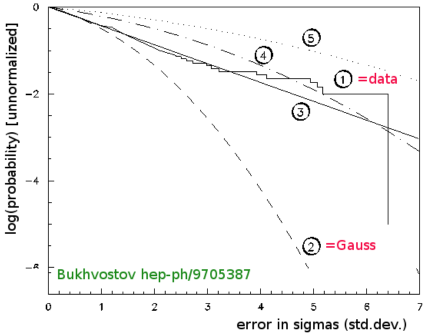
- Similarly, in contrast to $PDFs(x)$, it is very difficult to perform truly model independent extraction of $GPDs(x, \xi, t)$ (or $CFFs(\xi, t)$).
- Known GPD constraints don't decrease the dimensionality of the GPD domain space.

Problems with standard fitting approaches

1. Choice of fitting function introduces theoretical bias leading to systematic error which cannot be estimated (and is likely much larger for GPDs(x, η, t) than for PDFs(x).
2. Propagation of uncertainties from experiment to fitted function is difficult. Errors in actual experiments are not always Gaussian.



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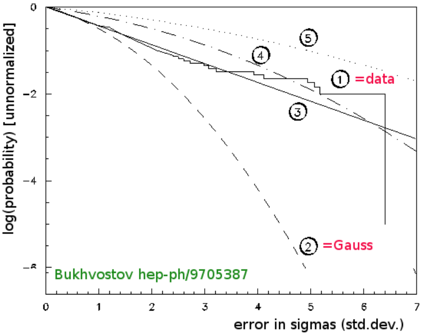
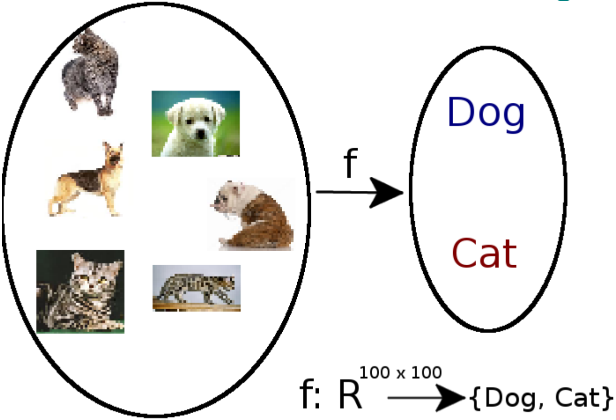


Introduction to neural networks: Cat-or-dog mapping†



† Homage to Vladimir Igorevich Arnold

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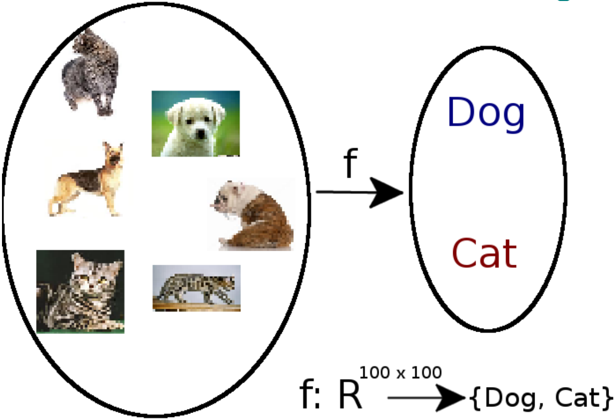
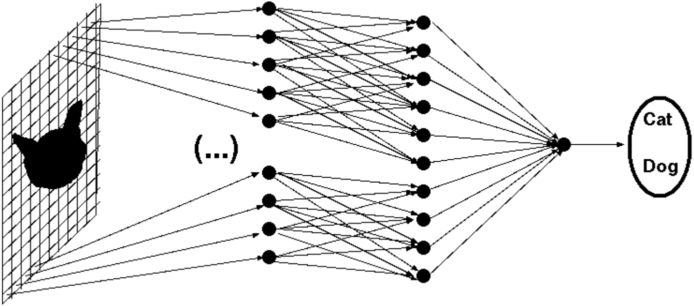


- How to represent function f by a computer algorithm?
- $\longrightarrow$ neural networks, learning machines, AI

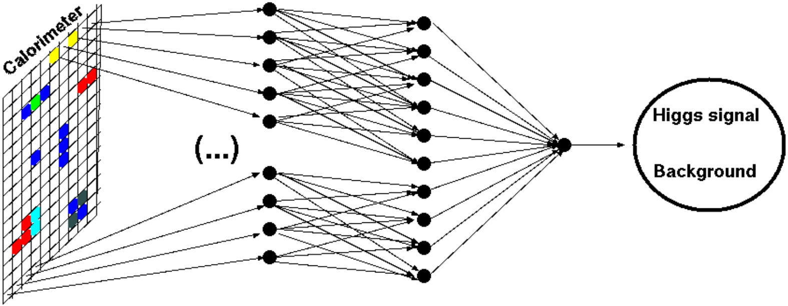
† Homage to Vladimir Igorevich Arnold

Cat-or-dog mapping by neural network



- Parameters ("weights") of neural network adjusted by "training" it on many samples
- Neural network becomes a representation of function f.
- Neural networks are capable of generalization: they successfully classify objects not seen during training

Neural networks in high-energy physics



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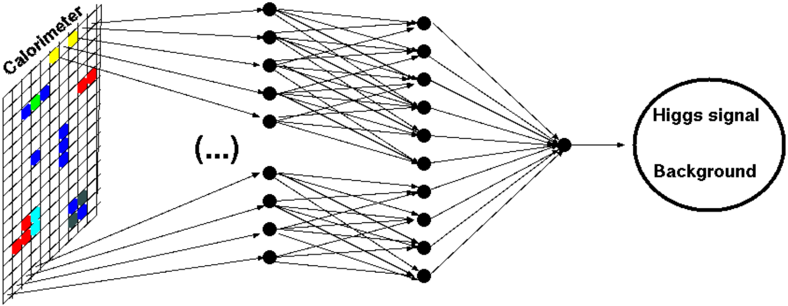
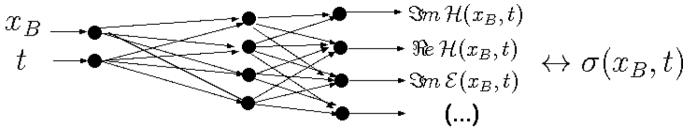


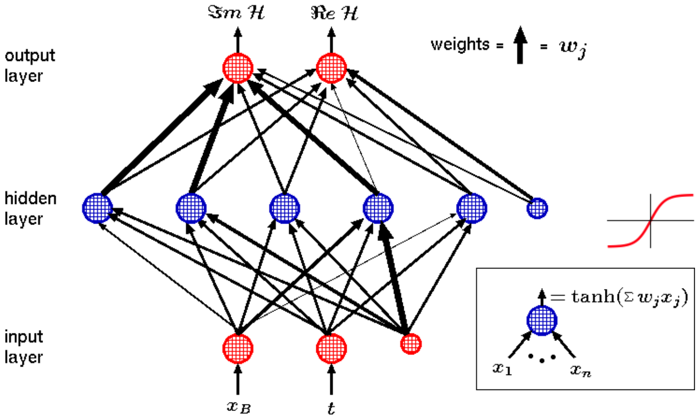
- Neural networks can be used
  - in place of triggers (hardware NN)
  - in place of simple “cuts” of detektor data (software NN)
- Used by CDF, D0, H1, BaBar, . . .
- Training usually done on Monte-Carlo simulated events
- Interpretation of NN behaviour is difficult so lot of testing is required before results can be trusted

Neural networks as a fitting tool



- Neural network now represents mapping $f : \mathbb{R}^2 \to \mathbb{R}^{n_\mathcal{F}}$.
- Classification problem is just a special case of optimization (χ^2 minimization) problem (where we have $\sigma(x_B, t) \in \mathbb{R}$ instead of *output* $\in \{\text{cat}, \text{dog}\}$).
- We can hope to be able to train neural networks to represent real underlying physical law
- NN approach is successfully applied to PDF fitting by [NNPDF] group and should be even more powerful in GPD fitting with GPDs being less-known functions of more variables.

Multilayer perceptron



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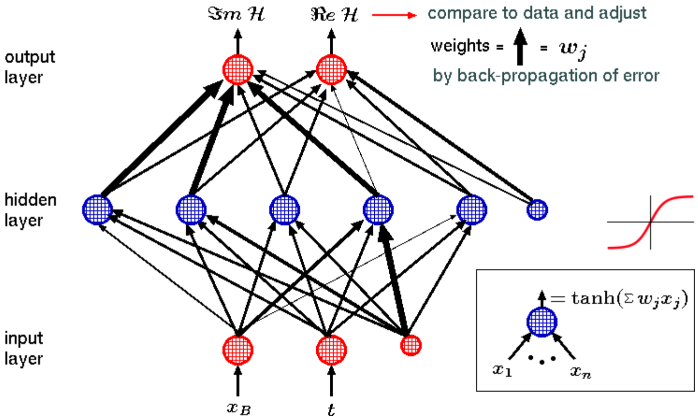

- Essentially a least-square fit of a complicated many-parameter function. $f(x) = \tanh(\sum w_i \tanh(\sum w_j \cdots))$ $\Rightarrow$ no theory bias

Function fitting by a neural net

- Theorem: Given enough neurons, any smooth function $f(x_1, x_2, \cdots)$ can be approximated to any desired accuracy. Single hidden layer is sufficient (but not always most efficient).

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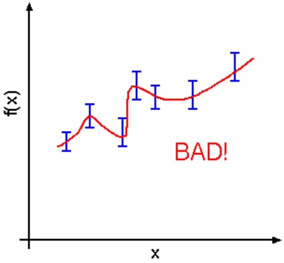

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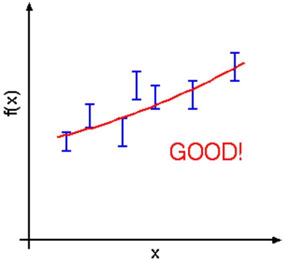

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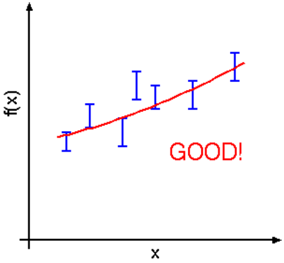
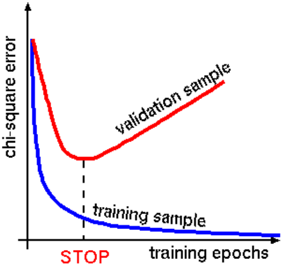
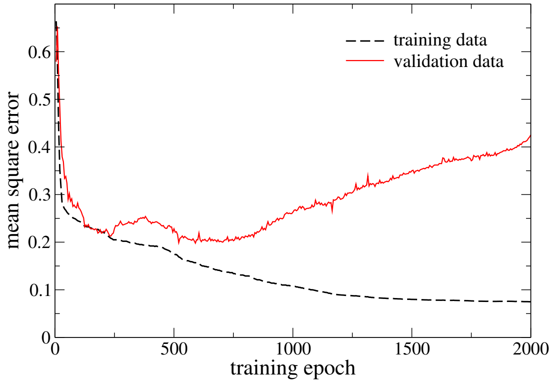
- Solution: Divide data (randomly) into two sets: *training* sample and *validation sample*. Stop training when error of validation sample starts increasing.

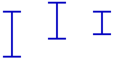




Example of a training with crossvalidation



Monte Carlo propagation of errors



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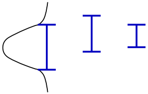

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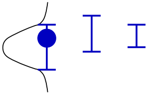

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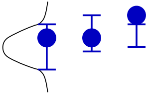

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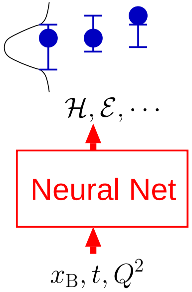

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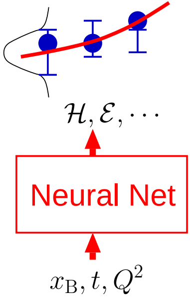
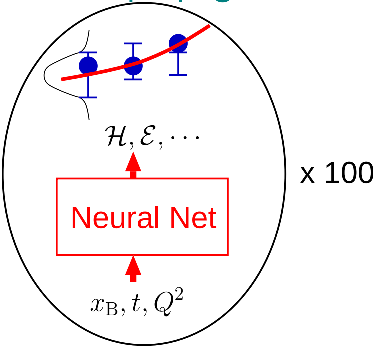
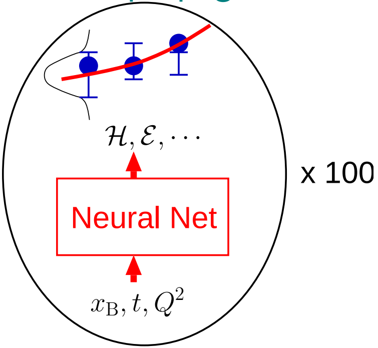


Monte Carlo propagation of errors



Monte Carlo propagation of errors



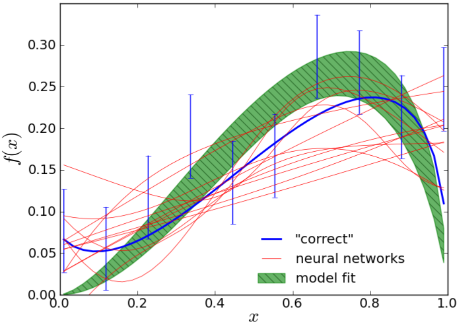
- Set of N_{rep} NNs defines a probability distribution in space of possible CFF functions:

$$\left\langle \mathcal{F}[\mathcal{H}] \right\rangle = \int \mathcal{D}\mathcal{H}\, \mathcal{P}[\mathcal{H}]\, \mathcal{F}[\mathcal{H}] = \frac{1}{N_{rep}} \sum_{k=1}^{N_{rep}} \mathcal{F}[\mathcal{H}^{(k)}] \,, \qquad (1)$$

- Experimental uncertainties and their correlations are preserved [Giele et al. '01]

Toy fitting example

- Fit to data generated according to function (which we pretend not to know).



- Fit with
  1. Standard Minuit fit with ansatz $f(x) = x^a(1-x)^b$
  2. Neural network fit

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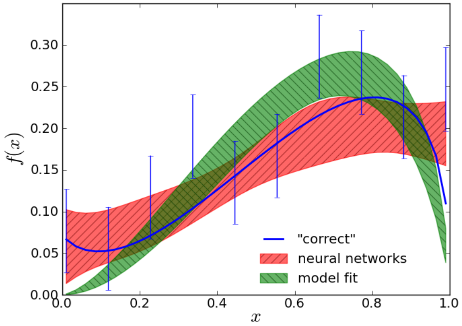

- Fit with
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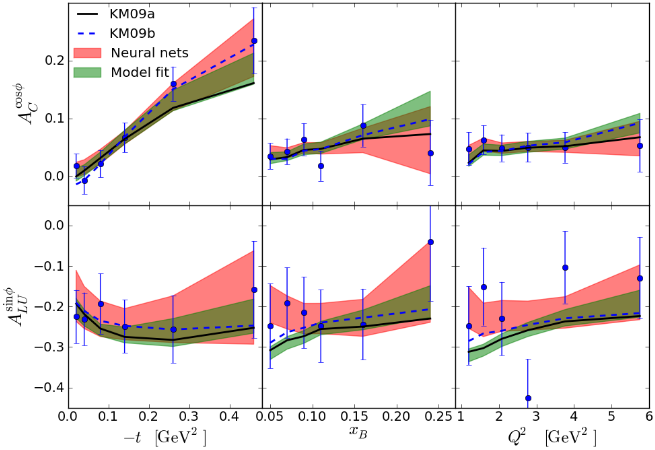
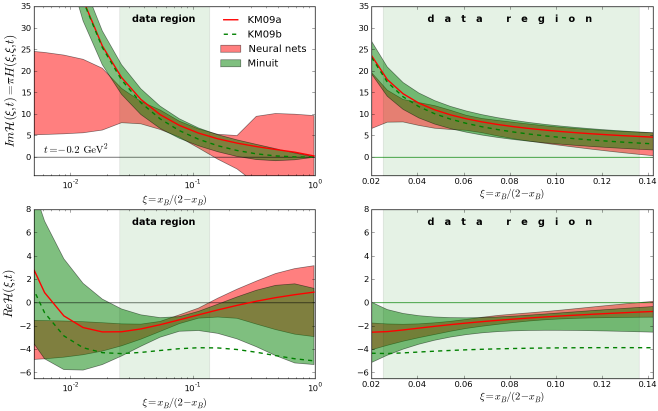
Fit to actual HERMES BSA+BCA data

- 50 neural nets with 13 neurons in a single hidden layer



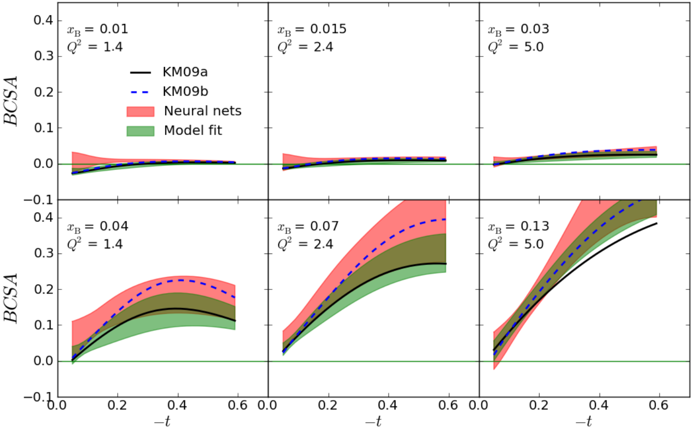
Resulting neural network CFFs



- interpolation in data region agrees with model fits
- extrapolation results in more realistic uncertainties

Prediction for COMPASS II BCSA

$$BCSA = \frac{\mathrm{d}\sigma_{\mu^{\downarrow +}} - \mathrm{d}\sigma_{\mu^{\uparrow -}}}{\mathrm{d}\sigma_{\mu^{\downarrow +}} + \mathrm{d}\sigma_{\mu^{\uparrow -}}} \qquad (E_\mu = 160\,\mathrm{GeV})$$



Summary

- Neural networks offer a powerful alternative approach to extraction of hadron structure information from measurements, enabling model-independent fits and facilitating error propagation from data to resulting structure functions.

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The End

Some properties of GPDs

- Forward limit ($\Delta \to 0$): $\Rightarrow$ GPD $\to$ PDF

$$F^q(x,0,0) = H^q(x,0,0) = \theta(x)q(x) - \theta(-x)\bar{q}(-x)$$

- Polynomiality:

$$\int_{-1}^{1} dx\, x^j H^q(x,\eta,t) = \sum_{k=0,\text{even}}^{j} (2\eta)^k A^q_{j+1,k}(t) \qquad \text{(even j)}$$

- Sum rules:

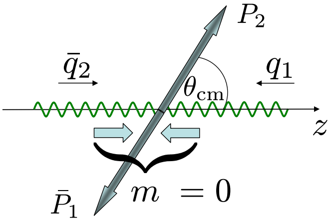
$$\int_{-1}^{1} dx \left\{ \begin{array}{l} H^q(x,\eta,t) \\ E^q(x,\eta,t) \end{array} \right. = \left\{ \begin{array}{ll} F_1^q(t) & \text{Dirac} \\ F_2^q(t) & \text{Pauli} \end{array} \right.$$

- "Ji's sum rule" (related to proton spin problem)

$$J^q = \frac{1}{2}\int_{-1}^{1} dx\, x \Big[H^q(x,\eta,t) + E^q(x,\eta,t) \Big]_{t\to 0} \qquad \text{[Ji '96]}$$

Modelling conformal moments of GPDs (I)

- How to model η-dependence of GPD's $H_j(\eta, t)$?
- Idea: consider crossed t-channel process $\gamma^*\gamma \to p\bar{p}$



When crossing back to DVCS channel we have:

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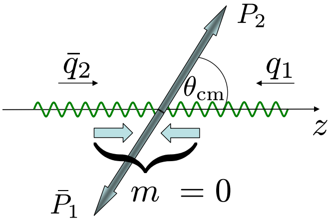


When crossing back to DVCS channel we have:

$$\cos\theta_{\rm cm} \to -\frac{1}{\eta}$$

- ... and dependence on $\theta_{\rm cm}$ in t-channel is given by SO(3) partial wave decomposition of $\gamma^*\gamma$ scattering

$$\mathcal{H}(\eta, \ldots) = \mathcal{H}^{(t)}(\cos\theta_{\rm cm} = -\frac{1}{\eta}, \ldots) = \sum_J (2J+1) f_J(\ldots) d^J_{0,\nu}(\cos\theta)$$

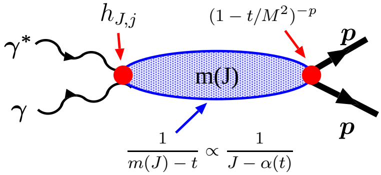
- $d^J_{0,\nu}$ — Wigner SO(3) functions (Legendre, Gegenbauer, ...)
  $\nu = 0, \pm 1$ — depending on hadron helicities

Modelling conformal moments of GPDs (II)

- OPE expansion of both $\mathcal{H}$ and $\mathcal{H}^{(t)}$ leads to

$$H_j(\eta, t) = \eta^{j+1}\, H_j^{(t)}(\cos\theta = -\frac{1}{\eta}, s^{(t)} = t)$$

- and t-channel partial waves are modelled as:



$$H_j(\eta, t) = \sum_J^{j+1} h_{J,j} \frac{1}{J - \alpha(t)} \frac{1}{\left(1 - \frac{t}{M^2(J)}\right)^p} \eta^{j+1-J} d^J_{0,\nu}$$

- Similar to “dual” parametrization [Polyakov, Shuvaev '02]

l-PW model — only leading partial wave

- Taking just a leading partial wave $J = j + 1$ gives ansatz:

$$\mathbf{H}_j(\xi, t, \mu_0^2) = \begin{pmatrix} N'_\Sigma \, F_\Sigma(t) \, \mathrm{B}\big(1 + j - \alpha_\Sigma(0), 8\big) \\ N'_\mathrm{G} \, F_\mathrm{G}(t) \, \mathrm{B}\big(1 + j - \alpha_\mathrm{G}(0), 6\big) \end{pmatrix}$$

$\alpha_a(t) = \alpha_a(0) + 0.15t$

$$F_a(t) = \frac{j + 1 - \alpha(0)}{j + 1 - \alpha(t)} \left(1 - \frac{t}{M_0^{a\,2}}\right)^{-p_a}$$

. . . corresponding in forward case to PDFs of form

$$\Sigma(x) = N'_\Sigma \, x^{-\alpha_\Sigma(0)} \, (1 - x)^7 \; ; \qquad G(x) = N'_\mathrm{G} \, x^{-\alpha_\mathrm{G}(0)} \, (1 - x)^5$$

I-PW model — only leading partial wave

- Taking just a leading partial wave $J = j + 1$ gives ansatz:

$$\mathbf{H}_j(\xi, t, \mu_0^2) = \begin{pmatrix} N'_\Sigma F_\Sigma(t)\, \mathrm{B}\big(1 + j - \alpha_\Sigma(0), 8\big) \\ N'_G F_G(t)\, \mathrm{B}\big(1 + j - \alpha_G(0), 6\big) \end{pmatrix}$$

$$\alpha_a(t) = \alpha_a(0) + 0.15t \qquad F_a(t) = \frac{j + 1 - \alpha(0)}{j + 1 - \alpha(t)} \left(1 - \frac{t}{M_0^{a\,2}}\right)^{-p_a}$$

...corresponding in forward case to PDFs of form

$$\Sigma(x) = N'_\Sigma\, x^{-\alpha_\Sigma(0)}\, (1 - x)^7 \; ; \qquad G(x) = N'_G\, x^{-\alpha_G(0)}\, (1 - x)^5$$

- $M_0^G = \sqrt{0.7}\,\mathrm{GeV}$ is fixed by the J/ψ production data
- Free parameters: N_Σ, $\alpha_\Sigma(0)$, M_0^Σ, N_G, $\alpha_G(0)$

For small ξ (small x_{Bj}) valence quarks are less important $\Rightarrow \Sigma \approx$ sea

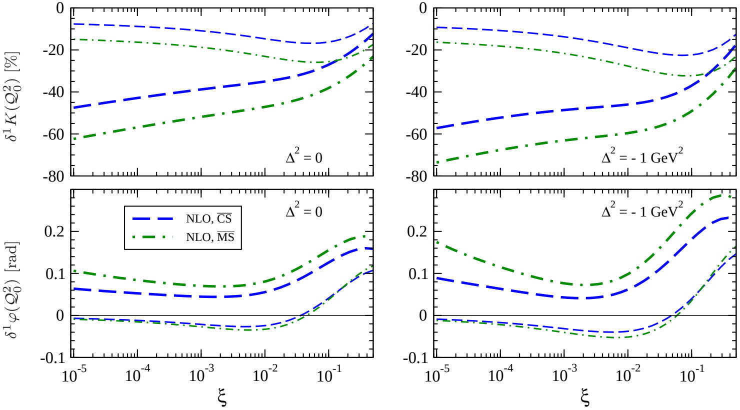
Inclusion of subleading PW — flexible models

- [K.K. and D. Müller '09]

$$\mathbf{H}_j(\eta, t) = \underbrace{\begin{pmatrix} N'_{\rm sea} F_{\rm sea}(t)\, \mathrm{B}\big(1+j-\alpha_{\rm sea}(0), 8\big) \\ N'_{\rm G} F_{\rm G}(t)\, \mathrm{B}\big(1+j-\alpha_{\rm G}(0), 6\big) \end{pmatrix}}_{\text{skewness } r\approx 1.6 \text{ (too large)}} + \underbrace{\begin{pmatrix} s_{\rm sea} \\ s_{\rm G} \end{pmatrix} \begin{pmatrix} \text{subleading partial waves, } \eta\text{-dependence!} \end{pmatrix}}_{\substack{<0 \\ \text{negative skewness}}}$$

- nl-PW — addition of second PW needed for good fits
- two new parameters: $s_{\rm sea}$ and $s_{\rm G}$
- nnl-PW — addition of third PW (doesn't improve fits but makes possible positive gluon GPDs at small $\mathcal{Q}^2$).

NLO corrections

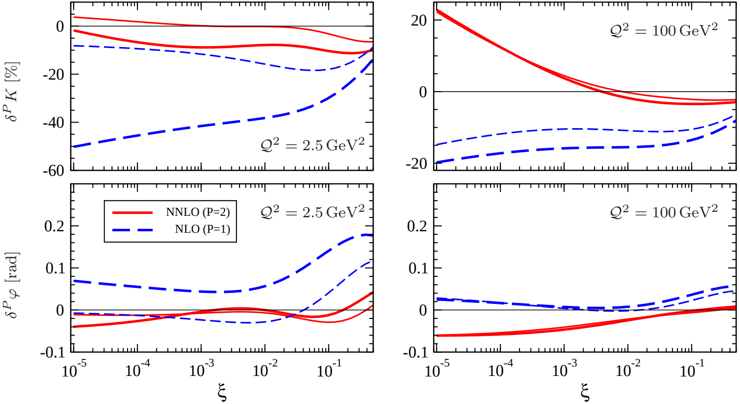


Thick lines:
"hard" gluon
$N_G = 0.4$
$\alpha_G(0) = \alpha_\Sigma(0) + 0.05$

Thin lines:
"soft" gluon
$N_G = 0.3$
$\alpha_G(0) = \alpha_\Sigma(0) - 0.02$

$$\delta^P K = \frac{\left|\mathcal{H}^{\mathrm{N}^P\mathrm{LO}}\right|}{\left|\mathcal{H}^{\mathrm{N}^{P-1}\mathrm{LO}}\right|} - 1\,, \qquad \delta^P \varphi = \arg\left(\frac{\mathcal{H}^{\mathrm{N}^P\mathrm{LO}}}{\mathcal{H}^{\mathrm{N}^{P-1}\mathrm{LO}}}\right)$$

NNLO corrections



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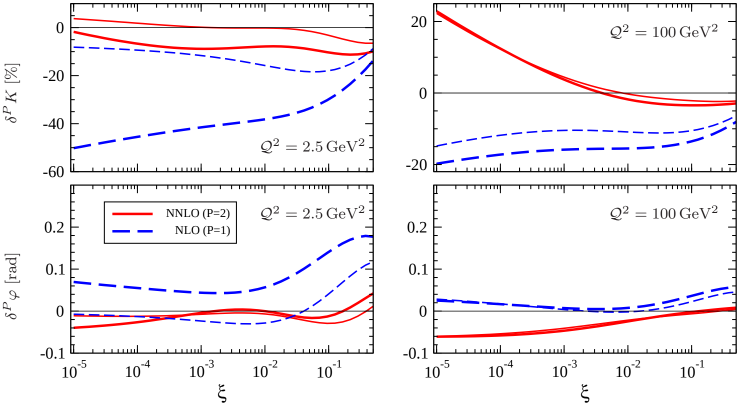

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Thin lines: "soft" gluon $N_G = 0.3$ $\alpha_G(0) = \alpha_\Sigma(0) - 0.02$

- breakdown at small-x_{Bj}, coming from $\alpha_s ln(1/x_{Bj})$ behaviour in evolution operator. Situation maybe worse for meson production [Diehl, Kugler, Ivanov, Szymanowski, Krasnikov]
  $\Rightarrow$ resummation needed

Beam charge asymmetry

$$BCA \equiv \frac{\mathrm{d}\sigma_{e^+} - \mathrm{d}\sigma_{e^-}}{\mathrm{d}\sigma_{e^+} + \mathrm{d}\sigma_{e^-}} = \frac{\mathcal{A}_{\mathrm{Interference}}}{|\mathcal{A}_{\mathrm{DVCS}}|^2 + |\mathcal{A}_{\mathrm{BH}}|^2} \overset{\mathrm{LO}}{\propto} F_1 \Re\mathrm{e}\mathcal{H} + \frac{|t|}{4M^2} F_2 \Re\mathrm{e}\mathcal{E}$$

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- Model E_{sea} as $(\mathcal{B}_{\mathrm{sea}}/N_{\mathrm{sea}})H_{\mathrm{sea}}$ and take $\mathcal{B}_{\mathrm{sea}} \equiv \int \mathrm{d}x \, x \, E_{\mathrm{sea}}$ as a parameter

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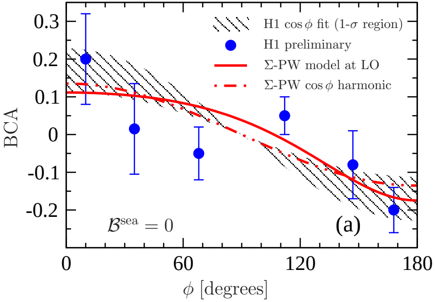
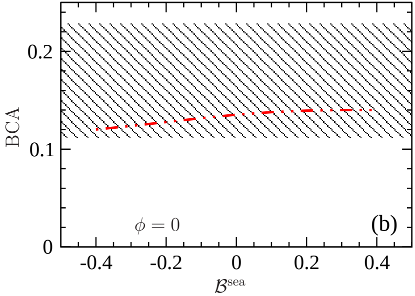
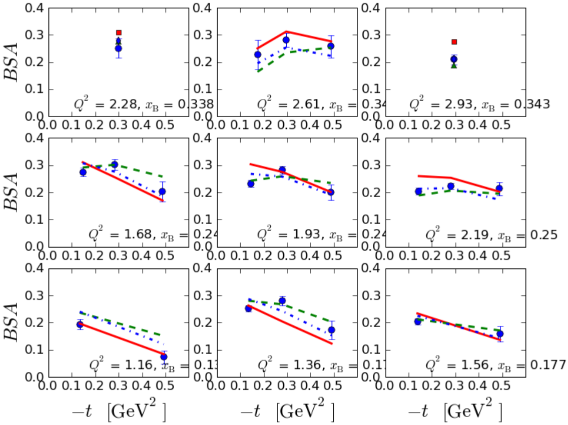




- We cannot extract $\mathcal{B}_{\rm sea}$ from H1 data

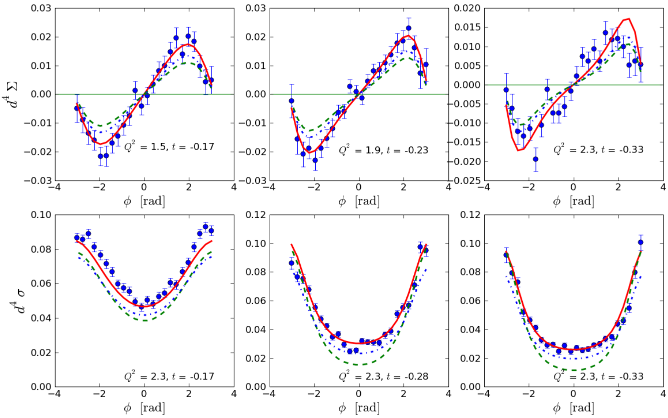
CLAS (2007)

- BSA. (Only data with $|t| \leq 0.3\,\mathrm{GeV}^2$ used for fits.)

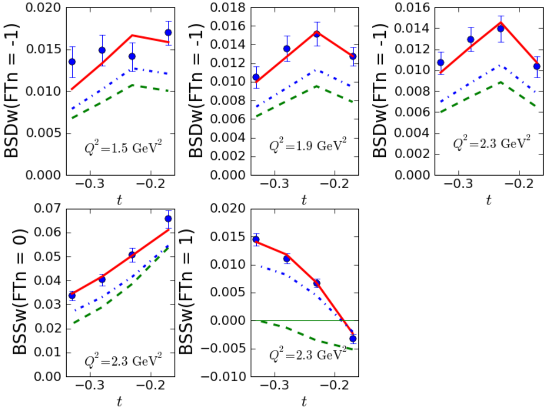


Hall A (2006)

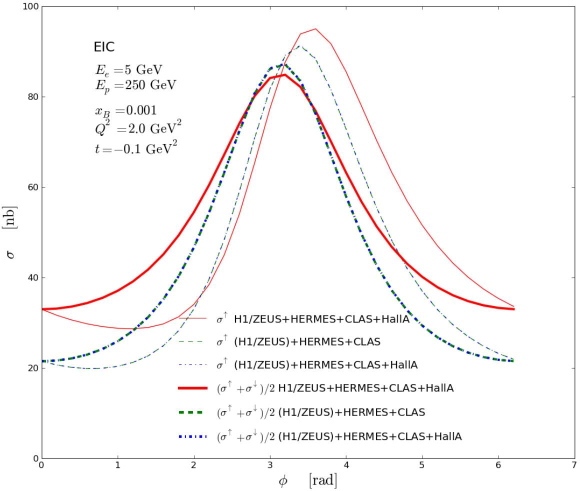
- Fit to unpolarized cross section $d\sigma/(dx_B dQ^2 dt d\phi)$ $\sim \Re\mathrm{e}\mathcal{H}$
- Fit is OK only with unusually large $\Re\mathrm{e}\mathcal{T}_{\mathrm{DVCS}}$ $(\to \tilde{\mathcal{H}})$



Hall A (2006) II



Prediction for EIC cross section



Assesment of uncertainties

- Theory predictions without appropriate uncertainties are of limited value.
- Usual procedure: calculate Hessian matrix of second derivatives of χ^2 w.r.t. parameters a_i at minimum χ^2_0 ...

$$H_{ij} = \frac{\partial^2 \chi^2}{\partial a_i \partial a_j} \; .$$

- ... and propagate errors to any quantity $f(a_i)$ via formula

$$(\Delta f)^2 = T^2 \sum_{ij} \frac{\partial f}{\partial a_i} H^{-1}_{ij} \frac{\partial f}{\partial a_j} \; .$$

- Textbook statistics instructs us to set tolerance parameter $T = 1$; this, however, usually underestimates uncertainties (for PDFs CTEQ has $T = 5 - 10$)