Drell-Yan process at foreward rapidities

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Outline

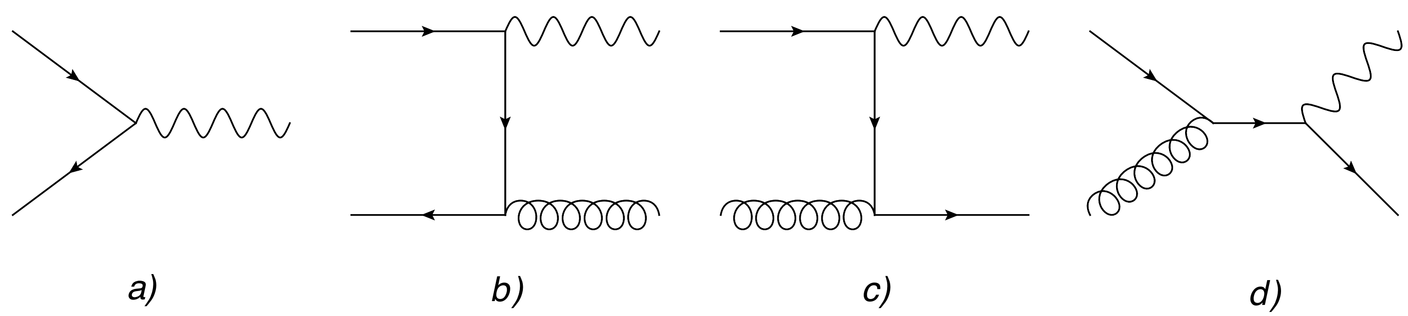
- Drell-Yan in the dipole model.
- Calculations for forward production of lepton pairs.
- Twist expansion.

In collaboration with K. Golec-Biernat and E. Lewandowska

Drell-Yan: collinear factorization

In the standard collinear factorization approach Drell-Yan process described by the fusion of the quark-antiquark and the production of the massive photon with subsequent decay. Higher orders involve gluon corrections.

M : invariant mass of the lepton pair



LO :

$$x_1 x_2 = M^2/s \equiv \tau$$

$$\frac{d^2\sigma^{LO}}{dM^2\,dx_F} = \frac{4\pi\alpha_{em}^2}{3N_c M^4}\frac{x_1 x_2}{x_1+x_2}\sum_f e_f^2\left\{q_f(x_1,M^2)\,\bar{q}_f(x_2,M^2) + \bar{q}_f(x_1,M^2)\,q_f(x_2,M^2)\right\},$$

NLO :

Altarelli, Ellis, Martinelli; Kubar-Andre, Paige

$$x_1 x_2 = \tau/z$$

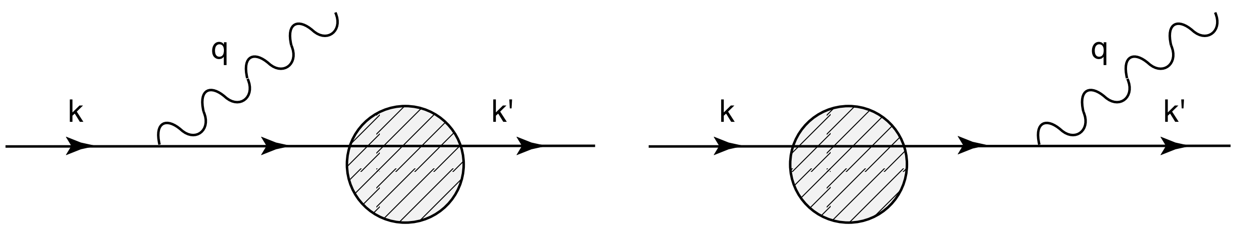
$$\frac{d^2\sigma^{NLO}}{dM^2\,dx_F} = \frac{4\pi\alpha_{em}^2}{3N_c M^4}\frac{\alpha_s(M^2)}{2\pi}\int_{z_{min}}^1 dz\,\frac{x_1 x_2}{x_1+x_2}\sum_f e_f^2\left\{q_f(x_1,M^2)\,\bar{q}_f(x_2,M^2)\,D_q(z)\right.$$
$$\left. + g(x_1,M^2)\left[q_f(x_2,M^2)+\bar{q}_f(x_2,M^2)\right]D_g(z) + (x_1 \leftrightarrow x_2)\right\}$$

Available at NNLO

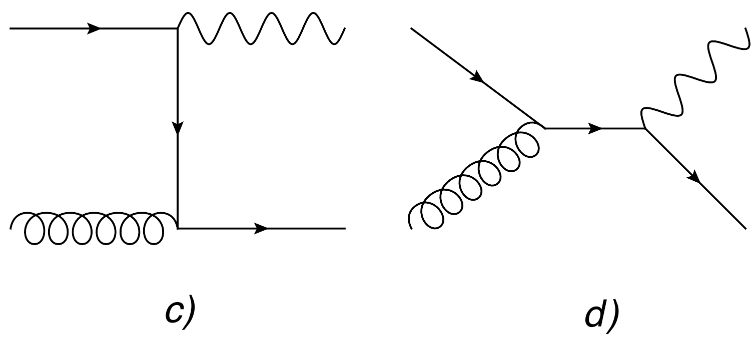
Matsuura, van der Marck, van Neerven, Hamberg; Blumlein, Ravindran

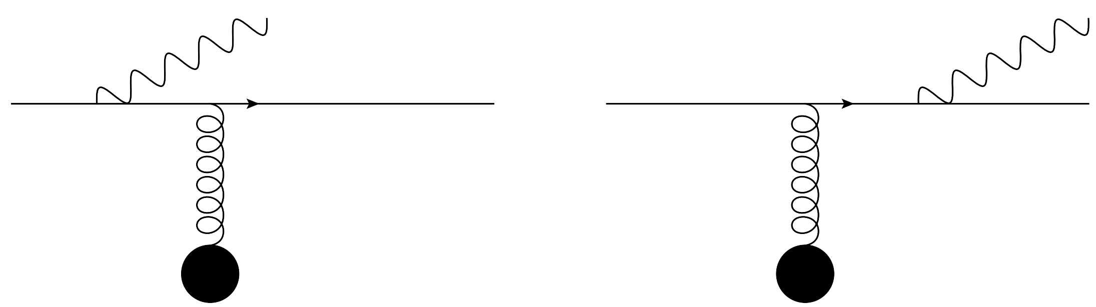
Dipole model for Drell-Yan

Drell-Yan process at high energy in the forward rapidity region



The quark from the projectile interacts with the field of the target, and radiates the massive photon (before or after the interaction). Photon decays into the leptons.





Enhanced (in the small x limit) diagrams are with the gluons of the target. Incoming quark is mostly valence.

Brodsky, Hebecker, Quack;
Kopeliovich, Raufeisen, Tarasov;
Kopeliovich, Tarasov, Schaefer;
in CGC formalism
Gelis, Jalilian-Marian;

What we mean by 'small x'?

In the parton model:

$$x_{1,2}$$

$$x_1 x_2 = M^2/s \equiv \tau$$

Light-cone momenta of partons in the partonic subprocess

$$x_1 = \frac{1}{2}(\sqrt{x_F^2 + 4\tau} + x_F)\,, \qquad x_2 = \frac{1}{2}(\sqrt{x_F^2 + 4\tau} - x_F)$$

$$x_1 \sim 1$$

in fact:

$$x_1 \sim x_F$$

$$x_2 = \frac{M^2}{s\,x_1} \ll 1$$

The hierarchy of scales : $M^2 \ll s$

Expect that on the target side the gluon density is the dominant

Dipole model for Drell-Yan

Drell-Yan in the dipole model at small x

$$\frac{d^2\sigma^{DY}_{T,L}}{dM^2\,dx_F} = \frac{\alpha_{em}}{6\pi M^2}\frac{1}{x_1+x_2}\sum_f \int_{x_1}^{1}\frac{dz}{z}\,F_2^f\left(\frac{x_1}{z},M^2\right)\,\sigma^f_{T,L}(qp\to\gamma^* X)\,.$$

Structure function of the incoming projectile

z Fraction of the energy of the quark taken by the photon

Radiation of the photon from the fast quark

r Photon - quark transverse separation

$$\sigma^f_{T,L}(qp\to\gamma^* X) = \int d^2r\,W^f_{T,L}(z,r,M^2,m_f)\,\sigma_{qq}(x_2,zr)\ ,$$

$$W^f_T = \frac{\alpha_{em}}{\pi^2}\left\{[1+(1-z)^2]\,\eta^2 K_1^2(\eta r) + m_f^2\,z^4 K_0^2(\eta r)\right\}\ ,$$

$$W^f_L = \frac{2\alpha_{em}}{\pi^2}\,M^2(1-z)^2 K_0^2(\eta r)$$

$$\eta^2 = (1-z)M^2 + z^2 m_f^2$$

As an example use the Golec Biernat and Wusthoff formula

$$\sigma_{qq}(x,r) = \sigma_0\left\{1-\exp(-r^2Q_s^2(x)/4)\right\}\ .$$

We will also use other models.

Recall dipole model for DIS

Dipole model for DIS

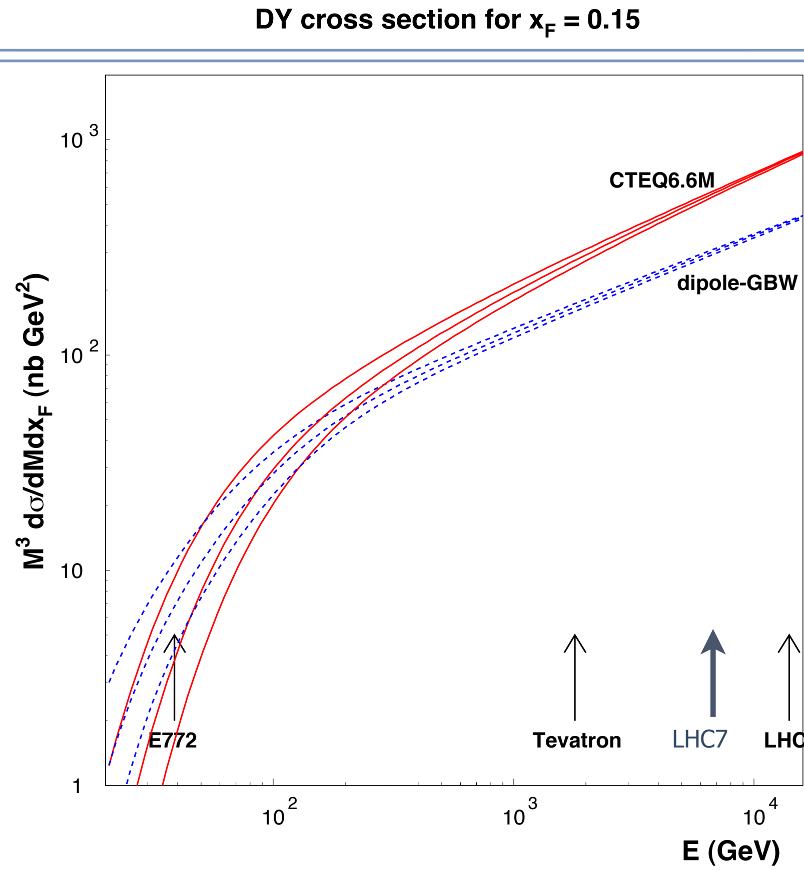
Cross section: $$\sigma_{T,L}(x,Q^2) = \int d^2r \int_0^1 dz |\Psi_{T,L}(z,r)|^2 \hat{\sigma}(x,r)$$

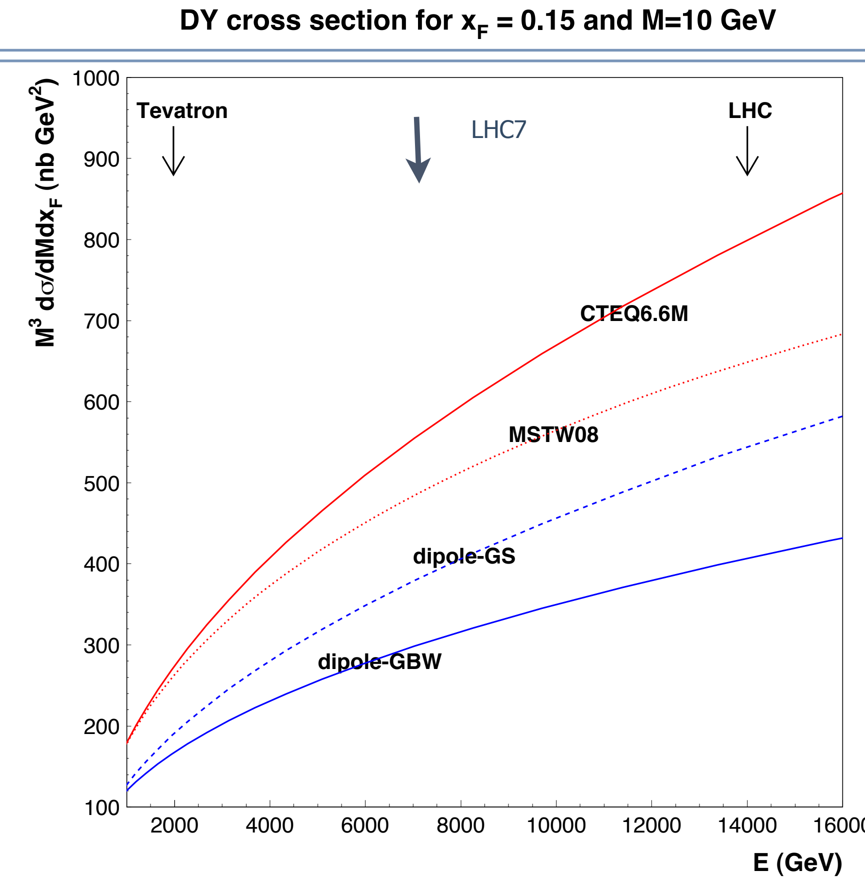
Wave function of the photon

Dipole cross section

In DY: although there is no physical dipole, the slow gluon 'sees' one as the interference of diagrams with emissions of the photon.

Predictions for LHC





Dilepton mass $M = 6, 8, 10$ GeV

dipole-GS (Golec-Sapeta)
DGLAP included

Large differences between collinear approaches

$x_2 \simeq 3 \cdot 10^{-6} - 10^{-5}$ typical values probed at energies 14-7 TeV

$y \sim 5 - 6$ range of rapidities

Dipole predictions systematically lower than the collinear calculations.

Twist expansion

What do we mean by 'twist expansion' ?

Classify different contributions by $\sim \left(\frac{1}{M^2}\right)^p$

Due to the presence of the nonlinear terms in the dipole cross section we classify these corrections by $\sim \left(\frac{Q_s^2(x)}{M^2}\right)^p$

Methods developed and applied to DIS structure functions:

Bartels, Golec-Biernat, Peters;
Bartels, Golec-Biernat, Motyka.

Twist expansion for Drell-Yan

First recall the method in DIS:

Cross section: $$\sigma_{T,L}(x,Q^2) = \int d^2r \int_0^1 dz |\Psi_{T,L}(z,r)|^2 \hat{\sigma}(x,r)$$

Take Mellin transform: $$f(r^2) = \int_{c-i\infty}^{c+i\infty} \frac{d\gamma}{2\pi i} (r^2)^\gamma \phi(\gamma)$$

$$\sigma_{T,L}(x,Q^2) = \sigma_0 \int_{1/2-i\infty}^{1/2+i\infty} \frac{d\gamma}{2\pi i} \left(\frac{Q_s^2(x)}{Q^2}\right)^\gamma H_{T,L}\left(\gamma, \frac{m_f^2}{Q^2}\right) G(\gamma)\,.$$

Photon wave function

dipole cross section

$$\sigma_0\, G(\gamma) \equiv \sigma_0 \int_0^\infty \frac{d\hat{r}^2}{\hat{r}^2} (\hat{r}^2)^{-\gamma} (1 - e^{-\hat{r}^2}) = -\sigma_0\, \Gamma(-\gamma)$$

Poles in γ control behavior in $1/Q^2$

Twist expansion for Drell-Yan

It is more complicated than in DIS, because of the convolution with the structure function of the forward projectile.

$$\frac{d^2\sigma_T^{DY}}{dM^2\,dx_F} = \frac{\alpha_{em}^2\sigma_0}{6\pi^2 M^2}\frac{1}{x_1+x_2}\int_{c-i\infty}^{c+i\infty}\frac{d\gamma}{2\pi i}\,G(\gamma)\widetilde{H}_T(\gamma)\left(\frac{Q_s^2(x_2)}{4M^2}\right)^{\gamma}$$
$$\times \boxed{\int_{x_1}^{1}\frac{dz}{z}\,F_2\!\left(\frac{x_1}{z},M^2\right)}\left[1+(1-z)^2\right]\left(\frac{z^2}{1-z}\right)^{\gamma}$$

Cannot directly perform integral over z (fraction of the light-cone momentum of the initial quark carried away by the photon), since it is weighted by the structure function of the projectile.

Two methods: fully analytical in terms of expansion in (1-x1).
Semi-analytical with exact results for twist contributions

Twist expansion: series

Define the function:

$$I_{T,\gamma}(x_1, z, M^2) = \frac{1}{z}\; F_2\Big(\frac{x_1}{z}, M^2\Big)\,[1+(1-z)^2](z^2)^\gamma$$

Expand it around z=1

$$\int_{x_1}^{1} dz \sum_{k\geq 0} I_{T,\gamma}^{(k)}(x_1, z=1, M^2)(1-z)^k \left(\frac{1}{1-z}\right)^\gamma = \sum_{k\geq 0} I_{T,\gamma}^{(k)}(x_1, z=1, M^2)(1-x_1)^{1-\gamma+k}\frac{1}{1-\gamma+k}$$

Expansion for the cross section:

$$\frac{d^2\sigma_T^{DY}}{dM^2\, dx_F} = \frac{\alpha_{em}^2\sigma_0}{6\pi^2 M^2}\frac{1}{x_1+x_2}\sum_{k\geq 0}\int_{c-i\infty}^{c+i\infty}\frac{d\gamma}{2\pi i}\,G(\gamma)\widetilde{H}_T(\gamma)\,\frac{I_{T,\gamma}^{(k)}(x_1, z=1, M^2)}{1-\gamma+k}\,(1-x_1)^{1+k}\left(\frac{Q_s^2(x_2)}{4M^2(1-x_1)}\right)^\gamma$$

Systematic expansion. Turns out very slowly convergent, because x1 is large but not large enough

$$x_1 \sim 0.1 - 0.2$$

In practice different method used to obtain exact results.

Twist expansion: explicit

Twist 2: contribution from $\gamma = 1$

$$\frac{d^2\sigma_T^{DY(\tau=2)}}{dM^2\, dx_F} = \Delta_{T,2}^{(0)} + \Delta_{T,2}^{(k>0)}$$

$$\Delta_{T,2}^{(0)} = \frac{\alpha_{em}^2\sigma_0}{6\pi^2M^2}\frac{F_2(x_1,M^2)}{x_1+x_2} \times 2\frac{Q_s^2(x_2)}{4M^2}\left[\frac{4}{3}\gamma_E - 1 + \frac{2}{3}\psi(\frac{5}{2}) - \frac{2}{3}\ln\frac{Q_s^2(x_2)}{4M^2(1-x_1)}\right]$$

$$\Delta_{T,2}^{(k>0)} = \frac{\alpha_{em}^2\sigma_0}{6\pi^2M^2}\frac{1}{x_1+x_2} \times \frac{4}{3}\left(\frac{Q_s^2(x_2)}{4M^2}\right)\int_{x_1}^{1} dz\, \frac{zF_2(\frac{x_1}{z},M^2)(1+(1-z)^2) - F_2(x_1,M^2)}{1-z}$$

First term contains the contribution from the double pole in the Mellin space (hence the logarithm). The result is exact twist 2 contribution.

Note the integrals over z over the structure function of the projectile.

Twist expansion: twist 4

Twist 4: contribution from $\gamma = 2$

Closed expressions found, though more cumbersome

$$\frac{d^2\sigma_T^{DY(\tau=4)}}{dM^2\,dx_F} = \Delta^{(1)}_{T,4} + \Delta^{(2)}_{T,4}$$

$$\Delta^{(1)}_{T,4} = \frac{\alpha_{em}^2\sigma_0}{6\pi^2 M^2}\frac{1}{x_1+x_2}\left(\frac{Q_s^2(x_2)}{4M^2}\right)^2 \times \frac{4}{15}\Big[F_2(x_1,M^2)\left(-63+36\,\gamma_E+18\,\psi(7/2)-18\log\left(\frac{Q_s^2(x_2)}{4M^2(1-x_1)}\right)\right) +x_1F_2'(x_1,M^2)\left(17-12\,\gamma_E-6\,\psi(7/2)+6\log\left(\frac{Q_s^2(x_2)}{4M^2(1-x_1)}\right)\right)\Big]$$

$$\Delta^{(2)}_{T,4} = \frac{\alpha_{em}^2\sigma_0}{6\pi^2 M^2}\frac{1}{x_1+x_2}\left(\frac{Q_s^2(x_2)}{4M^2}\right)^2 \times (-\frac{8}{5})\left[\delta\mathcal{F}(x_1,M^2) - \frac{F_2(x_1,M^2)}{1-x_1}\right]$$

Again regularized integral

$$\delta\mathcal{F}(x_1,M^2) = \int_{x_1}^1 dz\,\frac{z^3F_2(\frac{x_1}{z},M^2)(1+(1-z)^2) - F_2(x_1,M^2) - (1-z)[-3F_2(x_1,M^2)+x_1F_2'(x_1,M^2)]}{(1-z)^2}$$

The same can be computed for longitudinal component.

Twist expansion: longitudinal part

Expansion for the longitudinal part:

Twist 2:

$$\frac{d^2\sigma_L^{DY(\tau=2)}}{dM^2\,dx_F} = \frac{\alpha_{em}^2\sigma_0}{6\pi^2 M^2}\frac{2}{x_1+x_2}\frac{Q_s^2(x_2)}{4M^2}\times\frac{2}{3}\int_{x_1}^{1} dz\, z\, F_2\Big(\frac{x_1}{z}, M^2\Big)$$

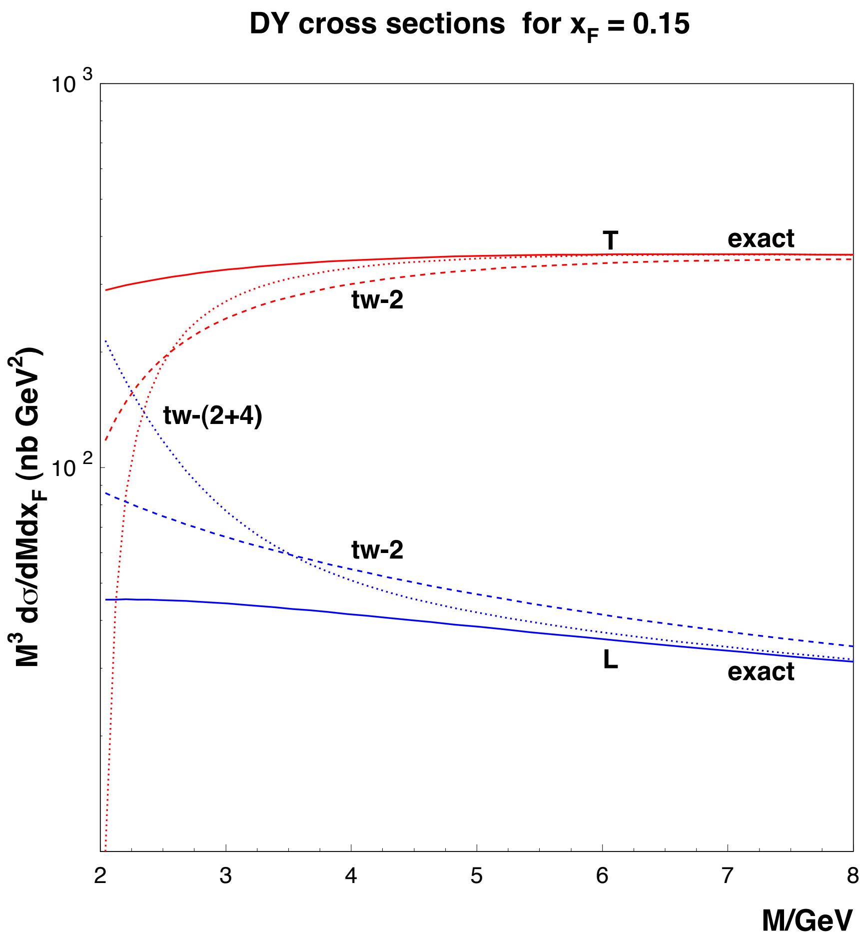
Twist 4:

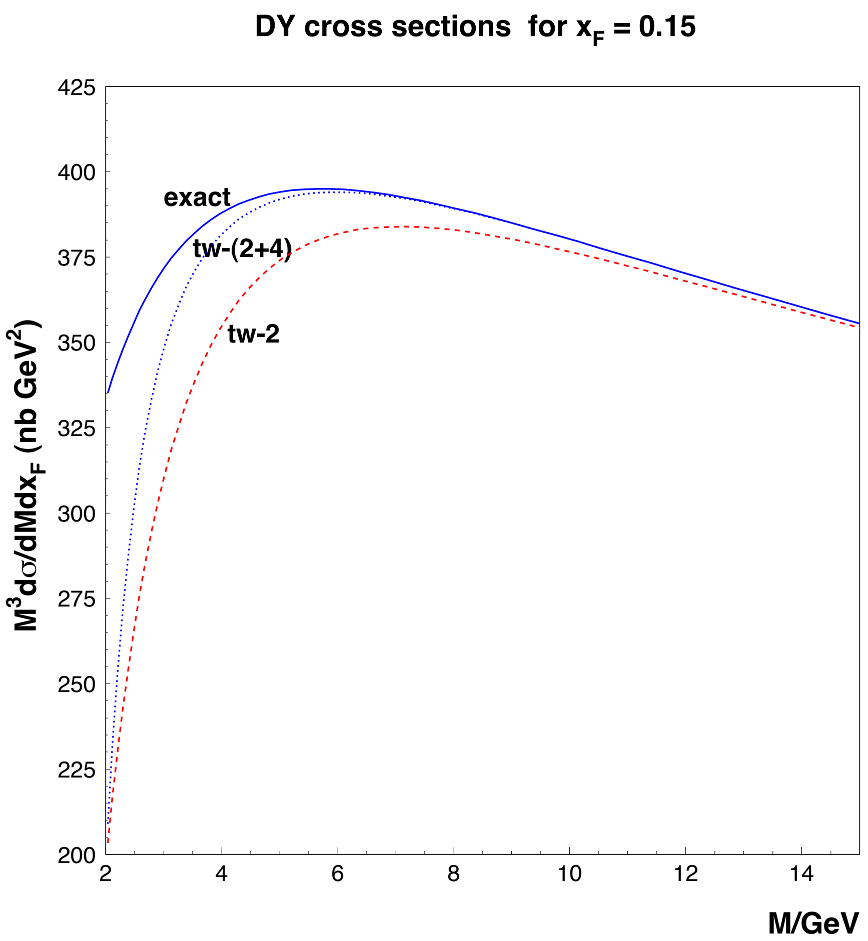
$$\Delta_{L,4}^{(1)} = \frac{\alpha_{em}^2\sigma_0}{6\pi^2 M^2}\frac{2F_2(x_1,M^2)}{x_1+x_2}\left(\frac{Q_s^2(x_2)}{4M^2}\right)^2\times\left(-\frac{16}{15}\right)\left[-3+2\gamma_E-\log\left(\frac{Q_s^2(x_2)}{4M^2(1-x_1)}\right)+\psi\left(\frac{7}{2}\right)\right]$$

$$\Delta_{L,4}^{(2)} = \frac{\alpha_{em}^2\sigma_0}{6\pi^2 M^2}\frac{2}{x_1+x_2}\left(\frac{Q_s^2(x_2)}{4M^2}\right)^2\times\left(-\frac{16}{15}\right)\int_{x_1}^{1} dz\,\frac{z^3F_2(\frac{x_1}{z},M^2)-F_2(x_1,M^2)}{1-z}$$

$$\frac{d^2\sigma_L^{DY(\tau=4)}}{dM^2\,dx_F} = \Delta_{L,4}^{(1)}+\Delta_{L,4}^{(2)}$$

Twist expansion for DY: results





$\sqrt{s} = 14\ \mathrm{TeV}$

- Twist expansion divergent for M<4.
- For higher masses M>6 twist 2 sufficient.
- For longitudinal twist 2 overestimates, for transverse part underestimates the exact result.
- The sum is better approximated by twist expansion.

Conclusions

- Suppression of the dipole model results with respect to the collinear approximation.
- However, large discrepancies between different models in the highest energy range.
- Twist expansion for the case of GBW formula can be constructed.
- More involved procedure for DY than in DIS. Semi-analytical results possible.
- Twist expansion divergent for invariant masses < 4-6 GeV.