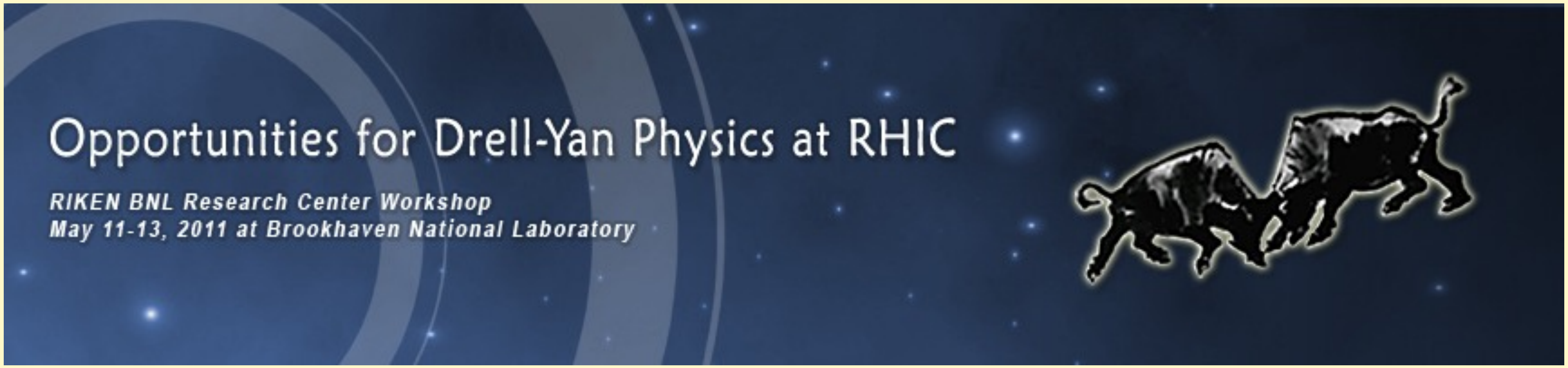
Gauge Links & Process dependence in Hadronic Reactions



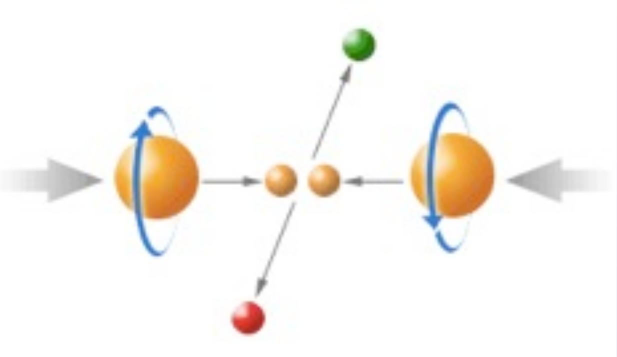
13 May 2011

Opportunities for Drell-Yan Physics at RHIC

Leonard Gamberg Penn State University

Phys.Lett. B696 2011 w/ Zhongbo Kang ***BNL***

Comments



- Single inclusive hadronic collisions largest/oldest observed TSSAs
- From theory viewpoint most challenging to understand-twist 3 power suppressed (as compared to SIDIS, DY and e^+e^-)
- Nonetheless there are connections
  - Operator rltn ETQS fnct 1st moment of Sivers

$$gT_F(x,x) = -\int d^2k_T \frac{|k_T^2|}{2M} f_{1T}^{\perp}(x,k_T^2) \quad + \quad \text{“UV”} \ldots$$

  - Connection btwn twist three approach and twist two in overlap regime
  - Same mechanism in both approaches ISI/FSI
  - Explore role parton model processes in tw-2&3 approaches ***LG & Z. Kang PLB 2011***

{\em Model Assumptions}

- “WTIM” consider hadronic processes taking into account ISI/FSI in gen. parton model
- Consider impact in three cases
  - Inclusive pion production at forward rapidity- Both Collins and Sivers can contribute
  - Direct photon - Sivers only, can be used to test sign change as in DY
  - Pion about a jet-Can disentangle Collins and Sivers
  - Inclusive jet-SIVERS - Only Sivers, can be used to test sign change as in DY

Comments cont ...

Similar studies performed for weighted k_T and unweighted

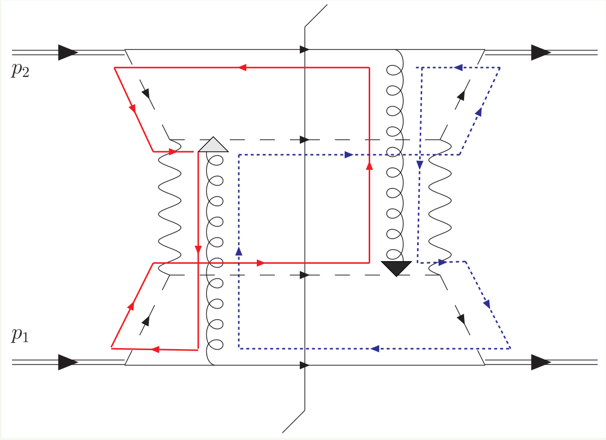
- photon Jet $p^{\uparrow}\, p \longrightarrow \gamma\, \mathrm{jet}\, X$ Bacchetta Bomhof, D'Alesio, Mulders, Murgia PRL 09
- 2-particle inclusive hadron production $p^{\uparrow}\, p \longrightarrow h_1\, h_2\, X$

Bacchetta Bomhof, Mulders, Piljman PRD05, Qiu Vogelsang Yuan PRD2007, Vogelsang Yuan 2007

Merits "PCQMR period "Pre-Collins Qiu Mulders Rogers"
1) two scale problem--TMD fact.
2) weigthed submits to transverse moments leads to gluonic pole factors & gluonic pole matrix elements--connection to twist three formalism

Problems/Challenges-"post CQMR period" **Collins Qiu PRD 2007 & Mulders Rogers 2010**

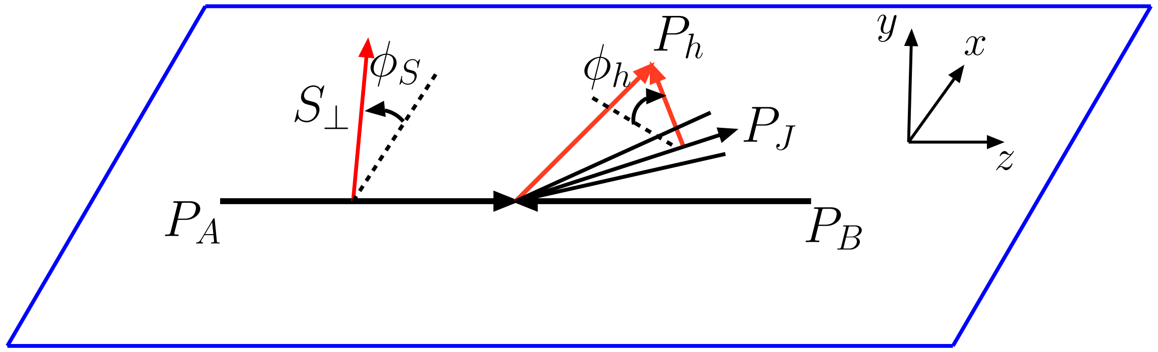
*) factorization violated



Other Hadronic Proccess

Azimuthal asymmetric distribution of hadrons inside a high energy jet in the transverse polarized nucleon-nucleon scattering, coming from the

$$p^{\uparrow}\, p \longrightarrow h_1\, \text{jet}\, X$$

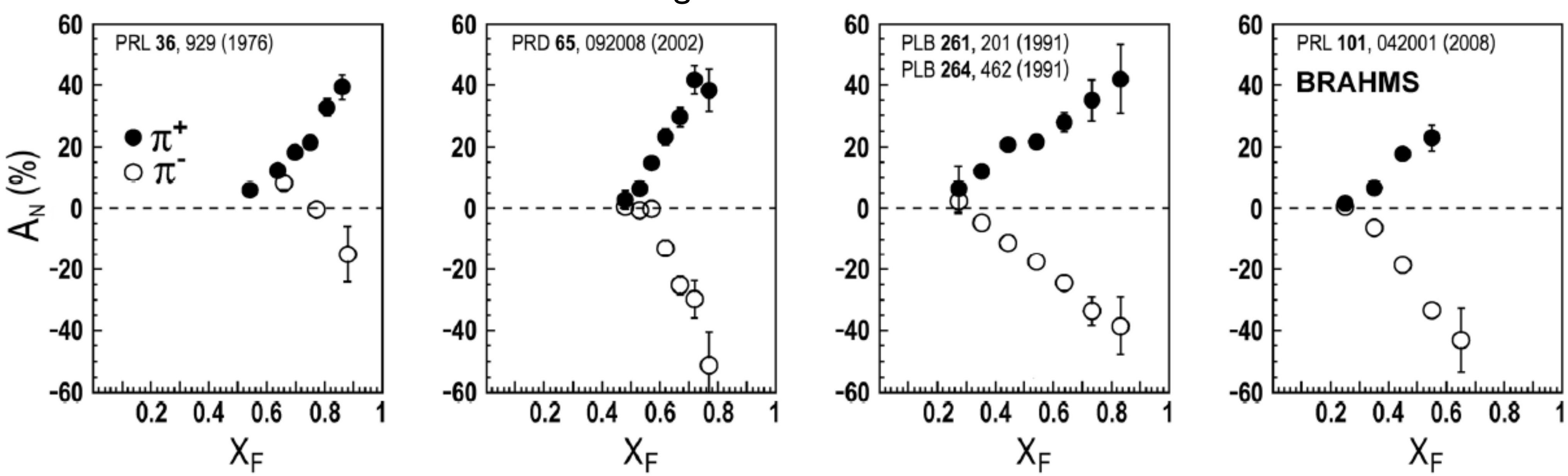


- Collins effect Yuan PRL 2008
- Separate Collins and Sivers!
  w/o ISI/FSI- D'Alesio, Murgia, Pisano PRD10,

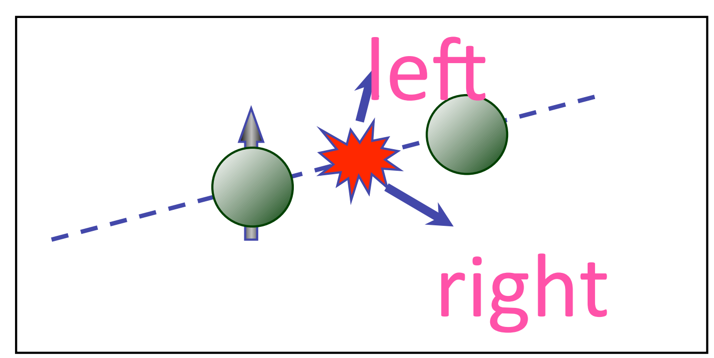
  w/ ISI/FSI D'Alesio, LG, Kang, Murgia, Pisano w/ ISI/FSI-in prep

Transverse Single-Spin Asymmetries: From Low to High Energies!

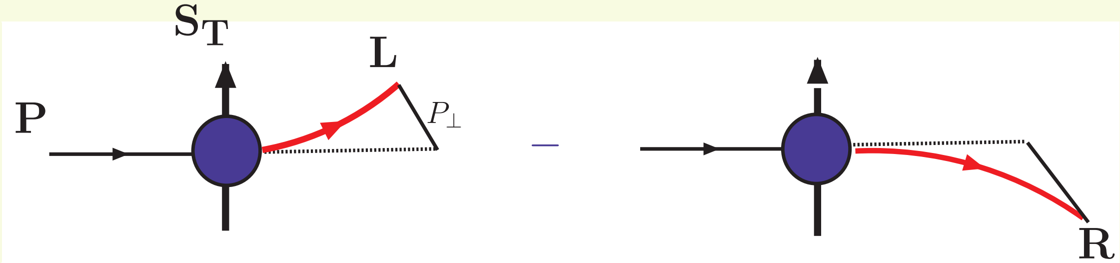
Pbeam=12 Gev/c	Pbeam=22 Gev/c	Pbeam=200 Gev/c	
ANL	BNL	FNAL	RHIC
√s=4.9 GeV	√s=6.6 GeV	√s=19.4 GeV	√s=62.4 GeV
Fixed target			Collider

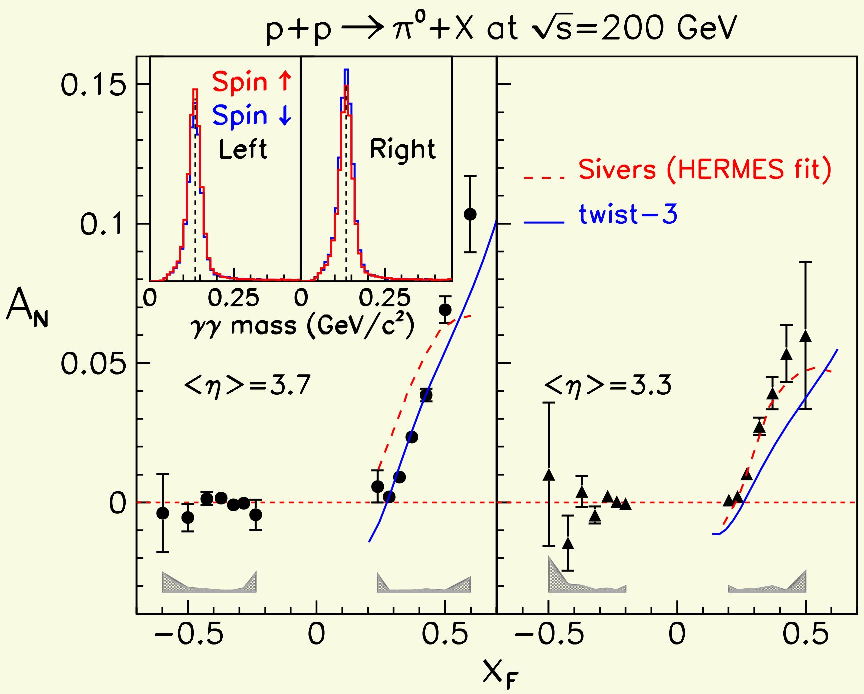


$$x_F = 2p_{long}/\sqrt{s}$$



Modern Era Transverse SSA's at $\sqrt{s}$ = 62.4 & 200 GeV at RHIC





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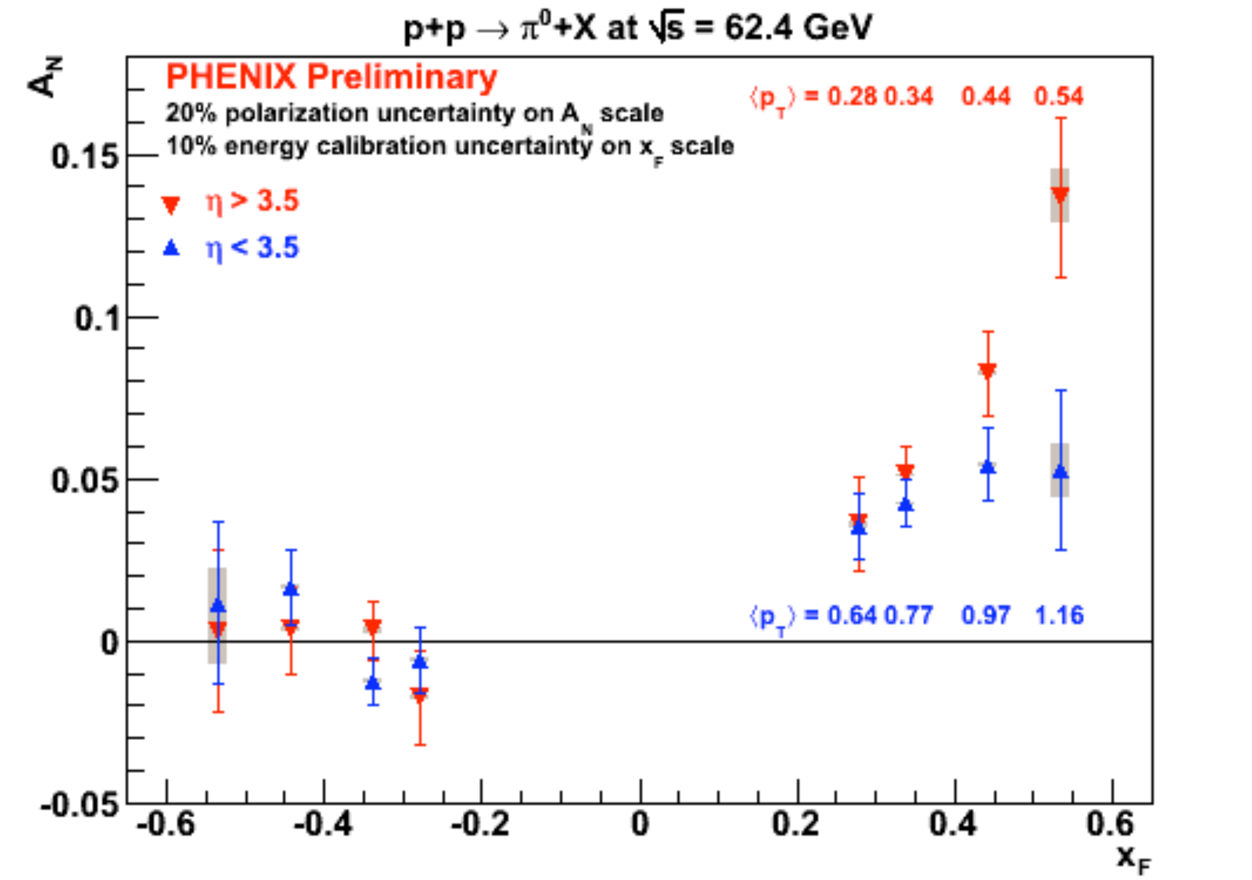
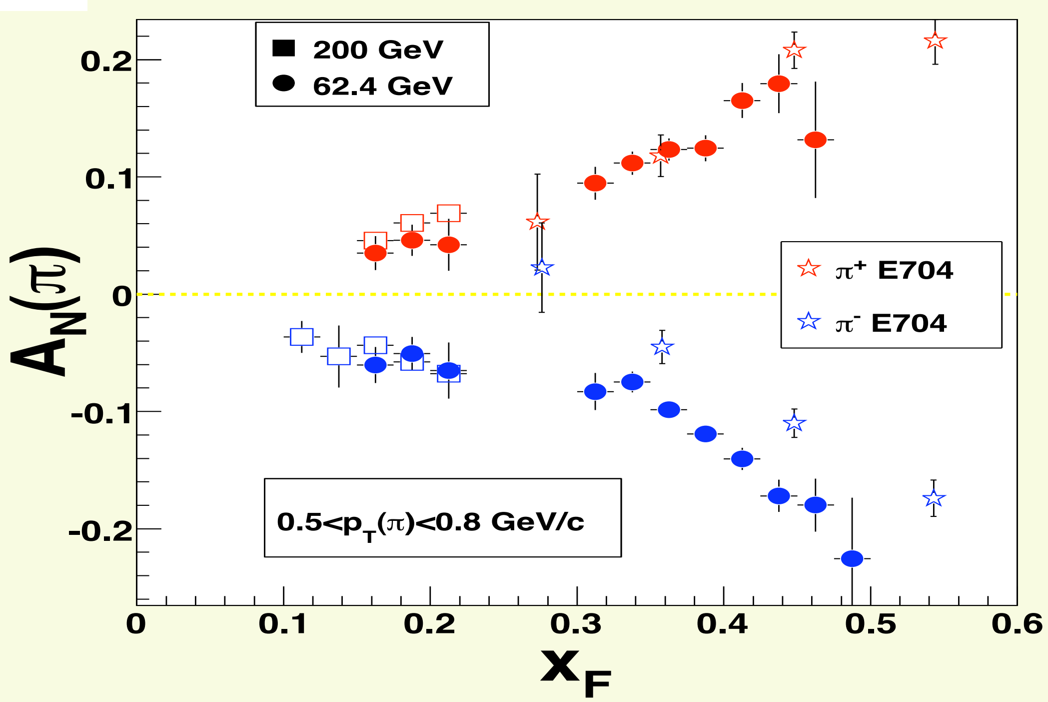
PRL101, 042001 (2008)





Outline

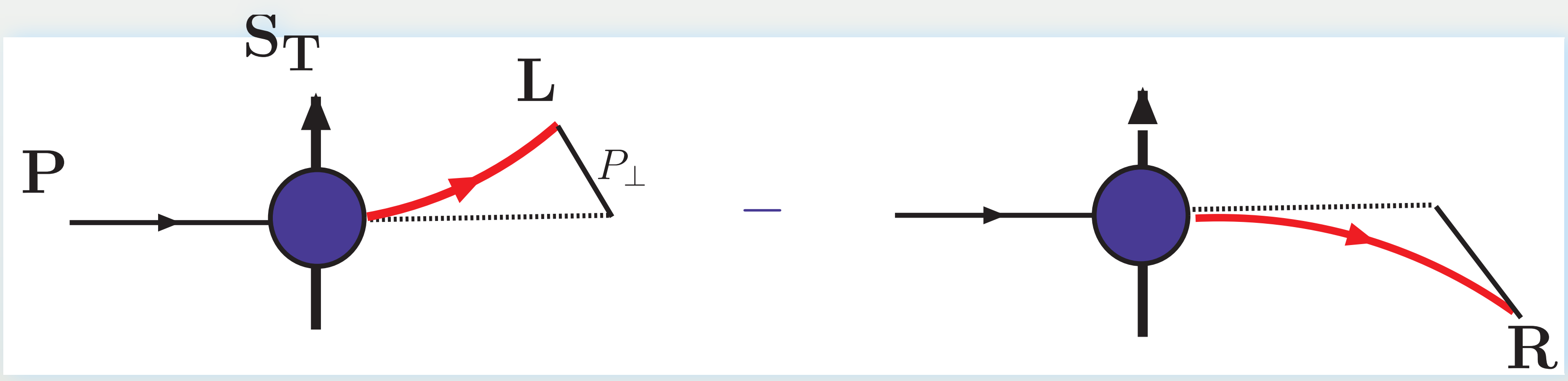
- **Reivew transverse structure spin Effects in TSSAs**
- **Gauge links-Color Gauge Inv.-“T-odd” TMDs**
- **T-odd PDFs via ISI/FSIs ... Phases & gauge link**

“QCD calc “ FSIs Gauge Links-Color Gauge Inv. “T-odd” TMDs

- Generalizing the Generalized Parton Model (GPM) color gauge invariance
- Some pheno results
- Connection to twist three

Transverse SPIN Observables SSA (TSSA) $P^{\uparrow}P \to \pi X$

- **Single Spin Asymmetry**

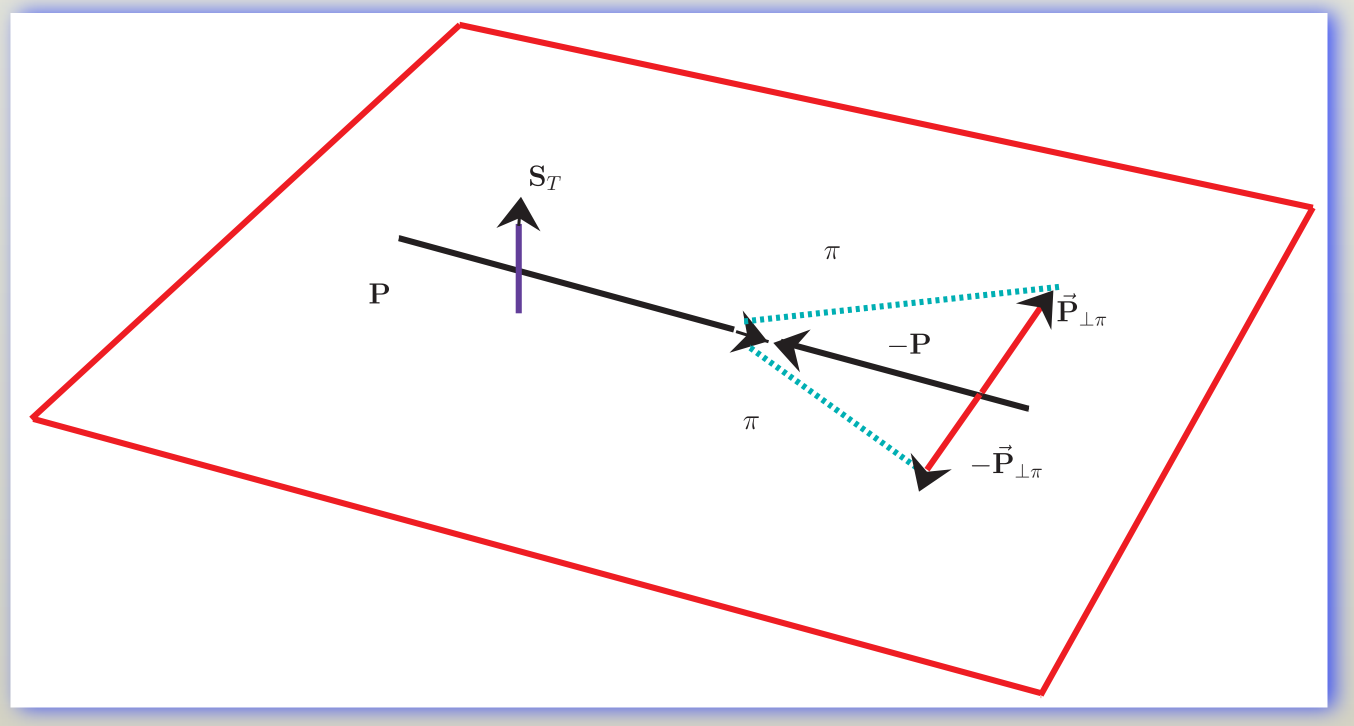


Parity Conserving interactions: SSAs Transverse Scattering plane

$$\Delta\sigma \sim iS_T \cdot (\mathbf{P} \times P_\perp^\pi)$$

- **Rotational invariance** $\sigma^{\downarrow}(x_F, \boldsymbol{p}_\perp) = \sigma^{\uparrow}(x_F, -\boldsymbol{p}_\perp)$
  $\Rightarrow$ ***Left-Right Asymmetry***

$$\boldsymbol{A}_N = \frac{\sigma^{\uparrow}(x_F,\boldsymbol{p}_\perp)-\sigma^{\uparrow}(x_F,-\boldsymbol{p}_\perp)}{\sigma^{\uparrow}(x_F,\boldsymbol{p}_\perp)+\sigma^{\uparrow}(x_F,-\boldsymbol{p}_\perp)} \equiv \Delta\sigma$$



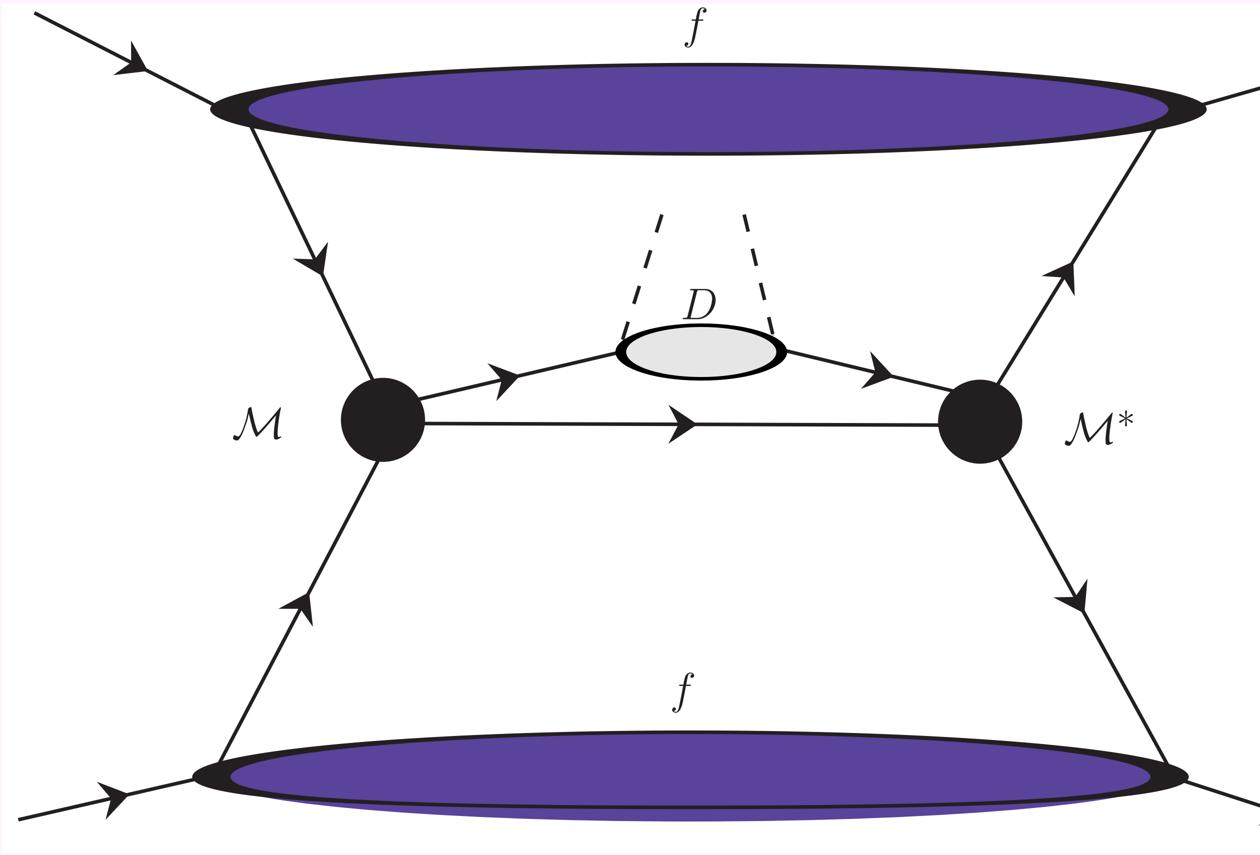
Collinear factorized QCD parton dynamics

$$\Delta\sigma^{pp^\uparrow \to \pi X} \sim f_a \otimes f_b \otimes \Delta\hat{\sigma} \otimes D^{q\to\pi}$$

$$\Delta\hat{\sigma} \equiv \hat{\sigma}^\uparrow - \hat{\sigma}^\downarrow$$

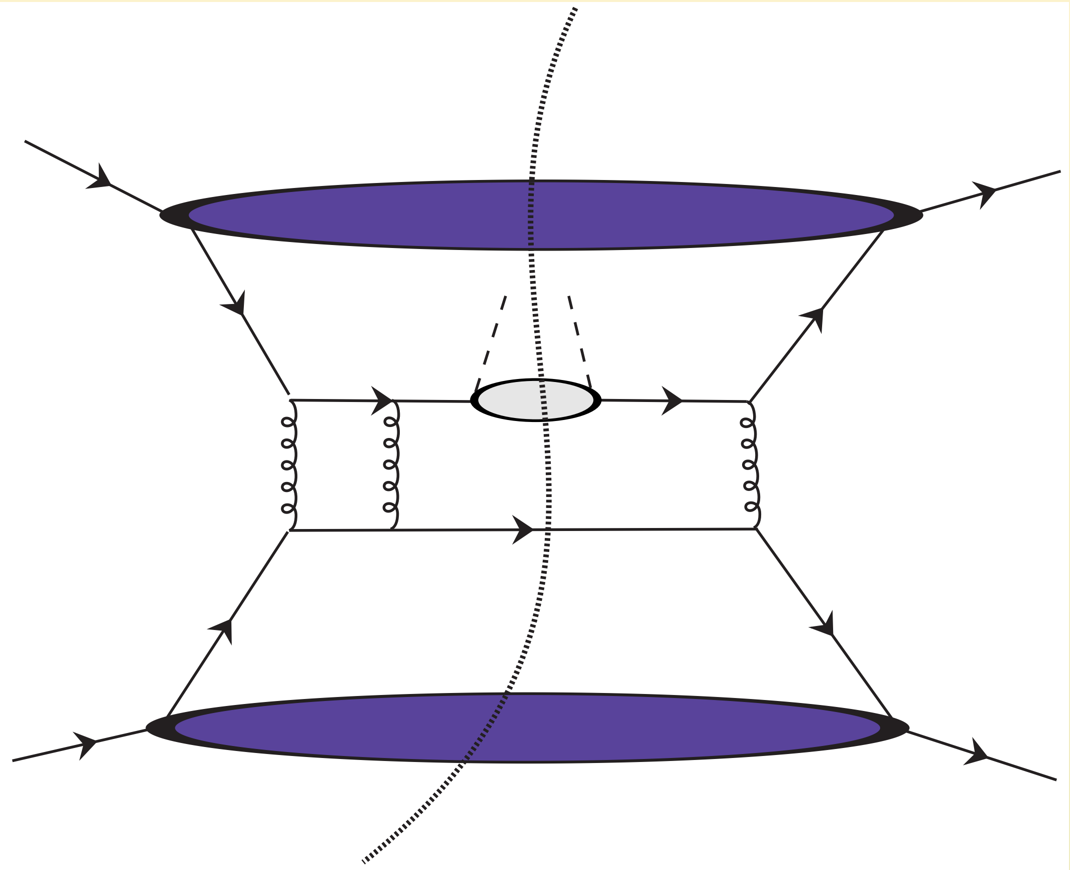
$$|\uparrow / \downarrow\rangle = (|+\rangle \pm i|-\rangle)$$

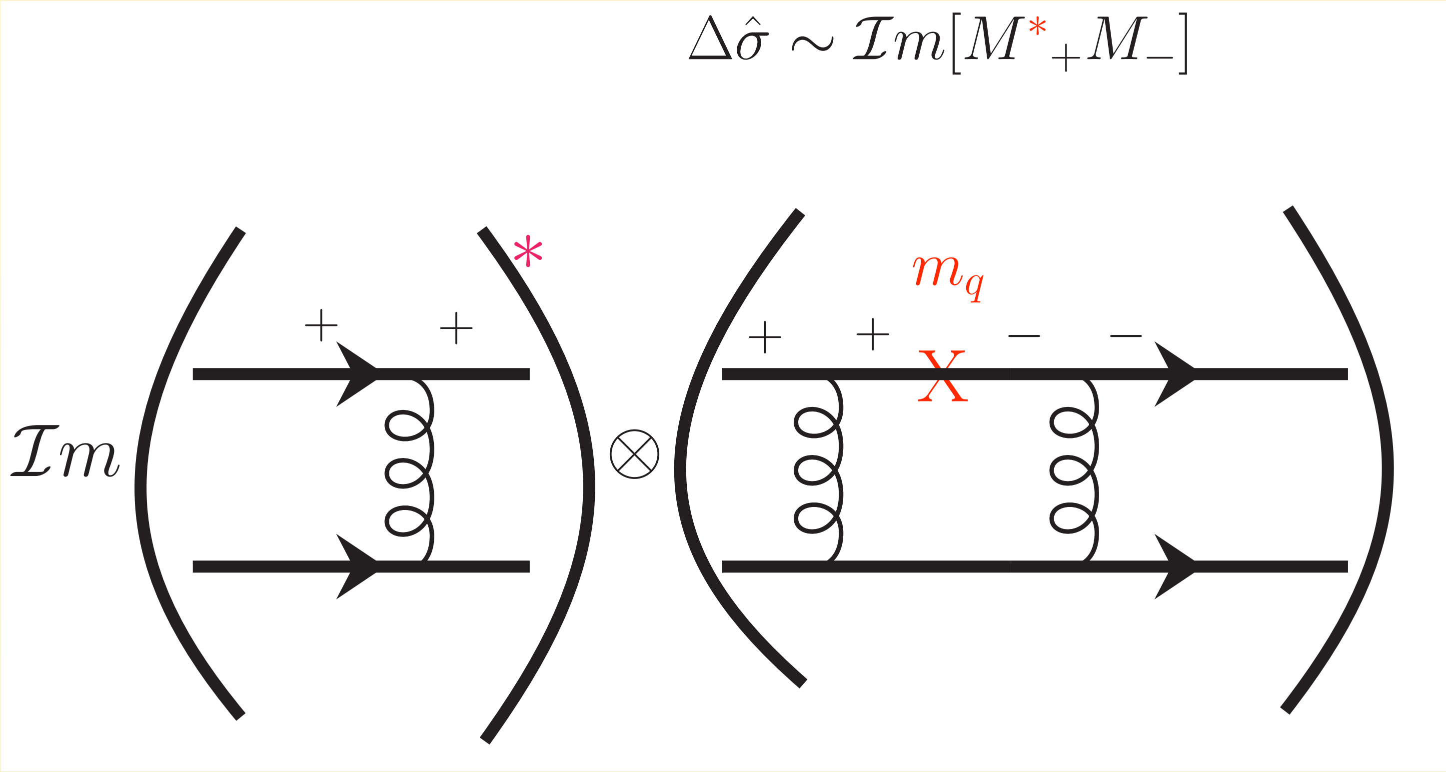
$$\hat{a}_N = \frac{\hat{\sigma}^\uparrow - \hat{\sigma}^\downarrow}{\hat{\sigma}^\uparrow + \hat{\sigma}^\downarrow} \sim \frac{\mathrm{Im}\left(\mathcal{M}^{+*}\mathcal{M}^{-}\right)}{|\mathcal{M}^+|^2 + |\mathcal{M}^-|^2}$$



⋆ **TSSA requires relative phase btwn *different* helicity amps**

Collinear Factorization Theorem & SSAs at Partonic level





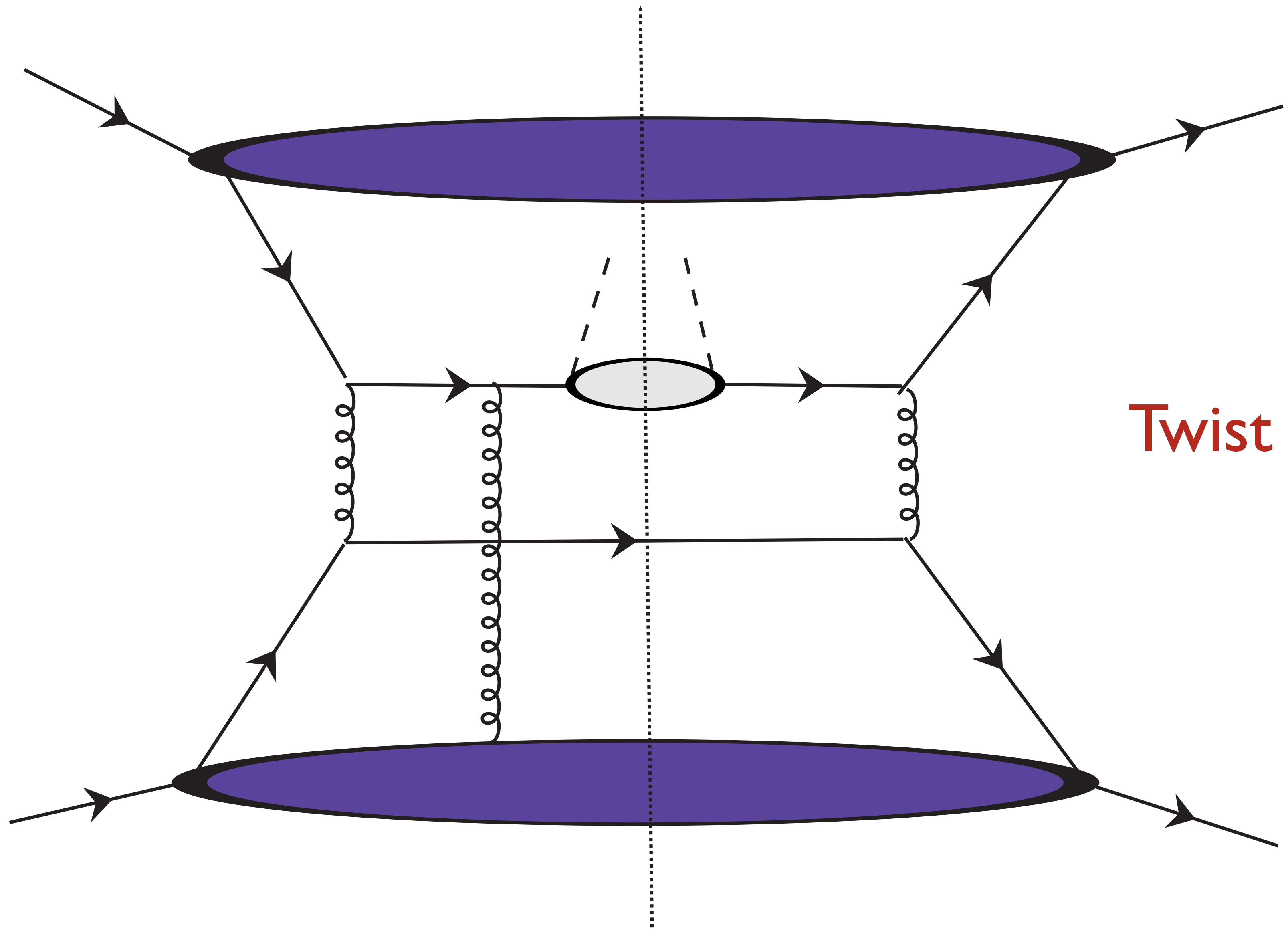
- Born amps are real -- need "loops"----> phases
- QCD interactions conserve helicity up to corrections $\mathcal{O}\left(\frac{m_q}{E_q}\right)$

Twist three and trivial in chiral limit

$$A_N \propto \frac{m_q}{E} \alpha_s$$ at the partonic level

Kane & Repko, PRL: 1978

Not the full story @ Twist 3 approach ETQS approach



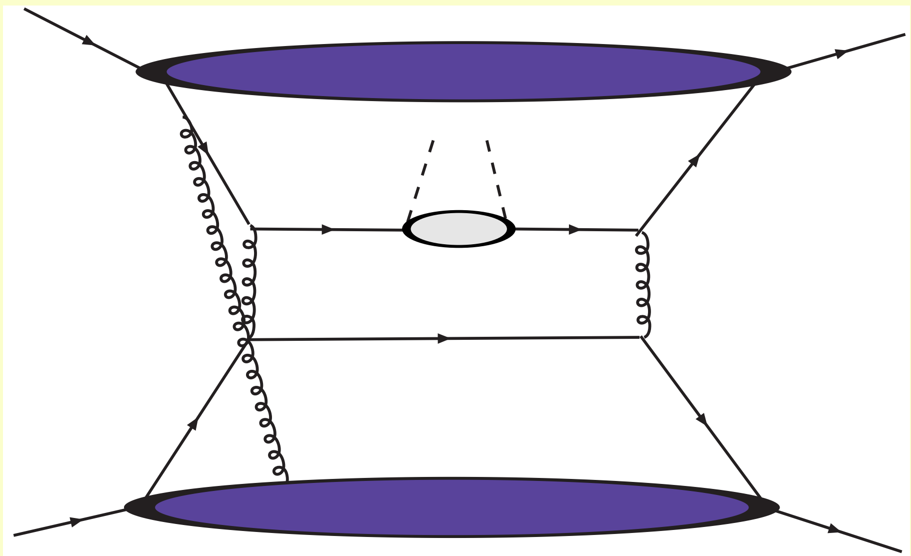
Twist three and non-trival?!

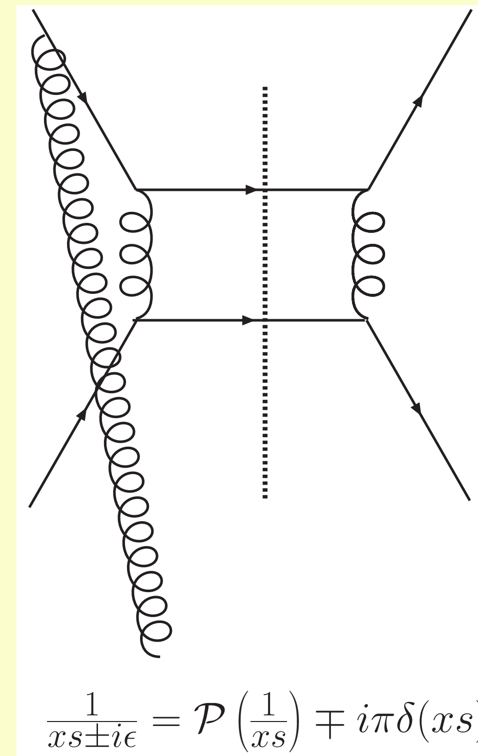
Phases in *soft* poles of propagator in hard subprocess Efremov & Teryaev :PLB 1982

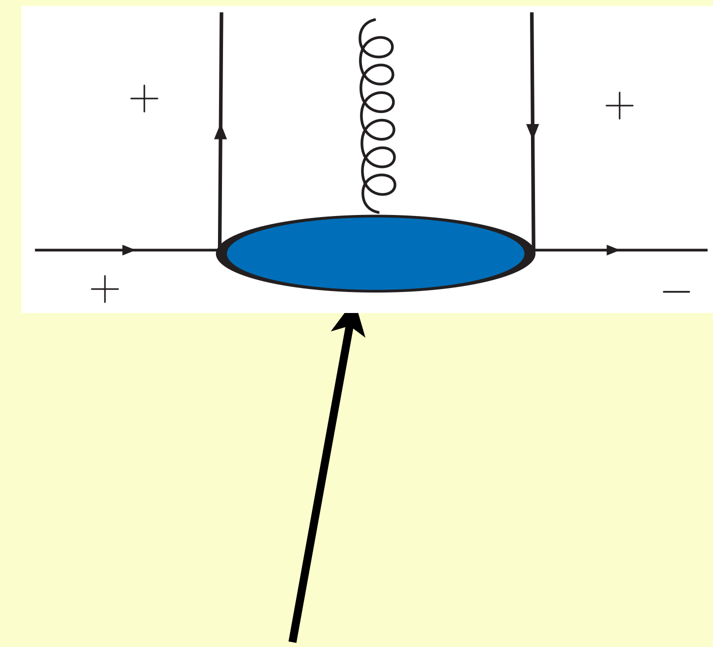
Factorization and Pheno: Qiu, Sterman 1991,1999..., Koike et al, 2000, ... 2010, Ji, Qiu, Vogelsang, Yuan, 2005 ... 2008 ..???, Yuan, Zhou 2008, 2009, Kang, Qiu, 2008, 2009 ...
Kouvaris Ji, Qiu, Vogelsang! 2006, Vogelsang and Yuan 2007

$Q \sim P_T >> \Lambda_{\rm qcd}$ Co-linear Twist 3 Mechanism

Phases in soft poles of prop in hard processes Efremov & Teryaev PLB 1982







- ⋆ **Get helicity flips and phases** $m_q \to \sim M_H$
- $\Delta\sigma \sim f_a \otimes T_F \otimes H_{ETQS} \otimes D^{q\to\pi}$

quark-gluon-quark correlator

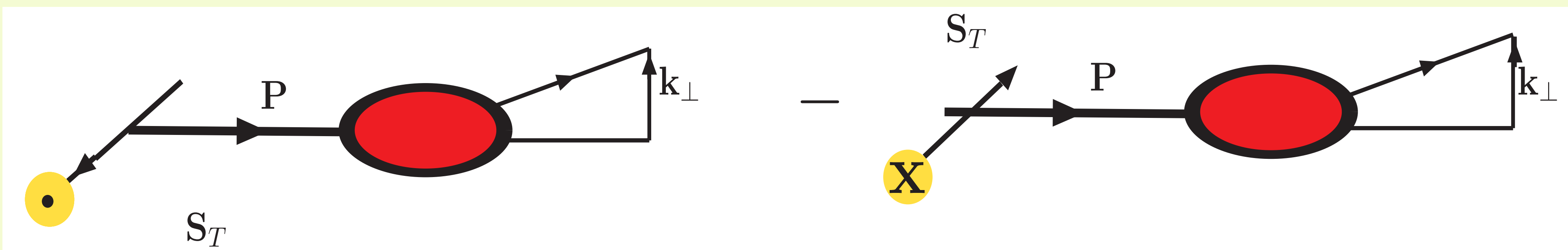
- Phases come from interference of two parton and three parton scattering amplitudes

Koike et al, 2000, ... 2010, Ji, Qiu, Vogelsang, Yuan, 2005 ... 2008 ..???, Yuan, Zhou 2008, 2009, Kang, Qiu, 2008, 2009 ... Kouvaris Ji, Qiu,Vogelsang! 2006, Vogelsang and Yuan 2007

TSSAs thru "T-odd" non-pertb. **spin-orbit** *correlations....*

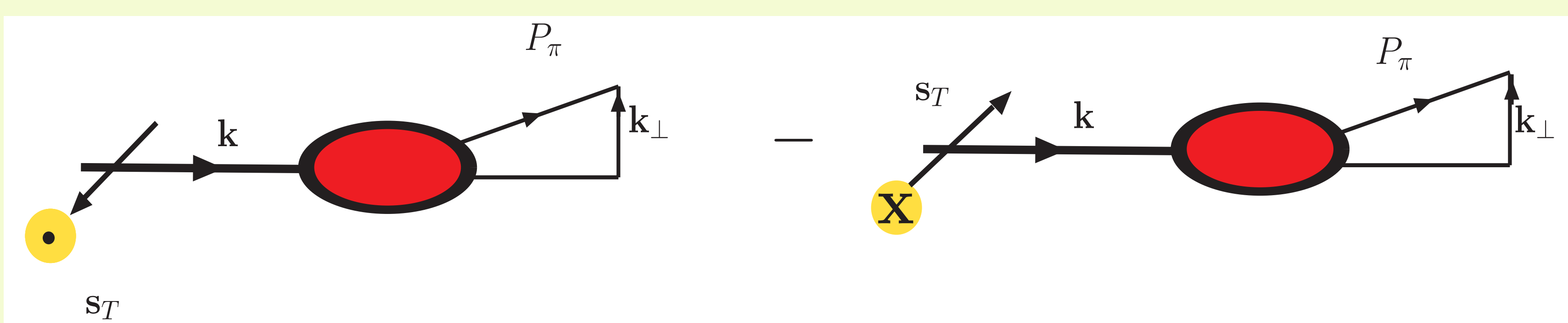
Sensitivity to $p_T \sim \mathbf{k}_T << \sqrt{Q^2}$

- **Sivers PRD: 1990** TSSA is associated w/ correlation *transverse* spin and momenta in initial state hadron



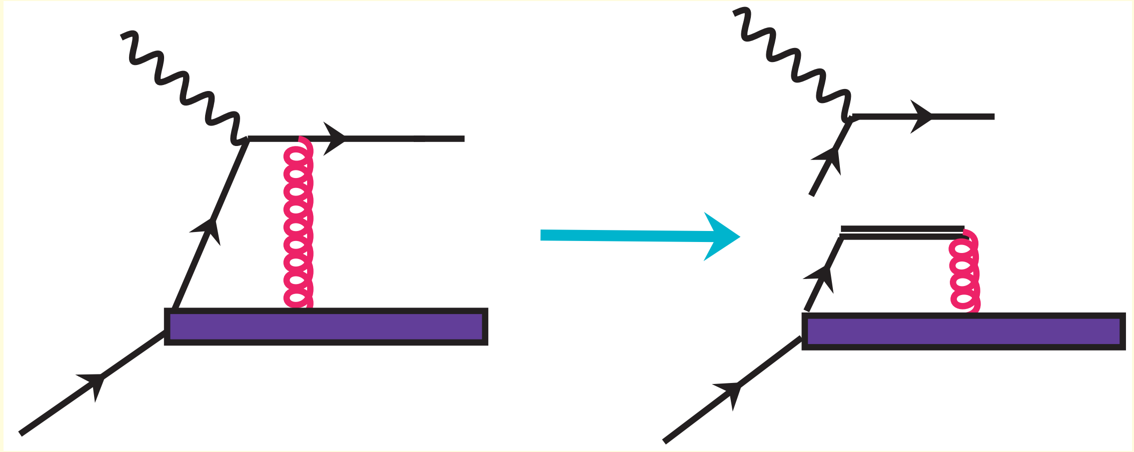
$$\Delta\sigma^{pp^\uparrow \to \pi X} \sim D \otimes f \otimes \Delta f^\perp \otimes \hat{\sigma}_{Born} \Longrightarrow \boldsymbol{\Delta f^\perp(x, k_\perp) = i S_T \cdot (P \times k_\perp)}\, f_{1T}^\perp(x, \boldsymbol{k}_\perp)$$

- **Collins NPB: 1993** TSSA is associated with *transverse* spin of fragmenting quark and transverse momentum of final state hadron



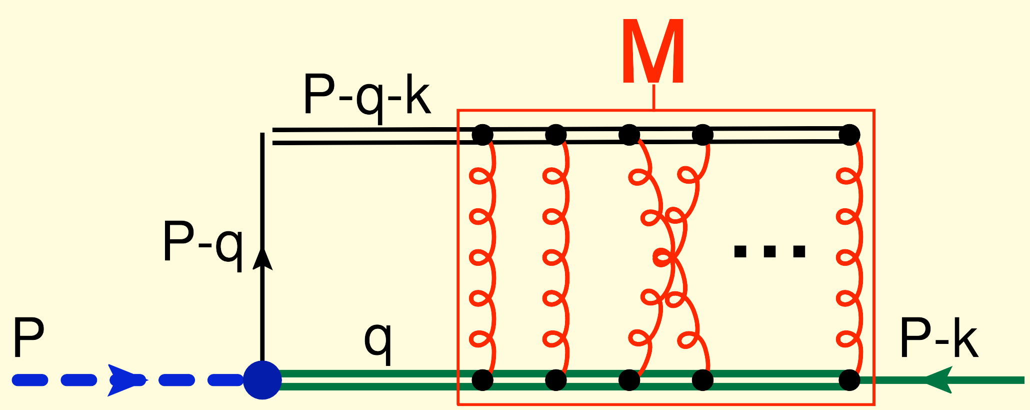
$$\Delta\sigma^{ep^\uparrow \to e\pi X} \sim \Delta D^\perp \otimes f \otimes \hat{\sigma}_{Born} \Longrightarrow \boldsymbol{\Delta D^\perp(x, p_\perp) = i s_T \cdot (P \times p_\perp)}\, H_1^\perp(x, \boldsymbol{p}_\perp)$$

FSI phases in TSSAs unsupressed



$$\Delta f^{\perp}(x, k_{\perp}) = iS_T \cdot (P \times k_{\perp}) \;\; f_{1T}^{\perp}(x, \boldsymbol{k}_{\perp})$$

- **Unsurpressed reaction mech. Boer PRD 1999 context of DY process RHIC**
- ***Brodsky Hwang Schmidt PLB 2002- SIDIS w/ transverse polarized target***
- ***Collins PLB 2002- Gauge link Sivers function doesn't vanish***
- Ji, Yuan PLB: 2002 -Sivers fnct. FSI emerge from Color Gauge-links
- **LG, Goldstein, Oganessyan 2002, 2003** PRD Boer-Mulders Fnct, and Sivers -spectator model
- **LG, M. Schlegel, PLB 2010** Boer-Mulders Fnct, and Sivers beyond summing the FSIs through gauge link



Factorization parton model

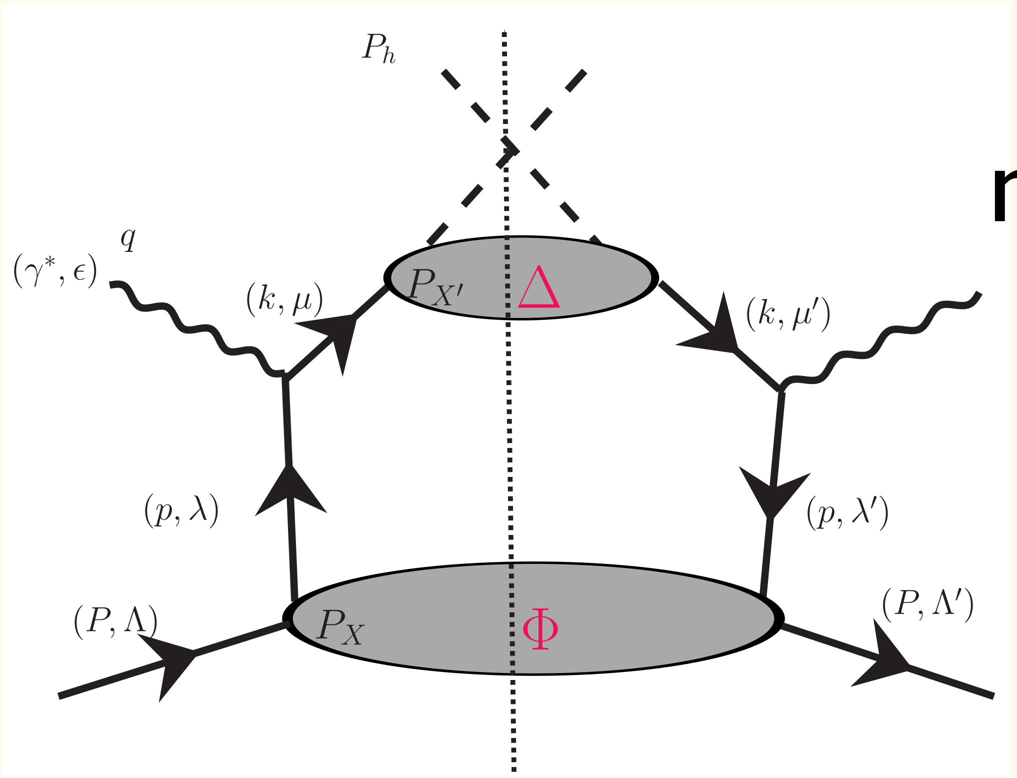
$$W^{\mu\nu}(q,P,S,P_h) \approx \sum_a e^2 \int \frac{d^2\mathbf{p}_T dp^- dp^+}{(2\pi)^4} \int \frac{d^2\mathbf{k}_T dk^- dk^+}{(2\pi)^4} \delta(p^+ - x_B P^+)\delta(k^- - \frac{P_h^-}{z})\delta^2(\mathbf{p}_T + \mathbf{q}_T - \mathbf{k}_T)$$

$$\times \mathrm{Tr}\left[\Phi(p,P,S)\gamma^\mu \Delta(k,P_h)\gamma^\nu\right]$$

$$W^{\mu\nu}(q,P,S,P_h) = \int \frac{d^2\mathbf{p}_T}{(2\pi)^4} \int \frac{d^2\mathbf{k}_T}{(2\pi)^4} \delta^2(\mathbf{p}_T - \frac{\mathbf{P}_{h\perp}}{z} - \mathbf{k}_T)\mathrm{Tr}\left[\left(\int dp^-\Phi\right)\gamma^\mu\left(\int dk^+\Delta\right)\gamma^\nu\right]$$

Small transverse momentum !!!

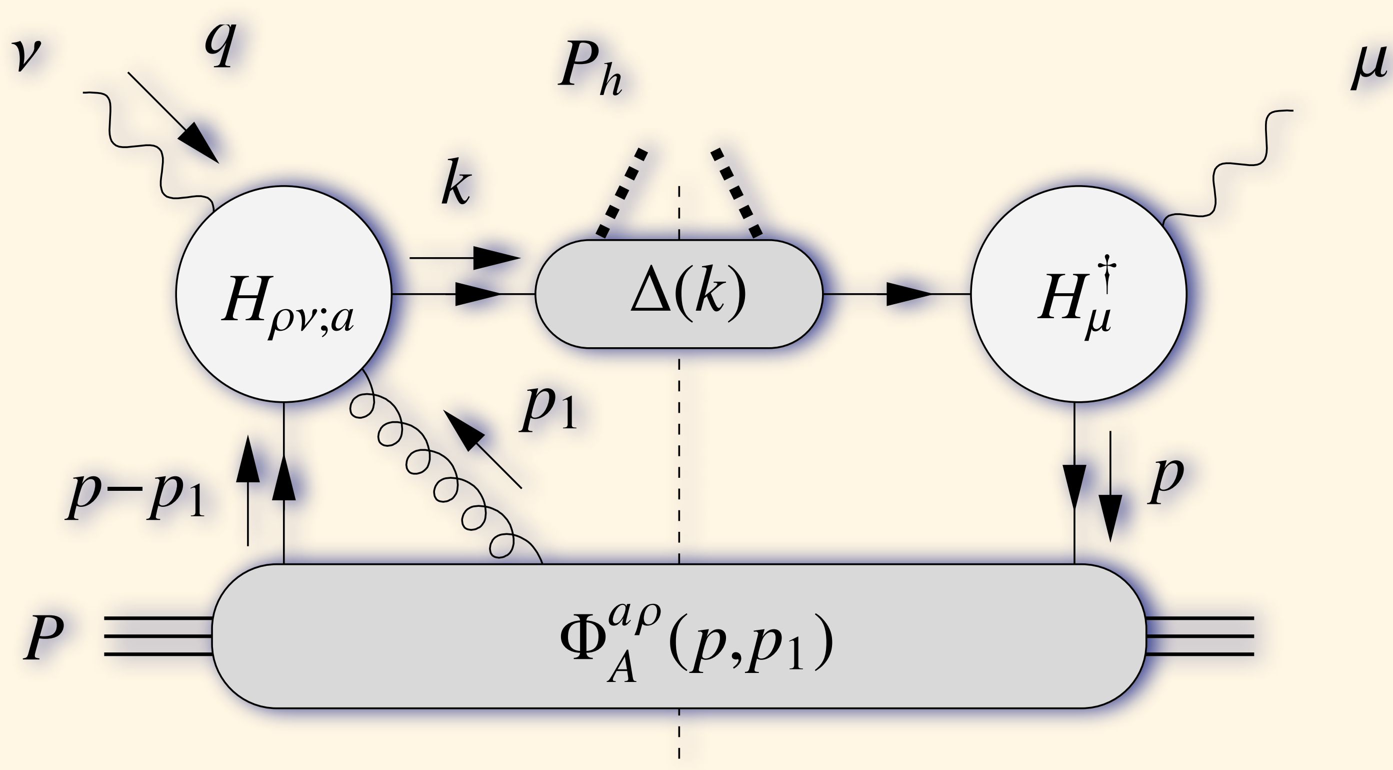
$$\Phi(x,\mathbf{p}_T,S) \equiv \int dp^- \Phi(p,P,S)\Big|_{p^+=x_B P^+}, \qquad \Delta(z,\mathbf{k}_T) \equiv \int dk^+ \Delta(k,P_h)\Big|_{k^-=\frac{P^-}{z_h}}$$



minimal requirement satisfy color gauge invariance

FSIs and TSSAs in Extend Parton Model-Gauge Links

- Obtained by summing the "leading order" gluons that implement ***color gauge invariance***?
- How is the correlator modified?



$$H_{\rho,\nu} = \gamma^\nu$$

“T-Odd” Effects From Color Gauge Inv.Via Gauge links

Gauge link determined re-summing gluon interactions btwn soft and hard

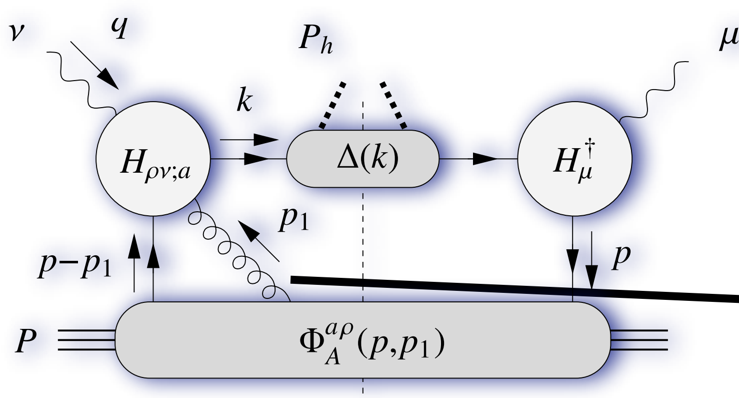
Efremov,Radyushkin *Theor. Math. Phys. 1981*

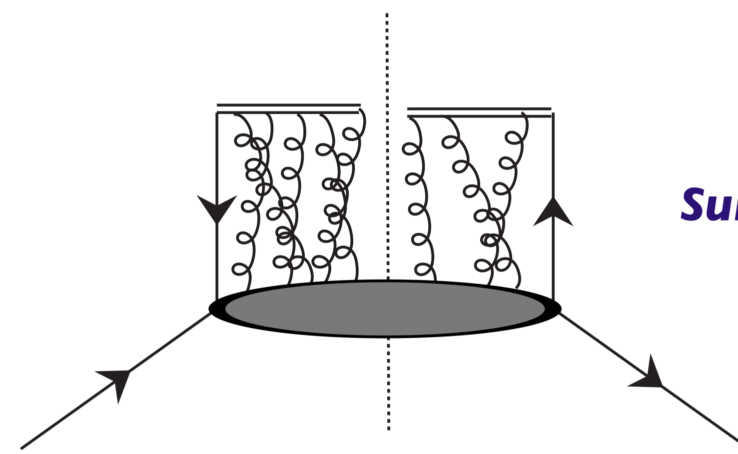
Belitsky, Ji, Yuan NPB 2003,

Boer, Bomhof, Mulders Pijlman, et al. 2003 - 2008- NPB, PLB, PRD

Vogelsang and Yuan PRD 2007

$$\Phi^{[\mathcal{U}[\mathcal{C}]]}(x,p_T)=\int\frac{d\xi^- d^2\xi_T}{2(2\pi)^3}e^{ip\cdot\xi}\langle P|\overline{\psi}(0)\mathcal{U}^{[C]}_{[0,\xi]}\psi(\xi^-,\xi_T)|P\rangle|_{\xi^+=0}$$

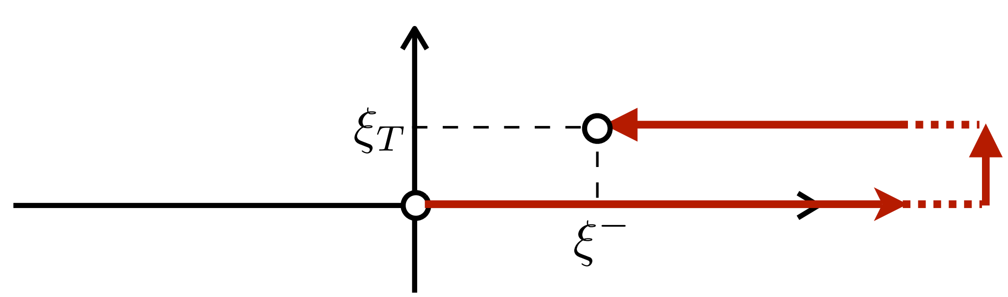




Summing gauge link with color
LG, M. Schlegel PLB 2010

- **The path $[C]$ is fixed by hard subprocess within hadronic process.**

$$\int d^4p d^4k\delta^4(p+q-k)\mathrm{Tr}\left[\Phi^{[U^{C}_{[\infty;\xi]}}(p)H^\dagger_\mu(p,k)\Delta(k)H_\nu(p,k)\right]$$



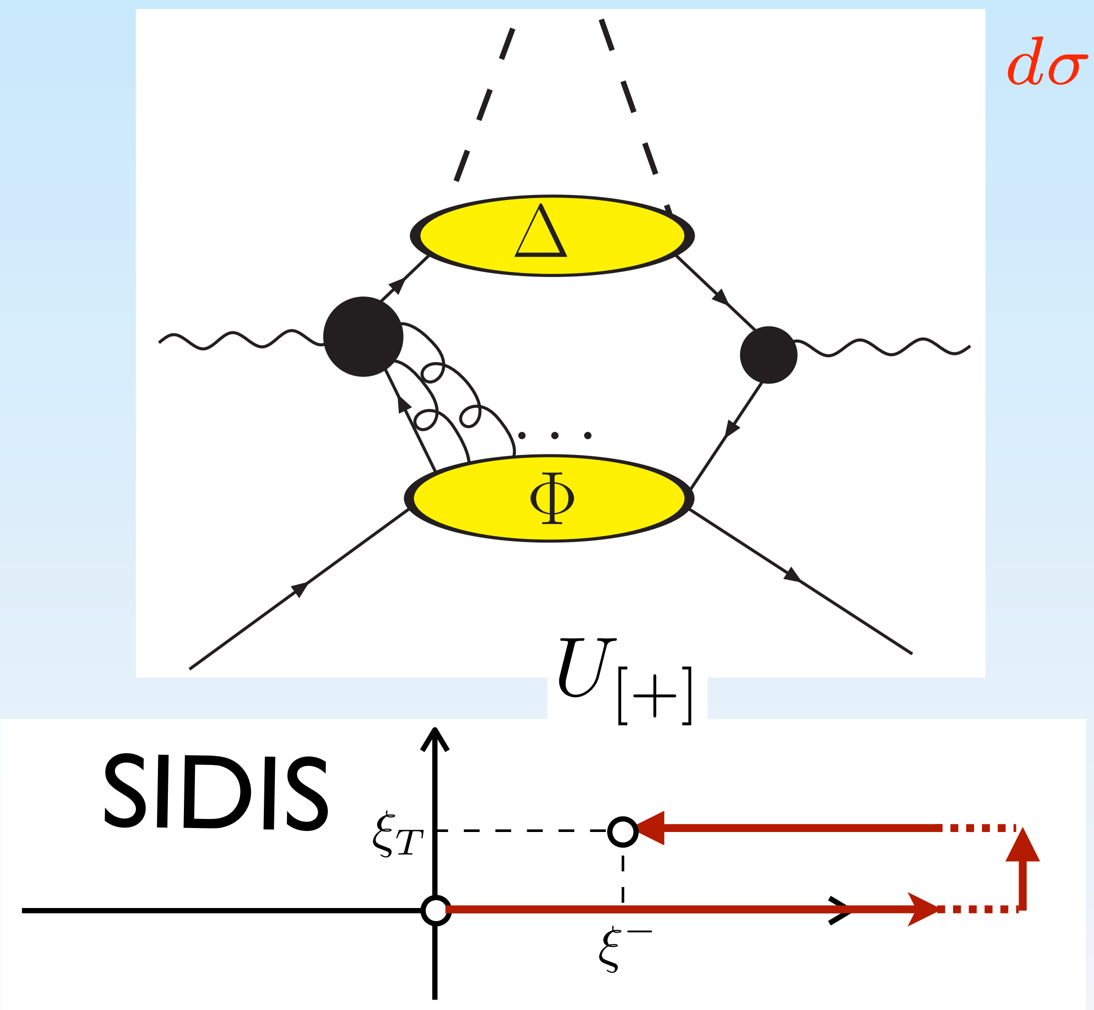
$U_{[+]}$

"Generalized Universality" Fund. Prediction of QCD Factorization

$$f_{1T\,sidis}^{\perp}(x,k_T) = -f_{1T\,DY}^{\perp}(x,k_T) \qquad p_T \sim \mathbf{k}_T << \sqrt{Q^2}$$

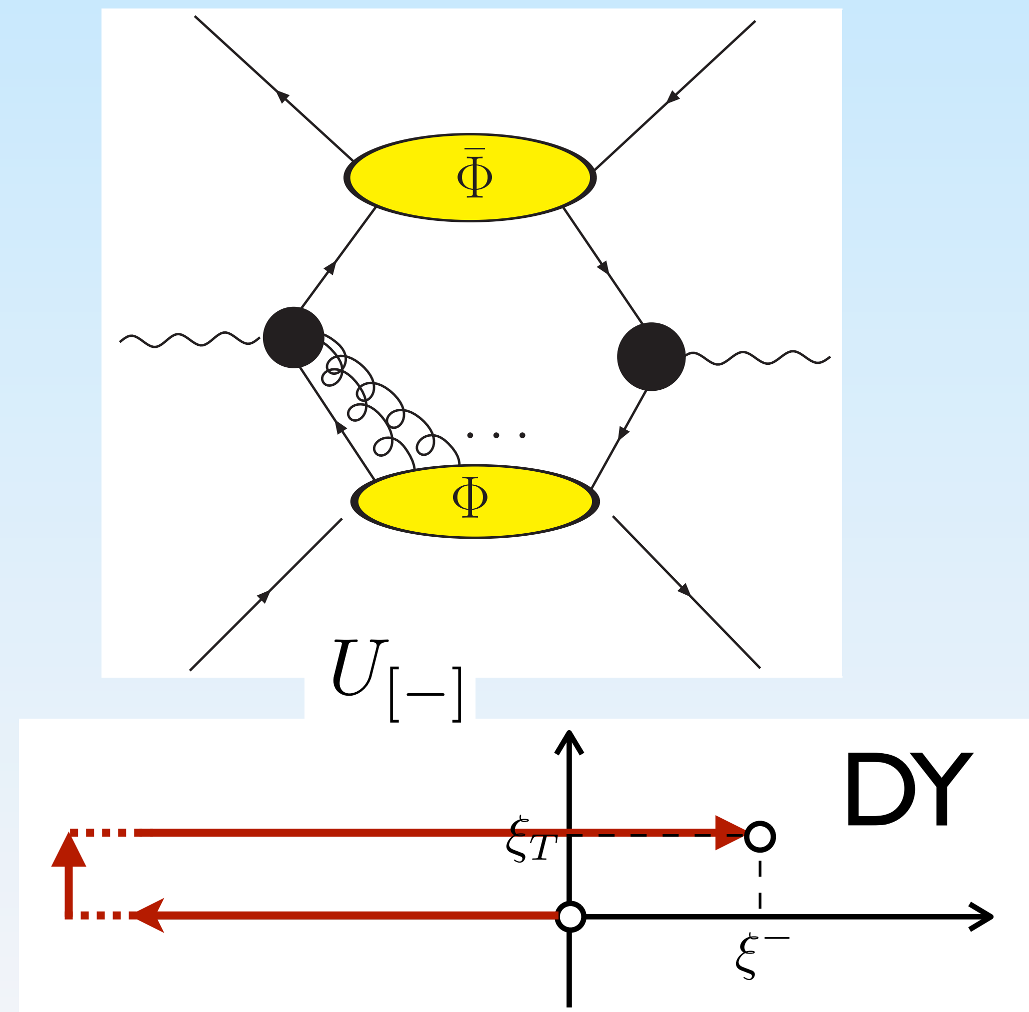
EIC conjunction with DY exp. E906-Fermi, RHIC II, Compass, JPARC

Process Dependence, Collins plb 02, Brodsky et al. NPB 02, Boer Mulders Pijlman Bomhoff 03, 04 ...



$$d\sigma = L_{\mu\nu} W^{\mu\nu} \Rightarrow$$

P&T



$$\Phi^{[+]*}(x,p_T) = i\gamma^1\gamma^3\Phi^{[-]}(x,p_T)i\gamma^1\gamma^3$$

Summary of Trans polz effects in QCD

- Realization that FSI and ISI btwn struck parton and target remnant provide necessary phases that lead to non-vanishing TSSAs

- Two scale factorization in terms of TMDs

$$p_T \sim \mathbf{k}_T << \sqrt{Q^2}$$

- One large scale factorization in terms twist 3 approach

$$Q \sim P_T >> \Lambda_{\rm qcd}$$

- Connection btwn two approaches overlap region. Unified picture Ji, Qiu, Vogelsang, Yuan PRL 2006 ... $\Lambda_{\rm QCD} << q_T << Q$

Generalizing the Generalized Parton Model GPM
(*see talk of Mauro Anselmino*)

- Feynman and Field Fox (PRD 77 & 78)-incorporate intrisic k_T
- Include Transverse spin pol. w/ intrisic k_t --Anselmino, Boglione, Murgia, Prokudin ...et al. *PLB 94*-see talk of Anselmino this wkshp.
- Pheno.... Torino group and other 1994-2011 inclusive processes
- Weighted and unweighted asymmetries in dijet, photon & jet (safer reactions) Bacchetta Bomhoff Mulders Pijlman 2005 PRD & w/ D'Alseio and Murgia 2007 PRL, Qiu, Vogelsang, Yuan PRD 2007
- Inclusive processes studied tw-3 formalism Kouvaris, Qiu, Vogelsang, Yuan PRD 2006, $pp \rightarrow \pi X \quad pp \rightarrow \gamma X$
- What happens when you adopt ansatz of GPM including dynamical reaction mechanism of FSI/ISI in inclusive processes
- take into accout ISI/FSI process dependent Sivers function
- Since this approach is twist three is there connection w/ twist 3 ?

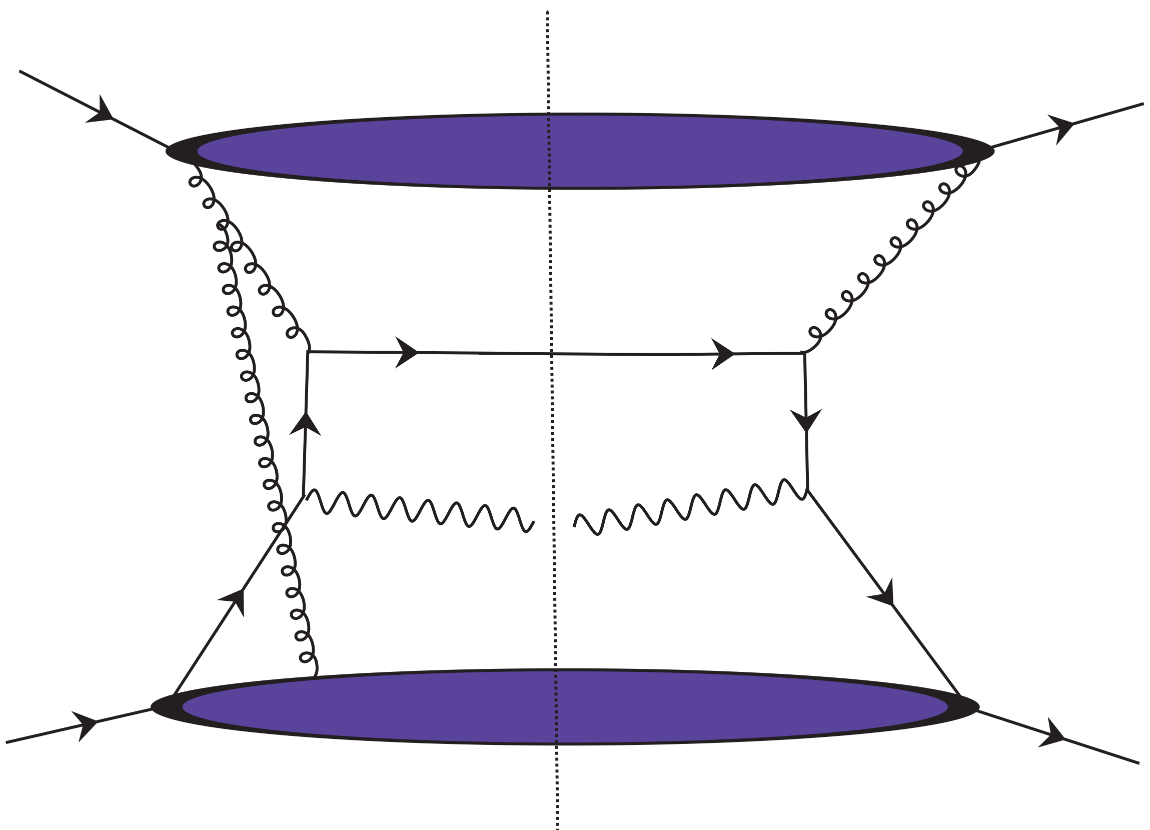
Method

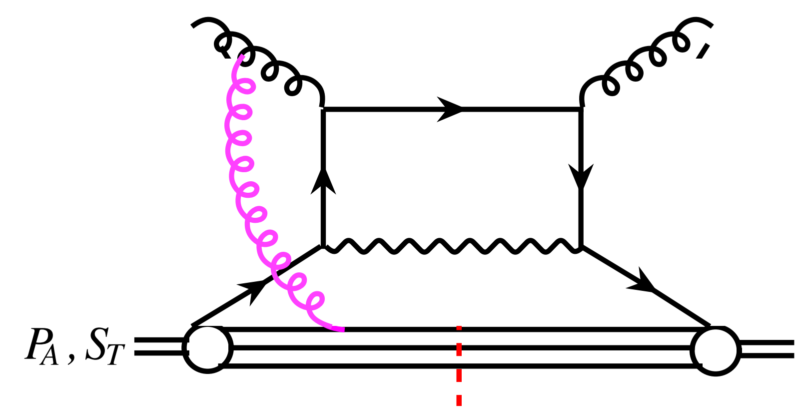
- Use diagramatic rather than helicity approach Bacchetta Bomhoff Mulders Pijlman 2005 PRD
- Has advantage of directly connecting to matrix elements of quark and gluon fields w/o rewriting into parton distributions w/ helicities.
- Allows inclusion of effects of ISI/FSI to determine color structure
- Color factors entirely due to color structure of the partonic subprocess

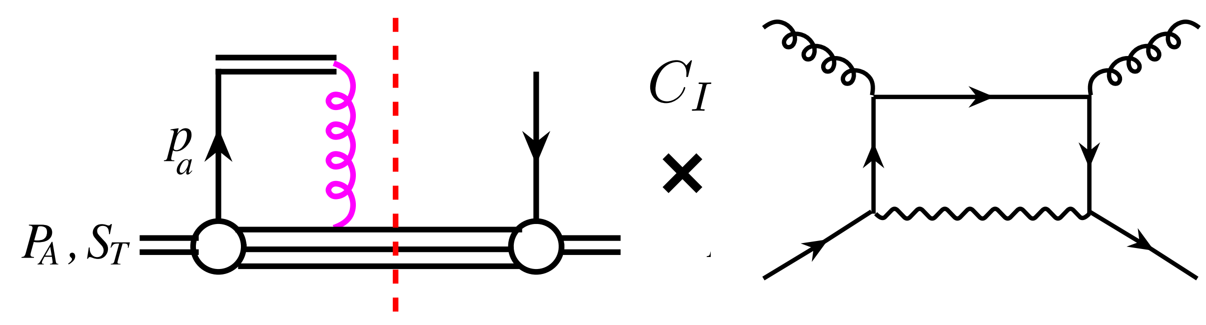
Consider direct Photon in GPM

$$\Delta\sigma^{pp^{\uparrow}\to\gamma X} \sim \Delta f_a \otimes f_b \otimes \Delta\hat{\sigma}$$

Factorize w/ leading 1 gluon exchange get GI & phase







Get Sivers function for this process to use in GPM

Spin Dependent Cross Section in GPM $pp \to \gamma X$

$$f_{q/A^\uparrow}(x,\vec{k}_T) = f_{q/A}(x,k_T^2) + \frac{1}{2}\Delta^N f_{q/A^\uparrow}(x,k_T^2)\vec{S}\cdot(\hat{P}\times\vec{k}_T)$$

A_N is defined by the ratio $A_N = E_\gamma \frac{d\Delta\sigma}{d^3P_\gamma}\Big/ E_\gamma \frac{d\sigma}{d^3P_\gamma}.$

$$E_\gamma \frac{d\Delta\sigma}{d^3P_\gamma} = \frac{\alpha_{em}\alpha_s}{S}\sum_{a,b}\int \frac{dx_a}{x_a} d^2k_{aT} \Delta^N f_{a/A}^{\rm DIS}(x_a,k_{aT})\frac{1}{2}S_A\cdot(\hat{P}_A\times\hat{k}_{aT}) \times \int \frac{dx_b}{x_b} d^2k_{bT} f_{b/B}(x_b,k_{bT}) H^U_{ab\to\gamma}(\hat{s},\hat{t},\hat{u})\delta(\hat{s}+\hat{t}+\hat{u}).$$

GPM Anselmino et al.

$$E_\gamma \frac{d\Delta\sigma}{d^3P_\gamma} = \frac{\alpha_{em}\alpha_s}{S}\sum_{a,b}\int \frac{dx_a}{x_a} d^2k_{aT} \Delta^N f_{a/A}^{ab\to\gamma}(x_a,k_{aT})\frac{1}{2}S_A\cdot(\hat{P}_A\times\hat{k}_{aT}) \times \int \frac{dx_b}{x_b} d^2k_{bT} f_{b/B}(x_b,k_{bT}) H_{ab\to\gamma}(\hat{s},\hat{t},\hat{u})\delta(\hat{s}+\hat{t}+\hat{u}),$$

GPM w/color

LG & Z. Kang

Phys.Lett. B696 2011

process-dependent Sivers function denoted as $\Delta^N f_{a/A}^{ab\to c}(x_a,k_{aT})$

how to get it factorize amplitude

Spin Dependent Cross Section in GPM $pp \to \pi X$

A_N is defined by the ratio $A_N \equiv E_h \frac{d\Delta\sigma}{d^3P_h} \Big/ E_h \frac{d\sigma}{d^3P_h}.$

$$E_h \frac{d\Delta\sigma}{d^3P_h} = \frac{\alpha_s^2}{S} \sum_{a,b,c} \int \frac{dx_a}{x_a} d^2k_{aT} \Delta^N f_{a/A}(x_a, k_{aT}) \frac{1}{2} S_A \cdot (\hat{P}_A \times \hat{k}_{aT}) \int \frac{dx_b}{x_b} d^2k_{bT} f_{b/B}(x_b, k_{bT})$$
$$\times \int \frac{dz_c}{z_c^2} D_{h/c}(z_c) H^U_{ab\to c}(\hat{s}, \hat{t}, \hat{u}) \delta(\hat{s} + \hat{t} + \hat{u}),$$

GPM Anselmino et al.

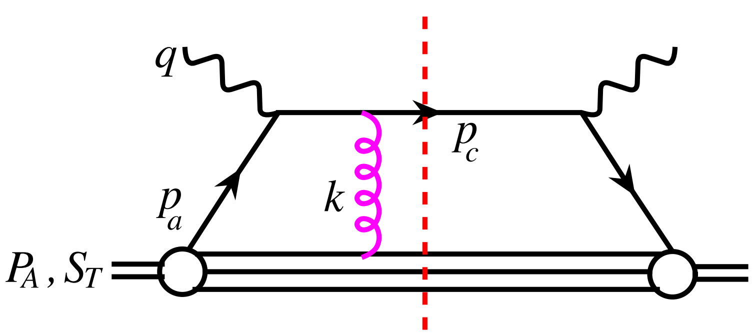
$$E_h \frac{d\Delta\sigma}{d^3P_h} = \frac{\alpha_s^2}{S} \sum_{a,b,c} \int \frac{dx_a}{x_a} d^2k_{aT} \Delta^N f^{ab\to c}_{a/A}(x_a, k_{aT}) \frac{1}{2} S_A \cdot (\hat{P}_A \times \hat{k}_{aT}) \int \frac{dx_b}{x_b} d^2k_{bT} f_{b/B}(x_b, k_{bT})$$
$$\times \int \frac{dz_c}{z_c^2} D_{h/c}(z_c) H^U_{ab\to c}(\hat{s}, \hat{t}, \hat{u}) \delta(\hat{s} + \hat{t} + \hat{u}),$$

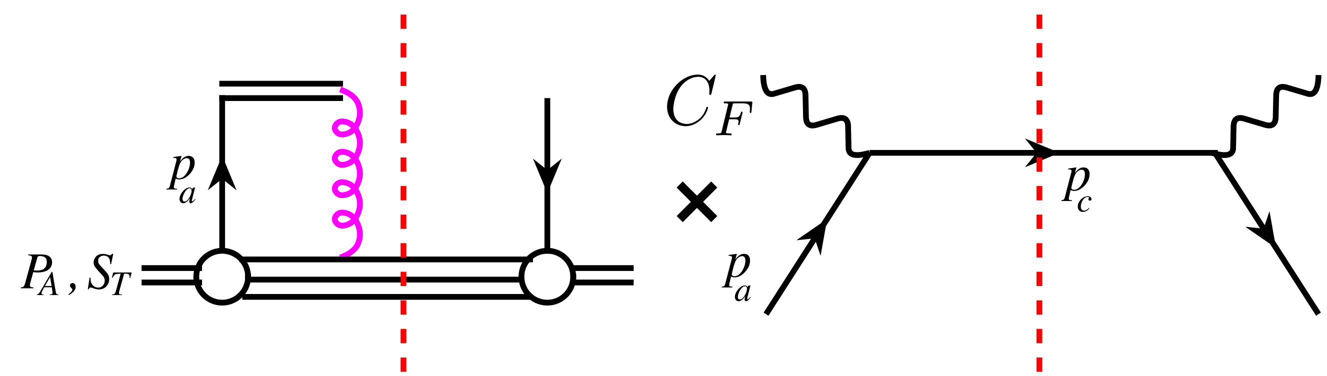
process-dependent Sivers function denoted as $\Delta^N f^{ab\to c}_{a/A}(x_a, k_{aT})$

how to get it ?

Classic example-same real pts opposite imaginary pts

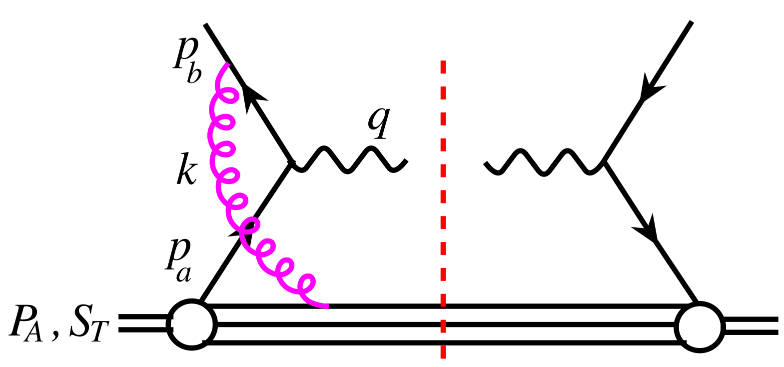
Final-state interaction in SIDIS

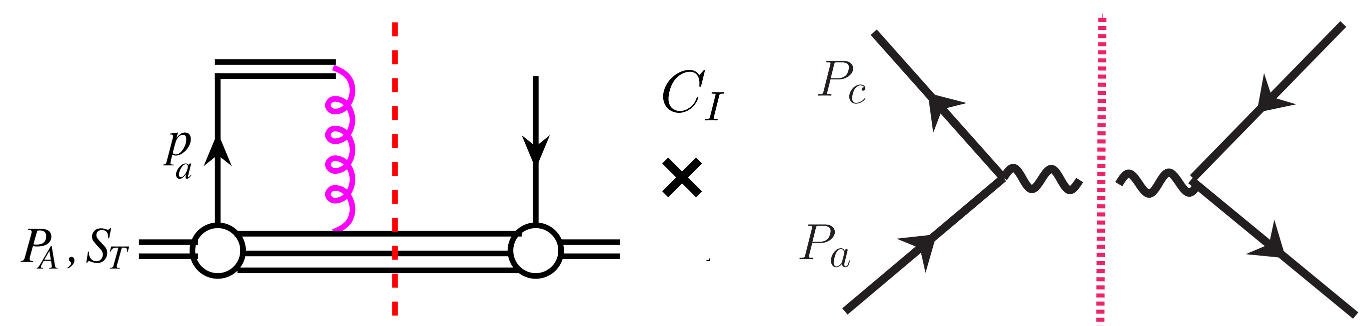




$$\bar{u}(p_c)(-ig)\gamma^- T^a \frac{i(\not{p}_c - \not{k})}{(p_c-k)^2 + i\epsilon} \approx \bar{u}(p_c)\left[\frac{g}{-k^+ + i\epsilon} T^a\right] \quad \rightarrow -\bar{u}(p_c) T^a i\pi\delta(k^+)$$

and initial-state interaction in DY



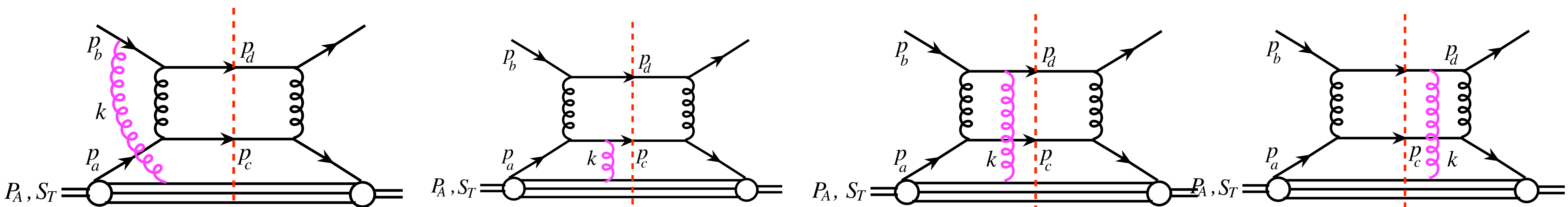


$$\bar{v}(p_b)(-ig)\gamma^- T^a \frac{-i(\not{p}_b + \not{k})}{(p_b+k)^2 + i\epsilon} \approx \bar{v}(p_b)\left[\frac{g}{-k^+ - i\epsilon} T^a\right], \quad \rightarrow v(p_b) T^a i\pi\delta(k^+)$$

Observation

- Crucial point: Sivers function in inclusive single particle production contains both ISI and FSI
- consider channel $qq' \to qq'$

One gluon exchange approx for ISI and FSI



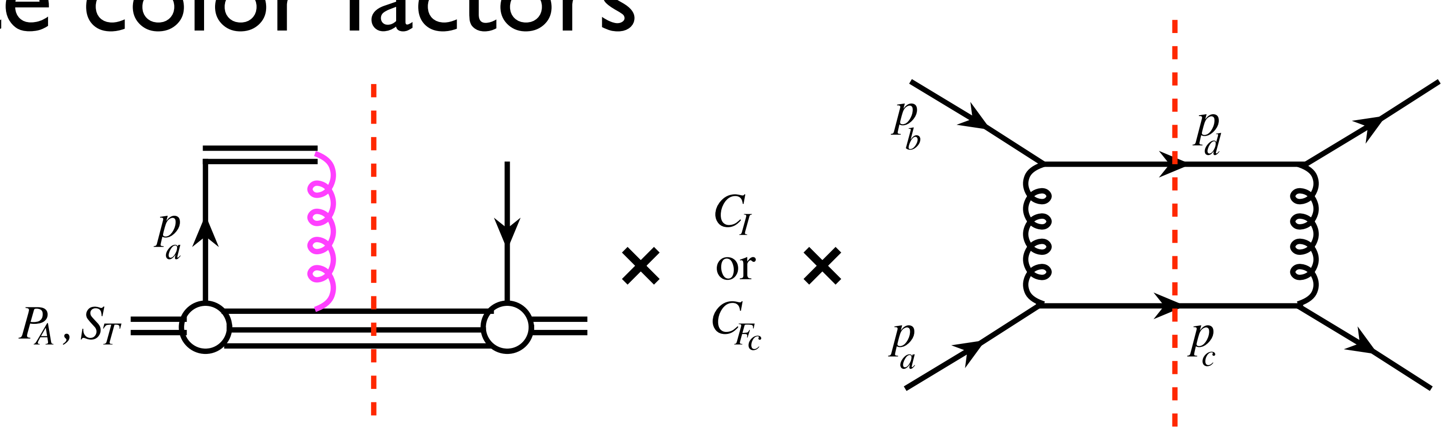
(a) (b) (c) (d)

interaction w/unobserved particle “d” vanishes after summing over both cuts

$$\left[\frac{-g}{-k^+ - i\epsilon} T^a\right] \qquad \left[\frac{g}{-k^+ + i\epsilon} T^a\right]$$

$$C_I = -\frac{1}{2N_c^2}, \qquad C_{F_c} = -\frac{1}{4N_c^2},$$

calculate color factors



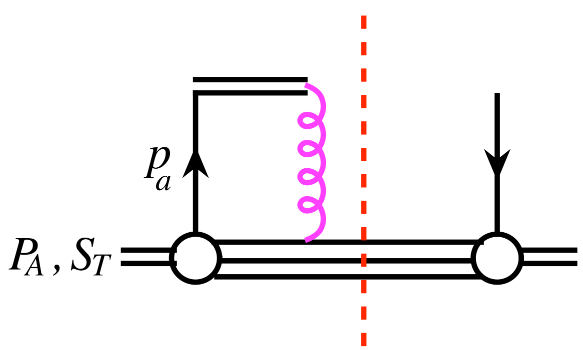
(a) (b)

Note unpolarized color factor

$$C_u = \frac{N_c^2 - 1}{4N_c^2}$$

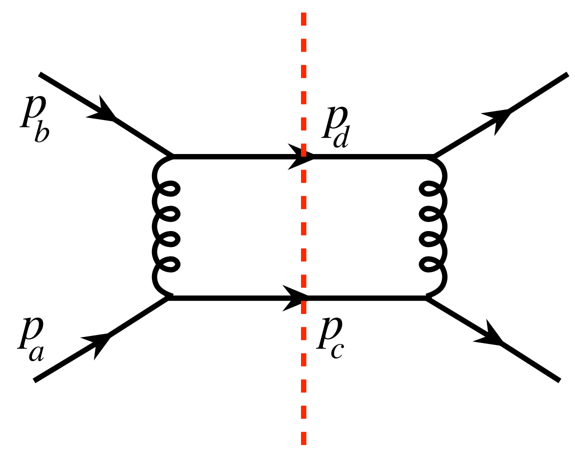
Comparing imag. pt of eikonal propagators for subprocess in SIDIS and inclusive single particle production

Sivers function probed in $qq' \to qq'$ process is related to those in SIDIS



(a)

$\times$ C_I or C_{F_c} $\times$



(b)

$$\Delta^N f_{a/A}^{qq' \to qq'} = \frac{C_I + C_{F_c}}{C_u} \Delta^N f_{a/A}^{\rm SIDIS}.$$

Alternatively one can move color factors "process dependence" to hard parts

$$E_h \frac{d\Delta\sigma}{d^3 P_h} = \frac{\alpha_s^2}{S} \sum_{a,b,c} \int \frac{dx_a}{x_a} d^2 k_{aT} \Delta^N f_{a/A}^{ab\to c}(x_a, k_{aT}) \frac{1}{2} S_A \cdot (\hat{P}_A \times \hat{k}_{aT}) \int \frac{dx_b}{x_b} d^2 k_{bT} f_{b/B}(x_b, k_{bT})$$
$$\times \int \frac{dz_c}{z_c^2} D_{h/c}(z_c) H^U_{ab\to c}(\hat{s}, \hat{t}, \hat{u}) \delta(\hat{s} + \hat{t} + \hat{u}),$$

$$E_h \frac{d\Delta\sigma}{d^3 P_h} = \frac{\alpha_s^2}{S} \sum_{a,b,c} \int \frac{dx_a}{x_a} d^2 k_{aT} \Delta^N f_{a/A}^{\mathrm{SIDIS}}(x_a, k_{aT}) \frac{1}{2} S_A \cdot (\hat{P}_A \times \hat{k}_{aT}) \int \frac{dx_b}{x_b} d^2 k_{bT} f_{b/B}(x_b, k_{bT})$$
$$\times \int \frac{dz_c}{z_c^2} D_{h/c}(z_c) H^{\mathrm{Inc}}_{ab\to c}(\hat{s}, \hat{t}, \hat{u}) \delta(\hat{s} + \hat{t} + \hat{u}),$$

In spirit of twist 3 approach

That is rearrange

$$\Delta^N f_{a/A}^{qq'\to qq'} H^U_{qq'\to qq'} = \frac{C_I + C_{F_c}}{C_u} \Delta^N f_{a/A}^{\mathrm{SIDIS}} H^U_{qq'\to qq'} = \Delta^N f_{a/A}^{\mathrm{SIDIS}} \left[C_I h_{qq'\to qq'} + C_{F_c} h_{qq'\to qq'} \right],$$

where hard partonic c.s. w/o color factors

$$h_{qq'\to qq'} = 2\frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2}.$$

In spirit of twist 3 approach

Then "modified" GPM is

$$E_h \frac{d\Delta\sigma}{d^3P_h} = \frac{\alpha_s^2}{S} \sum_{a,b,c} \int \frac{dx_a}{x_a} d^2k_{aT} \Delta^N f_{a/A}^{\mathrm{SIDIS}}(x_a, k_{aT}) \frac{1}{2} S_A \cdot (\hat{P}_A \times \hat{k}_{aT}) \int \frac{dx_b}{x_b} d^2k_{bT} f_{b/B}(x_b, k_{bT})$$
$$\times \int \frac{dz_c}{z_c^2} D_{h/c}(z_c) H_{ab\to c}^{\mathrm{Inc}}(\hat{s}, \hat{t}, \hat{u}) \delta(\hat{s} + \hat{t} + \hat{u}),$$

$$H_{qq'\to qq'}^{\mathrm{Inc}} \equiv H_{qq'\to qq'}^{\mathrm{Inc-I}} + H_{qq'\to qq'}^{\mathrm{Inc-F}},$$

where,

$$H_{qq'\to qq'}^{\mathrm{Inc-I}} = C_I h_{qq'\to qq'}, \qquad H_{qq'\to qq'}^{\mathrm{Inc-F}} = C_{F_c} h_{qq'\to qq'}$$

hard partonic c.s. w/o color factors

$$h_{qq'\to qq'} = 2\frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2}.$$

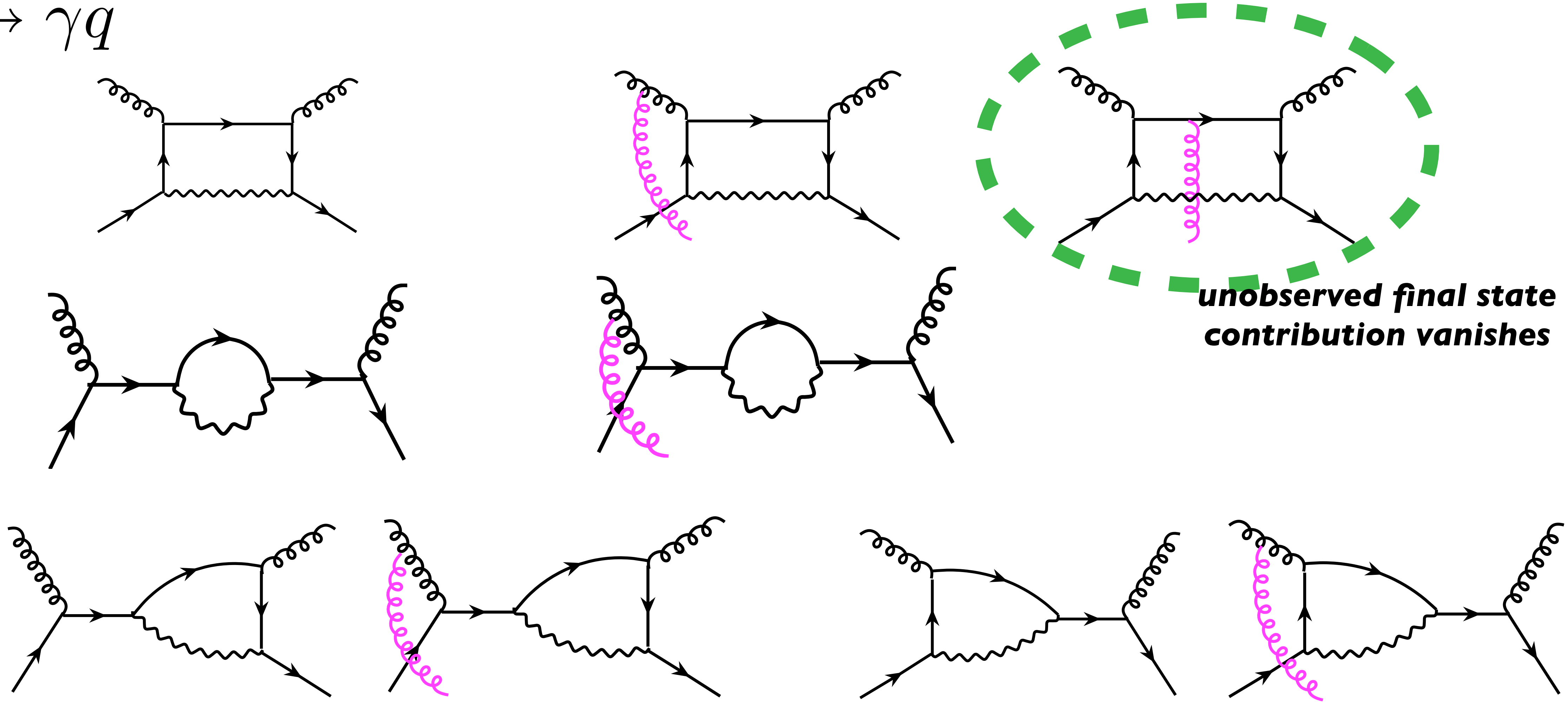
Color modification of hard cross sections due to "phases"

$qg \to \gamma q$

t-channel

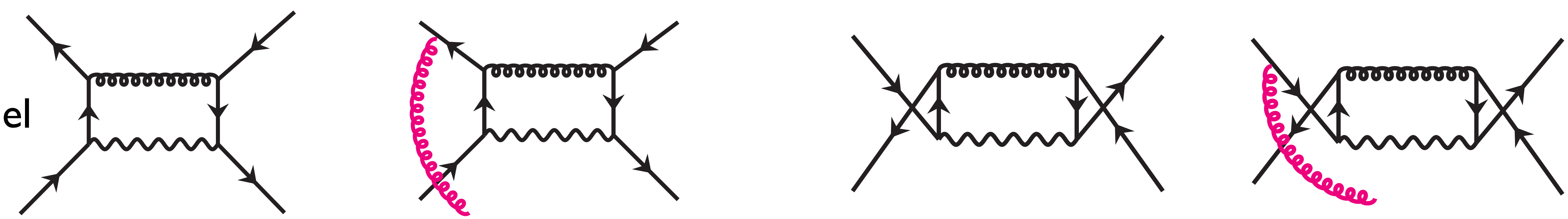
s-channel

s-t interference



$\bar{q}q \to \gamma g$

t & u-channel



etc ....

t-u interference

The contributions for $pp \to \gamma X$

the various contributing partonic subprocesses are given by

$$\boxed{\begin{aligned} H^{\rm Inc}_{qg\to\gamma q} &= -H^{\rm Inc}_{\bar{q}g\to\gamma\bar{q}} = -\frac{N_c}{N_c^2-1}e_q^2\left[-\frac{\hat{t}}{\hat{s}}-\frac{\hat{s}}{\hat{t}}\right] \\ H^{\rm Inc}_{q\bar{q}\to\gamma g} &= -H^{\rm Inc}_{\bar{q}q\to\gamma g} = \frac{1}{N_c^2}e_q^2\left[\frac{\hat{t}}{\hat{u}}+\frac{\hat{u}}{\hat{t}}\right] \end{aligned}}$$

The contributions for $pp \to \pi X$

the various contributing partonic subprocesses are given by

$$H^{\mathrm{Inc-I}}_{qq' \to qq'} = -H^{\mathrm{Inc-I}}_{\bar{q}\bar{q}' \to \bar{q}\bar{q}'} = -\frac{1}{N_c^2}\left[\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}\right]$$

$$H^{\mathrm{Inc-F}}_{qq' \to qq'} = -H^{\mathrm{Inc-F}}_{\bar{q}\bar{q}' \to \bar{q}\bar{q}'} = -\frac{1}{2N_c^2}\left[\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}\right]$$

$$H^{\mathrm{Inc-I}}_{q\bar{q}' \to q\bar{q}'} = -H^{\mathrm{Inc-I}}_{\bar{q}q' \to \bar{q}q'} = -\frac{N_c^2-2}{2N_c^2}\left[\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}\right]$$

$$H^{\mathrm{Inc-F}}_{q\bar{q}' \to q\bar{q}'} = -H^{\mathrm{Inc-F}}_{\bar{q}q' \to \bar{q}q'} = -\frac{1}{2N_c^2}\left[\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}\right]$$

$$H^{\mathrm{Inc-I}}_{qq' \to q'q} = -H^{\mathrm{Inc-I}}_{\bar{q}\bar{q}' \to \bar{q}'\bar{q}} = -\frac{1}{N_c^2}\left[\frac{\hat{s}^2+\hat{t}^2}{\hat{u}^2}\right]$$

$$H^{\mathrm{Inc-F}}_{qq' \to q'q} = -H^{\mathrm{Inc-F}}_{\bar{q}\bar{q}' \to \bar{q}'\bar{q}} = \frac{N_c^2-2}{2N_c^2}\left[\frac{\hat{s}^2+\hat{t}^2}{\hat{u}^2}\right]$$

$$H^{\mathrm{Inc-I}}_{q\bar{q}' \to \bar{q}'q} = -H^{\mathrm{Inc-I}}_{\bar{q}q' \to q'\bar{q}} = -\frac{N_c^2-2}{2N_c^2}\left[\frac{\hat{s}^2+\hat{t}^2}{\hat{u}^2}\right]$$

$$H^{\mathrm{Inc-F}}_{q\bar{q}' \to \bar{q}'q} = = -H^{\mathrm{Inc-F}}_{\bar{q}q' \to q'\bar{q}} = \frac{1}{N_c^2}\left[\frac{\hat{s}^2+\hat{t}^2}{\hat{u}^2}\right]$$

$$H^{\mathrm{Inc-I}}_{qq \to qq} = -H^{\mathrm{Inc-I}}_{\bar{q}\bar{q} \to \bar{q}\bar{q}} = -\frac{1}{N_c^2}\left[\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}+\frac{\hat{s}^2+\hat{t}^2}{\hat{u}^2}\right]+\frac{N_c^2+1}{N_c^3}\frac{\hat{s}^2}{\hat{t}\hat{u}}$$

$$H^{\mathrm{Inc-F}}_{qq \to qq} = -H^{\mathrm{Inc-F}}_{\bar{q}\bar{q} \to \bar{q}\bar{q}} = -\frac{1}{2N_c^2}\left[\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}\right]+\frac{N_c^2-2}{2N_c^2}\left[\frac{\hat{s}^2+\hat{t}^2}{\hat{u}^2}\right]+\frac{1}{N_c^3}\frac{\hat{s}^2}{\hat{t}\hat{u}}$$

$$H^{\mathrm{Inc-I}}_{q\bar{q} \to q'\bar{q}'} = -H^{\mathrm{Inc-I}}_{\bar{q}q \to \bar{q}'q'} = \frac{1}{2N_c^2}\left[\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}\right]$$

$$H^{\mathrm{Inc-F}}_{q\bar{q} \to q'\bar{q}'} = -H^{\mathrm{Inc-F}}_{\bar{q}q \to \bar{q}'q'} = \frac{N_c^2-2}{2N_c^2}\left[\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}\right]$$

$$H^{\mathrm{Inc-I}}_{q\bar{q} \to \bar{q}'q'} = -H^{\mathrm{Inc-I}}_{\bar{q}q \to q'\bar{q}'} = \frac{1}{2N_c^2}\left[\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}\right]$$

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$$H^{\mathrm{Inc-F}}_{q\bar{q} \to q\bar{q}} = -H^{\mathrm{Inc-F}}_{\bar{q}q \to \bar{q}q} = -\frac{1}{2N_c^2}\left[\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}\right]+\frac{N_c^2-2}{2N_c^2}\left[\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}\right]+\frac{1}{N_c^3}\frac{\hat{u}^2}{\hat{s}\hat{t}}$$

$$H^{\mathrm{Inc-I}}_{q\bar{q} \to \bar{q}q} = -H^{\mathrm{Inc-I}}_{\bar{q}q \to q\bar{q}} = -\frac{N_c^2-2}{2N_c^2}\left[\frac{\hat{s}^2+\hat{t}^2}{\hat{u}^2}\right]+\frac{1}{2N_c^2}\left[\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}\right]-\frac{1}{N_c^3}\frac{\hat{t}^2}{\hat{s}\hat{u}}$$

$$H^{\mathrm{Inc-F}}_{q\bar{q} \to \bar{q}q} = -H^{\mathrm{Inc-F}}_{\bar{q}q \to q\bar{q}} = \frac{1}{N_c^2}\left[\frac{\hat{s}^2+\hat{t}^2}{\hat{u}^2}+\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}\right]-\frac{N_c^2+1}{N_c^3}\frac{\hat{t}^2}{\hat{s}\hat{u}}$$

$$H^{\mathrm{Inc-I}}_{qg \to qg} = -H^{\mathrm{Inc-I}}_{\bar{q}g \to \bar{q}g} = \frac{1}{2(N_c^2-1)}\left[-\frac{\hat{s}}{\hat{u}}-\frac{\hat{u}}{\hat{s}}\right]+\frac{N_c^2}{2(N_c^2-1)}\left[\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}\frac{\hat{u}}{\hat{s}}\right]$$

$$H^{\mathrm{Inc-F}}_{qg \to qg} = -H^{\mathrm{Inc-F}}_{\bar{q}g \to \bar{q}g} = \frac{1}{2N_c^2(N_c^2-1)}\left[-\frac{\hat{s}}{\hat{u}}-\frac{\hat{u}}{\hat{s}}\right]-\frac{1}{N_c^2-1}\left[\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}\right]$$

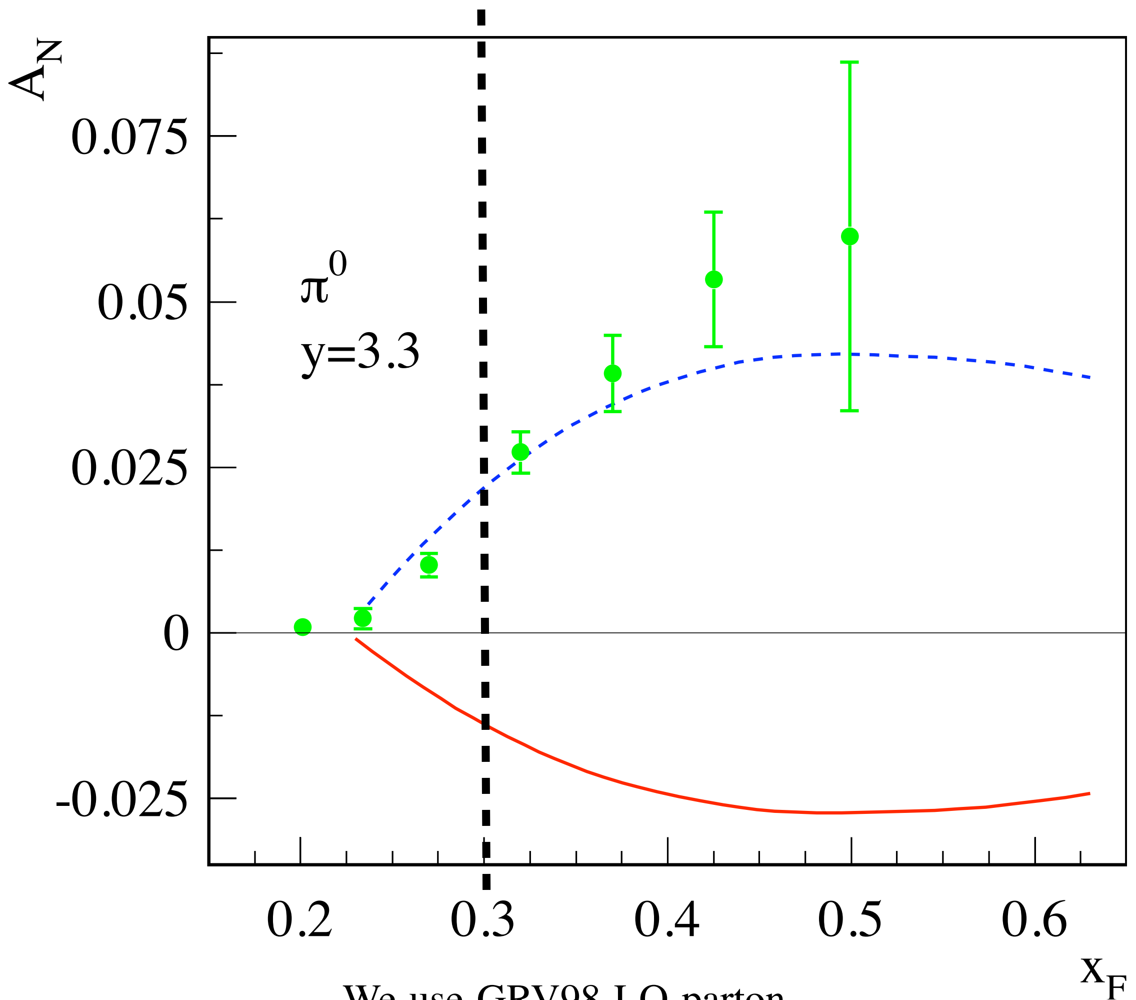
$$H^{\mathrm{Inc-I}}_{qg \to gq} = -H^{\mathrm{Inc-I}}_{\bar{q}g \to g\bar{q}} = \frac{1}{2(N_c^2-1)}\left[-\frac{\hat{s}}{\hat{t}}-\frac{\hat{t}}{\hat{s}}\right]+\frac{N_c^2}{2(N_c^2-1)}\left[\frac{\hat{s}^2+\hat{t}^2}{\hat{u}^2}\frac{\hat{t}}{\hat{s}}\right]$$

$$H^{\mathrm{Inc-F}}_{qg \to gq} = -H^{\mathrm{Inc-F}}_{\bar{q}g \to g\bar{q}} = -\frac{1}{2(N_c^2-1)}\left[-\frac{\hat{s}}{\hat{t}}-\frac{\hat{t}}{\hat{s}}\right]-\frac{N_c^2}{2(N_c^2-1)}\left[\frac{\hat{s}^2+\hat{t}^2}{\hat{u}^2}\frac{\hat{s}}{\hat{t}}\right]$$

$$H^{\mathrm{Inc-I}}_{q\bar{q} \to gg} = -H^{\mathrm{Inc-I}}_{\bar{q}q \to gg} = -\frac{1}{2N_c^3}\left[\frac{\hat{u}}{\hat{t}}+\frac{\hat{t}}{\hat{u}}\right]-\frac{1}{N_c}\left[\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}\right]$$

$$H^{\mathrm{Inc-F}}_{q\bar{q} \to gg} = -H^{\mathrm{Inc-F}}_{\bar{q}q \to gg} = -\frac{1}{2N_c}\left[\frac{\hat{u}}{\hat{t}}+\frac{\hat{t}}{\hat{u}}\right]+\frac{N_c}{2}\left[\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}\frac{\hat{u}}{\hat{t}}\right]$$

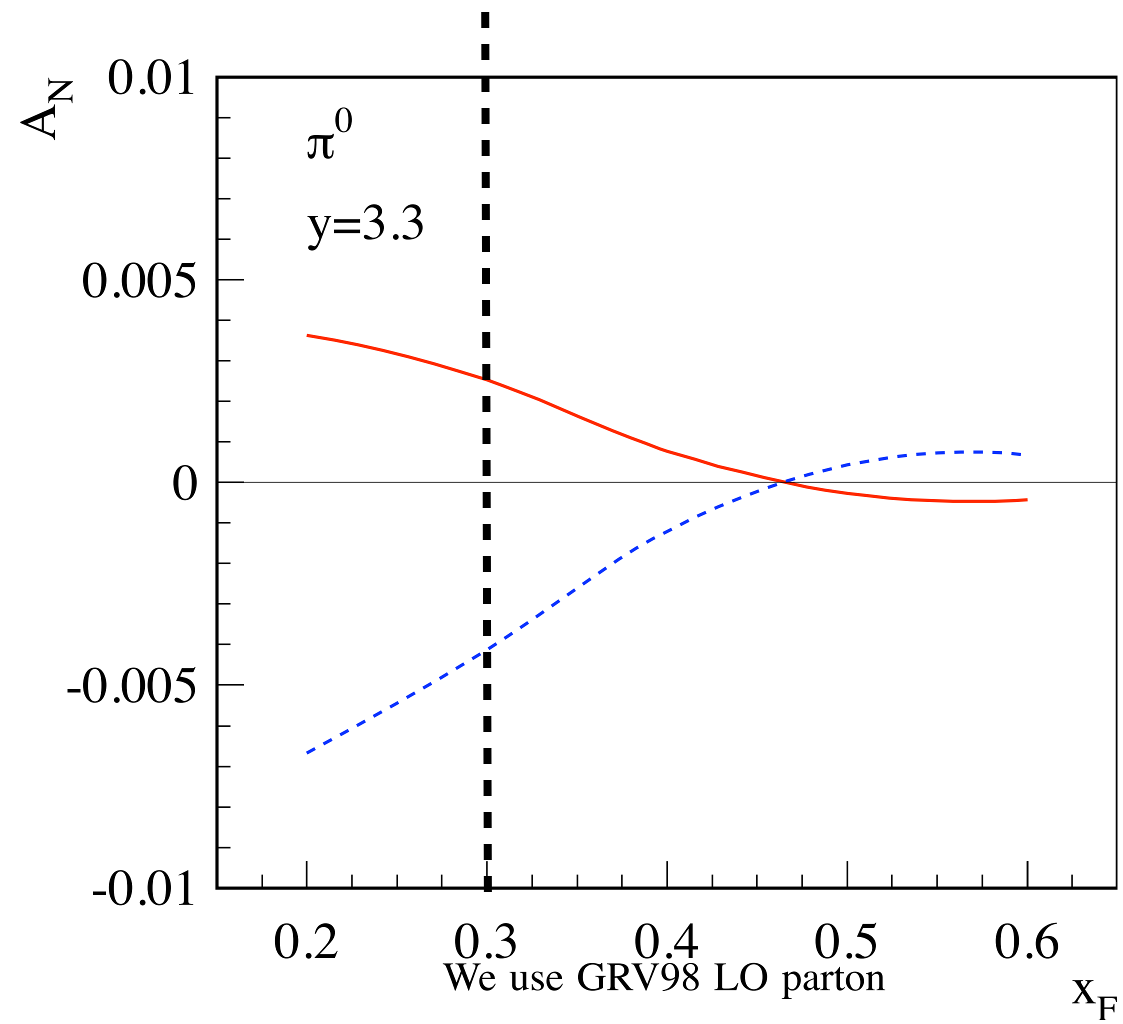
Based on old parameterization



We use GRV98 LO parton

the old Sivers function from [4], and Kretzer fragmentation function [5].

Based on new parameterization

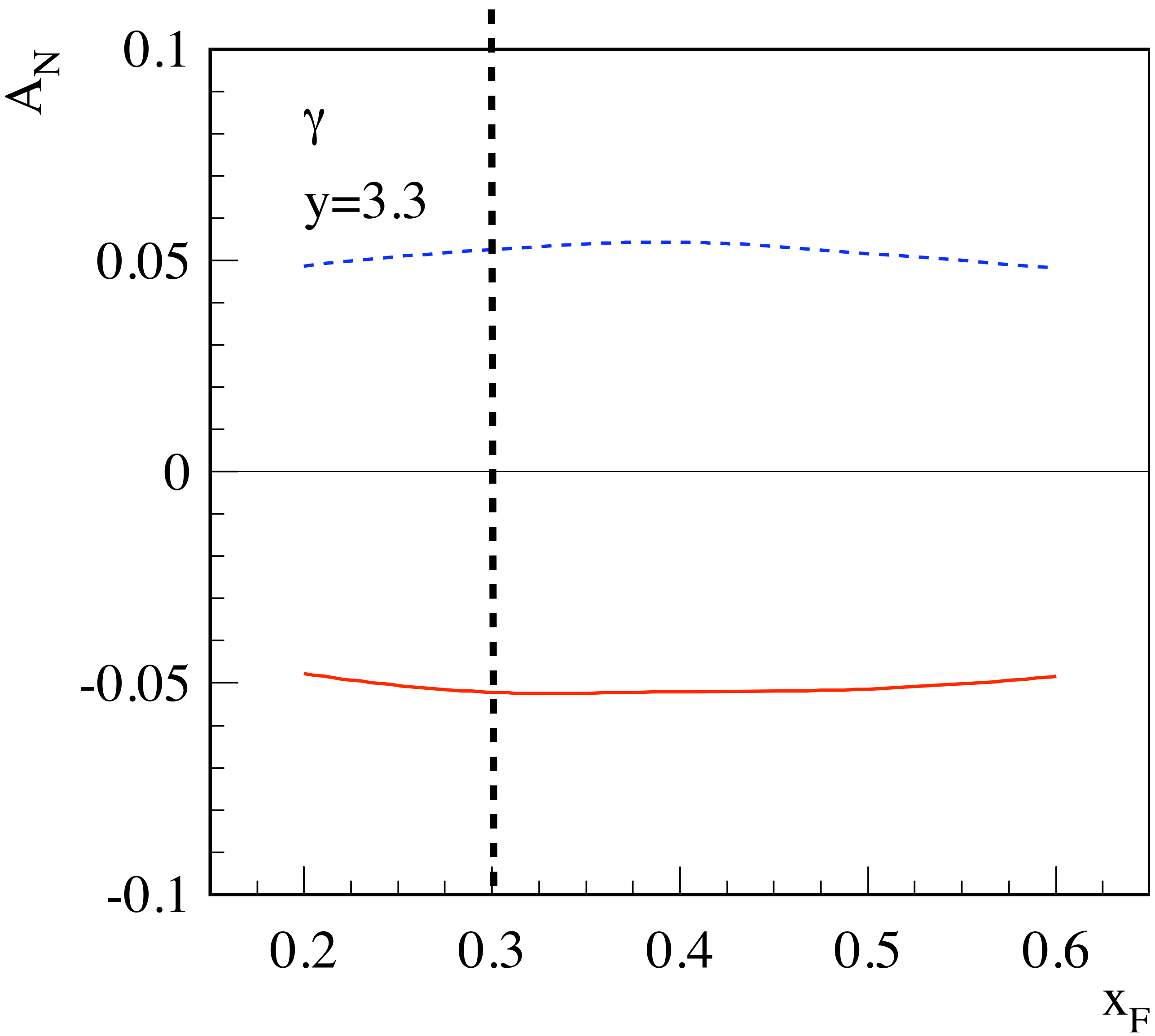


We use GRV98 LO parton

, the latest Sivers function from [2], and DSS fragmentation function [3].

$$\Delta\sigma^{pp^{\uparrow}\to\pi X} \sim \Delta f_a \otimes f_b \otimes \Delta\hat{\sigma} \otimes D^{q\to\pi}$$

Based on old parameterization



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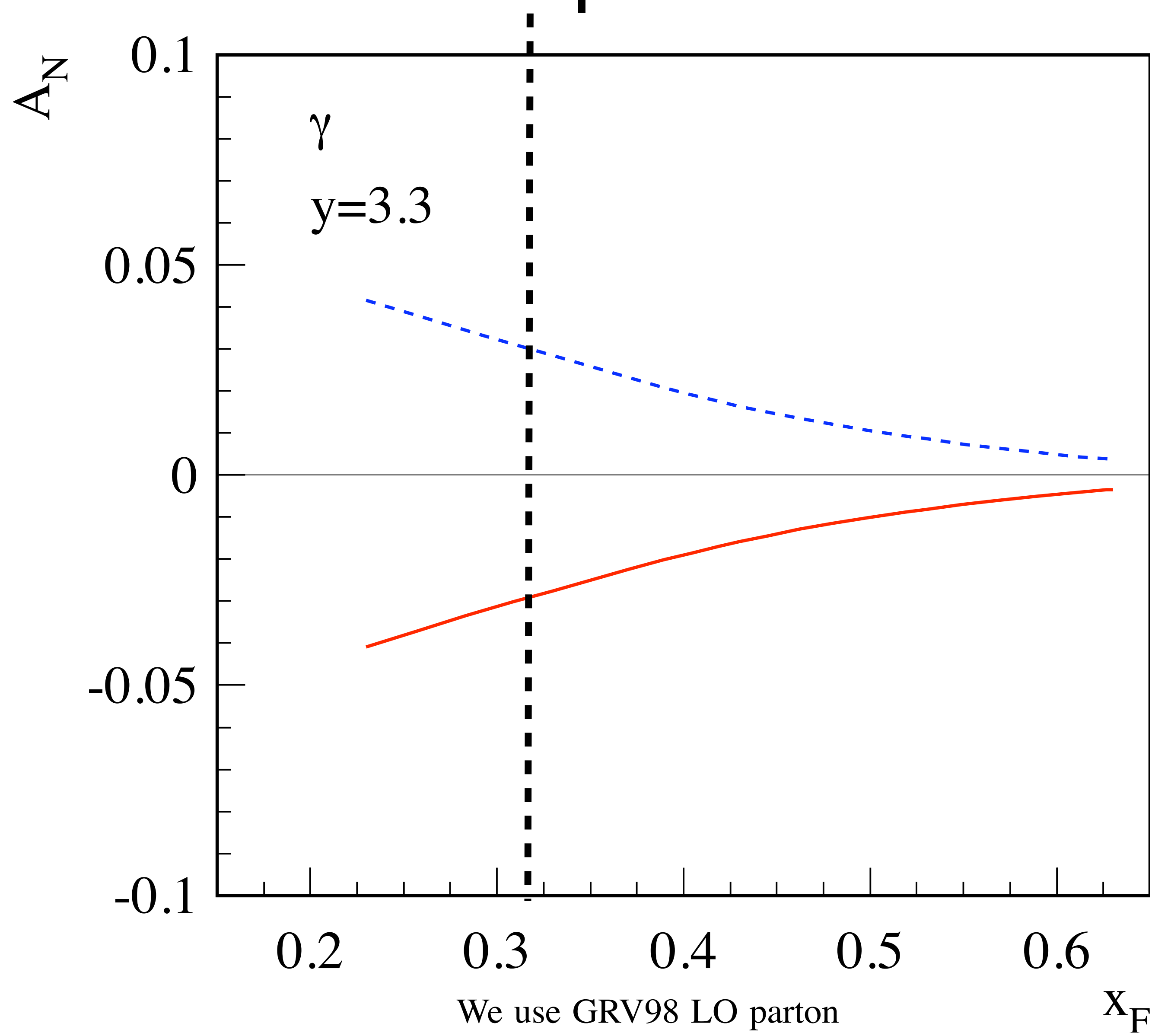
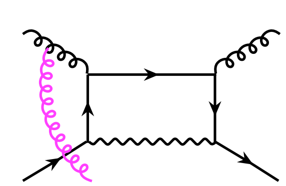
Based on new parameterization



We use GRV98 LO parton
, the latest Sivers function from [2], and DSS fragmentation function [3].

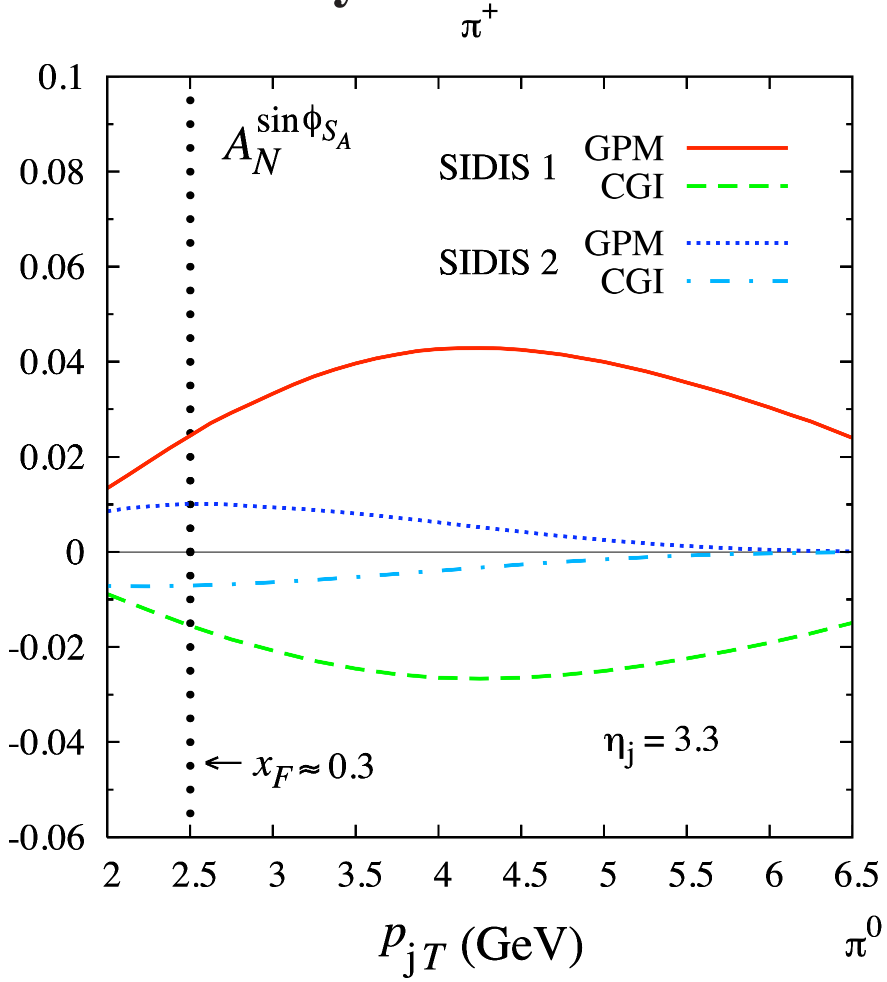
$$H^{\text{Inc}}_{qg\to\gamma q} = -\frac{N_c}{N_c^2-1} e_q^2 \left[-\frac{\hat{t}}{\hat{s}} - \frac{\hat{s}}{\hat{t}} \right]$$



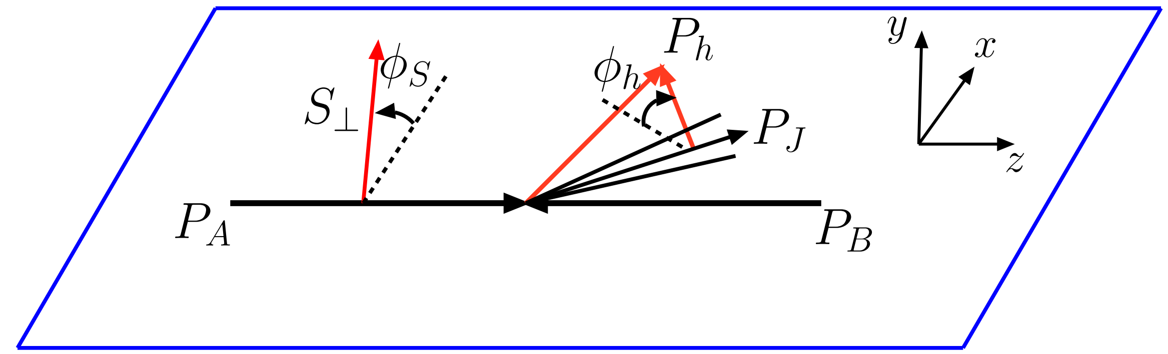
ISI drives result

- In this connection see Kouvaris, Qiu, Vogelsang and Yuan PRD 2006

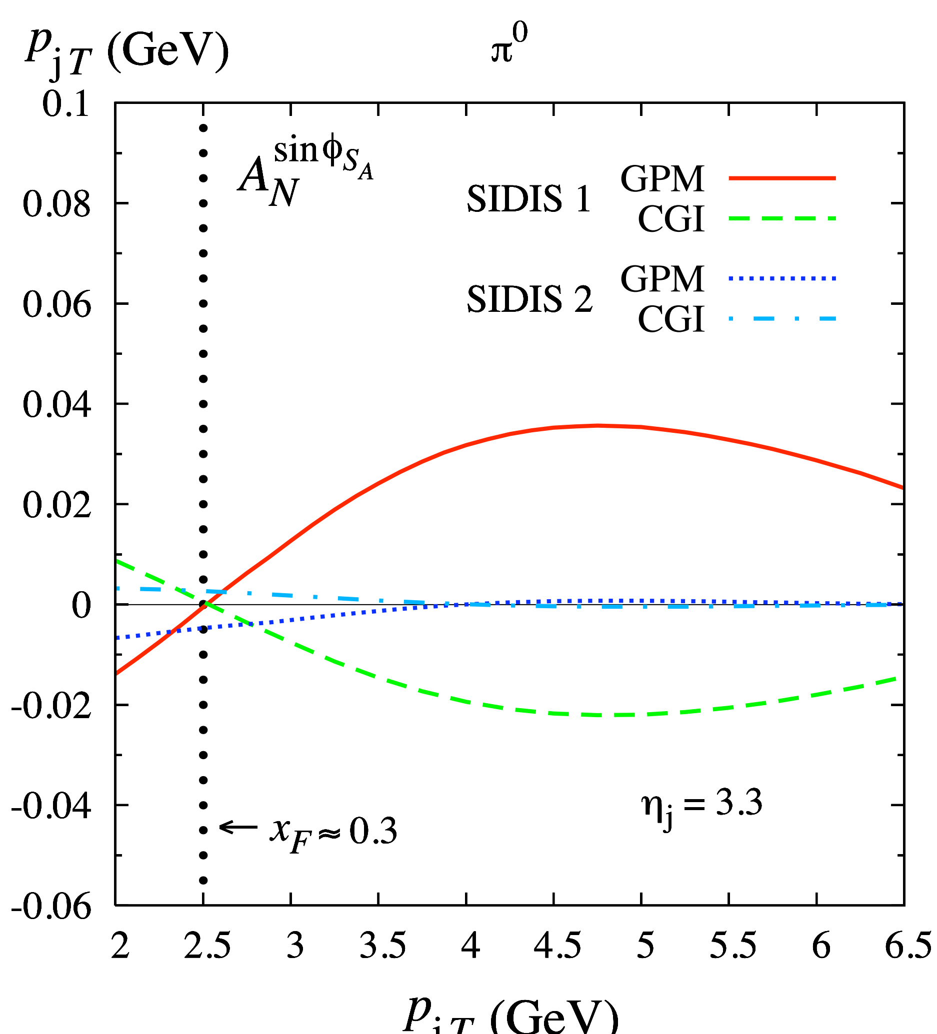
Azimuthal asymmetries for hadron distributions inside a jet in hadronic collisions

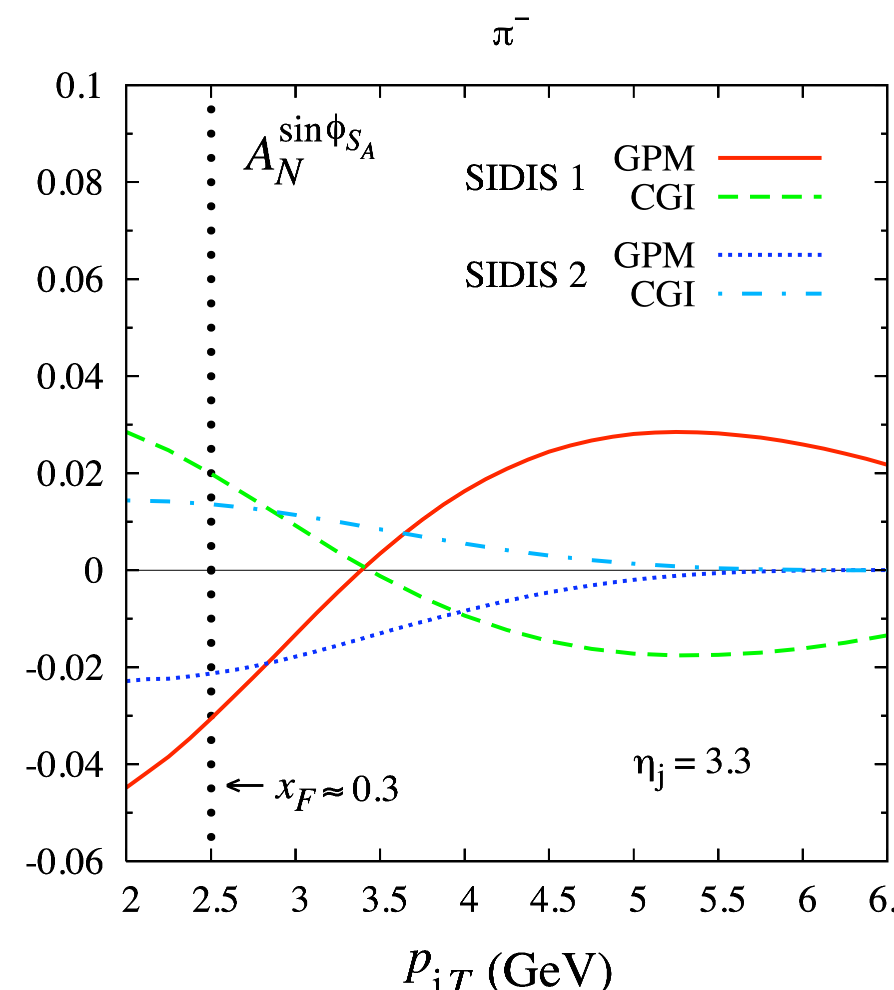


$$\sqrt{s} = 200\,\mathrm{GeV}$$



$$x_F = \frac{2P_T}{\sqrt{s}} \sinh(\eta)$$





Observations

- Hard amplitudes squared have same form in Mandelstam variables as twist-3 $\hat{s},\ \hat{t},\ \hat{u}$

see Kouvaris,Qiu ,Vogelsang, and Yuan PRD 2006

- However $\hat{s},\ \hat{t},\ \hat{u}$ depend on k_T in GPM whereas in twist-3 approach there has been collinear expansion on hard and soft factors

- We have shown that GPM expanded with respect to k_T results in twist-3 result {\em almost }

Collinear Expansion in GPM

- Implement delta function
- now "s" and "t" depend on k_{aT}
- expand k_{aT} and study contribution from Sivers function and hard cross section

$$E_h \frac{d\Delta\sigma}{d^2P_h} = \frac{\alpha_s}{s}\sum_{abc}\int d^2k_{aT}\frac{1}{M}\epsilon^{\alpha S_T n\hat{n}}k_{aT\alpha}\frac{1}{x_a}f_{1T}^{\perp\,\text{sidis}}(x_a,k_{aT}^2)\bigg|_{x_a=X+\frac{2P_{hT}\cdot k_{aT}/z}{x_b s+T/z}}$$
$$\times\int\frac{dx_b}{x_b}\int\frac{dz}{z^2}H^{\text{Inc}}_{ab\to c}(\hat{s},\hat{t},\hat{u})\frac{1}{x_b s+T/z}$$

that is.... in GPM

$$\hat{s} = (p_a + p_b)^2 = x_a x_b S + \quad \mathcal{O}(k_T^2)$$

$$\hat{t} = (x_a P_A + k_{aT} - \frac{P_h}{z})^2 = \frac{x_a}{z} T - \frac{2 P_{hT} \cdot k_{aT}}{z}$$

$$\hat{u} = (p_b - p_c)^2 = (x_b P_B - \frac{P_h}{z})^2 = \frac{x_b}{z} U$$

$$\delta(\hat{s} + \hat{t} + \hat{u}) = \frac{1}{x_b s + \frac{T}{z}} \delta(x_a - X - \frac{2 P_{hT} \cdot K_{aT}}{x_b s + \frac{T}{z}})$$

$$x = -x_b U / (z_c x_b S + T)$$

Collinear twist three

$$E_h \frac{d\Delta\sigma}{d^3P_h} = \frac{\alpha_s^2}{s}\sum_{abc}\int \frac{dz_c}{z_c^2} D_{h/c}(z_c)\frac{\epsilon^{P_h S_T n\bar{n}}}{z_c\tilde{u}}\frac{1}{x}\left(T_F(x,x) - x\frac{d}{dx}T_F(x,x)\right)$$
$$\times \int \frac{dx_b}{x_b} f_{b/B}(x_b)\int H^{\rm Inc}_{ab\to c}(\tilde{s},\tilde{t},\tilde{u})\frac{1}{x_b s + T/z_c}$$

same as Kouvaris, Qiu , Vogelsang, and Yuan PRD 2006

- Twist 3 and twist 2 approach connection????

we have another term ....comes from $H^{\rm Inc}_{ab\to c}(\hat{s},\hat{t},\hat{u})$

Conclusions

- Generalize GPM w/ color--can then perform global analysis
- Elephant in the room is break down of factorization for these processes
- Appears to be connection between generalized parton model at twist 3 and twist 3 approach
- Estimate mismatch-investigating LG Z. Kang
- TMD fact. is assumed in both GPM and GGPM is this a reasonable pheno. approximation?
- Direct photon driven by same ISI factor as in DY