

Theoretical Perspectives on Drell-Yan Production Measurements

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Outline

Almost all talks so far addressed “theoretical perspectives on Drell-Yan production measurements”

□ Drell-Yan production measurements:

Drell-Yan type observables

□ Theoretical perspectives:

Opportunities and challenges

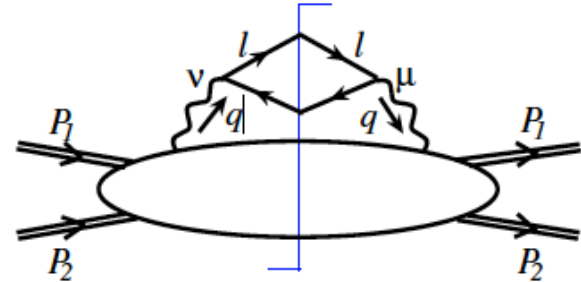
Drell-Yan offers much more than the sign change

□ Summary and outlook

Drell-Yan production measurements

□ Process:

$$A(P_1) + B(P_2) \longrightarrow V[\rightarrow \ell^+ \ell^-(q)] + X$$



□ Observables ($V(q) \rightarrow \gamma^*(q)$):

$$\frac{d\sigma}{d^4 q d\Omega} = \frac{\alpha_{\text{em}}^2}{2(2\pi)^4 S^2 Q^2} [W_T(1 + \cos^2\theta) + W_L(1 - \cos^2\theta) + W_\Delta(\sin(2\theta) \cos\phi) + W_{\Delta\Delta}(\sin^2\theta \cos(2\phi))].$$

✧ Single-scale cross sections ($Q \gg 1/\text{fm}$):

$$q^2 \equiv Q^2 \quad \frac{d\sigma}{dQ^2}, \quad \frac{d\sigma}{dQ^2 dy}$$

✧ Two-scale cross sections ($Q \gg q_T, Q \sim q_T, Q \ll q_T$):

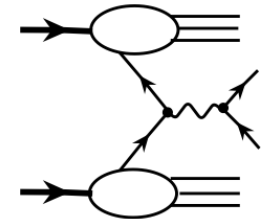
$$\frac{d\sigma}{dQ^2 dq_T^2}, \quad \frac{d\sigma}{dQ^2 dq_T^2 dy}, \quad \frac{d\sigma}{dQ^2 dq_T^2 dy d\Omega}$$

✧ Asymmetries \propto Difference of cross sections

Drell-Yan mechanism in parton model

□ Drell-Yan lepton-pair production:

$$\begin{aligned} \frac{d\sigma_{A+B \rightarrow \ell\bar{\ell}(Q^2)+X}}{dQ^2} &= \sigma_0 \sum_q e_q^2 \int dx \phi_{q/A}(x) \int dx' \phi_{\bar{q}/B}(x') \delta(Q^2 - xx' s_{AB}) + q \leftrightarrow \bar{q} \\ &= \frac{\sigma_0}{s_{AB}} \sum_q e_q^2 \mathcal{F}_{q\bar{q}}(\tau = Q^2/s_{AB}), \\ \sigma_0 &= \sigma_{q\bar{q} \rightarrow \ell\bar{\ell}(Q^2)}^{\text{incl}} \end{aligned}$$



Effective flux: $\mathcal{F}_{q\bar{q}}(\tau) = \int dx \phi_{q/A}(x) \int dx' \phi_{\bar{q}/B}(x') \delta(\tau - xx') + q \leftrightarrow \bar{q}$

□ Predictions:

✧ No free parameter for production rate!

✧ Normalized Drell-Yan angular distribution

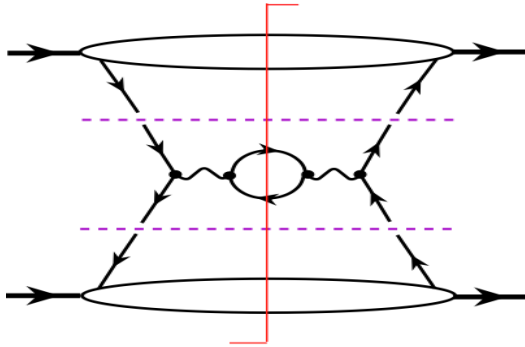
$$\frac{dN}{d\Omega} \equiv \left(\frac{d\sigma}{d^4q} \right)^{-1} \frac{d\sigma}{d^4q d\Omega} = \frac{3}{4\pi} \left(\frac{1}{\lambda + 3} \right) \left[1 + \lambda \cos^2\theta + \mu \sin(2\theta) \cos\phi + \frac{\nu}{2} \sin^2\theta \cos(2\phi) \right]$$

✧ Transversely polarized virtual photon: $1 + \cos^2\theta$ distribution

✧ Lam-Tung relation: $1 - \lambda - 2\nu = 0$

Drell-Yan mechanism in QCD

□ Leading order in QCD:



$$I = \frac{1}{2} [\gamma^+ \gamma^- + \gamma^- \gamma^+]$$

⇐ all γ structure: $\gamma^\alpha, \gamma^\alpha \gamma^5, \sigma^{\alpha\beta}$ (or $\gamma^5 \sigma^{\alpha\beta}$), I, γ^5

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□ Leading power distributions: $\langle P, s | \bar{\psi}_q(0) \mathcal{O} \psi_q(y^-) | P, s \rangle$

$$\mathcal{O} = \frac{\gamma \cdot n}{P \cdot n}, \frac{\gamma \cdot n \gamma^5}{P \cdot n}, \frac{\gamma \cdot n \gamma_\perp^\sigma}{P \cdot n} \quad \Leftrightarrow \quad q(x), \Delta q(x), \delta q(x) \text{ (or } h_1(x))$$

□ Transversity distribution:

$$h_1(x) \propto \langle P, S_\perp | \bar{\psi}(0) \frac{\gamma \cdot n \gamma_\perp^\sigma}{P \cdot n} \psi(y n) | P, S_\perp \rangle$$

□ Asymmetries – collinear factorization:

$$A_{LL} \propto \sum_q e_q^2 \Delta q(x) \Delta \bar{q}(x') \quad A_{TT} \propto \sum_q e_q^2 h_{1q}(x) h_{1\bar{q}}(x') \quad A_L \propto \sum_q (c_v * c_a) \Delta q(x) \bar{q}(x')$$

$$A_N \propto \sum_q e_q^2 T_q(x, x) \bar{q}(x') \quad A_{LT} \propto \sum_q e_q^2 \Delta q(x) \tilde{T}_{\bar{q}}(x')$$

From parton model to QCD

□ Parton model – big K-factor:

$$K \equiv \frac{(d\sigma/dQ^2)_{\text{PM}}}{(d\sigma/dQ^2)_{\text{exp}}} \gtrsim 2$$

- ✧ Parton model = leading order QCD without DGLAP evolution
- ✧ Leading order QCD calculation has the same size K-factor

□ QCD calculation at NLO and higher:

$$K \equiv \frac{(d\sigma/dQ^2)_{\text{NLO}}}{(d\sigma/dQ^2)_{\text{exp}}} = 1$$

- ✧ Normalization uncertainty in QCD global fit is limited by systematic error of individual experiment
- ✧ High order corrections are sensitive to if the virtual photon's invariant mass is space-like or time-like

$$\log(q_{\text{DIS}}^2) \rightarrow \log(-q_{\text{DIS}}^2) + \log(-1)$$

Factorized Drell-Yan cross section

□ TMD factorization ($q_{\perp} \ll Q$):

$$\frac{d\sigma_{AB}}{d^4q} = \sigma_0 \int d^2k_{a\perp} d^2k_{b\perp} d^2k_{s\perp} \delta^2(q_{\perp} - k_{a\perp} - k_{b\perp} - k_{s\perp}) \mathcal{F}_{a/A}(x_A, k_{a\perp}) \mathcal{F}_{b/B}(x_B, k_{b\perp}) \mathcal{S}(k_{s\perp}) + \mathcal{O}(q_{\perp}/Q)$$
$$x_A = \frac{Q}{\sqrt{s}} e^y \quad x_B = \frac{Q}{\sqrt{s}} e^{-y}$$

The soft factor, \mathcal{S} , is universal, could be absorbed into the definition of TMD parton distribution

□ Collinear factorization ($q_{\perp} \sim Q$, $q_{\perp} \gg Q$):

$$\frac{d\sigma_{AB}}{d^4q} = \int dx_a f_{a/A}(x_a, \mu) \int dx_b f_{b/B}(x_b, \mu) \frac{d\hat{\sigma}_{ab}}{d^4q}(x_a, x_b, \alpha_s(\mu), \mu)$$

□ Spin dependence:

The factorization arguments are independent of the spin states of the colliding hadrons

➡ same formula with different distributions for γ^* , W/Z , H^0 ...

TMD vs collinear factorization

- TMD factorization and collinear factorization cover different regions of kinematics:

$$\text{Collinear: } Q_1 \dots Q_n \gg \Lambda_{\text{QCD}}$$

$$\text{TMD: } Q_1 \gg Q_2 \sim \Lambda_{\text{QCD}}$$

- ✧ One complements the other, but, cannot replace the other!
- ✧ Predictive power of both formalisms relies on the validity of their own factorization

Consistency check – overlap region – perturbative region

- “Formal” operator relation between TMD distributions and collinear factorized distributions:

$$\text{spin-averaged: } \int d^2 k_{\perp} \Phi_f^{\text{SIDIS}}(x, k_{\perp}) + \text{UVCT}(\mu_F^2) = \phi_f(x, \mu_F^2)$$

$$\text{Transverse-spin: } \frac{1}{M_P} \int d^2 k_{\perp} \vec{k}_{\perp}^2 q_T(x, k_{\perp}) + \text{UVCT}(\mu_F^2) = T_F(x, x, \mu_F^2)$$

But, TMD factorization is only valid for low k_T – TMD PDFs at low k_T

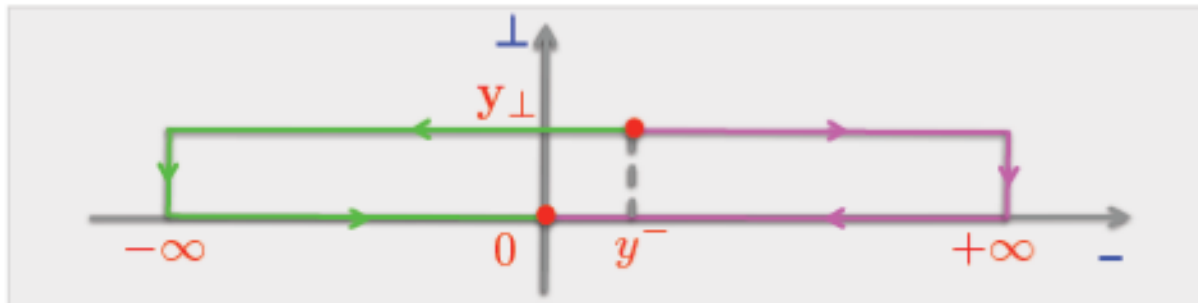
The sign change of Sivers function

□ TMD quark distribution:

See talks by Collins, Kang, ...

$$f_{q/h^\uparrow}(x, \mathbf{k}_\perp, \vec{S}) = \int \frac{dy^- d^2 y_\perp}{(2\pi)^3} e^{ixp^+ y^- - i\mathbf{k}_\perp \cdot \mathbf{y}_\perp} \langle p, \vec{S} | \bar{\psi}(0^-, \mathbf{0}_\perp) \text{ Gauge link } \frac{\gamma^+}{2} \psi(y^-, \mathbf{y}_\perp) | p, \vec{S} \rangle$$

- **SIDIS:** $\Phi_n^\dagger(\{+\infty, 0\}, \mathbf{0}_\perp) \Phi_{n_\perp}^\dagger(+\infty, \{\mathbf{y}_\perp, \mathbf{0}_\perp\}) \Phi_n(\{+\infty, y^-\}, \mathbf{y}_\perp)$
- **DY:** $\Phi_n^\dagger(\{-\infty, 0\}, \mathbf{0}_\perp) \Phi_{n_\perp}^\dagger(-\infty, \{\mathbf{y}_\perp, \mathbf{0}_\perp\}) \Phi_n(\{-\infty, y^-\}, \mathbf{y}_\perp)$



- For a fixed spin state:

$$f_{q/h^\uparrow}^{\text{SIDIS}}(x, \mathbf{k}_\perp, \vec{S}) \neq f_{q/h^\uparrow}^{\text{DY}}(x, \mathbf{k}_\perp, \vec{S})$$

□ Parity + Time-reversal invariance:

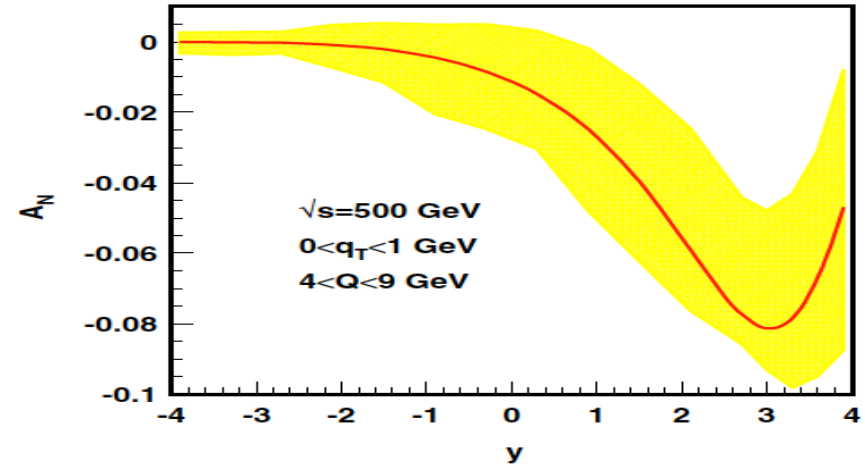
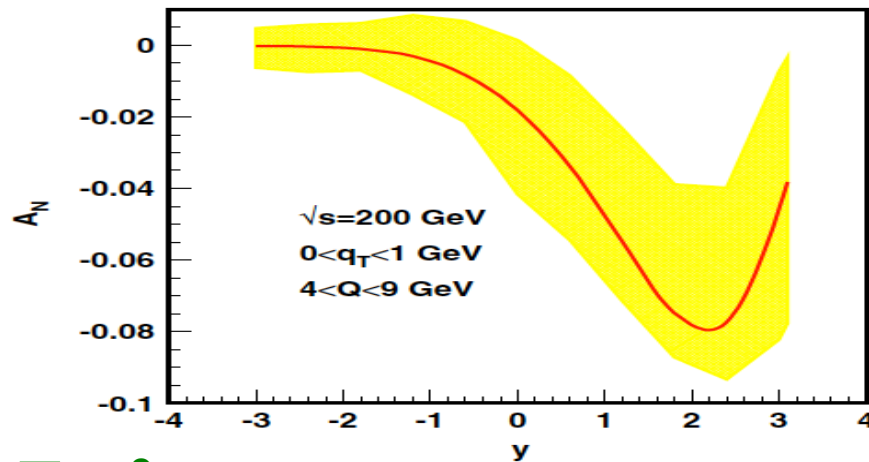
$$\longrightarrow f_{q/h^\uparrow}^{\text{Sivers}}(x, k_\perp)^{\text{SIDIS}} = -f_{q/h^\uparrow}^{\text{Sivers}}(x, k_\perp)^{\text{DY}}$$

It is a critical test of TMD factorization approach

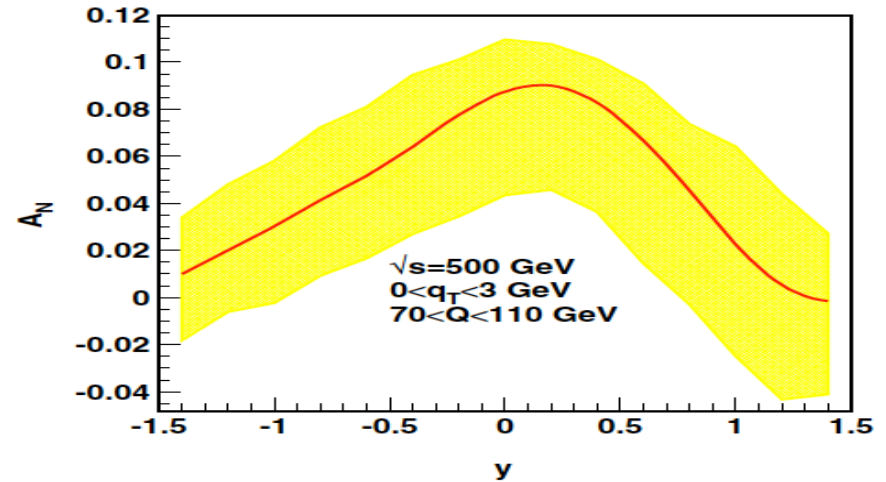
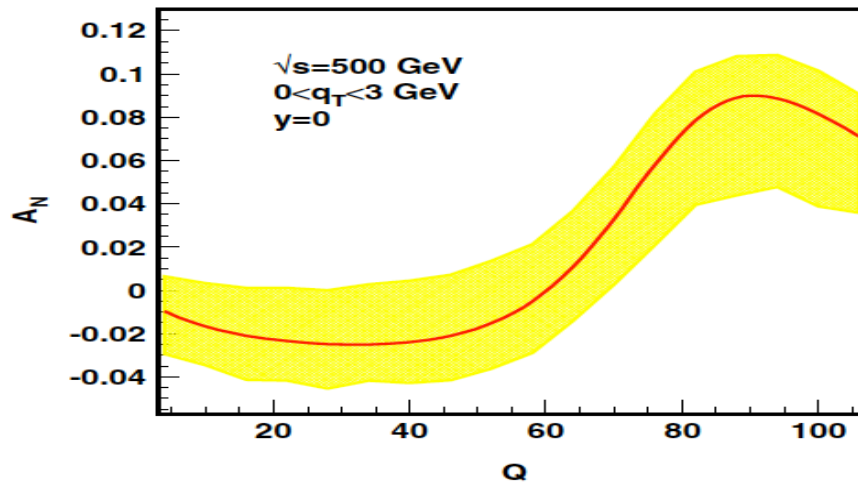
Test of the modified universality

□ Drell-Yan: $A_N^{\sin(\phi-\phi_s)} = -A_N$

See talks by Anselmino, Rogers, ...



□ Z^0 :



Theoretical challenge: Q-dependence of Sivers function?

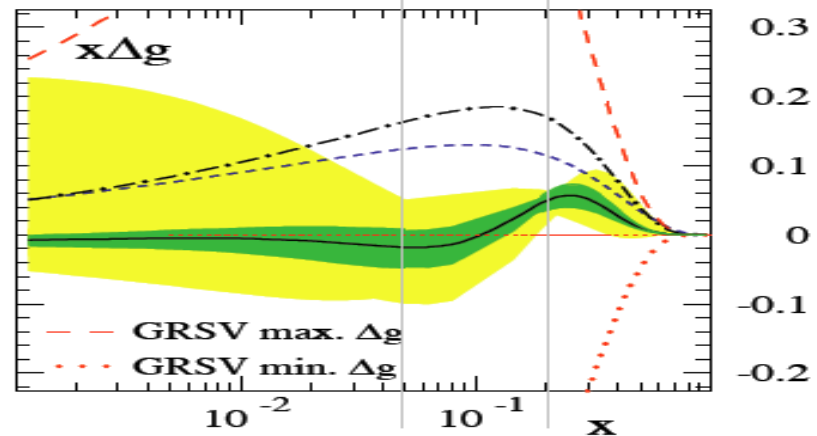
The sign “mismatch”

□ Asymmetry could have a node:

See talks by Kang

Sign change of $\Delta g(x)$:

$$A(s) \propto \sigma(s) - \sigma(-s)$$

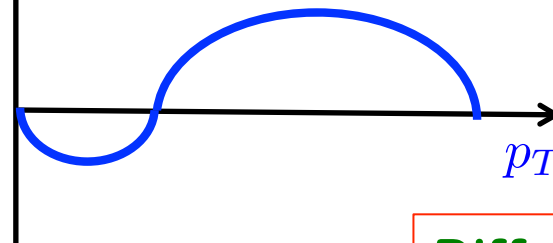


□ A_N of Drell-Yan p_T distribution:

We could have:

A_N

Collinear region ($p_T \sim Q$)



TMD region ($p_T \ll Q$)

Difference of two Gaussians

Important measurement for understanding A_N of hadronic pion

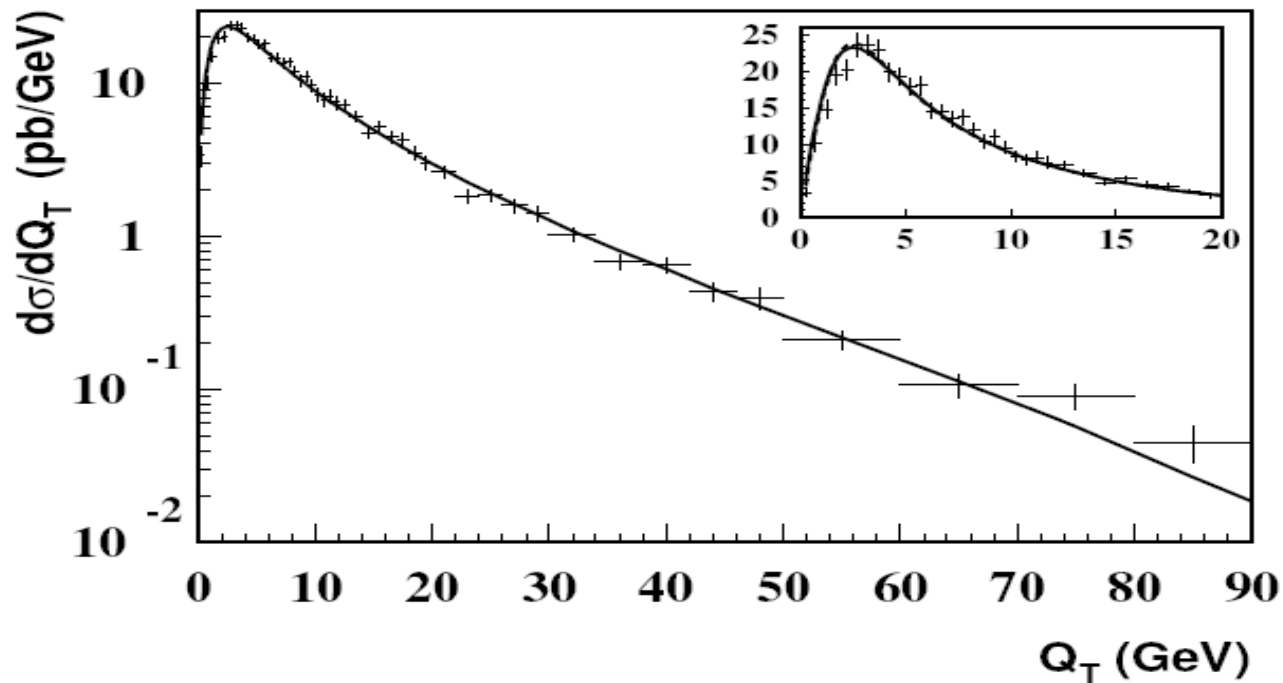
Unpolarized Drell-Yan cross section

- The denominator of the Asymmetry:

$$\frac{d\sigma}{d^4q} \quad \frac{d\sigma}{d^4q d\Omega}$$

- Angular integrated Drell-Yan is under control:

- Fermilab CDF data on Z at $\sqrt{S} = 1.8$ TeV



CSS resummation formalism – small role of nonperturbative physics

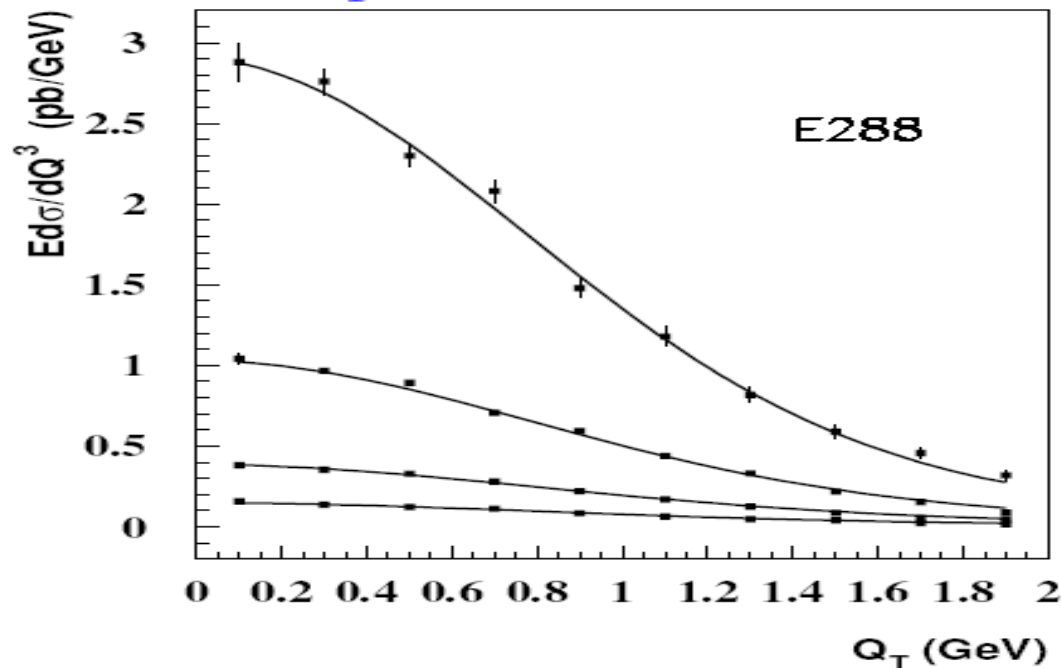
Unpolarized Drell-Yan cross section

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- Angular integrated Drell-Yan is under control:

- Fermilab E288 data at $p_{\text{beam}} = 400 \text{ GeV}$



Qiu and Zhang, 2001

CSS resummation formalism – importance of power corrections

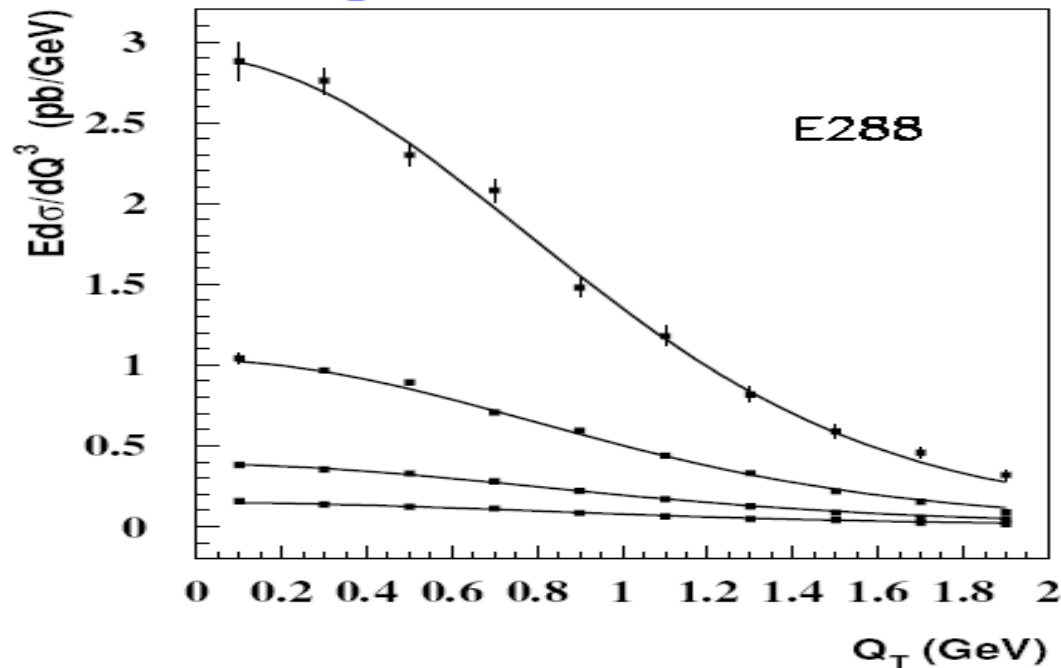
Unpolarized Drell-Yan cross section

□ The denominator of the Asymmetry:

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□ Angular integrated Drell-Yan is under control:

- Fermilab E288 data at $p_{\text{beam}} = 400 \text{ GeV}$



But, Drell-Yan lepton angular distribution needs work!

Lam-Tung relation

Normalized Drell-Yan lepton angular distribution:

$$\frac{dN}{d\Omega} \equiv \left(\frac{d\sigma}{d^4q} \right)^{-1} \frac{d\sigma}{d^4q d\Omega} = \frac{3}{4\pi} \left(\frac{1}{\lambda + 3} \right) \left[1 + \lambda \cos^2\theta + \mu \sin(2\theta) \cos\phi + \frac{\nu}{2} \sin^2\theta \cos(2\phi) \right]$$

Lam-Tung relation:

$$1 - \lambda - 2\nu = 0$$

Collinear factorization:

$$\lambda = \frac{W_T - W_L}{W_T + W_L} \approx \frac{W_T^{\text{Resum}} - W_L^{\text{Resum}}}{W_T^{\text{Resum}} + W_L^{\text{Resum}}} = \frac{1 - \frac{1}{2} Q_{\perp}^2 / Q^2}{1 + \frac{3}{2} Q_{\perp}^2 / Q^2}$$

$$\nu = \frac{2W_{\Delta\Delta}}{W_T + W_L} \approx \frac{2W_{\Delta\Delta}^{\text{Resum}}}{W_T^{\text{Resum}} + W_L^{\text{Resum}}} = \frac{Q_{\perp}^2 / Q^2}{1 + \frac{3}{2} Q_{\perp}^2 / Q^2}$$

Still valid after resummation of large logs

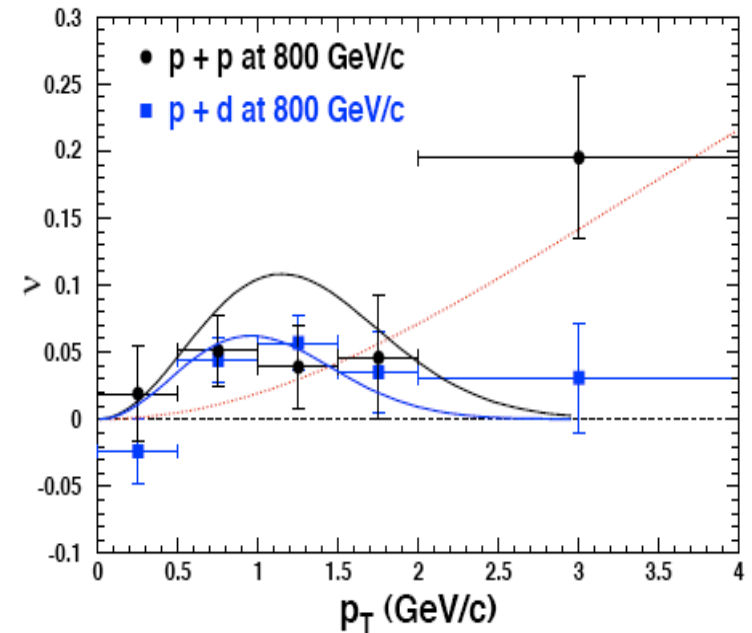
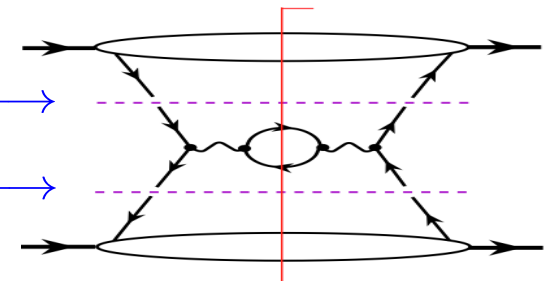
TMD factorization:

- Boer - Mulder function - $\cos(2\Phi)$:

$$h_1^{\perp \text{DY}}(x) = -h_1^{\perp \text{SIDIS}}(x)$$

$$\sigma^{+\alpha} k_{\perp\alpha}$$

$$\sigma^{+\beta} k'_{\perp\beta}$$

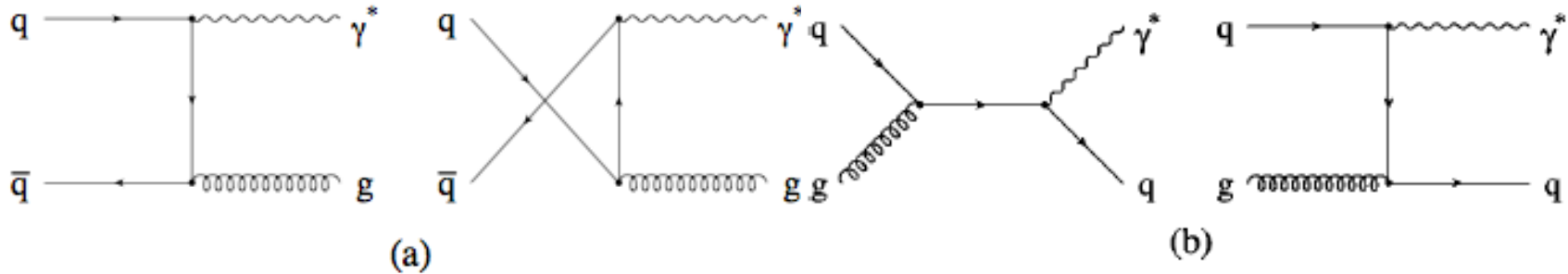


Berger, Qiu, Rodriguez, 2007

Theory challenge: Q-dependence of BM function, $\cos(\Phi)$, ...

High p_T and low mass Drell-Yan

□ Clean probe of gluon without final-state interaction



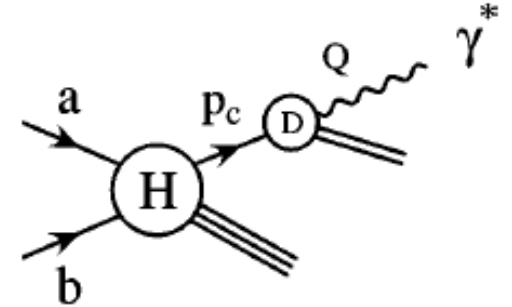
Compton subprocess dominates when $q_T > Q/2$

Berger, Qiu, Zhang, PRD 2002

□ Complementary to prompt photon

$$\frac{d\sigma_{AB \rightarrow \ell^+ \ell^- (Q) X}}{dQ^2 dQ_T^2 dy} = \left(\frac{\alpha_{em}}{3\pi Q^2} \right) \frac{d\sigma_{AB \rightarrow \gamma^* (Q) X}}{dQ_T^2 dy}$$

$\frac{10^{-3}}{Q^2}$



□ Idea: low Q^2 – increases the rate, and high p_T – reliable pQCD calculation

IF $p_T \gg Q_s$, Collinear factorization is as good as that of prompt photon

Kang, Qiu, Vogelsang, PRD 2009

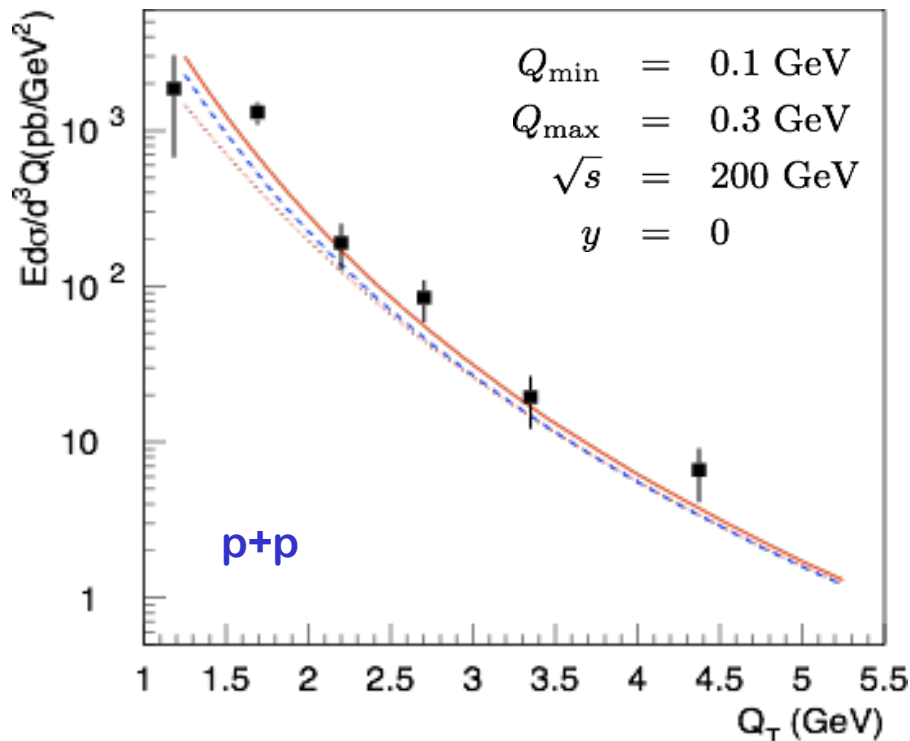
Very low mass Drell-Yan ($p_T > Q \sim 1/\text{fm}$)

Kang, Qiu, Vogelsang, PRD 2009

□ Invariant cross section:

$$E \frac{d\sigma_{AB \rightarrow \ell^+ \ell^- (Q) X}}{d^3Q} \equiv \int_{Q_{\min}^2}^{Q_{\max}^2} dQ^2 \frac{1}{\pi} \frac{d\sigma_{AB \rightarrow \ell^+ \ell^- (Q) X}}{dQ^2 dQ_T^2 dy}$$

□ Role of non-perturbative fragmentation function:



Data from PHENIX: arXiv:0804.4168

✧ Input FF:

$$D(z, \mu_0) = D^{\text{QED}}(z) + \kappa D^{\text{NP}}(z)$$

✧ QED alone (dotted):

$$\kappa = 0 \text{ at } \mu_0 = 1 \text{ GeV}$$

✧ QED + hadronic input (solid):

$$\kappa = 1 \text{ at } \mu_0 = 1 \text{ GeV}$$

Hadronic component of fragmentation is very important at low Q_T

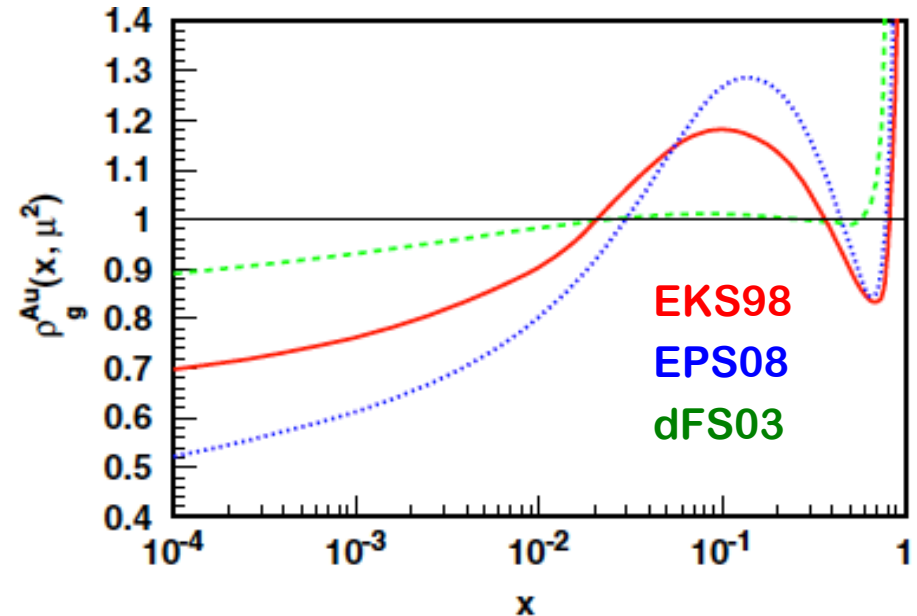
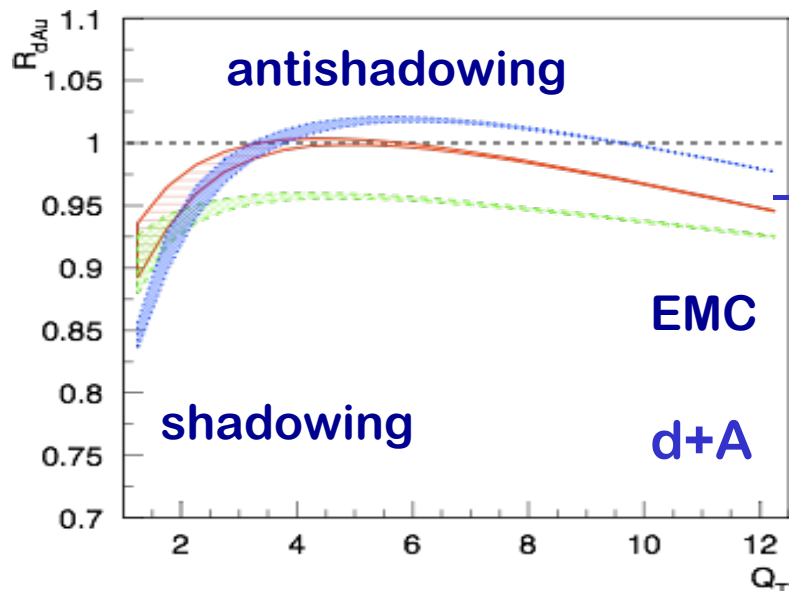
Excellent probe of gluon distribution

□ Nuclear modification factor:

Kang, Qiu, Vogelsang, PRD 2009

$$R_{dAu} \equiv \frac{1}{\langle N_{coll} \rangle} \frac{d^2 N^{dAu} / dQ_T dy}{d^2 N^{pp} / dQ_T dy} \stackrel{\text{min. bias}}{=} \frac{\frac{1}{2A} d^2 \sigma^{dAu} / dQ_T dy}{d^2 \sigma^{pp} / dQ_T dy}$$

□ RHIC kinematics – if dominated by single scattering:

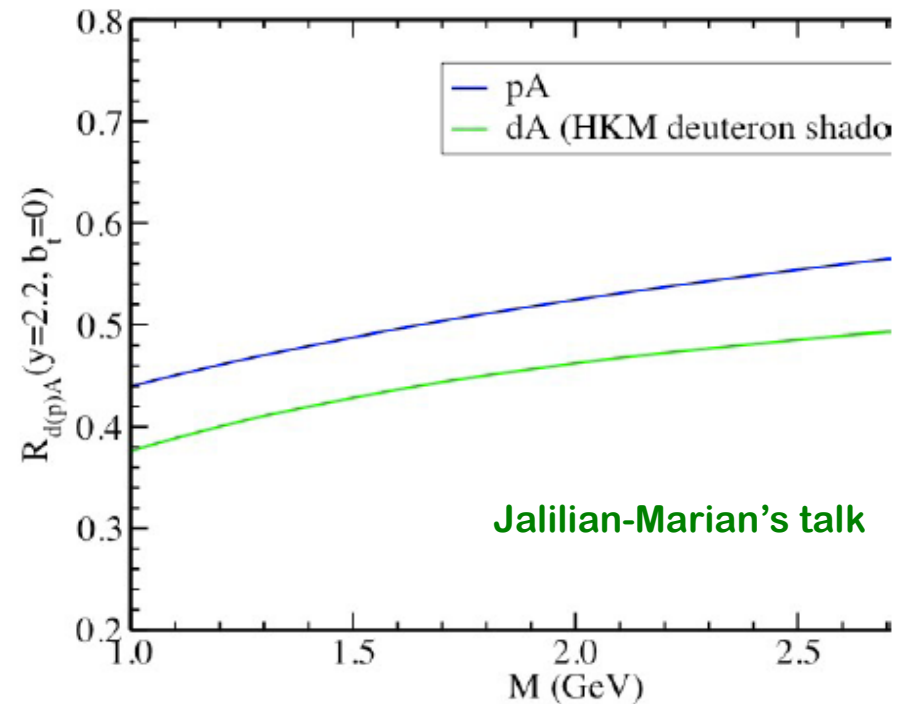
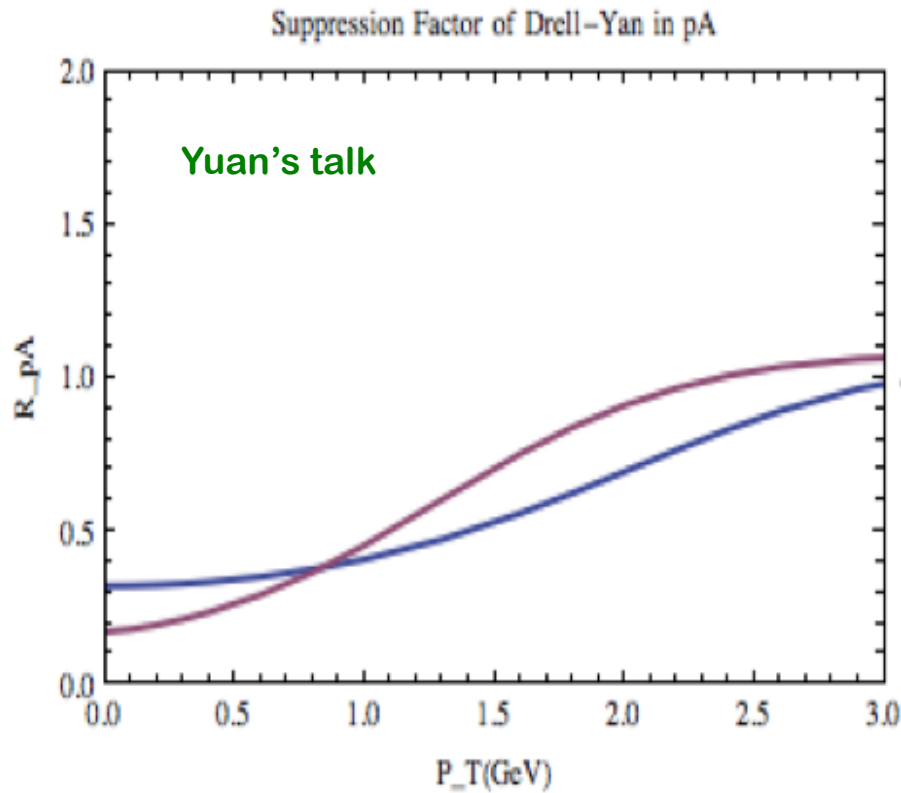


- The band is given by $\kappa=1$ (top lines) and $\kappa=0$ (bottom lines)
- Ratio follows the feature of gluon distribution if turns off isospin
- No suppression if removing isospin effect

Saturation and CGC physics

□ Forward rapidity ($y \gg 0$):

If $Q_T \sim Q_s$, collinear factorization fails

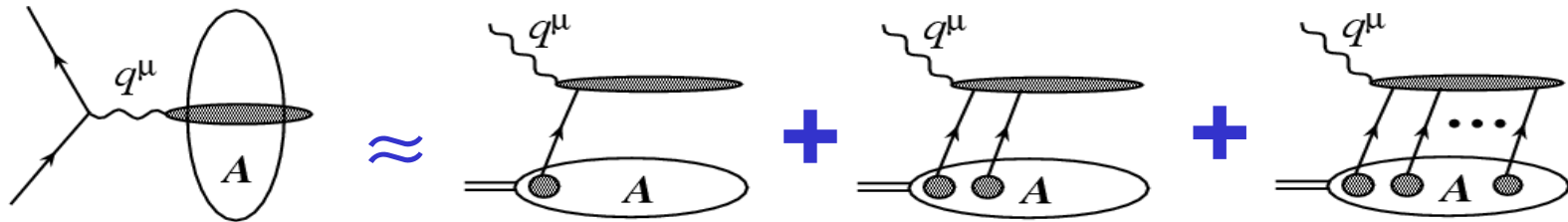


Nuclear shadowing cannot produce such suppression!

Theory challenge: Role of p_T ?

Another sign change

□ Power correction to DIS – single scale:

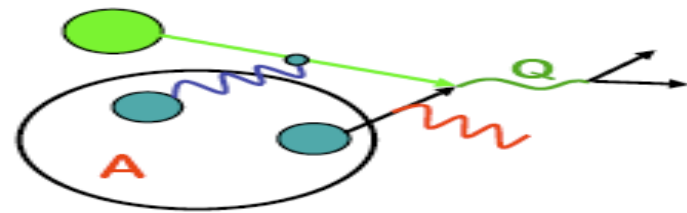


$$F_{eA}(x, Q^2) = F_{eA}^{\text{LP}}(x, Q^2) + \frac{1}{Q^2} F_{eA}^{\text{NLP}}(x, Q^2) + \dots$$

Negative - suppression

□ Power correction to inclusive DY – single scale:

$$\frac{d\sigma_{pA}}{dQ^2} = \frac{d\sigma_{pA}^{\text{LP}}}{dQ^2} + \frac{1}{Q^2} \frac{d\sigma_{pA}^{\text{NLP}}}{dQ^2} + \dots$$

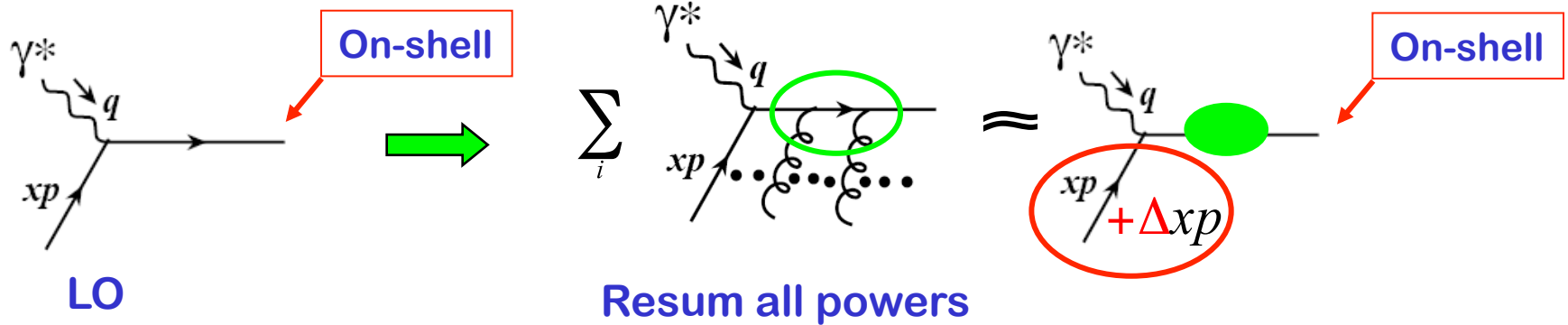


Positive - enhancement

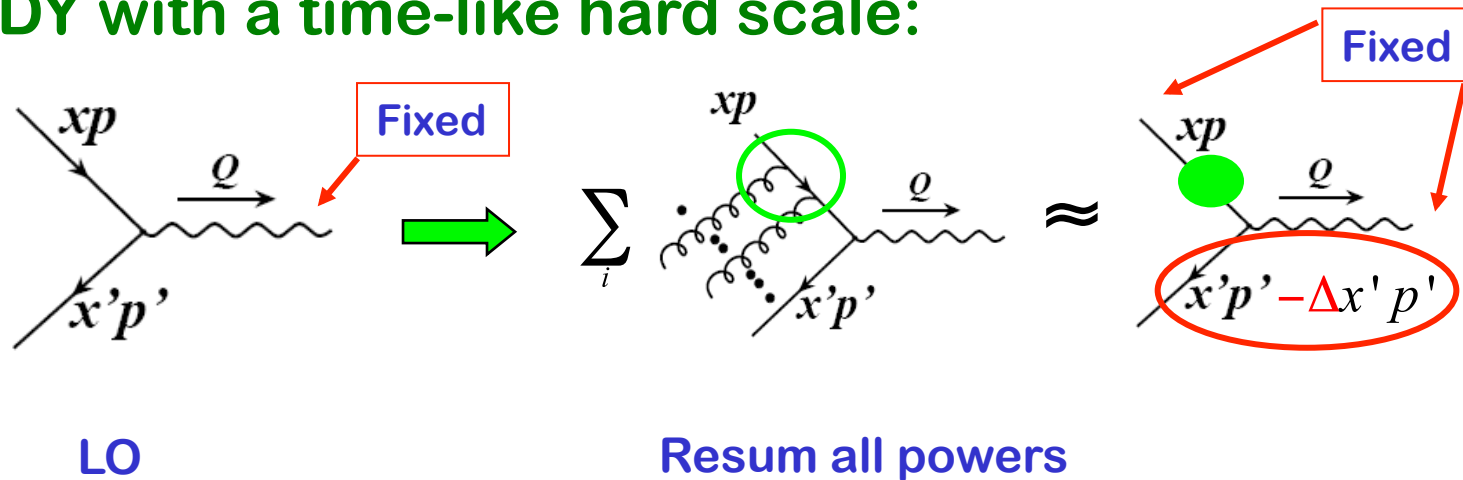
Compton gives negative contribution in CO factorization

Another sign change

DIS with a space-like hard scale:

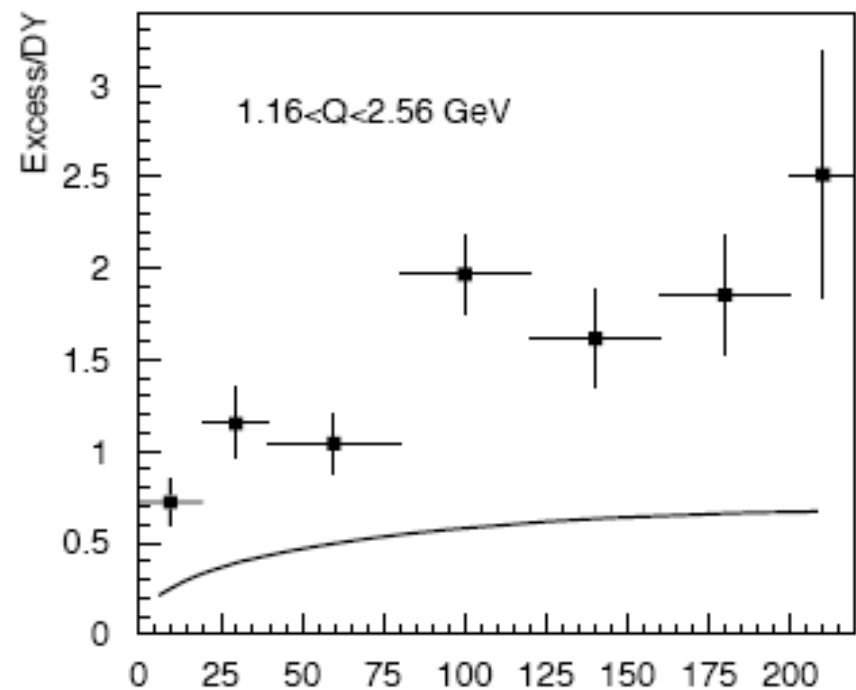
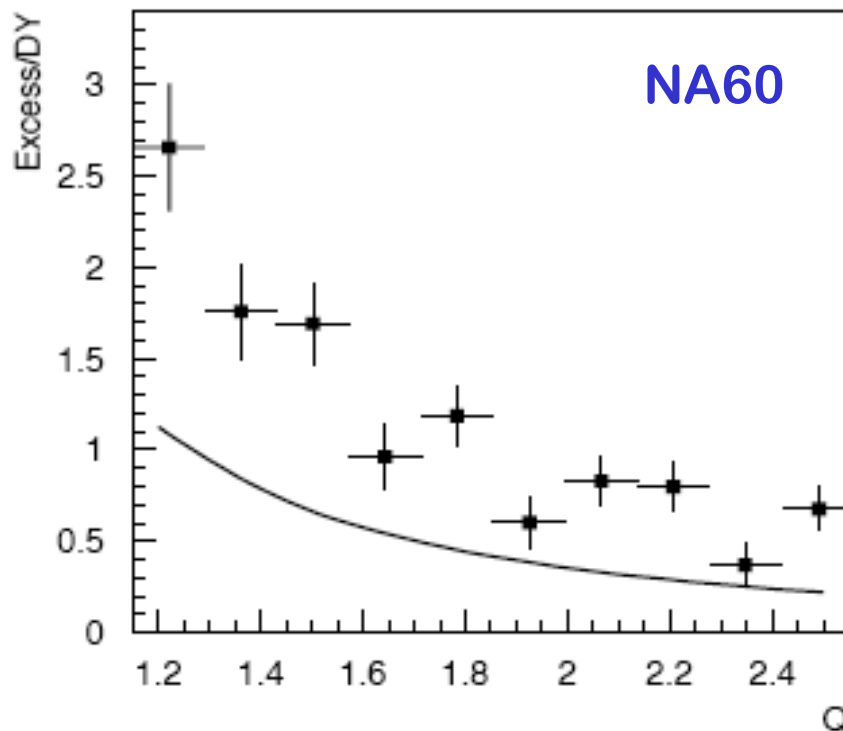


DY with a time-like hard scale:



Inclusive low mass Drell-Yan

□ Power correction to inclusive total rate: $d\sigma^{AA}/dQ^2$



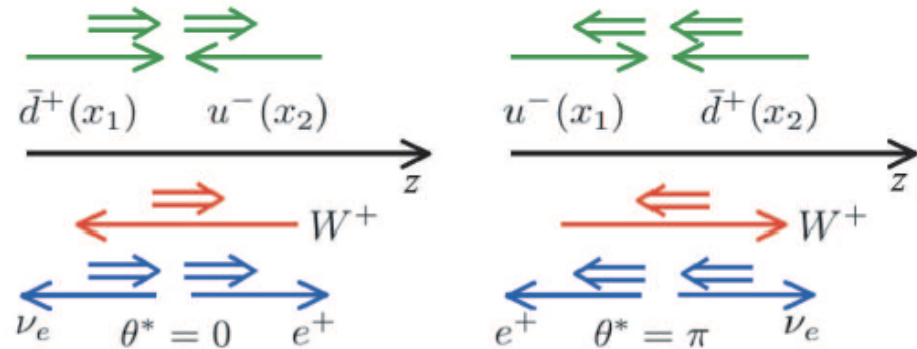
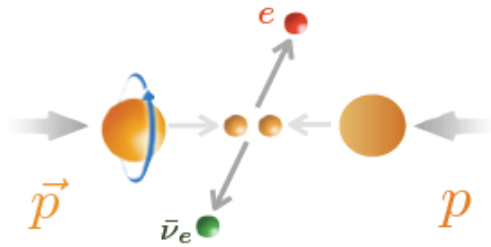
✧ Power correction from cold nuclear matter enhances the dilepton production - consistent with NA60 data

Theory challenge: A-dependence of DY's p_T and y distributions

See talks by Mueller, Yuan, Jalilian-Marian, Reimer, Peng, ...

Drell-Yan with parity violation

□ W's are left-handed:



□ Flavor separation:

Lowest order:

$$A_L^{W^+} = -\frac{\Delta u(x_1)\bar{d}(x_2) - \Delta\bar{d}(x_1)u(x_2)}{u(x_1)\bar{d}(x_2) + \bar{d}(x_1)u(x_2)}$$

$$x_1 = \frac{M_W}{\sqrt{s}}e^{y_W}, \quad x_2 = \frac{M_W}{\sqrt{s}}e^{-y_W}$$

Forward W^+ (backward e^+):

$$A_L^{W^+} \approx -\frac{\Delta u(x_1)}{u(x_1)} < 0$$

Backward W^+ (forward e^+):

$$A_L^{W^+} \approx -\frac{\Delta\bar{d}(x_2)}{\bar{d}(x_2)} < 0$$

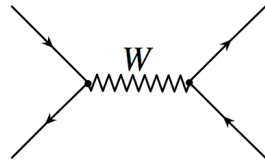
□ Complications:

High order, W's p_T -distribution at low p_T

Challenge in predicting A_L of lepton

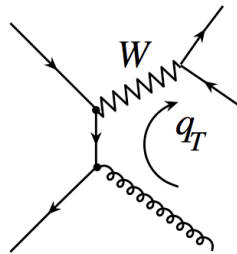
- ❑ RHIC experiments measure decay lepton not the W 's:
- ❑ Fixed order pQCD calculation:

LO:



$$\propto \delta^2(q_T)$$

NLO:



$$\propto \frac{1}{q_T^2} \Rightarrow \infty \text{ as } q_T^2 \rightarrow 0$$

Leptons not from W decay – background – hard for theorists

- ❑ All order resummation is needed:

CSS formalism – implemented in RHICBOS – only diagonal contribution

Resummation for the lepton angular distribution needed!

Scale dependence of the polarized sea

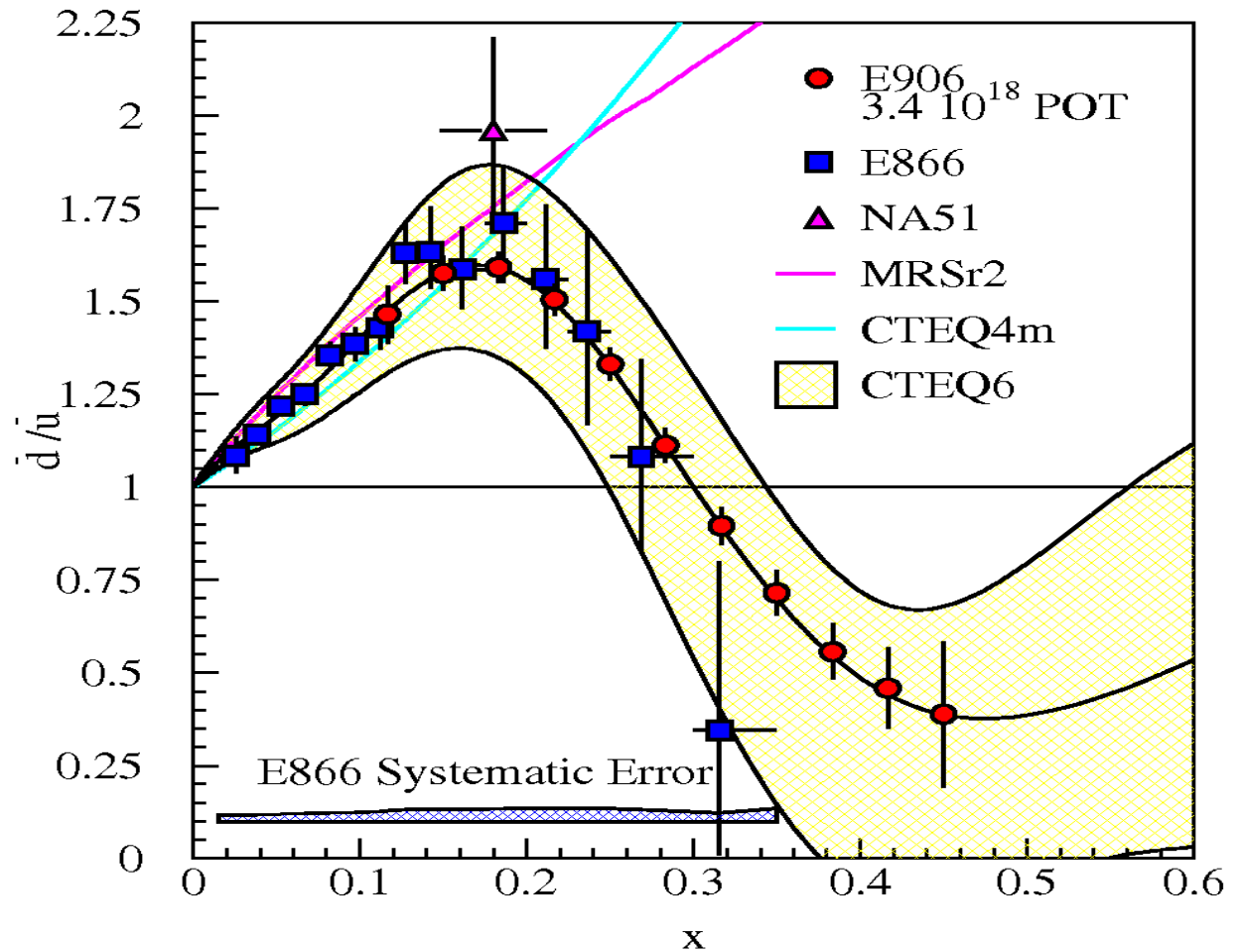
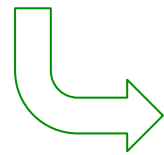
$$\Delta\bar{q}(\mu = M_W) \implies \Delta\bar{q}(\mu = Q \sim \text{GeV's})_{\text{SIDIS}}$$

Sea quark asymmetry from Drell-Yan

□ Flavor asymmetry of the sea:

See talks by Reimer, Peng, ...

$$\frac{\sigma^{pd}}{2\sigma^{pp}} \approx \frac{1}{2} \left(1 + \frac{\bar{d}(x_2)}{\bar{u}(x_2)} \right)$$



Theory challenge: Why there is a node? Role of pion cloud? ...

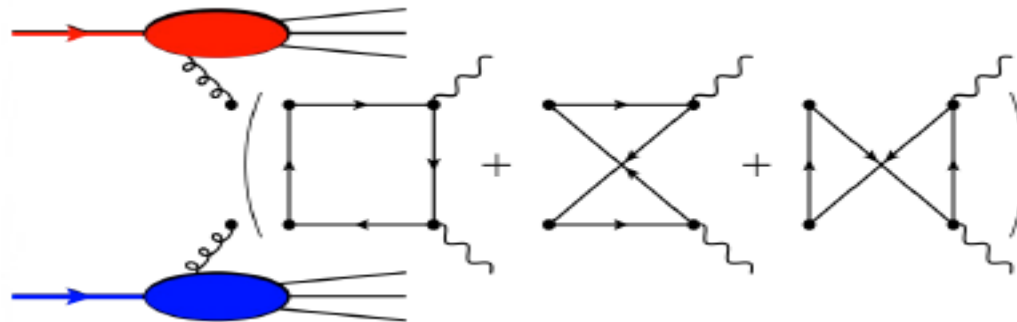
TMD gluon distribution

□ Gluon Sivers function:

Need TMD factorizable observables!

□ RHIC: momentum imbalance of two isolated photons:

$$A(P_1) + B(P_2) \longrightarrow \gamma(p_1) + \gamma(p_2) + X$$



See talk by Schlegel

□ Future EIC:

$$\ell(k) + B(P) \longrightarrow D(p_1) + \bar{D}(p_2) + X$$

Diehl and Xiao

$$\ell(k) + B(P) \longrightarrow J/\psi(P_T) + X$$

Summary and outlook

- Drell-Yan process is one of the oldest hard process proposed to test QCD – it still a very good one!
- The proof of QCD factorization for Drell-Yan is solid (LP + NLP for collinear, LP for TMD)
- The test of the sign change of the Sivers function is a critical test of TMD factorization!
- Drell-Yan could provide much more than the sign change of Sivers function

Thank you!

Backup transparencies