TMD fracture functions in SIDIS and DY

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- Introduction
- Polarized SIDIS
  - CFR
  - TFR (based on M.Anselmino, V.Barone and AK, **arXiv:1102.4214; PLB 699 (2011) 108)**
  - DSIDIS: TFR+CFR
- Polarized SIDY
- Discussion

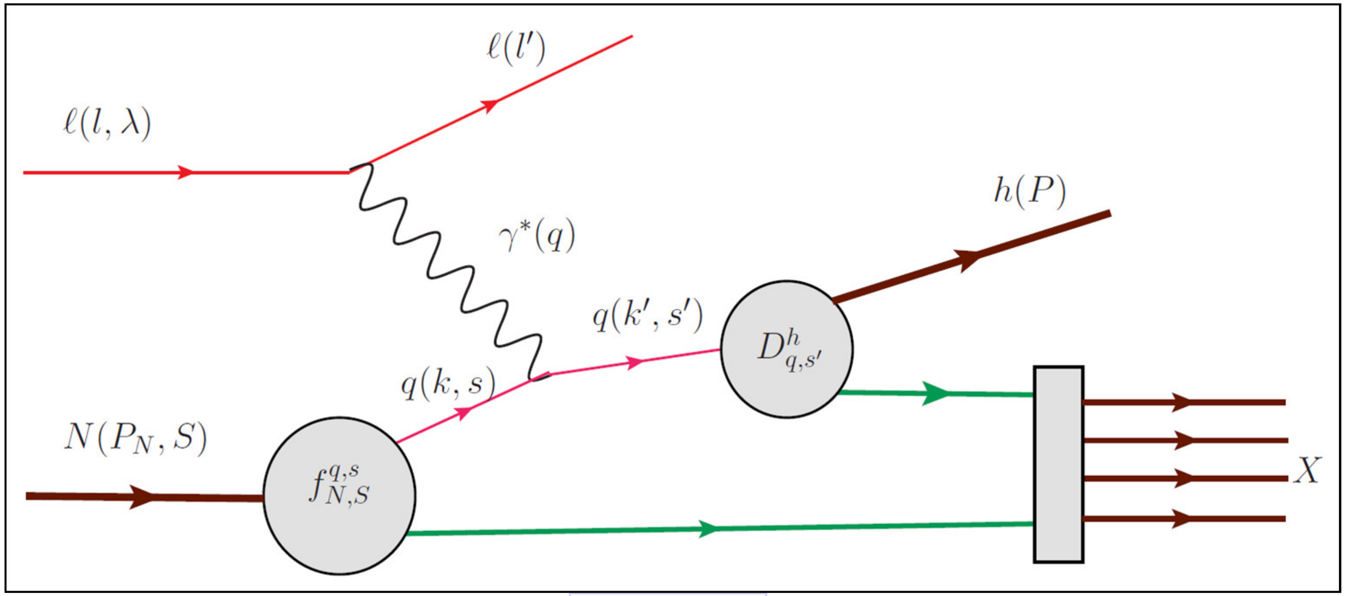
Introduction

- DIS: only lepton probe
  - No access to quark k_T at LO
- SIDIS: lepton and final hadron probe
  - We need excellent understanding of hadronization and factorization
    - Exploited in CFR: $x_F > 0$
    - For better understanding one have to exploit also TFR: $x_F < 0$
- New objects – Spin and Transverse Momentum Dependent Fracture Functions
- Can be studied also in Semi Inclusive DY processes
  - $H_a + H_b \rightarrow \mu^+ + \mu^- + h + X$

STMD DFs

Nucleon Polarization \ Quark polarization	U	L	T
U	$f_1^q(x,k_T^2)$		$\frac{\epsilon_T^{ij}k_T{}^j}{M}h_1^{\perp q}(x,k_T^2)$
L		$S_L g_{1L}^q(x,k_T^2)$	$S_L\frac{\mathbf{k}_T}{M}h_{1L}^{\perp q}(x,k_T^2)$
T	$\frac{\mathbf{k}_T\times\mathbf{S}_T}{M}f_{1T}^{\perp q}(x,k^2)$	$\frac{\mathbf{k}_T\cdot\mathbf{S}_T}{M}g_{1T}^{\perp q}(x,k_T^2)$	$\mathbf{S}_T h_{1T}^q(x,k_T^2)$ $\frac{\mathbf{k}_T}{M}\frac{(\mathbf{k}_T\cdot\mathbf{S}_T)_T}{M}h_{1T}^{\perp q}(x,k_T^2)$

SIDIS: CFR



$$x_F > 0$$

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h(P)+X}}{dx\,dQ^2\,d\phi_S\,dz\,d^2P_T} = f_{q,s/N,S} \otimes \frac{d\sigma^{\ell(l,\lambda)+q(k,s)\to\ell(l')+q(k',s')}}{dQ^2} \otimes D^{h_1}_{q,s'}$$

$$D^{h_1}_{q,s'}(z,\mathbf{p}_T) = D_1(z,p_T^2) + \frac{\mathbf{p}_T \times \mathbf{s}'_T}{m_h} H_1(z,p_T^2)$$

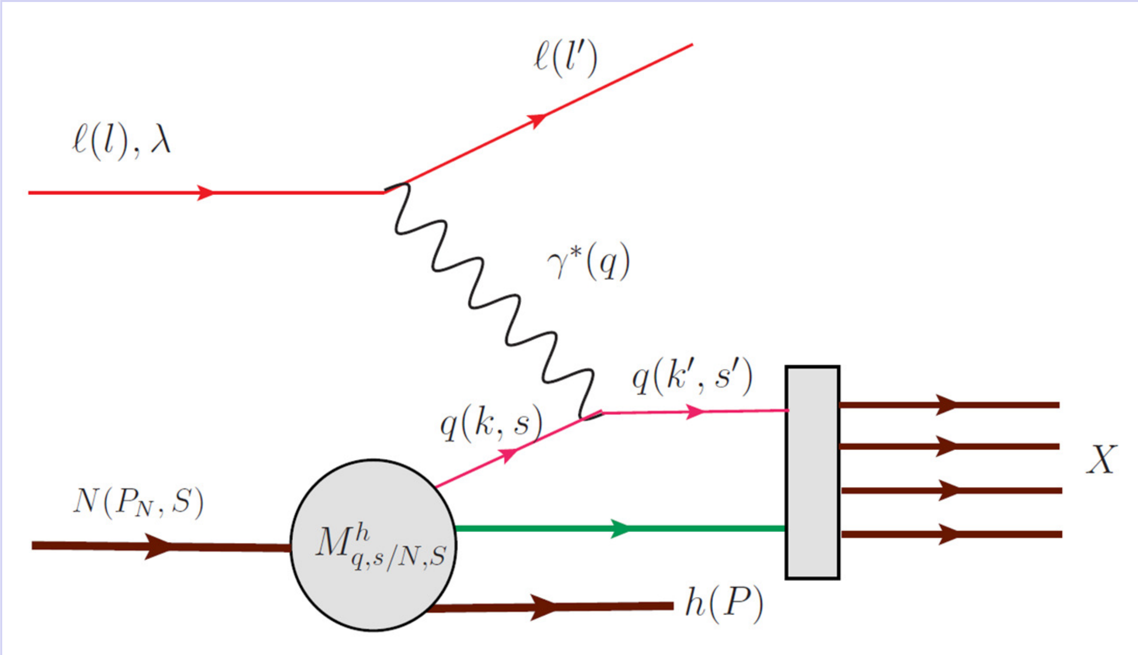
LO cross section in SIDIS CFR

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h(P)+X}(x_F>0)}{dx\,dQ^2\,d\phi_S\,dz\,d^2P_T}=\frac{\alpha^2 x}{yQ^2}\left(1+(1-y)^2\right)\times$$

$$\times\left[\begin{array}{l} F_{UU,T}+D_{nn}(y)F_{UU}^{\cos 2\phi_h}\cos(2\phi_h)+ \\ S_L D_{nn}(y)F_{UL}^{\sin 2\phi_h}\sin(2\phi_h)+\lambda S_L D_{ll}(y)F_{LL}+ \\ S_T\left(F_{UT,T}^{\sin(\phi_h-\phi_S)}\sin(\phi_h-\phi_S)+D_{nn}(y)\left(\begin{array}{l} F_{UT}^{\sin(\phi_h+\phi_S)}\sin(\phi_h+\phi_S)+ \\ F_{UT}^{\sin(3\phi_h-\phi_S)}\sin(3\phi_h-\phi_S)\end{array}\right)\right)+ \\ \lambda S_T D_{ll}(y)F_{LT}^{\cos(\phi_h-\phi_S)}\cos(\phi_h-\phi_S)\end{array}\right]$$

8 terms out of 18 Structure Functions, 6 azimuthal modulations

SIDIS: TFR



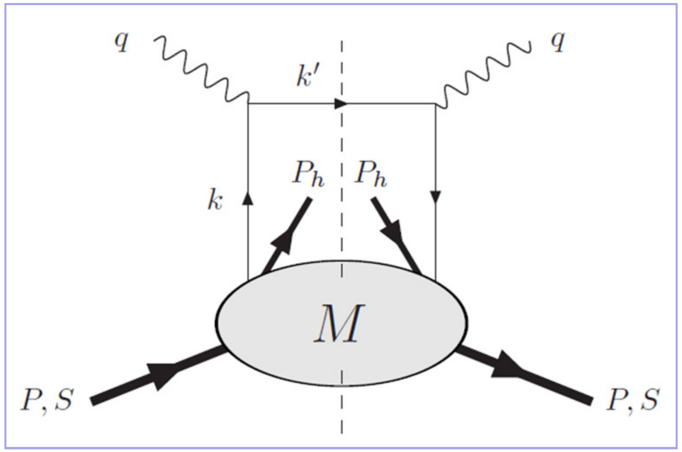
$$x_F < 0$$

M.Anselmino, V.Barone and A.K., arXiv:1102.4214; PL B699 (2011) 108

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h_1(P_1)+h_2(P_2)+X}}{dx\,dQ^2\,d\phi_S\,dz\,d^2P_{T1}\,d\zeta\,d^2P_{T2}} = M^{h_2}_{q,s/N,S} \otimes \frac{d\sigma^{\ell(l,\lambda)+q(k,s)\to\ell(l')+q(k',s')}}{dQ^2}$$

where $\zeta = \dfrac{P^-}{P_N^-} \approx x_F(1-x)$

Quark correlator



$$\mathcal{M}^{[\Gamma]}(x_B,\vec{k}_\perp,\zeta,\vec{P}_{h\perp}) = \frac{1}{4\zeta}\int\frac{d\xi^+ d^2\xi_\perp}{(2\pi)^6} e^{\mathrm{i}(x_B P^- \xi^+ - \vec{k}_\perp\cdot\vec{\xi}_\perp)} \sum_X \int \frac{d^3 P_X}{(2\pi)^3 2E_X} \times$$

$$\times \langle P,S \,|\, \bar{\psi}(0)\Gamma \,|\, P_h, S_h; X\rangle\langle P_h, S_h; X \,|\, \psi(\xi^+, 0, \vec{\xi}_\perp) \,|\, P,S\rangle$$

$$\Gamma = \gamma^-, \quad \gamma^-\gamma_5, \quad i\sigma^{i-}\gamma_5$$

STMD Fracture Functions for spinless hadron production

Nucleon Polarization \ Quark polarization	U	L	T
U	$\hat{M}$	$\dfrac{\mathbf{k}_T \times \mathbf{P}_T}{m_N m_h} \Delta \hat{M}^{\perp h}$	$\dfrac{\epsilon_T^{ij} P_T^{\,j}}{m_h} \Delta_T \hat{M}^{h} + \dfrac{\epsilon_T^{ij} k_T^{\,j}}{m_N} \Delta_T \hat{M}^{\perp}$
L	$\dfrac{S_L (\mathbf{k}_T \times \mathbf{P}_T)}{m_N m_h} \hat{M}_L^{\perp h}$	$S_L \Delta \hat{M}_L$	$\dfrac{S_L \mathbf{P}_T}{m_h} \Delta_T \hat{M}_L^{h} + \dfrac{S_L \mathbf{k}_T}{m_N} \Delta_T \hat{M}_L^{\perp}$
T	$\dfrac{\mathbf{P}_T \times \mathbf{S}_T}{m_h} \hat{M}_T^{h} + \dfrac{\mathbf{k}_T \times \mathbf{S}_T}{m_N} \hat{M}_T^{\perp}$	$\dfrac{\mathbf{P}_T \cdot \mathbf{S}_T}{m_h} \Delta \hat{M}_T^{h} + \dfrac{\mathbf{k}_T \cdot \mathbf{S}_T}{m_N} \Delta \hat{M}_T^{\perp}$	$\mathbf{S}_T \Delta_T \hat{M}_T + \dfrac{\mathbf{P}_T (\mathbf{P}_T \cdot \mathbf{S}_T)}{m_h^2} \Delta_T \hat{M}_T^{hh} + \dfrac{\mathbf{k}_T (\mathbf{k}_T \cdot \mathbf{S}_T)}{m_N^2} \Delta_T \hat{M}_T^{\perp\perp} + \dfrac{\mathbf{P}_T (\mathbf{k}_T \cdot \mathbf{S}_T) - \mathbf{k}_T \cdot (\mathbf{P}_T \cdot \mathbf{S}_T)}{m_N m_h} \Delta_T \hat{M}_T^{\perp h}$

Integrals of STMD fracture functions

Trentadue & Veneziano (1994)

$$M(x,\zeta) = \int d^2P_T \int d^2k_T\, \hat{M}(x,k_T^2,\zeta,P_T^2,\mathbf{k}_T\cdot\mathbf{P}_T)$$

$$\int d^2k_T M^h_{q/N,S}(x,k_T^2,\zeta,P_T^2,\mathbf{k}_T\cdot\mathbf{P}_T) = M(x,\zeta,P_T^2) + \frac{\mathbf{P}_T\times\mathbf{S}_T}{m_h} M_T^h(x,\zeta,P_T^2)$$

$$M(x,\zeta,P_T^2) = \int d^2k_T \hat{M}(x,k_T^2,\zeta,P_T^2,\mathbf{k}_T\cdot\mathbf{P}_T)$$

$$M_T^h(x,\zeta,P_T^2) = \int d^2k_T \left(\hat{M}_T^h + \frac{m_h}{m_N}\frac{\mathbf{k}_T\cdot\mathbf{P}_T}{P_T^2}\hat{M}_T^\perp \right)$$

Extended fracture function Grazini, Trentadue & Veneziano (1997)

$$\int d^2k_T \Delta M^h_{q,s_L/N,S}(x,k_T^2,\zeta,P_T^2,\mathbf{k}_T\cdot\mathbf{P}_T) = S_L\Delta M_L(x,\zeta,P_T^2) + \frac{\mathbf{P}_T\cdot\mathbf{S}_T}{m_h}\Delta M_T^h(x,\zeta,P_T^2)$$

$$\Delta M_L(x,\zeta,P_T^2) = \int d^2k_T \Delta\hat{M}_L(x,k_T^2,\zeta,P_T^2,\mathbf{k}_T\cdot\mathbf{P}_T)$$

$$\Delta M_T^h(x,\zeta,P_T^2) = \int d^2k_T \left(\Delta\hat{M}_T^h + \frac{m_h}{m_N}\frac{\mathbf{k}_T\cdot\mathbf{P}_T}{P_T^2}\Delta\hat{M}_T^\perp \right)$$

Sum Rules

$$\sum_h \int \zeta d\zeta \int d^2 P_T \hat{M} = (1-x) f_1(x, k_T^2)$$

$$\sum_h \int \zeta d\zeta \int d^2 P_T \left(\hat{M}_T^{\perp} + \frac{m_N}{m_h} \frac{\mathbf{k}_T \cdot \mathbf{P}}{k_T^2} \hat{M}_T^{h} \right) = -(1-x) f_{1T}^{\perp}(x, k_T^2)$$

$$\sum_h \int \zeta d\zeta \int d^2 P_T \Delta\hat{M}_L = (1-x) g_{1L}(x, k_T^2)$$

$$\sum_h \int \zeta d\zeta \int d^2 P_T \left(\Delta\hat{M}_T^{\perp} + \frac{m_N}{m_h} \frac{\mathbf{k}_T \cdot \mathbf{P}_T}{k_T^2} \Delta\hat{M}_T^{h} \right) = (1-x) g_{1T}(x, k_T^2)$$

$$\sum_h \int \zeta d\zeta \int d^2 P_T \left(\Delta_T\hat{M}_L^{\perp} + \frac{m_N}{m_h} \frac{\mathbf{k}_T \cdot \mathbf{P}_T}{k_T^2} \Delta_T\hat{M}_L^{h} \right) = (1-x) h_{1L}^{\perp}(x, k_T^2)$$

$$\sum_h \int \zeta d\zeta \int d^2 P_T \left(\Delta_T\hat{M}^{\perp} + \frac{m_N}{m_h} \frac{\mathbf{k}_T \cdot \mathbf{P}_T}{k_T^2} \Delta_T\hat{M}^{h} \right) = -(1-x) h_1^{\perp}(x, k_T^2)$$

$$\sum_h \int \zeta d\zeta \int d^2 P_T \left(\Delta_T\hat{M}_T^{\perp\perp} + \frac{m_N^2}{m_h^2} \frac{2\left(\mathbf{k}_T \cdot \mathbf{P}_T\right)^2 - k_T^2 P_T^2}{k_T^4} \Delta_T\hat{M}_T^{hh} \right) = (1-x) h_{1T}^{\perp}(x, k_T^2)$$

$$\sum_h \int \zeta d\zeta \int d^2 P_T \left(\Delta\hat{M}_T + \frac{k_T^2}{2m_N^2} \Delta_T\hat{M}_T^{\perp\perp} + \frac{P_T^2}{2m_h^2} \Delta\hat{M}_T^{hh} \right) = (1-x) h_1(x, k_T^2)$$

LO cross-section in TFR

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h(P)+X}(x_F<0)}{dx\,dQ^2\,d\phi_S\,d\zeta\,d^2P_T}=\frac{\alpha^2 x}{yQ^4}\left(1+(1-y)^2\right)\sum_q e_q^2\times$$

$$\times\left[\begin{array}{l}M\left(x,\zeta,P_T^2\right)-S_T\frac{P_T}{m_h}M_T^h\left(x,\zeta,P_T^2\right)\sin(\phi_h-\phi_S)+\\ \lambda D_{ll}(y)\left(S_L\Delta M_L\left(x,\zeta,P_T^2\right)+S_T\frac{P_T}{m_h}\Delta M_T^h\left(x,\zeta,P_T^2\right)\cos(\phi_h-\phi_S)\right)\end{array}\right]$$

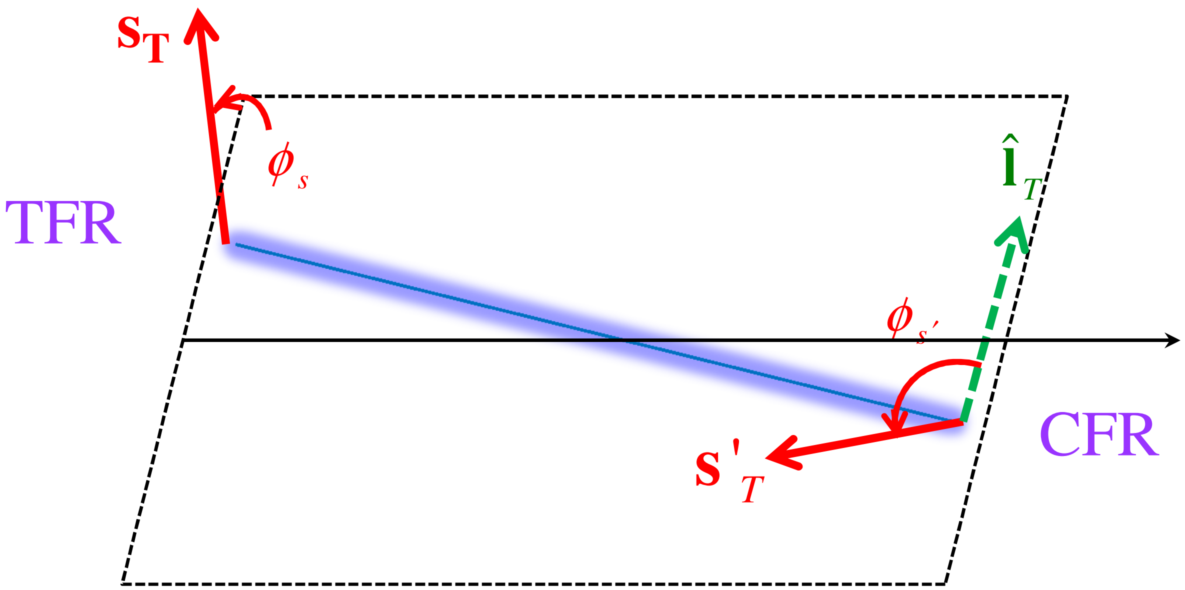
Only 4 terms out of

18 Structure Functions,

2 azimuthal modulations

Nucleon Polarization	Quark polarization: U	L	T
U	$M\left(x,\zeta,P_T^2\right)$		
L		$\Delta M_L\left(x,\zeta,P_T^2\right)$	
T	$M_T^h\left(x,\zeta,P_T^2\right)$	$\Delta M_T^h\left(x,\zeta,P_T^2\right)$	

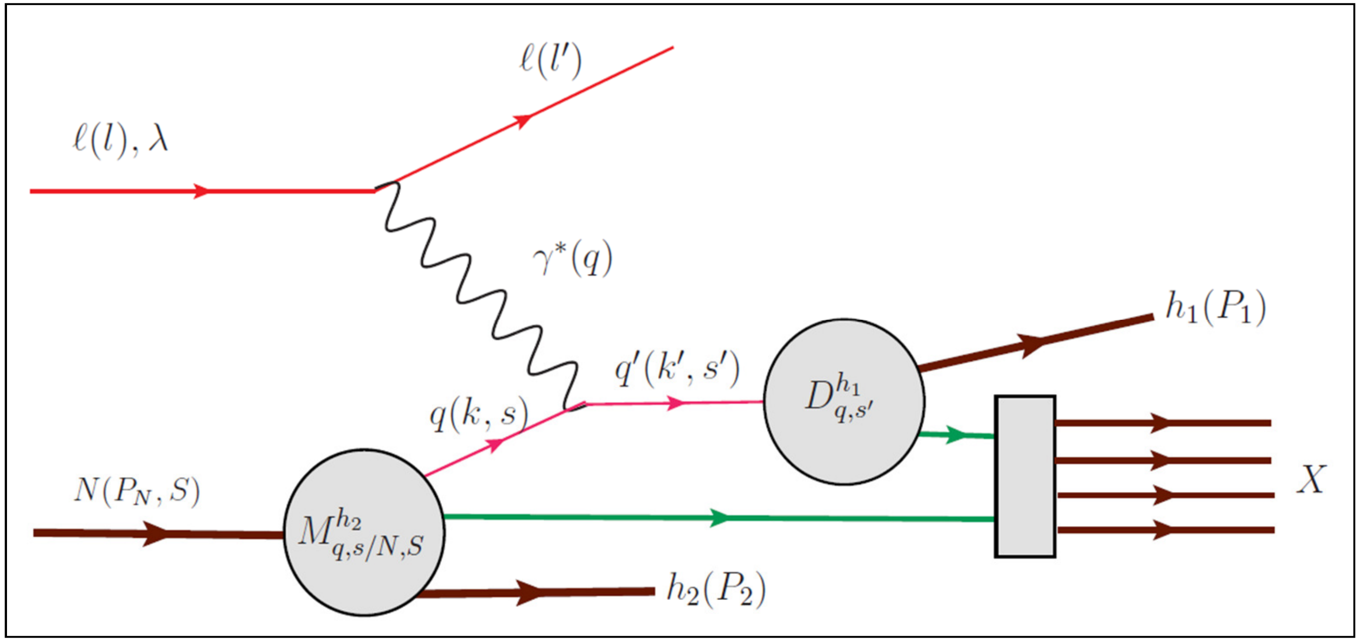
Hadronization in SIDIS



$$\frac{d\sigma^{\ell(l,\lambda)+q(k,s)\to\ell(l')+q(k',s')}}{dQ^2} = e_q^2 \frac{2\pi\alpha^2}{\breve{s}^2}\frac{1}{Q^4}\Big((\breve{s}^2+\breve{u}^2)(1+s_L s'_L)+(\breve{s}^2-\breve{u}^2)\lambda(s_L+s'_L)$$
$$-2\breve{s}\breve{u}(\mathbf{s}_T\cdot\mathbf{s}'_T)-4\breve{u}(\mathbf{s}_T\cdot\mathbf{l}_T)(\mathbf{s}'_T\cdot\mathbf{l}'_T)-4\breve{s}(\mathbf{s}_T\cdot\mathbf{l}'_T)(\mathbf{s}'_T\cdot\mathbf{l}_T)\Big)$$

$$s'_T = D_{nn}(y)s_T, \quad D_{nn}(y)=\frac{2(1-y)}{1+(1-y)^2}, \quad \phi_{s'}=\pi-\phi_s$$

DSIDIS: TFR & CFR



$$x_{F2} < 0, \quad x_{F1} > 0$$

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h_1(P_1)+h_2(P_2)+X}}{dx\,dQ^2\,d\phi_S\,dz\,d^2P_{T1}\,d\zeta\,d^2P_{T2}} = M^{h_2}_{q,s/N,S} \otimes \frac{d\sigma^{\ell(l,\lambda)+q(k,s)\to\ell(l')+q(k',s')}}{dQ^2} \otimes D^{h_1}_{q,s'}$$

$$D^{h_1}_{q,s'}(z,\mathbf{p}_T) = D_1(z,p_T^2) + \frac{\mathbf{p}_T \times \mathbf{s}'_T}{m_h} H_1(z,p_T^2)$$

$$\mathbf{s}'_T = D_{nn}(y)\mathbf{s}^R_T$$

DSIDIS cross-section

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h_1(P_1)+h_2(P_2)+X}}{dx\,dQ^2\,d\phi_S\,dz\,d^2P_{T1}\,d\zeta\,d^2P_{T2}} =$$

$$= \frac{\alpha^2 x}{Q^4 y}\left(1+(1-y)^2\right)\begin{pmatrix} M_{q/N,S}^{h_2} \otimes D_{1q}^{h_1} + \lambda D_{ll}(y)\Delta M_{q,s_L/N,S}^{h_2} \otimes D_{1q}^{h_1} \\ +\Delta_T M_{q,\mathbf{s}_T/N,S}^{h_2} \otimes \frac{\mathbf{p}_T \times \mathbf{s}'_T}{m_{h_1}} H_{1q}^{h_1} \end{pmatrix}$$

$$= \frac{\alpha^2 x}{Q^4 y}\left(1+(1-y)^2\right)\begin{pmatrix} \sigma_{UU} + S_L\sigma_{UL} + S_T\sigma_{UT} + \\ \lambda D_{ll}\left(\sigma_{LU} + S_L\sigma_{LL} + S_T\sigma_{UT}\right) \end{pmatrix}$$

σ_{UU}

$$\sigma_{UU} = F_0^{\hat{M}\cdot D_1} - D_{nn}\left(\begin{array}{l} \dfrac{P_{T1}^2}{m_1 m_N} F_{kp1}^{\Delta_T \hat{M}^{\perp}\cdot H_1}\cos(2\phi_1) \\ + \dfrac{P_{T1}P_{T2}}{m_1 m_2} F_{p1}^{\Delta_T \hat{M}^{h}\cdot H_1}\cos(\phi_1+\phi_2) \\ + \left(\dfrac{P_{T2}^2}{m_1 m_N} F_{kp2}^{\Delta_T \hat{M}^{\perp}\cdot H_1} + \dfrac{P_{T2}^2}{m_1 m_2} F_{p2}^{\Delta_T \hat{M}^{h}\cdot H_1} \right)\cos(2\phi_2) \end{array} \right)$$

		Quark polarization		
		U	**L**	**T**
Nucleon Polarization	**U**	*		*
	L			
	T			

σ_{UL}

$$
\begin{aligned}
\sigma_{UL} = & -\frac{P_{T1}P_{T2}}{m_2 m_N} F_{k1}^{\hat{M}_L^{\perp h}\cdot D_1} \sin(\phi_1 - \phi_2) \\
& + D_{nn} \left(\frac{P_{T1}^2}{m_1 m_N} F_{kp1}^{\Delta_T \hat{M}_L^{\perp}\cdot H_1} \sin(2\phi_1) + \frac{P_{T1}P_{T2}}{m_1 m_2} F_{p1}^{\Delta_T \hat{M}_L^{h}\cdot H_1} \sin(\phi_1 + \phi_2) + \left(\frac{P_{T2}^2}{m_1 m_N} F_{kp2}^{\Delta_T \hat{M}_L^{\perp}\cdot H_1} + \frac{P_{T2}^2}{m_1 m_2} F_{p2}^{\Delta_T \hat{M}_L^{h}\cdot H_1} \right) \sin(2\phi_2) \right)
\end{aligned}
$$

		Quark polarization		
		U	L	T
Nucleon Polarization	U			
	L	*		*
	T			

σ_{UT}

$$
\begin{aligned}
\sigma_{UT} = & -\frac{P_{T1}}{m_N} F_{k1}^{\hat{M}_T^{\perp}\cdot D_1} \sin(\phi_1 - \phi_S) \\
& -\left(\frac{P_{T2}}{m_2} F_0^{\hat{M}_T^{h}\cdot D_1} + \frac{P_{T2}}{m_N} F_{k2}^{\hat{M}_T^{\perp}\cdot D_1} \right) \sin(\phi_2 - \phi_S) \\
& + D_{nn}(y) \left[\left(\frac{P_{T1}}{m_1} F_{p1}^{\Delta_T \hat{M}_T \cdot H_1} + \frac{P_{T1} P_{T2}^2}{2 m_1 m_2^2} F_{p1}^{\Delta_T \hat{M}_T^{hh} \cdot H_1} - \frac{P_{T1} P_{T2}^2}{2 m_1 m_2 m_N} F_{kp3}^{\Delta_T \hat{M}_T^{\perp h} \cdot H_1} \right.\right. \\
& \qquad \left. + \frac{P_{T1}^3}{2 m_1 m_N^2} F_{kkp1}^{\Delta_T \hat{M}_T^{\perp\perp} \cdot H_1} + \frac{P_{T1} P_{T2}^2}{2 m_1 m_N^2} F_{kkp4}^{\Delta_T \hat{M}_T^{\perp\perp} \cdot H_1} + \frac{P_{T1}}{m_1 m_N^2} F_{kkp5}^{\Delta_T \hat{M}_T^{\perp\perp} \cdot H_1} \right) \sin(\phi_1 + \phi_S) \\
& \qquad + \left(\frac{P_{T2}}{m_1} F_{p2}^{\Delta_T \hat{M}_T \cdot H_1} + \frac{P_{T2}^3}{2 m_1 m_2^2} F_{p2}^{\Delta_T \hat{M}_T^{hh} \cdot H_1} + \frac{P_{T1}^2 P_{T2}}{2 m_1 m_2 m_N} F_{kp1}^{\Delta_T \hat{M}_T^{\perp h} \cdot H_1} + \frac{P_{T2}}{m_1 m_2 m_N} F_{kp4}^{\Delta_T \hat{M}_T^{\perp h} \cdot H_1} \right. \\
& \qquad \left. + \frac{P_{T1}^2 P_{T2}}{2 m_1 m_N^2} F_{kkp2}^{\Delta_T \hat{M}_T^{\perp\perp} \cdot H_1} + \frac{P_{T2}^3}{2 m_1 m_N^2} F_{kkp3}^{\Delta_T \hat{M}_T^{\perp\perp} \cdot H_1} \frac{P_{T2}}{m_1 m_N^2} F_{kkp6}^{\Delta_T \hat{M}_T^{\perp\perp} \cdot H_1} \right) \sin(\phi_2 + \phi_S) \\
& \qquad + \frac{P_{T1}^3}{2 m_1 m_N^2} F_{kkp1}^{\Delta_T \hat{M}_T^{\perp\perp} \cdot H_1} \sin(3\phi_1 - \phi_S) \\
& \qquad + \left(\frac{P_{T2}^3}{2 m_1 m_2^2} F_{p2}^{\Delta_T \hat{M}_T^{hh} \cdot H_1} + \frac{P_{T2}^3}{2 m_1 m_N^2} F_{kkp3}^{\Delta_T \hat{M}_T^{\perp\perp} \cdot H_1} \right) \sin(3\phi_2 - \phi_S) \\
& \qquad + \left(\frac{P_{T1} P_{T2}^2}{2 m_1 m_2^2} F_{p1}^{\Delta_T \hat{M}_T^{hh} \cdot H_1} + \frac{P_{T1} P_{T2}^2}{2 m_1 m_N^2} F_{kkp4}^{\Delta_T \hat{M}_T^{\perp\perp} \cdot H_1} \right) \sin(\phi_1 + 2\phi_2 - \phi_S) \\
& \qquad - \frac{P_{T1}^2 P_{T2}}{2 m_1 m_2 m_N} F_{kp1}^{\Delta_T \hat{M}_T^{\perp h} \cdot H_1} \sin(2\phi_1 - \phi_2 + \phi_S) \\
& \qquad - \frac{P_{T1} P_{T2}^2}{2 m_1 m_2 m_N} F_{kp3}^{\Delta_T \hat{M}_T^{\perp h} \cdot H_1} \sin(\phi_1 - 2\phi_2 - \phi_S) \\
& \qquad \left. + \frac{P_{T1}^2 P_{T2}}{2 m_1 m_N^2} F_{kkp2}^{\Delta_T \hat{M}_T^{\perp\perp} \cdot H_1} \sin(2\phi_1 + \phi_2 - \phi_S) \right]
\end{aligned}
$$

Nucleon Polarization	Quark polarization: U	L	T
U			
L			
T	*		*

σ_{LU}, σ_{LL}, σ_{LT}

$$\sigma_{LU} = -\frac{P_{T1}P_{T2}}{m_2 m_N} F_{k1}^{\Delta\hat{M}^{\perp h}\cdot D_1} \sin(\phi_1 - \phi_2)$$

Nucleon Polarization \ Quark polarization	U	L	T
U		*	
L			
T			

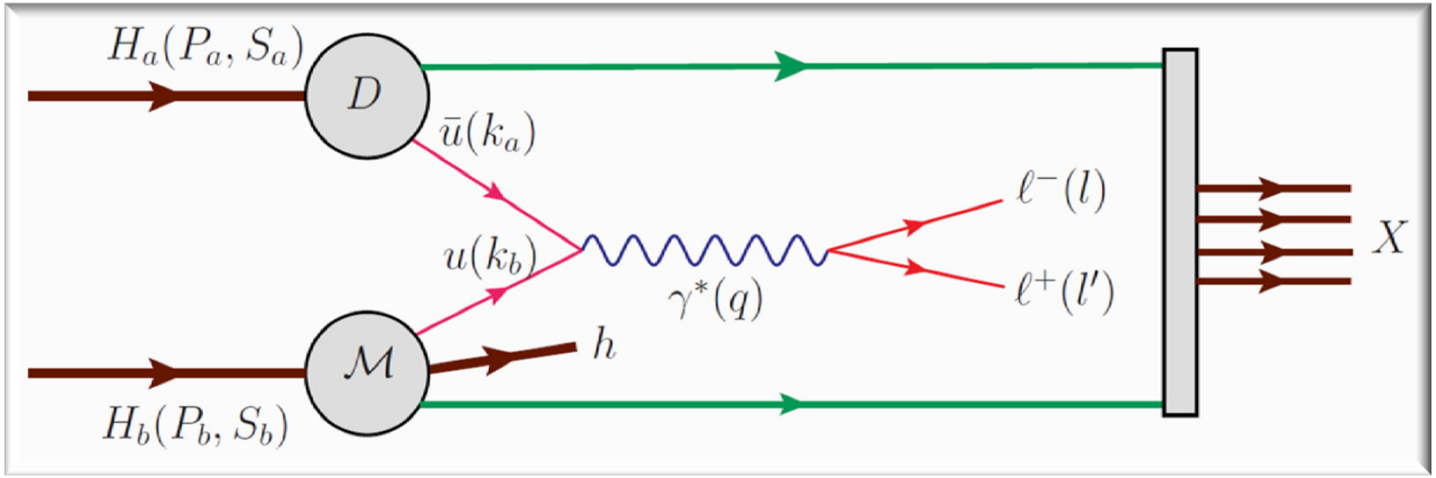
$$\sigma_{LL} = F_0^{\Delta\hat{M}_L \cdot D_1}$$

Nucleon Polarization \ Quark polarization	U	L	T
U			
L		*	
T			

$$\sigma_{LT} = \frac{P_{T1}}{m_N} F_{k1}^{\Delta\hat{M}_T^{\perp}\cdot D_1} \cos(\phi_1 - \phi_S) + \left(\frac{P_{T2}}{m_2} F_0^{\Delta\hat{M}_T^{h}\cdot D_1} + \frac{P_{T2}}{m_N} F_{k2}^{\Delta\hat{M}_T^{\perp}\cdot D_1} \right) \cos(\phi_2 - \phi_S)$$

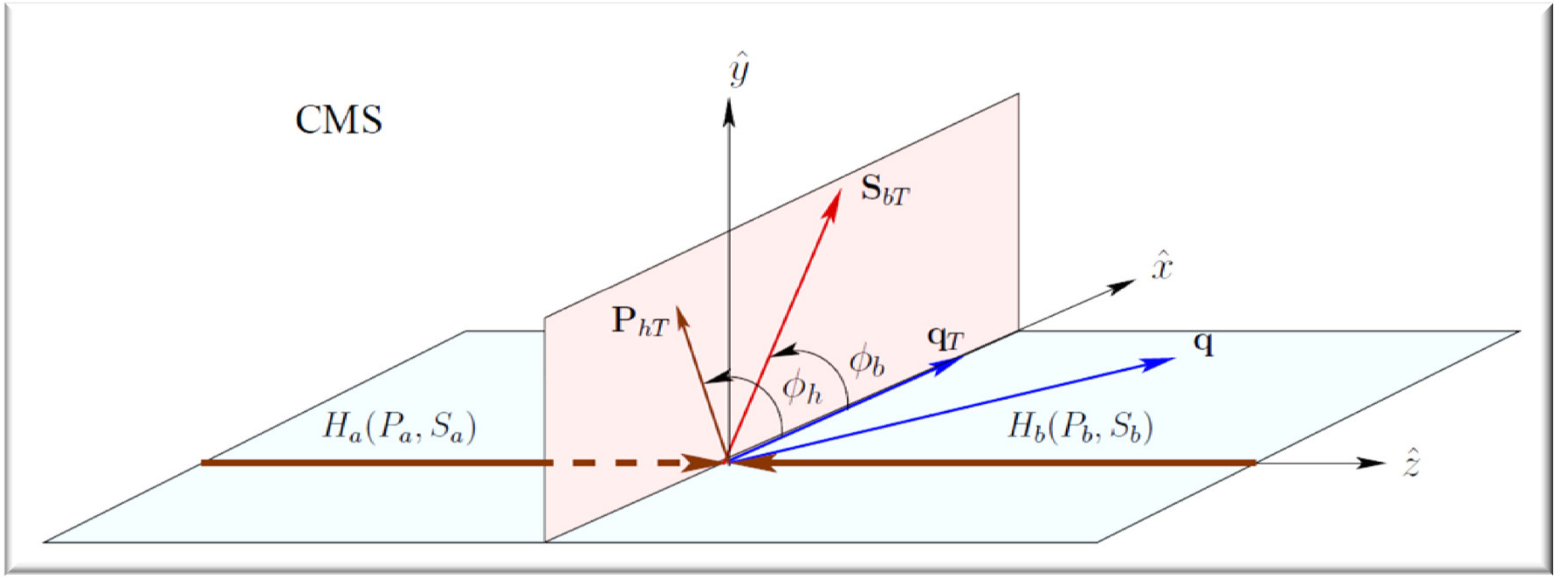
Nucleon Polarization \ Quark polarization	U	L	T
U			
L			
T		*	

Polarized SIDY processes

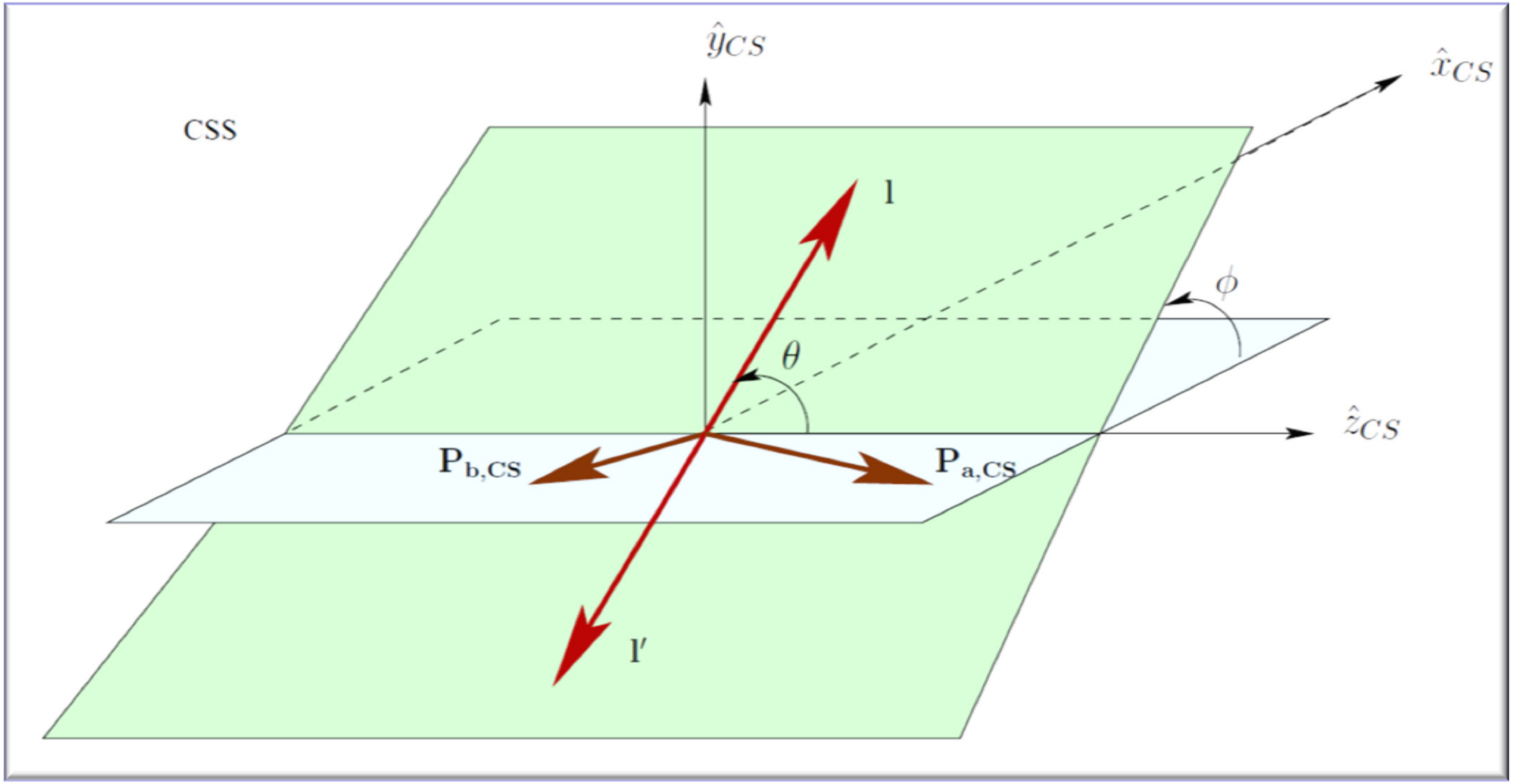


Kinematics as in

Arnold.Metz.Schlegel, PhysRevD.79.034005



Collins Soper system



SIDY cross section

$$\frac{d\sigma}{d^4q\,d\Omega\,d\zeta\,d^2P_T}=\frac{\alpha_{em}^2x_ax_b}{2q^4}\frac{1}{N_c}\sum_q e_q^2\int d^2\vec{k}_{aT}\,d^2\vec{k}_{bT}\,\delta^{(2)}\left(\vec{q}_T-\vec{k}_{aT}-\vec{k}_{bT}\right)\times$$

$$\times\begin{pmatrix}\left(1+\cos^2\theta\right)\left(\Phi^{q[\gamma^+]}\overline{\mathcal{M}}^{q[\gamma^-]}+\Phi^{q[\gamma^+\gamma_5]}\overline{\mathcal{M}}^{q[\gamma^-\gamma_5]}\right)\\ +\sin^2\theta\begin{pmatrix}\cos 2\phi\left(\delta^{i1}\delta^{j1}-\delta^{i2}\delta^{j2}\right)\\ +\sin 2\phi\left(\delta^{i1}\delta^{j2}+\delta^{i2}\delta^{j1}\right)\end{pmatrix}\Phi^{q[i\sigma^{i+}\gamma_5]}\overline{\mathcal{M}}^{q[i\sigma^{j-}\gamma_5]}\\ +\{\Phi\leftrightarrow\overline{\Phi},\overline{\mathcal{M}}\leftrightarrow\mathcal{M}\}+\mathcal{O}(1/q)\end{pmatrix}$$

$$=\frac{\alpha_{em}^2x_ax_b}{2q^4}\begin{pmatrix}\sigma_{UU}+S_{bL}\sigma_{UL}+S_{bT}\sigma_{UT}\\ +S_{aL}\sigma_{LU}+S_{aL}S_{bL}\sigma_{LL}+S_{aL}S_{bT}\sigma_{LT}\\ +S_{aT}\sigma_{TU}+S_{aT}S_{bL}\sigma_{TL}+S_{aT}S_{bT}\sigma_{TT}\end{pmatrix}$$

σ_{UU}

$$\sigma_{UU} = \left(1+\cos^2\theta\right) F_{UU} - \sin^2\theta \begin{bmatrix} F_{UU}^{\cos(2\phi)} \cos(2\phi) \\ + F_{UU}^{\cos(2\phi-\phi_h)} \cos(2\phi-\phi_h) \\ + F_{UU}^{\cos(2\phi-2\phi_h)} \cos(2\phi-2\phi_h) \end{bmatrix}$$

$$F_{UU} = F_0^{f_1\cdot\hat{M}}$$

$$F_{UU}^{\cos(2\phi)} = \frac{q_T^2}{M_a M_b} F_{ab2}^{h_1^\perp\cdot\Delta_T\hat{M}^\perp}$$

$$F_{UU}^{\cos(2\phi-\phi_h)} = \frac{P_T q_T}{m M_a} F_{a2}^{h_1^\perp\cdot\Delta_T\hat{M}^h}$$

$$F_{UU}^{\cos(2\phi-2\phi_h)} = \frac{P_T^2}{m M_a} F_{a1}^{h_1^\perp\cdot\Delta_T\hat{M}^h} + \frac{P_T^2}{M_a M_b} F_{ab1}^{h_1^\perp\cdot\Delta_T\hat{M}^\perp}$$

		Quark polarization		
		U	L	T
Nucleon Polarization	U	*		*
	L			
	T			

σ_{UL}

$$\sigma_{UL} = \left(1+\cos^2\theta\right) F_{UL}^{\sin(\phi_h)} \sin(\phi_h) - \sin^2\theta \begin{bmatrix} F_{UL}^{\sin(2\phi)} \sin(2\phi) \\ +F_{UL}^{\sin(2\phi-\phi_h)} \sin(2\phi-\phi_h) \\ +F_{UL}^{\sin(2\phi-2\phi_h)} \sin(2\phi-2\phi_h) \end{bmatrix}$$

		Quark polarization		
		U	L	T
Nucleon Polarization	U			
	L	*		*
	T			

σ_{UT}

$$\sigma_{UT} = \left(1+\cos^2\theta\right)\left[\begin{array}{l} F_{UT}^{\sin(\phi_b-\phi_h)}\sin\left(\phi_b-\phi_h\right) \\ +F_{UT}^{\sin(\phi_b)}\sin\left(\phi_b\right) \end{array}\right]$$

$$-\sin^2\theta\left[\begin{array}{l} F_{UT}^{\sin(2\phi+\phi_b)}\sin\left(2\phi+\phi_b\right) \\ +F_{UT}^{\sin(2\phi-\phi_b+\phi_h)}\sin\left(2\phi-\phi_b+\phi_h\right) \\ +F_{UT}^{\sin(2\phi+\phi_b-\phi_h)}\sin\left(2\phi+\phi_b-\phi_h\right) \\ +F_{UT}^{\sin(2\phi+\phi_b-3\phi_h)}\sin\left(2\phi+\phi_b-3\phi_h\right) \\ +F_{UT}^{\sin(2\phi-\phi_b-2\phi_h)}\sin\left(2\phi-\phi_b-2\phi_h\right) \\ +F_{UT}^{\sin(2\phi+\phi_b-2\phi_h)}\sin\left(2\phi+\phi_b-2\phi_h\right) \\ +F_{UT}^{\sin(2\phi-\phi_b-\phi_h)}\sin\left(2\phi-\phi_b-\phi_h\right) \\ +F_{UT}^{\sin(2\phi-\phi_b)}\sin\left(2\phi-\phi_b\right) \end{array}\right]$$

		Quark polarization		
		U	**L**	**T**
Nucleon Polarization	**U**			
	L			
	T	*		*

σ_{LU}

$$\sigma_{LU} = \left(1+\cos^2\theta\right) F_{LU}^{\sin(\phi_h)} \sin(\phi_h)$$

$$-\sin^2\theta \left[\begin{array}{l} F_{LU}^{\sin(2\phi-2\phi_h)} \sin(2\phi-2\phi_h) \\ +F_{LU}^{\sin(2\phi-\phi_h)} \sin(2\phi-\phi_h) \\ +F_{LU}^{\sin(2\phi)} \sin(2\phi) \end{array} \right]$$

		Quark polarization		
		U	L	T
Nucleon Polarization	U		*	*
	L			
	T			

σ_{LL}

$$\sigma_{LL} = \left(1+\cos^2\theta\right) F_{LL}$$

$$+\sin^2\theta \left[F_{LL}^{\cos(2\phi-2\phi_h)} \cos\left(2\phi-2\phi_h\right) + F_{LL}^{\cos(2\phi-\phi_h)} \cos\left(2\phi-\phi_h\right) + F_{LL}^{\cos(2\phi)} \cos(2\phi) \right]$$

		Quark polarization		
		U	L	T
Nucleon Polarization	U			
	L		*	*
	T			

σ_{LT}

$$\sigma_{LT} = \left(1+\cos^2\theta\right)\left[\begin{array}{l} F_{LT}^{\cos(\phi_b-\phi_h)}\cos\left(\phi_b-\phi_h\right) \\ +F_{LT}^{\cos(\phi_b)}\cos\left(\phi_b\right) \end{array}\right]$$

$$+\sin^2\theta\left[\begin{array}{l} F_{LT}^{\cos(2\phi-\phi_b+\phi_h)}\cos\left(2\phi-\phi_b+\phi_h\right) \\ +F_{LT}^{\cos(2\phi+\phi_b)}\cos\left(2\phi+\phi_b\right)+ \\ F_{LT}^{\cos(2\phi+\phi_b-\phi_h)}\cos\left(2\phi+\phi_b-\phi_h\right) \\ +F_{LT}^{\cos(2\phi+\phi_b-3\phi_h)}\cos\left(2\phi+\phi_b-3\phi_h\right) \\ +F_{LT}^{\cos(2\phi-\phi_b-2\phi_h)}\cos\left(2\phi-\phi_b-2\phi_h\right) \\ +F_{LT}^{\cos(2\phi+\phi_b-2\phi_h)}\cos\left(2\phi+\phi_b-2\phi_h\right) \\ +F_{LT}^{\cos(2\phi-\phi_b-\phi_h)}\cos\left(2\phi-\phi_b-\phi_h\right) \\ +F_{LT}^{\cos(2\phi-\phi_b)}\cos\left(2\phi-\phi_b\right) \end{array}\right]$$

		Quark polarization		
		U	**L**	**T**
Nucleon Polarization	**U**			
	L			
	T		*	*

σ_{TU}

$$\sigma_{TU} = \left(1+\cos^2\theta\right)\begin{bmatrix} F_{TU}^{\sin(\phi_a-\phi_h)}\sin(\phi_a-\phi_h) \\ +F_{TU}^{\sin(\phi_a)}\sin(\phi_a) \\ +F_{TU}^{\sin(\phi_a+\phi_h)}\sin(\phi_a+\phi_h) \\ +F_{TU}^{\sin(\phi_a-2\phi_h)}\sin(\phi_a-2\phi_h) \end{bmatrix}$$

$$+\sin^2\theta\begin{bmatrix} F_{TU}^{\sin(2\phi-\phi_a-\phi_h)}\sin(2\phi-\phi_a-\phi_h) \\ +F_{TU}^{\sin(2\phi-\phi_a)}\sin(2\phi-\phi_a) \\ +F_{TU}^{\sin(2\phi+\phi_a-3\phi_h)}\sin(2\phi+\phi_a-3\phi_h)+ \\ F_{TU}^{\sin(2\phi+\phi_a-\phi_h)}\sin(2\phi+\phi_a-\phi_h) \\ +F_{TU}^{\sin(2\phi+\phi_a-2\phi_h)}\sin(2\phi+\phi_a-2\phi_h) \\ +F_{TU}^{\sin(2\phi+\phi_a)}\sin(2\phi+\phi_a) \end{bmatrix}$$

		Quark polarization		
		U	**L**	**T**
Nucleon Polarization	**U**	*	*	*
	L			
	T			

σ_{TL}

$$\sigma_{TL} = \left(1+\cos^2\theta\right)\left[\begin{array}{l} F_{TL}^{\cos(\phi_a-\phi_h)}\cos(\phi_a-\phi_h) \\ +F_{TL}^{\cos(\phi_a)}\cos(\phi_a) \\ +F_{TL}^{\cos(\phi_a+\phi_h)}\cos(\phi_a+\phi_h) \\ +F_{TL}^{\cos(\phi_a-2\phi_h)}\cos(\phi_a-2\phi_h) \end{array}\right]$$

$$+\sin^2\theta\left[\begin{array}{l} F_{TL}^{\cos(2\phi+\phi_a-3\phi_h)}\cos(2\phi+\phi_a-3\phi_h) \\ +F_{TL}^{\cos(2\phi+\phi_a-2\phi_h)}\cos(2\phi+\phi_a-2\phi_h) \\ +F_{TL}^{\cos(2\phi+\phi_a)}\cos(2\phi+\phi_a) \\ +F_{TL}^{\cos(2\phi-\phi_a-\phi_h)}\cos(2\phi-\phi_a-\phi_h) \\ +F_{TL}^{\cos(2\phi-\phi_a)}\cos(2\phi-\phi_a) \end{array}\right]$$

		Quark polarization		
		U	L	T
Nucleon Polarization	U			
	L		*	*
	T			

σ_{TT}

$$\sigma_{TT} = \left(1+\cos^2\theta\right)\begin{bmatrix} F_{TT}^{\cos(\phi_a-\phi_b)}\cos(\phi_a-\phi_b) \\ +F_{TT}^{\cos(\phi_a+\phi_b-2\phi_h)}\cos(\phi_a+\phi_b-2\phi_h) \\ +F_{TT}^{\cos(\phi_a-\phi_b+\phi_h)}\cos(\phi_a-\phi_b+\phi_h) \\ +F_{TT}^{\cos(\phi_a+\phi_b-\phi_h)}\cos(\phi_a+\phi_b-\phi_h) \\ +F_{TT}^{\cos(\phi_a+\phi_b)}\cos(\phi_a+\phi_b) \\ +F_{TT}^{\cos(\phi_a-\phi_b-\phi_h)}\cos(\phi_a-\phi_b-\phi_h) \end{bmatrix}$$

$$+\sin^2\theta\begin{bmatrix} F_{TT}^{\cos(2\phi-\phi_a-\phi_b-\phi_h)}\cos(2\phi-\phi_a-\phi_b-\phi_h) \\ +F_{TT}^{\cos(2\phi-\phi_a-\phi_b+\phi_h)}\cos(2\phi-\phi_a-\phi_b+\phi_h) \\ +F_{TT}^{\cos(2\phi+\phi_a+\phi_b)}\cos(2\phi+\phi_a+\phi_b) \\ +F_{TT}^{\cos(2\phi+\phi_a-\phi_b+\phi_h)}\cos(2\phi+\phi_a-\phi_b+\phi_h)+ \\ F_{TT}^{\cos(2\phi-\phi_a+\phi_b)}\cos(2\phi-\phi_a+\phi_b) \\ +F_{TT}^{\cos(2\phi-\phi_a+\phi_b-2\phi_h)}\cos(2\phi-\phi_a+\phi_b-2\phi_h) \\ +F_{TT}^{\cos(2\phi+\phi_a+\phi_b-4\phi_h)}\cos(2\phi+\phi_a+\phi_b-4\phi_h) \\ +F_{TT}^{\cos(2\phi+\phi_a-\phi_b-3\phi_h)}\cos(2\phi+\phi_a-\phi_b-3\phi_h) \\ +F_{TT}^{\cos(2\phi-\phi_a-\phi_b)}\cos(2\phi-\phi_a-\phi_b) \\ +F_{TT}^{\cos(2\phi+\phi_a-\phi_b-2\phi_h)}\cos(2\phi+\phi_a-\phi_b-2\phi_h) \\ +F_{TT}^{\cos(2\phi+\phi_a+\phi_b-2\phi_h)}\cos(2\phi+\phi_a+\phi_b-2\phi_h) \\ +F_{TT}^{\cos(2\phi+\phi_a-\phi_b-\phi_h)}\cos(2\phi+\phi_a-\phi_b-\phi_h) \\ +F_{TT}^{\cos(2\phi+\phi_a-\phi_b)}\cos(2\phi+\phi_a-\phi_b) \end{bmatrix}$$

		Quark polarization		
		U	L	T
Nucleon Polarization	U			
	L			
	T	*	*	*

CONCLUSIONS

- New members appeared in the polarized TMDs family -- 16 LO STMD fracture functions
- For hadron produced in the TFR, only 4 k_T-integrated fracture functions of unpolarized and longitudinally polarized quarks are probed.
  - SSA contains only a Sivers-type modulation $\sin(\varphi_h-\varphi_S)$ but no Collins-type $\sin(\varphi_h+\varphi_S)$ or $\sin(3\varphi_h-\varphi_S)$. The eventual observation of Collins-type asymmetry will indicate that LO factorized approach fails and long range correlations between the struck quark polarization and P_T of produced in TFR hadron might be important.
- DSIDIS cross section at LO contains 2 azimuthal independent and 20 azimuthally modulated terms.
- SIDY cross section at LO contains 2 azimuthal independent, 20 lepton azimuth independent and 52 lepton azimuth dependent terms
- The ideal place to test the fracture functions factorization and measure these new nonperturbative objects are JLab12 and EIC facilities with full coverage of phase space and polarized SIDY
- To do
  - Factorization proof (SIDIS, DSIDIS, SIDY).
    - Structure of Wilson lines. SIDIS ←→ DY universality: sign changes of some fracture functions? Higher twist. Polarized hadron production. Phenomenology: parameterizations , simple models. Other processes: $P\uparrow + P \rightarrow \pi + X$, $P\uparrow + P \rightarrow \pi + \text{jet} + X$, ….