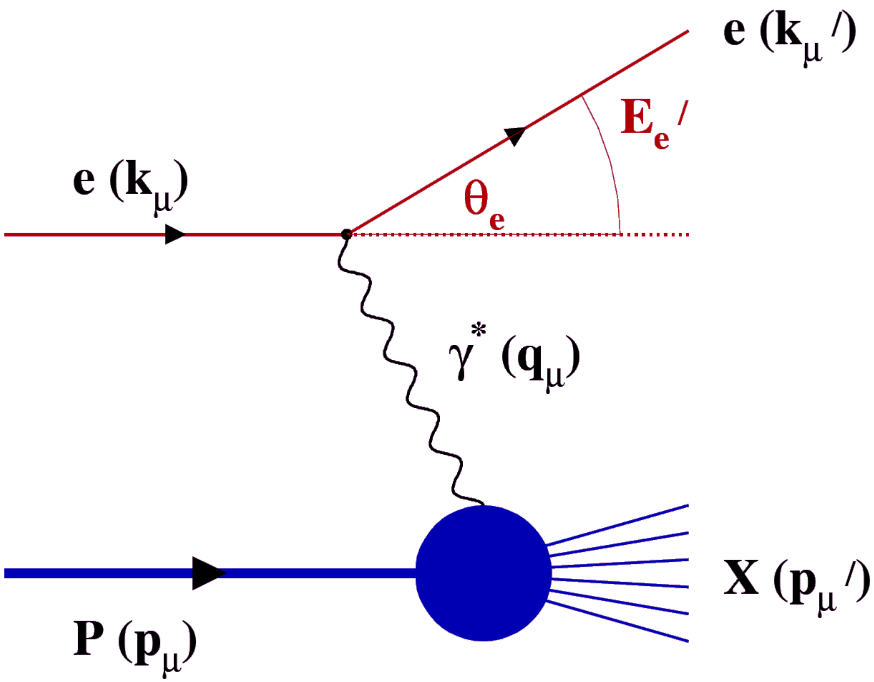
From
DIS to DY
through the
Color Glass

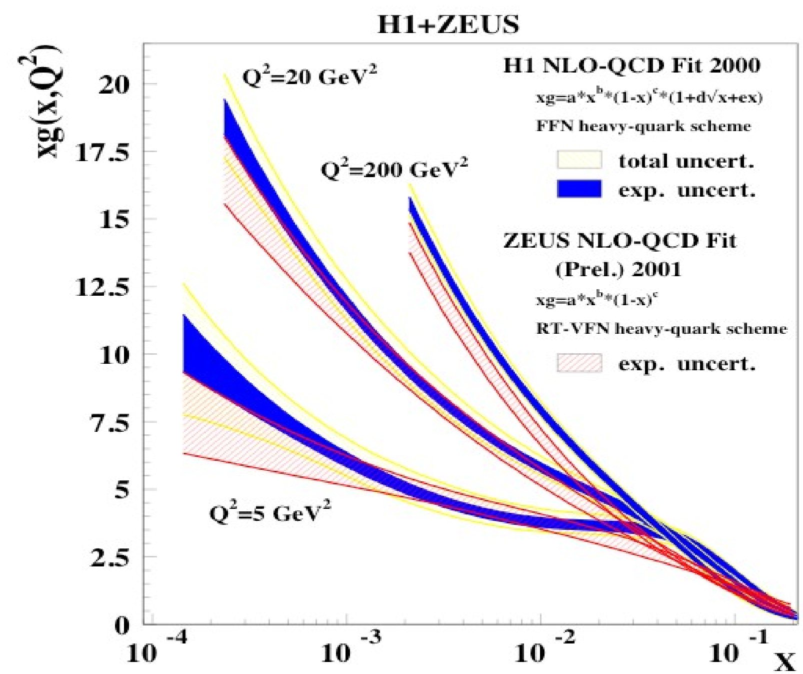
Jamal Jalilian-Marian
Baruch College, New York NY

based on (old) work with F. Gelis

A hadron at small x

DIS: $\mathbf{e\,p \to e\,X}$





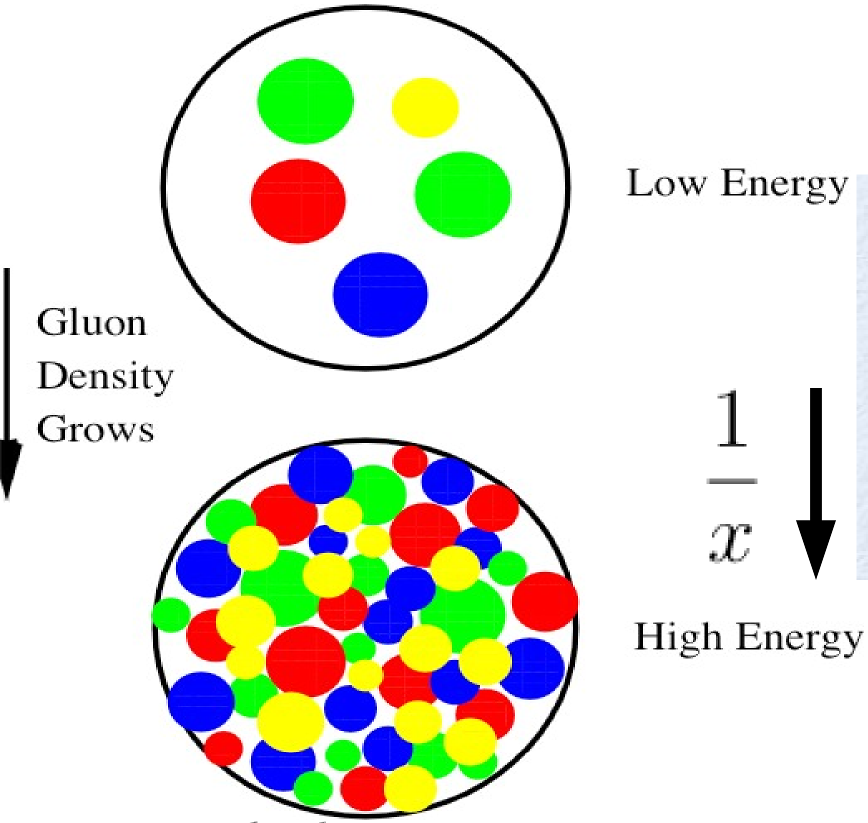
$$x = \frac{p^+}{P^+}$$ is the fraction of hadron energy carried by a parton

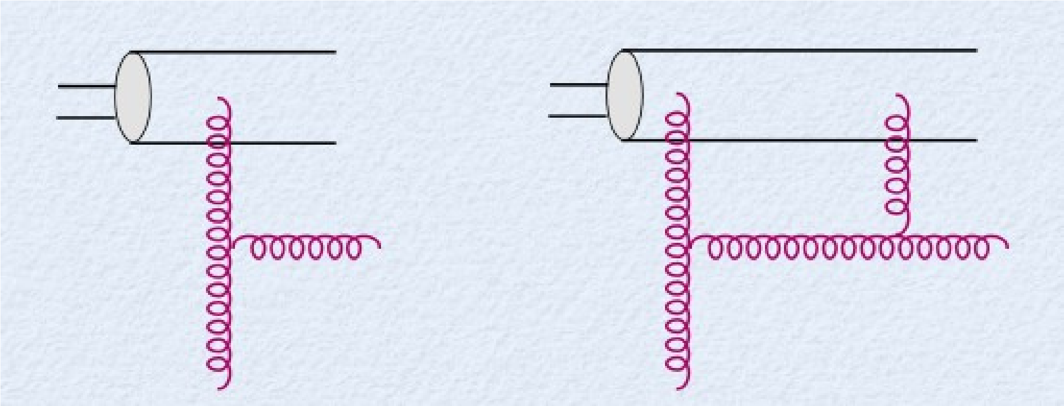
there are a lot of gluons at small x

Gluon saturation

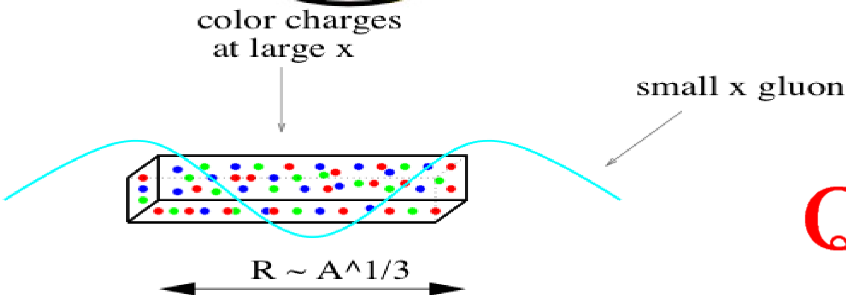
Gribov-Levin-Ryskin

“attractive” bremsstrahlung vs. “repulsive” recombination





$$\frac{\alpha_s\, xG(x, b_t, Q^2)}{S_\perp\, Q^2} \sim 1$$

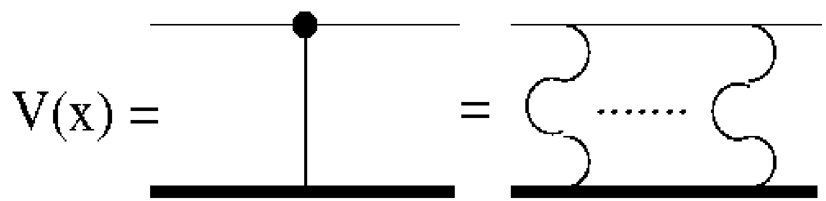


$$Q_s^2(x, b_t, A) \sim A^{1/3}\,\left(\frac{1}{x}\right)^{0.3}$$

CGC: QCD at high gluon density

effective degrees of freedom: Wilson line $V(x_t)$

CGC observables: $< \mathrm{Tr}\, V \cdots\cdots V^\dagger >$ with $\mathrm{V(x_t)} = \hat{\mathrm{P}} e^{ig \int dx^- A_a^+ t_a}$



$$\mathbf{A_a^\mu(x_t, x^-)} \sim \delta^{\mu +}\, \delta(\mathbf{x}^-)\, \alpha_\mathbf{a}(\mathbf{x_t}) \qquad \alpha^\mathbf{a}(\mathbf{k_t}) = \mathbf{g}\, \rho^\mathbf{a}(\mathbf{k_t})/\mathbf{k_t^2}$$

gluon distribution: $\mathrm{xG(x, Q^2)} \sim \int^{Q^2} \frac{d^2k_t}{k_t^2}\, \phi(\mathrm{x, k_t})$ with $\phi(\mathbf{x}, \mathbf{k_t^2}) \sim < \rho_\mathbf{a}^\star(\mathbf{k_t})\, \rho_\mathbf{a}(\mathbf{k_t}) >$

averaging over color charges ρ

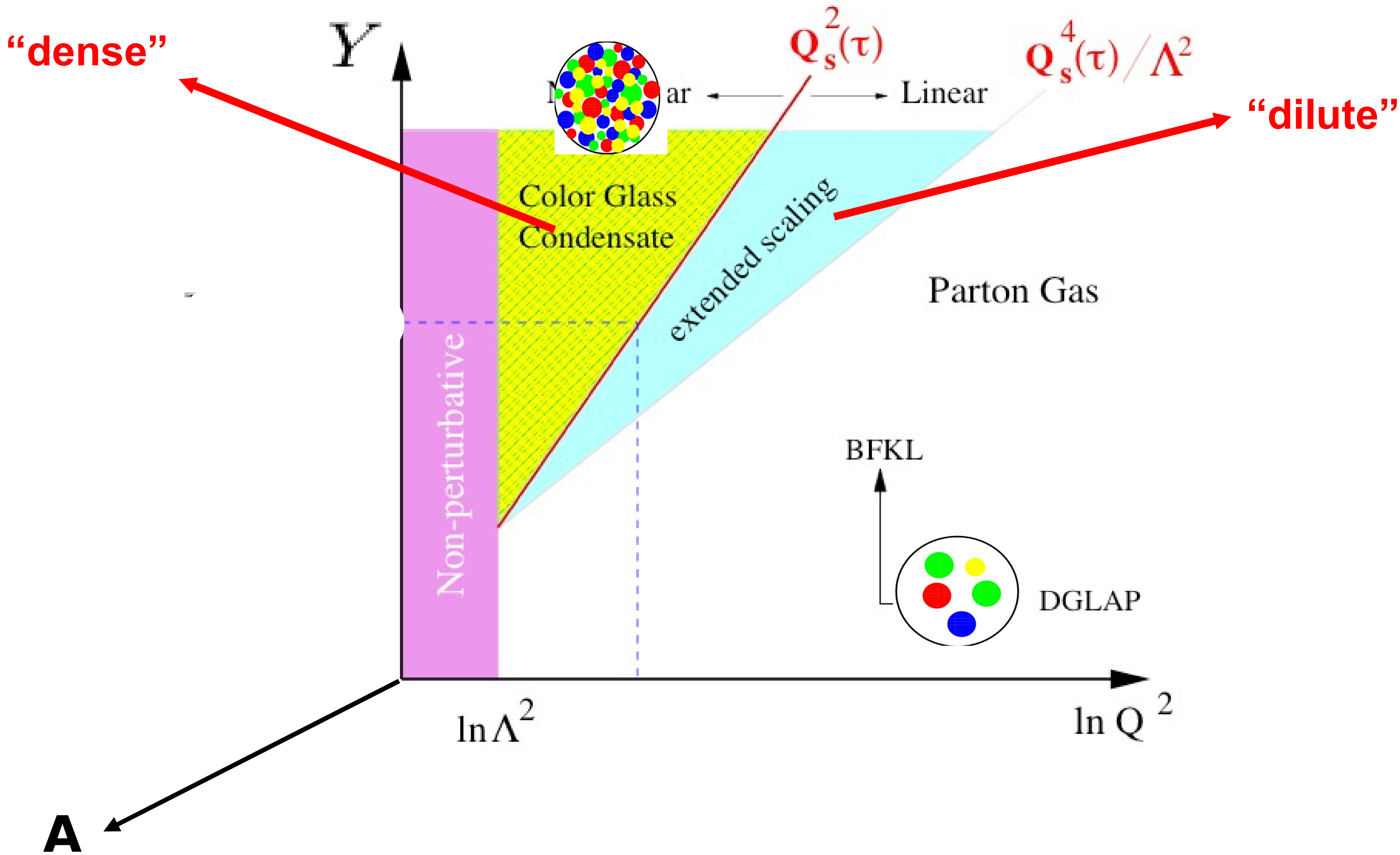
two main effects:

"multiple scatterings" → p_t broadening, *Cronin effect*

evolution with ln (1/x) → suppression

"Leading twist" nuclear shadowing (single gluon exchange limit)

Road Map of QCD Phase Space

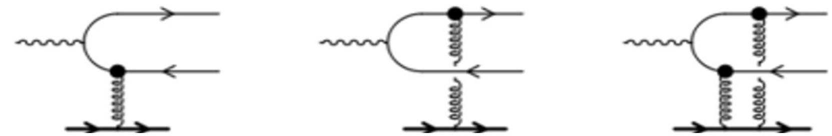


MV
PRD59 (1999) 094002

DIS at small x

Gelis,Jalilian-Marian
PRD67 (2003) 074019

consider $\gamma^\star\, \mathrm{T} \to \mathrm{q}\,\bar{\mathrm{q}}\,\mathrm{X}$



$$M^\mu(k;q,p) \;=\; \frac{i}{2}\int \frac{d^2 l_t}{(2\pi)^2}\, d^2x_t d^2y_t e^{il_t\cdot x_t} e^{i(p_t+q_t-k_t-l_t)\cdot y_t}\, \bar{u}(q)\,\Gamma^\mu(k;q,p)\,v(p)\,\left[V(x_t)\,V^\dagger(y_t)-1\right]$$

cross section:

averaging over color charges ρ

$$\mathbf{2p_0\,2q_0\,\frac{d\sigma}{d^3q\,d^3p} = \frac{1}{(2\pi)^5}\,\frac{1}{k^-}\,2\pi\delta(k^- p^- - q^-)\,\langle M^\mu M^{\nu\,\star}\rangle\,\epsilon_\mu(k)\epsilon^\star_\nu(k)}$$

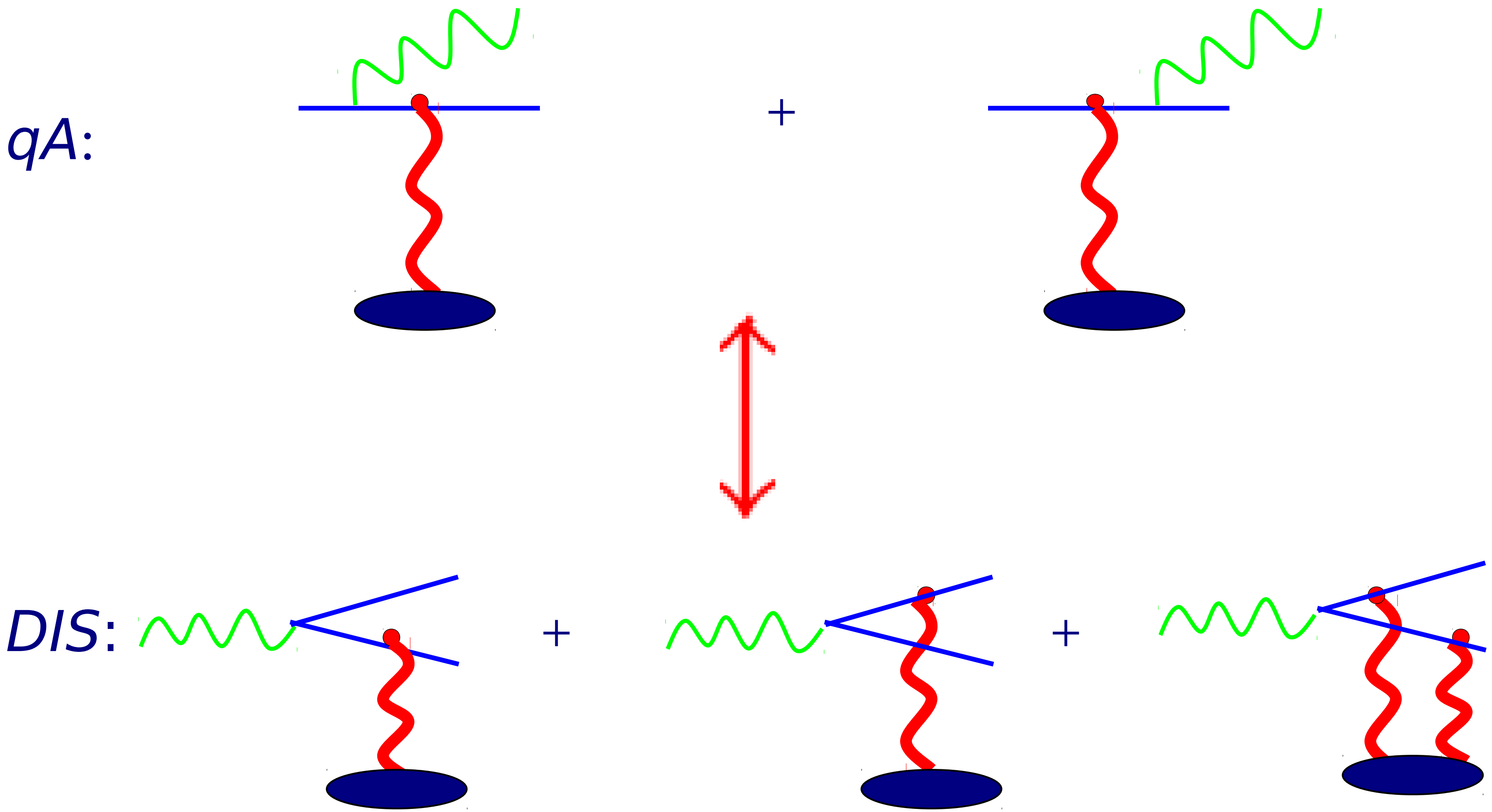
to get the DIS total cross section, <u>integrate over quark, anti-quark momenta</u>

$$\sigma^{\gamma^\star\,\mathrm{T}\to\mathrm{X}} = \int_0^1 \mathrm{d}z \int \mathrm{d^2x_t\,d^2y_t}\,|\Psi|^2\, \underbrace{\frac{1}{\mathrm{N_c}}\left\langle \mathrm{Tr}\,\left[1-\mathrm{V}(\mathrm{x_t})\,\mathrm{V}^\dagger(\mathrm{y_t})\right]\right\rangle}_{\textit{dipole cross section}\ T(x_g,r_t,b_t)}$$

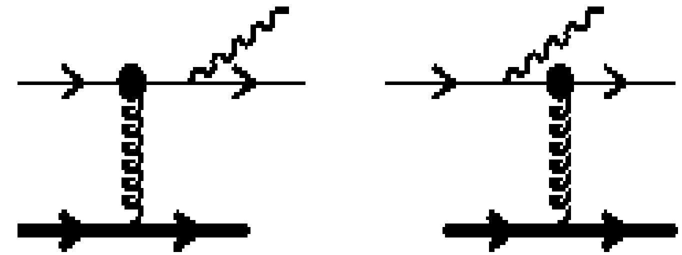
$\mathbf{T(x_g,r_t,b_t)}$
satisfies the JIMWLK/BK eqs.

From DIS to DY: crossing symmetry (LO)

$$\gamma^\star \mathbf{A} \rightarrow \mathbf{q}\,\bar{\mathbf{q}}\,\mathbf{X} \quad \leftrightarrow \quad \mathbf{q}\,\mathbf{A} \rightarrow \mathbf{q}\,\gamma^\star\,\mathbf{X}$$



DY at small x



$$\mathbf{q\,T \rightarrow q\,\gamma^{\star}\,X}$$

$$\mathbf{M^{\mu}(p;k,q) = i \int d^2x_t e^{i(q_t+k_t-p_t)\cdot x_t}\, \bar{u}(q)\, \bar{\Gamma}^{\mu}(k;q,p)\, u(p)\; [V(x_t) - 1]}$$

$$= \frac{i}{2}\int \frac{d^2l_t}{(2\pi)^2}\, d^2x_t d^2y_t e^{il_t\cdot x_t} e^{i(q_t+k_t-p_t-l_t)\cdot y_t}\, \bar{u}(q)\, \Gamma^{\mu}(-k;q,-p)\, u(p)$$

$$\left[V(x_t)\, V^{\dagger}(y_t) - 1\right]\; \underbrace{V(y_t)}_{\text{extra: unitary matrix}}$$

same as DIS

cross section

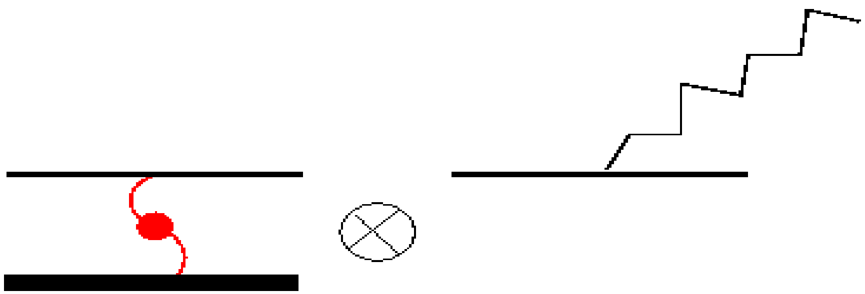
$$\frac{d\sigma}{dz\, d^2k_t\, d\log M^2\, d^2b_t} = \frac{2\alpha_{em}^2}{3\pi}\int \frac{d^2l_t}{(2\pi)^4}\, d^2r_t e^{il_t\cdot r_t}\, T(x_g, b_t, r_t)\Bigg\{ \left[\frac{1+(1-z)^2}{z}\right] \frac{z^2\, l_t^2}{[k_t^2+(1-z)M^2][(k_t-zl_t)^2+(1-z)M^2]}$$

$$- \quad z(1-z)M^2\left[\frac{1}{[k_t^2+(1-z)M^2]} - \frac{1}{[(k_t-zl_t)^2+(1-z)M^2]}\right]^2 \Bigg\}$$

Gelis,Jalilian-Marian
PRD66 (2002) 094014

pQCD limit (real photon)

near side: collinear divergence $\theta \rightarrow 0$

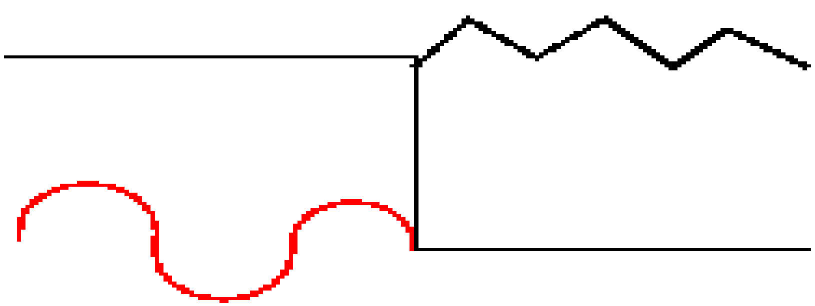


$$\sigma_{\mathbf{dipole}} \otimes \mathbf{D}_{\gamma/\mathbf{q}}$$

$(\mathbf{1}-\mathbf{z})\,\mathrm{M}^{\mathbf{2}}$ screens the divergence

away side: $\theta \rightarrow \pi$

Baier-Mueller-Schiff, NPA741 (2004) 358



$$\mathrm{p_t} >> \mathrm{Q_s}$$

DY at small x: Lam-Tung relation

$$\mathbf{d\sigma_{DY} = \frac{1}{(2\pi)^4}\frac{\alpha_{em}^2}{M^2}\frac{d^2k_{1t}\,dk_1^-}{k_1^-}\frac{d^2k_{2t}\,dk_2^-}{k_2^-}e_q^2\,L_{\mu\nu}\,W^{\mu\nu}}$$

$$\mathbf{L_{\mu\nu} = g_{\mu\nu} - \frac{k_1^\mu k_2^\nu + k_1^\nu k_2^\mu}{k_1\cdot k_2}}$$

leptonic tensor

$$\mathbf{W^{\mu\nu} = \int d^2b_t\,d^2r_t\,d^2q_t\,e^{i(q_t+k_t)\cdot r_t}\,\Gamma(p;q,k)\,T(x_g,r_t,b_t)}$$

hadronic tensor

projectors

$$\begin{aligned} P_{TL}^{\mu\nu} &\equiv -g^{\mu\nu} \\ P_{L12}^{\mu\nu} &\equiv \frac{P_1^\mu P_2^\nu + P_1^\nu P_2^\mu}{E_1E_2} \\ P_{L1}^{\mu\nu} &\equiv \frac{P_1^\mu P_1^\nu}{E_1^2} \\ P_{L2}^{\mu\nu} &\equiv \frac{P_2^\mu P_2^\nu}{E_2^2} \end{aligned}$$

dilepton rest frame

$$\begin{aligned} T_{TL} &\equiv P_{TL}^{\mu\nu}\,W_{\mu\nu} \\ T_{L12} &\equiv P_{L12}^{\mu\nu}\,W_{\mu\nu} \\ T_{L1} &\equiv P_{L1}^{\mu\nu}\,W_{\mu\nu} \\ T_{L2} &\equiv P_{L2}^{\mu\nu}\,W_{\mu\nu} \end{aligned}$$

$$R_{CS} \equiv \frac{1}{2}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{2\cos^2\delta} & \frac{1}{2\cos^2\delta} & -\frac{1}{2\cos^2\delta} \\ 0 & -\frac{1}{\sin 2\delta} & \frac{1}{\sin 2\delta} & 0 \\ 1 & \frac{1+\cos^2\delta}{\sin^2 2\delta} & -\frac{1+\cos^2\delta}{\sin^2 2\delta} & \frac{1-3\cos^2\delta}{\sin^2 2\delta} \end{bmatrix}$$

$$\begin{aligned} cos^2\delta &= \frac{M^2}{M^2+k_t^2} \\ sin^2\delta &= \frac{k_t^2}{M^2+k_t^2} \end{aligned}$$

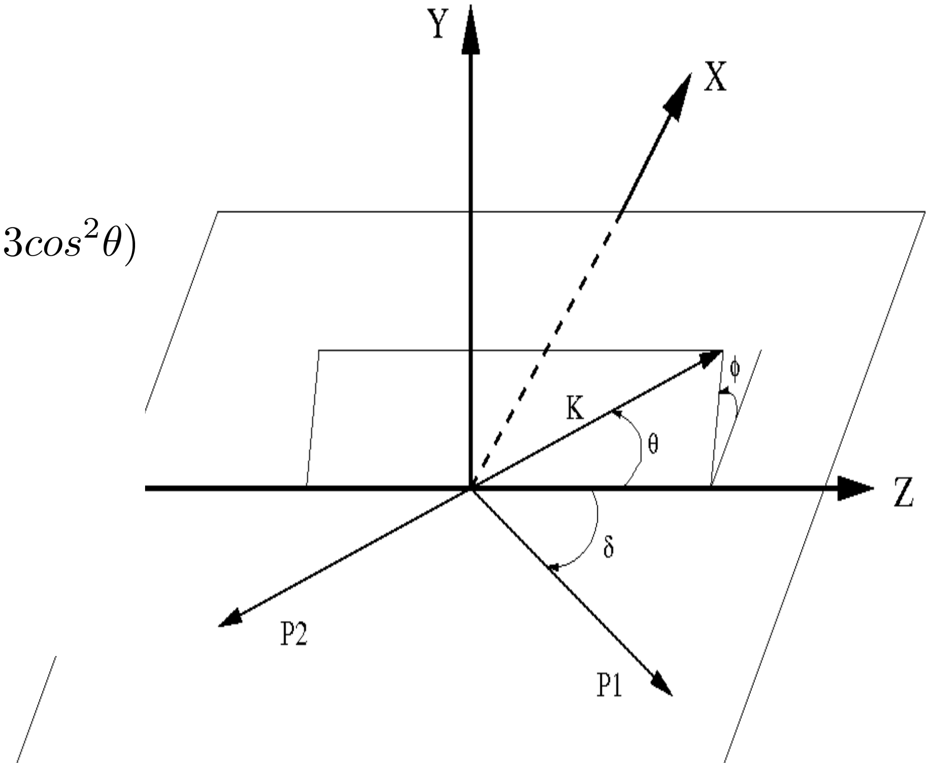
Collins-Soper structure functions

$$\begin{bmatrix} W_{TL} \\ W_L \\ W_\Delta \\ W_{\Delta\Delta} \end{bmatrix} \equiv R_{CS}\begin{bmatrix} T_{TL} \\ T_{L1} \\ T_{L2} \\ T_{L12} \end{bmatrix}$$

Gelis, Jalilian-Marian
PRD76 (2007) 074015

Lam-Tung relation

$$\frac{\mathbf{dN}}{\mathbf{d\Omega}} \equiv \left[\frac{\mathbf{d}\sigma}{\mathbf{d^4k}}\right]^{-1} \frac{\mathbf{d}\sigma}{\mathbf{d\Omega d^4k}}$$



$$\begin{aligned}\frac{dN}{d\Omega} &= \frac{3}{16\pi\, W_{TL}}\Big[W_{TL}(1+cos^2\theta)+\frac{1}{2}W_L(1-3cos^2\theta) \\ &+ W_\Delta\, sin2\theta\, cos\phi + W_{\Delta\Delta} sin^2\theta\, cos2\phi\Big] \\ &= \frac{3}{16\pi}\Big[1+cos^2\theta+\frac{1}{2}\,A_0\,(1-3cos^2\theta) \\ &+ A_1\, sin2\theta\, cos\phi + \frac{1}{2}\,A_2\, sin^2\theta\, cos2\phi\Big]\end{aligned}$$

with

$$A_0 = \frac{W_L}{W_{TL}}$$

$$A_1 = \frac{W_\Delta}{W_{TL}}$$

$$A_2 = \frac{2W_{\Delta\Delta}}{W_{TL}}$$

Lam-Tung relation

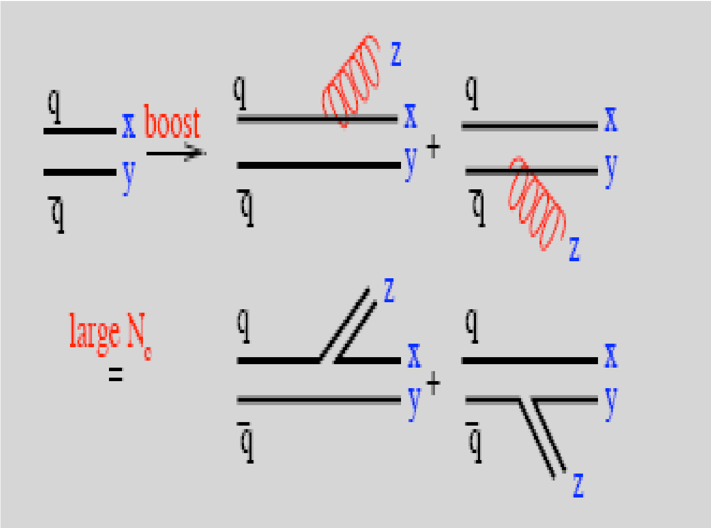
$$\boxed{\begin{aligned} W_{\Delta\Delta} &= \frac{1}{2} W_L \\ A_0 &= A_2 \end{aligned}}$$

W's (or A's) are given in terms of <u>dipole cross section</u> $T(x_g, b_t, r_t)$

Lam-Tung relation is satisfied in the dilute (BFKL) region but *violated in the dense region*

Evolution of a dipole (2-point function): BK

$$\frac{d}{dy} < \mathrm{Tr} V_x^\dagger V_y >= -\frac{\bar{\alpha}_s}{2\pi} \int d^2z \, \frac{(x-y)^2}{(x-z)^2 (y-z)^2} \times$$
$$\left[< \mathrm{Tr} V_x^\dagger V_y > - \frac{1}{N_c} < \mathrm{Tr} V_x^\dagger V_z \, \mathrm{Tr} V_z^\dagger V_y > \right]$$



$$\frac{d}{dy} S_4(r, \bar{r} : s) \simeq \frac{d}{dy} [S_2(s - \bar{r}) \, S_2(r - s)] + O(\frac{1}{N_c^2})$$

DIS F2,FL
DY in pA are sensitive
to dipoles only

Dijet production probes quadrupoles

A quadrupole ***is not*** *the same as dipole X dipole*
AD-JJM, 2011

NLO BK:
B-KW-G-BC (2007-2008)

Evolution of the 2-point function

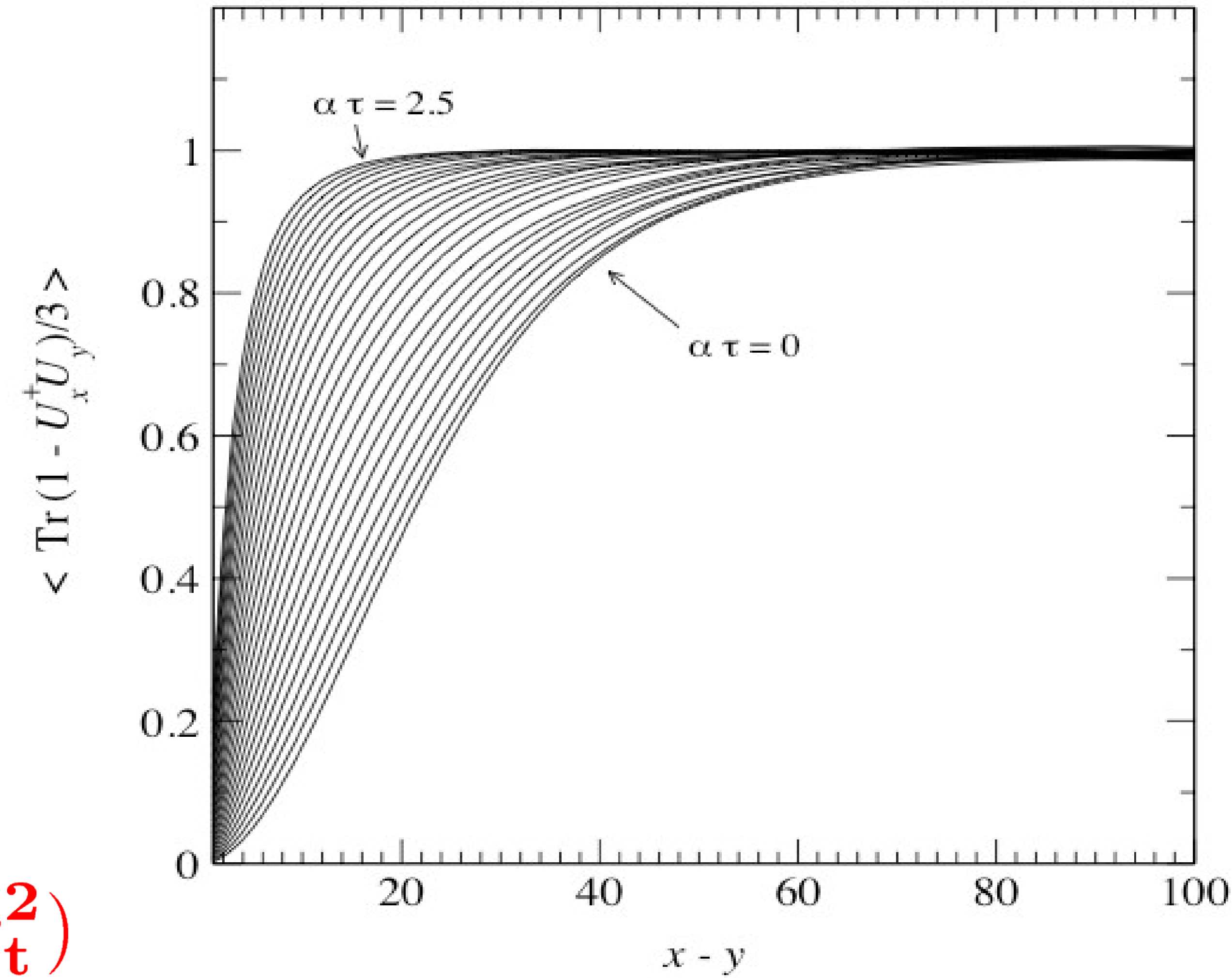
saturation region

dilute region

$$\sim [\mathbf{r_t^2\, Q_s^2}]^\gamma$$

pQCD region

$$\sim \mathbf{r_t^2\, xG(x, 1/r_t^2)}$$



RW, NPA739 (2004) 183

Models of the dipole cross section

$$N(x,r) = 1 - \exp\{-\frac{1}{4}(\frac{C_F}{N_c} r^2 Q_s^2)^{\gamma}\}$$

Kharzeev, Kovchegov, Tuchin (2004) KKT

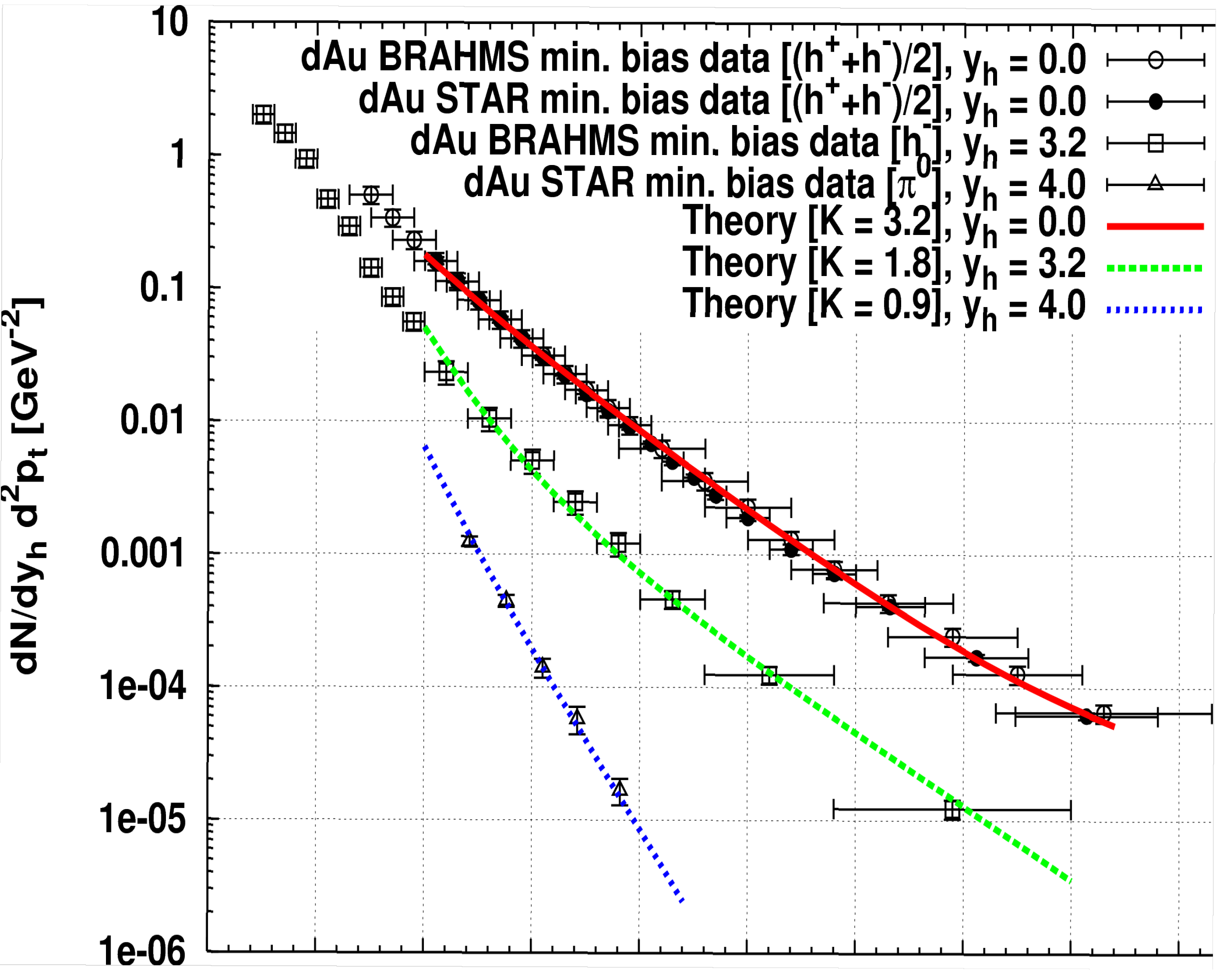
$$\gamma(Y,r) = \frac{1}{2}\left(1 + \frac{\xi(Y,r)}{\xi(Y,r) + \sqrt{2\xi(Y,r)} + 7\zeta(3)c}\right) \qquad \xi = \frac{\ln[1/(r^2 Q_{s0}^2)]}{(\lambda/2)(Y - Y_0)}$$

Dumitru, Hayashigaki, Jalilian-Marian (2006) DHJ

$$\gamma(Y,r) = \gamma_s + (1-\gamma_s)\frac{|\log\frac{1}{r^2 Q_s^2}|}{\lambda Y + |\log\frac{1}{r^2 Q_s^2}| + d\sqrt{Y}}$$

Also, IIM, BUW, RS,........ Earlier models: GBW, BGBK (γ =1)

Single inclusive hadrons in dA



DHJ:NPA770 (2006)

spectrum at y=4 was a true prediction

also
BUW:PRD77 (2008)

Running coupling BK

- Output: Modified evolution kernel:

⇒ Leading order: $$\frac{\partial S(\underline{x},\underline{y};Y)}{\partial Y} = \int d^2z\, K^{LO}(\underline{r},\underline{r}_1,\underline{r}_2)\, \left[S(\underline{x},\underline{z})\, S(\underline{z},\underline{y}) - S(\underline{x},\underline{y})\right]$$

↓

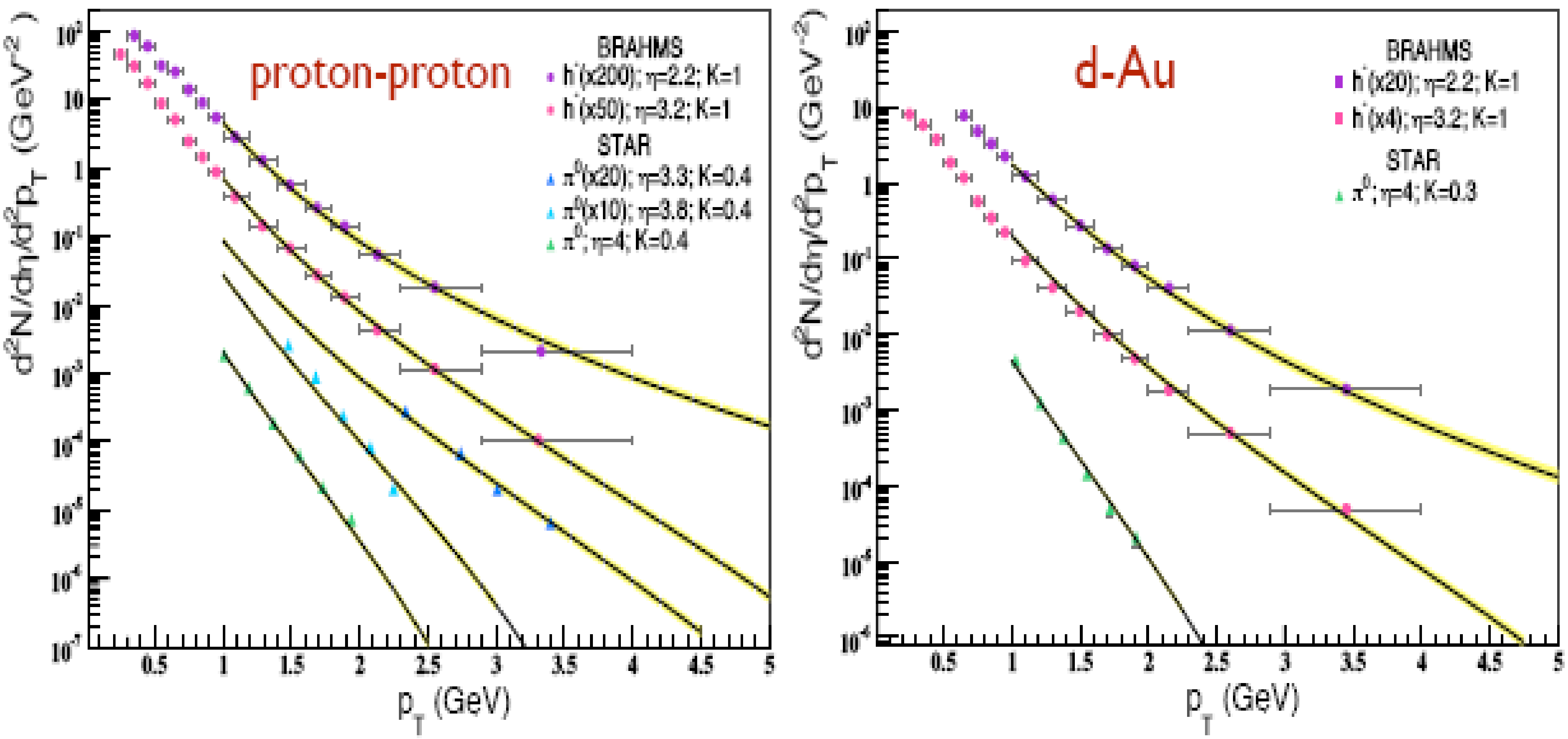
⇒ Running coupling: $$\frac{\partial S(\underline{x},\underline{y};Y)}{\partial Y} = \int d^2z\, \tilde{K}(\underline{r},\underline{r}_1,\underline{r}_2)\, \left[S(\underline{x},\underline{z})\, S(\underline{z},\underline{y}) - S(\underline{x},\underline{y})\right]$$

$$\tilde{K}_{Bal}(\underline{r},\underline{r}_1,\underline{r}_2) = \frac{N_c\,\alpha_s(r^2)}{2\pi^2}\left[\frac{r^2}{r_1^2\, r_2^2} + \frac{1}{r_1^2}\left(\frac{\alpha_s(r_1^2)}{\alpha_s(r_2^2)} - 1\right) + \frac{1}{r_2^2}\left(\frac{\alpha_s(r_2^2)}{\alpha_s(r_1^2)} - 1\right)\right]$$

numerical solution, Albacete-Kovchegov PRD 75 (2007) 125021, ...

beware of <u>initial conditions</u> !

dA at RHIC



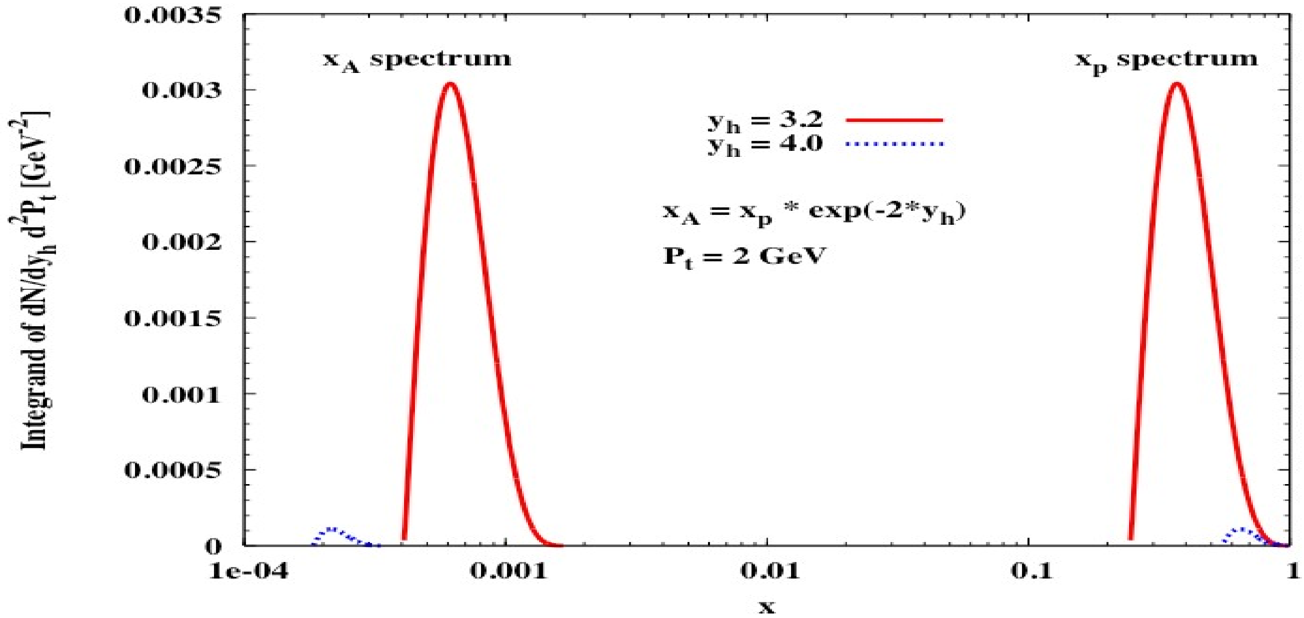
Solution to the running coupling BK equation

J. Albacete + C. Marquet

PLB687 (2010) 174

dA at RHIC

Kinematics:
projectile x
is very large



how important
is cold matter
energy loss?

Kopeliovich, Frankfurt and Strikman
Neufeld-Vitev-Zhang

how about centrality dependence?
Q_s (b_t) ala KLN is too rough:
Woods-Saxon, fluctuations

Dilepton production: k_t integrated

$$\frac{d\sigma^{d(p)\,A\to l^+ l^- X}}{d^2b_t\, dM^2\, dx_F} = \frac{\alpha_{em}^2}{6\pi^2}\frac{1}{x_q + x_g}\int_{x_q}^{1} dz \int dr_t^2\, \frac{1-z}{z^2}$$

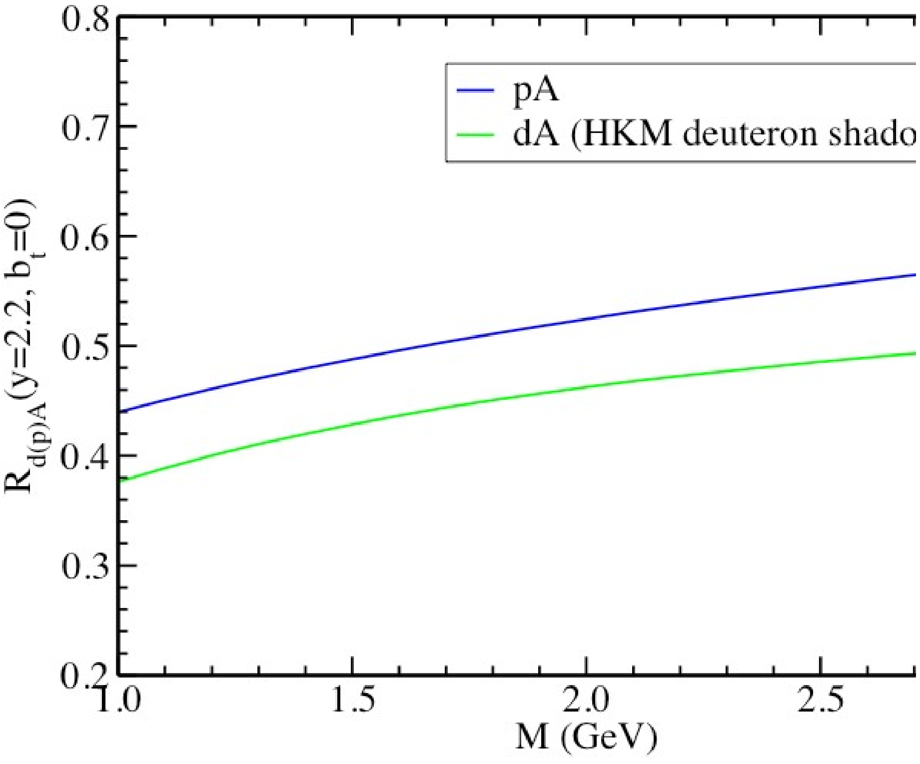
F. Gelis & JJM 02, JJM 04

$$F_2^{d(p)}(x_q/z)\,\gamma(x_g, b_t, r_t)$$

$$\Big[[1+(1-z)^2]\, K_1^2[\frac{\sqrt{1-z}}{z} M r_t]$$

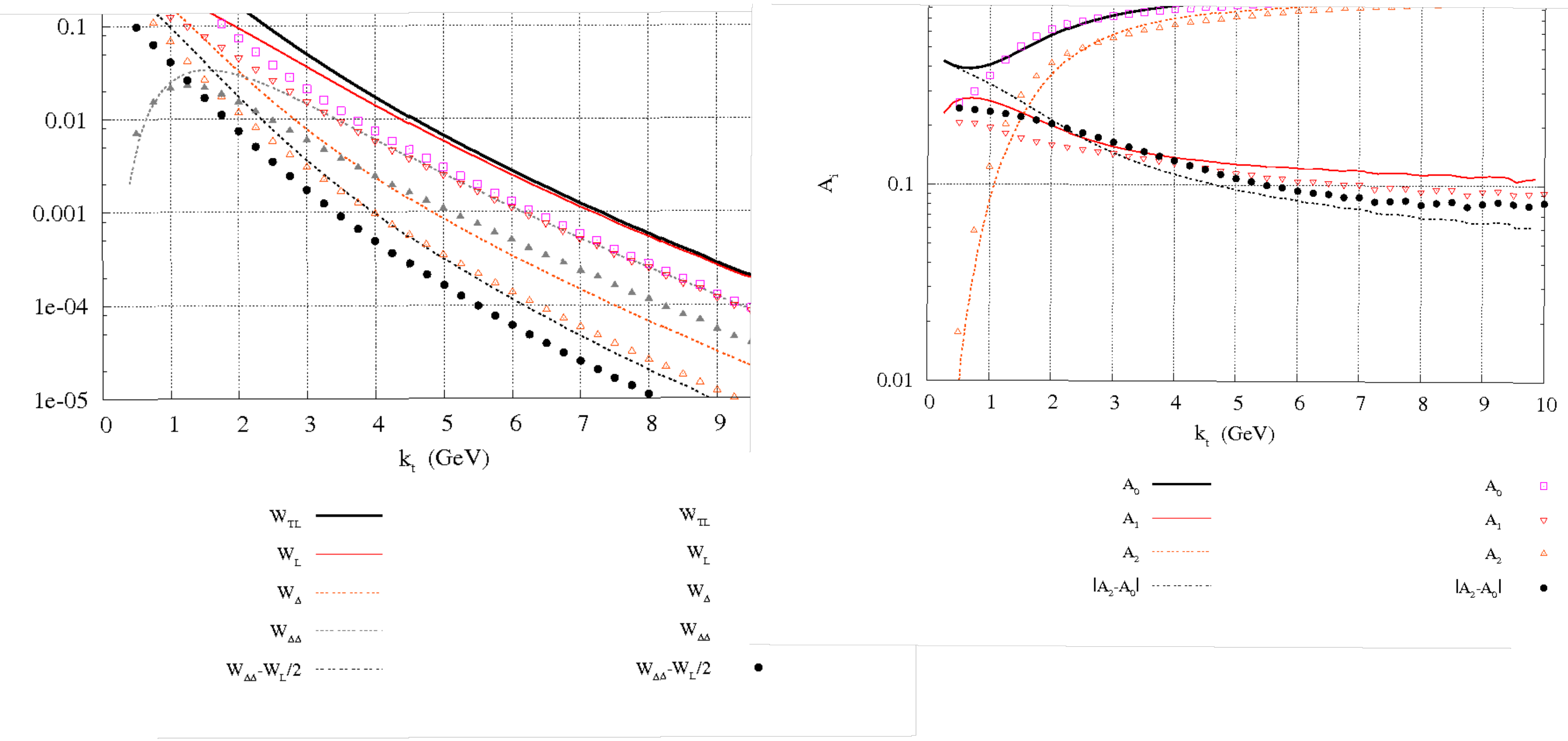
$$+2(1-z)\, K_0^2[\frac{\sqrt{1-z}}{z} M r_t] \Big]$$

$$x_F \equiv \frac{M}{\sqrt{s}}[e^y - e^{-y}]$$



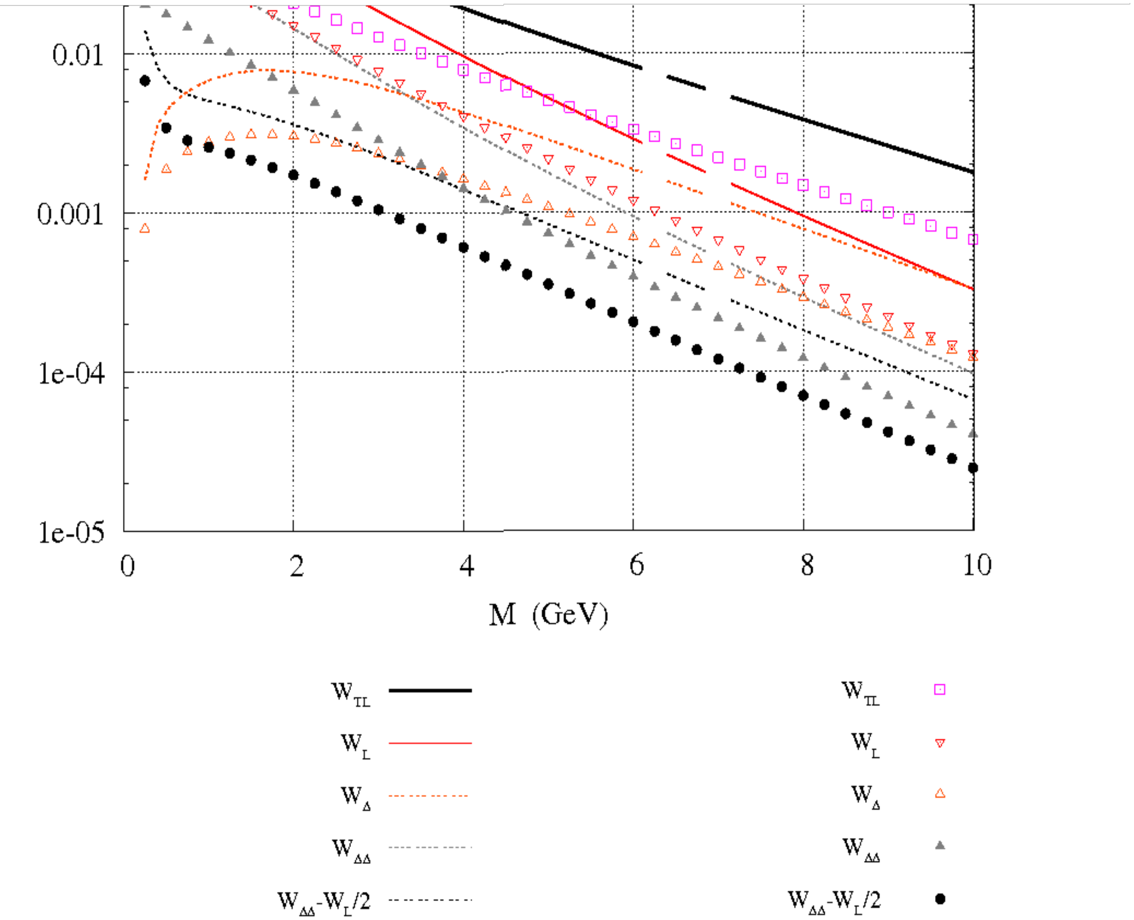
RHIC: M = 2 GeV, y = 2

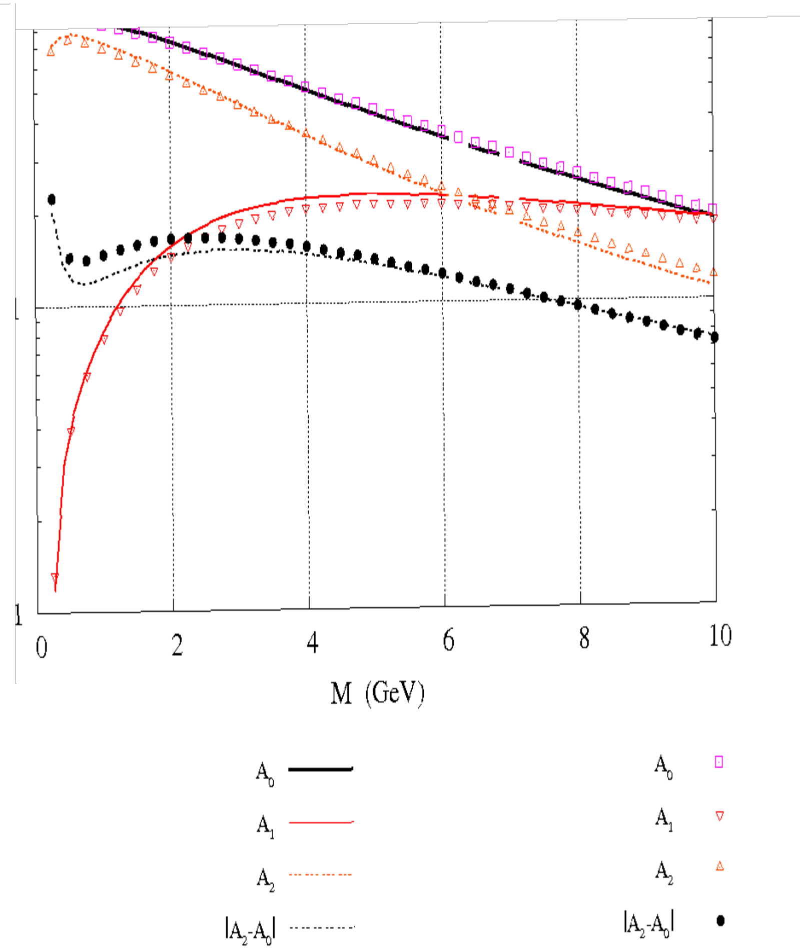
Gelis,Jalilian-Marian
PRD76 (2007) 074015



Lines: (fixed coupling) BK
Points: DHJ

RHIC: k_t = 3 GeV, y = 2



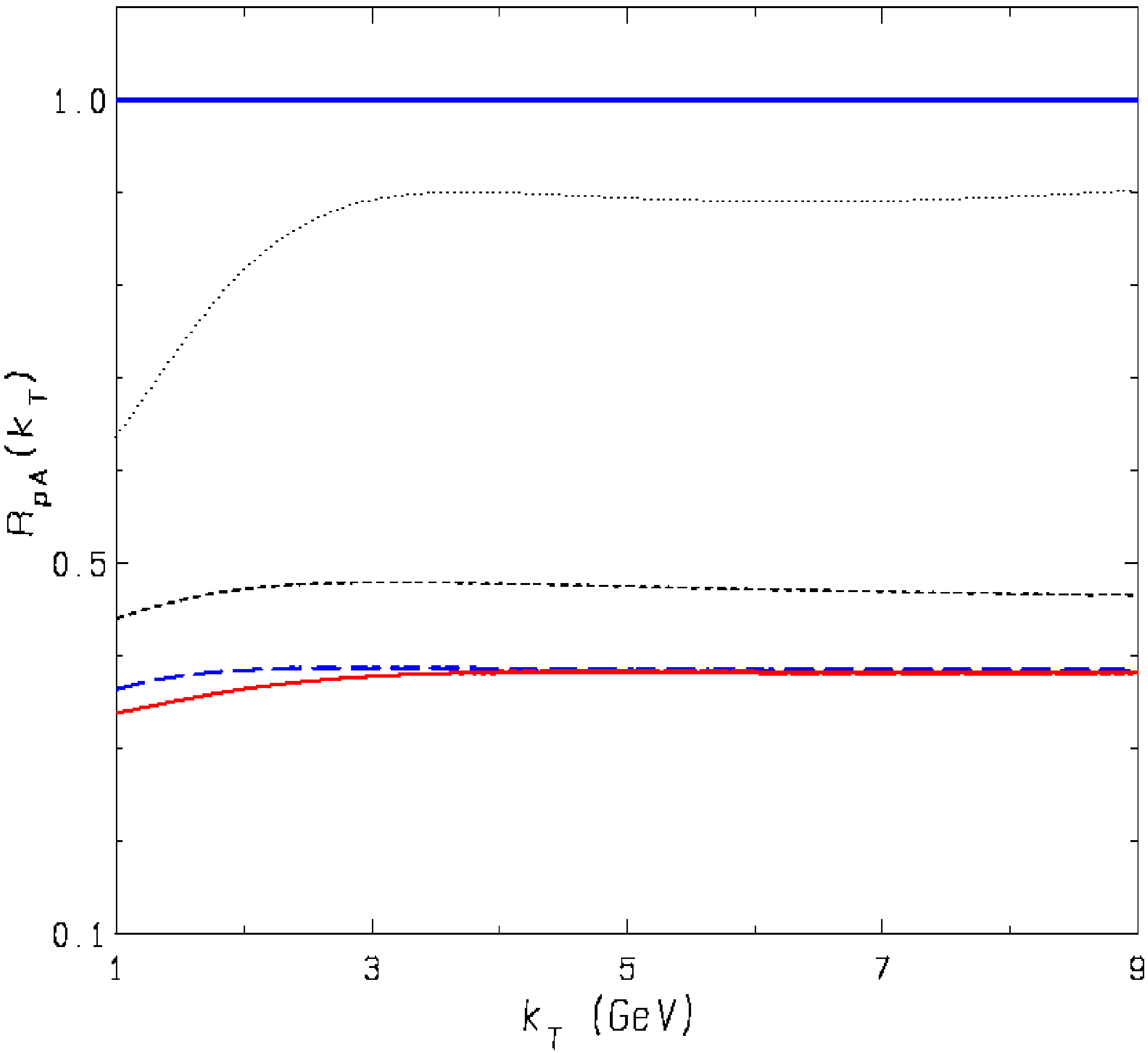


Lines: (fixed coupling) BK
Points: DHJ

Dilepton production: k_t dependence

Baier-Mueller-Schiff,
NPA741 (2004) 358

M = 2, 4 GeV
y = 0.5, 1.5, 3



The role of initial conditions

McLerran-Venugopalan (93)

$$< \mathbf{O}(\rho) > \equiv \int \mathbf{D}[\rho]\, \mathbf{O}(\rho)\, \mathbf{W}[\rho]$$

$$\mathbf{W}[\rho] \simeq \mathbf{e}^{-\int \mathbf{d^2 x_t} \frac{\rho^{\mathbf{a}}(\mathbf{x_t})\rho^{\mathbf{a}}(\mathbf{x_t})}{\mathbf{2}\,\mu^{\mathbf{2}}}} \qquad \mu^{\mathbf{2}} \equiv \frac{\mathbf{g^2 A}}{\mathbf{S}_\perp}$$

$$\mathbf{S}(\mathbf{y_t}, \mathbf{z_t}) \equiv \frac{\mathbf{1}}{\mathbf{N_c}} < \mathbf{Tr}\, \mathbf{V_y^\dagger}\, \mathbf{V_z} > \sim \mathbf{e}^{-\#\,(\mathbf{y_t}-\mathbf{z_t})^{\mathbf{2}}\, \mathbf{Q_s^2}}$$

how about higher order terms in ρ ?

$$\mathbf{W}[\rho] \simeq \mathbf{e}^{-\int \mathbf{d^2 x_t} \left[\frac{\rho^{\mathbf{a}}(\mathbf{x_t})\rho^{\mathbf{a}}(\mathbf{x_t})}{\mathbf{2}\,\mu^{\mathbf{2}}} - \frac{\mathbf{d^{abc}}\,\rho^{\mathbf{a}}(\mathbf{x_t})\rho^{\mathbf{b}}(\mathbf{x_t})\rho^{\mathbf{c}}(\mathbf{x_t})}{\kappa_{\mathbf{3}}} + \frac{\mathbf{F^{abcd}}\,\rho^{\mathbf{a}}(\mathbf{x_t})\rho^{\mathbf{b}}(\mathbf{x_t})\rho^{\mathbf{c}}(\mathbf{x_t})\rho^{\mathbf{d}}(\mathbf{x_t})}{\kappa_{\mathbf{4}}} \right]}$$

these higher order terms may make the single inclusive spectra steeper and give leading N_c correlations (ridge)

AD+JJM+EP, in progress

CGC:QCD at high gluon density

Effective action + RG

Saturation scale:
a semi-hard scale generated dynamically

Degrees of freedom: Wilson lines

Structure functions, single inclusive production probe
dipoles: rcBK

Di-jet correlations probe quadrupoles

DY in p(d)A at RHIC and LHC
cleaner probe of CGC