

Workshop on "Drell-Yan Opportunities at RHIC"
May 11th-13th, 2011 - BNL

What semi-inclusive DIS has taught us about TMDs

TMDs*)

- *) transverse-momentum-dependent (TMD) -- aka unintegrated -- PDFs and FFs
- go beyond the approximation of collinear moving partons
- importance realized, e.g., in
 - late 70s [Cahn]: transverse momentum of partons lead to cosine modulations in unpolarized semi-inclusive DIS
 - 90s [Sivers]: transverse motion of unpolarized partons in transversely polarized nucleons lead to transverse single-spin asymmetries
- complimentary to generalized parton distribution that probe transverse spatial d.o.f.

Spin-Momentum Structure of the Nucleon

$$\frac{1}{2}\text{Tr}\left[(\gamma^+ + \lambda\gamma^+\gamma_5)\Phi\right] = \frac{1}{2}\left[f_1 + S^i\epsilon^{ij}k^j\frac{1}{m}f_{1T}^\perp + \lambda\Lambda g_1 + \lambda S^i k^i\frac{1}{m}g_{1T}\right]$$

$$\frac{1}{2}\text{Tr}\left[(\gamma^+ - s^j i\sigma^{+j}\gamma_5)\Phi\right] = \frac{1}{2}\left[f_1 + S^i\epsilon^{ij}k^j\frac{1}{m}f_{1T}^\perp + s^i\epsilon^{ij}k^j\frac{1}{m}h_1^\perp + s^i S^i h_1\right. \\ \left.+ s^i(2k^i k^j - k^2\delta^{ij})S^j\frac{1}{2m^2}h_{1T}^\perp + \Lambda s^i k^i\frac{1}{m}h_{1L}^\perp\right]$$

quark pol.

nucleon pol.		U	L	T
	U	f_1		$h_1^\perp$
	L		g_{1L}	$h_{1L}^\perp$
	T	$f_{1T}^\perp$	g_{1T}	$h_1, h_{1T}^\perp$

Twist-2 TMDs

- functions in black survive integration over transverse momentum
- functions in green box are chirally odd
- functions in red are naive T-odd

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quark pol.

helicity

nucleon pol.

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_{1T}^\perp, h_1$

● functions in black survive integration
Boer-Mulders $h_1^\perp$ $h_{1T}^\perp$ h_1

● functions in green box are chirally odd

● functions in red are naive T-odd

Sivers

Twist-2 TMDs

worm-gear

transversity

Mulders-Tangerman*

*aka Pretzelosity

TMDs - Probabilistic interpretation

Proton goes out of the screen/ photon goes into the screen

  nucleon with transverse or longitudinal spin

  parton with transverse or longitudinal spin

 parton transverse momentum

$$f_1 = \text{[Diagram: Circle with a red circle inside]}$$

$$g_1 = \text{[Diagram: Circle with a black dot and a red circle inside]} - \text{[Diagram: Circle with a black dot and a red circle with a cross inside]}$$

$$h_1 = \text{[Diagram: Circle with a red circle and a red arrow pointing right]} - \text{[Diagram: Circle with a red circle and a red arrow pointing left]}$$

$$f_{1T}^\perp = \text{[Diagram: Circle with a red circle and a blue arrow pointing down]} - \text{[Diagram: Circle with a red circle and a blue arrow pointing up]}$$

$$h_1^\perp = \text{[Diagram: Circle with a red circle, a red arrow pointing right, and a blue arrow pointing down]} - \text{[Diagram: Circle with a red circle, a red arrow pointing right, and a blue arrow pointing up]}$$

$$g_{1T} = \text{[Diagram: Circle with a red circle, a red dot, and a blue arrow pointing right]} - \text{[Diagram: Circle with a red circle, a red dot, and a blue arrow pointing left]}$$

$$h_{1L}^\perp = \text{[Diagram: Circle with a black dot, a red circle, and a blue arrow pointing right]} - \text{[Diagram: Circle with a black dot, a red circle, and a blue arrow pointing left]}$$

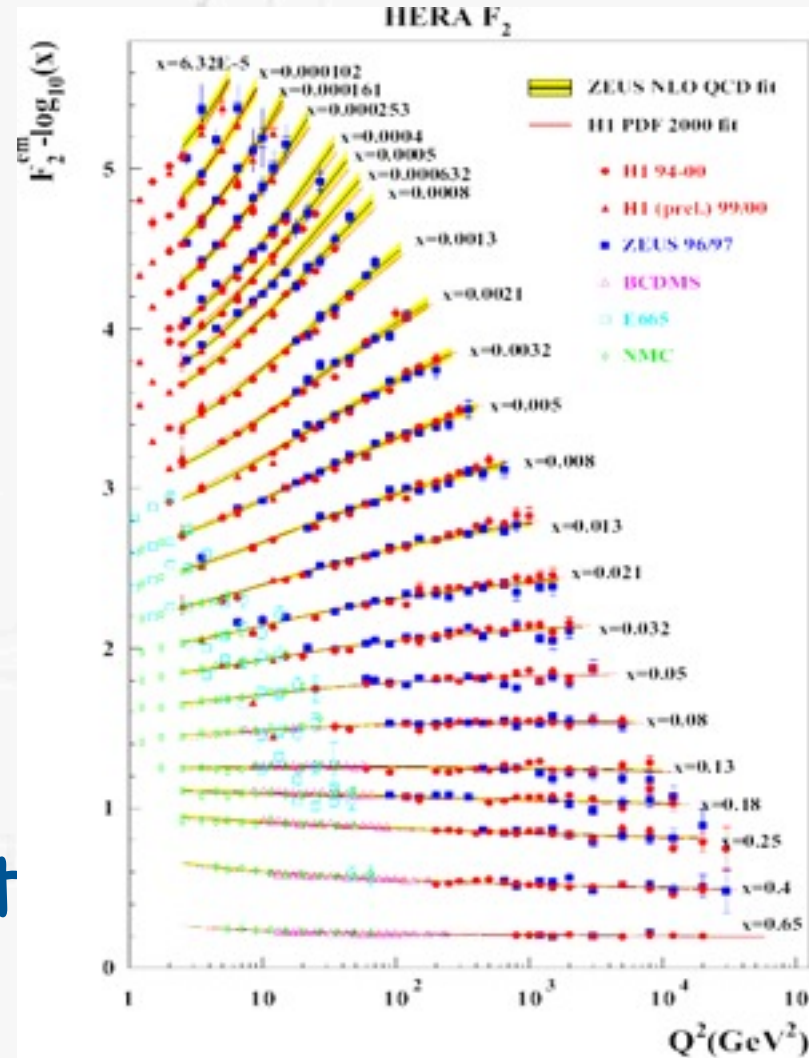
$$h_{1T}^\perp = \text{[Diagram: Circle with a red circle, a red arrow pointing right, and a blue arrow pointing right]} - \text{[Diagram: Circle with a red circle, a red arrow pointing left, and a blue arrow pointing right]}$$

[courtesy of A. Bacchetta]

Momentum density

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1, h_{1T}^\perp$

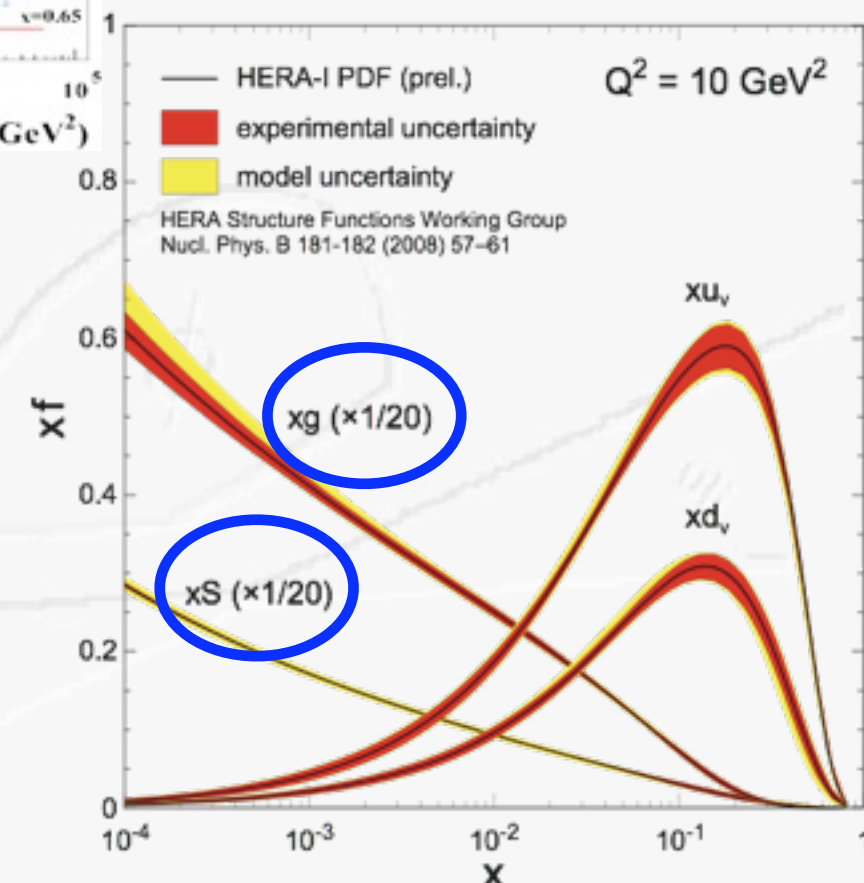
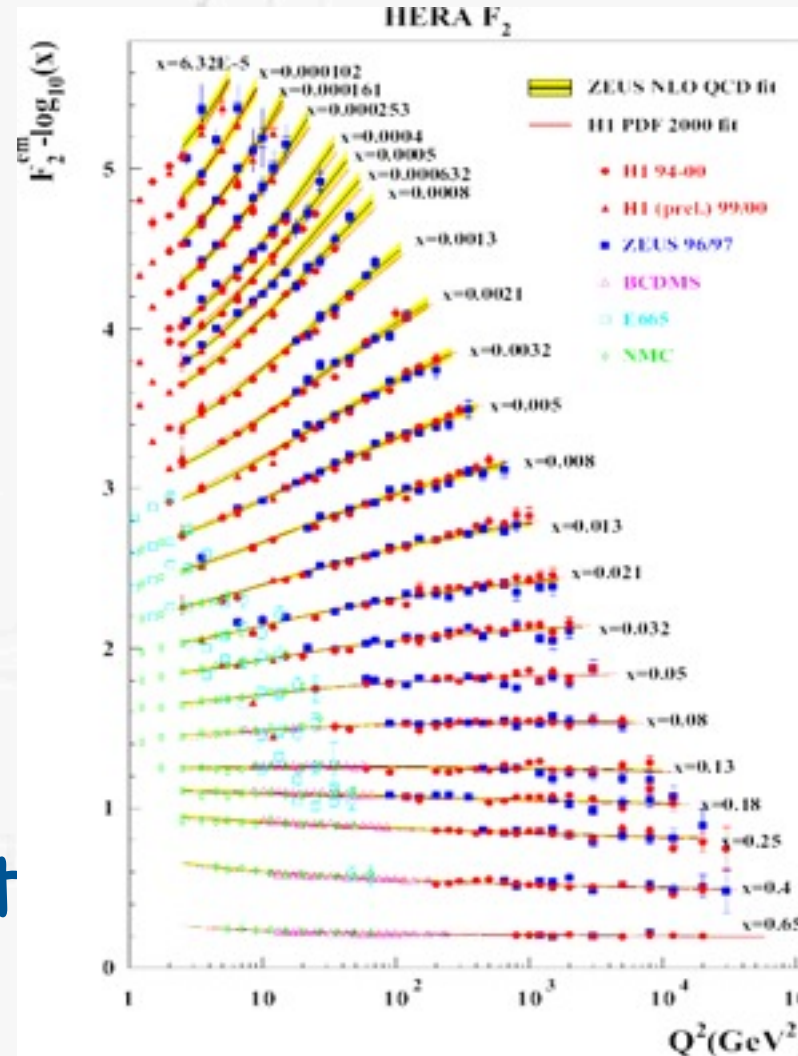
- plenty of data available
- but only for integrated version of f_1
- some efforts to get unintegrated gluon density
- spin asymmetries involve unintegrated f_1 in denominator
- need multiplicities and fragmentation functions not only binned in z but also in $P_{h\perp}$



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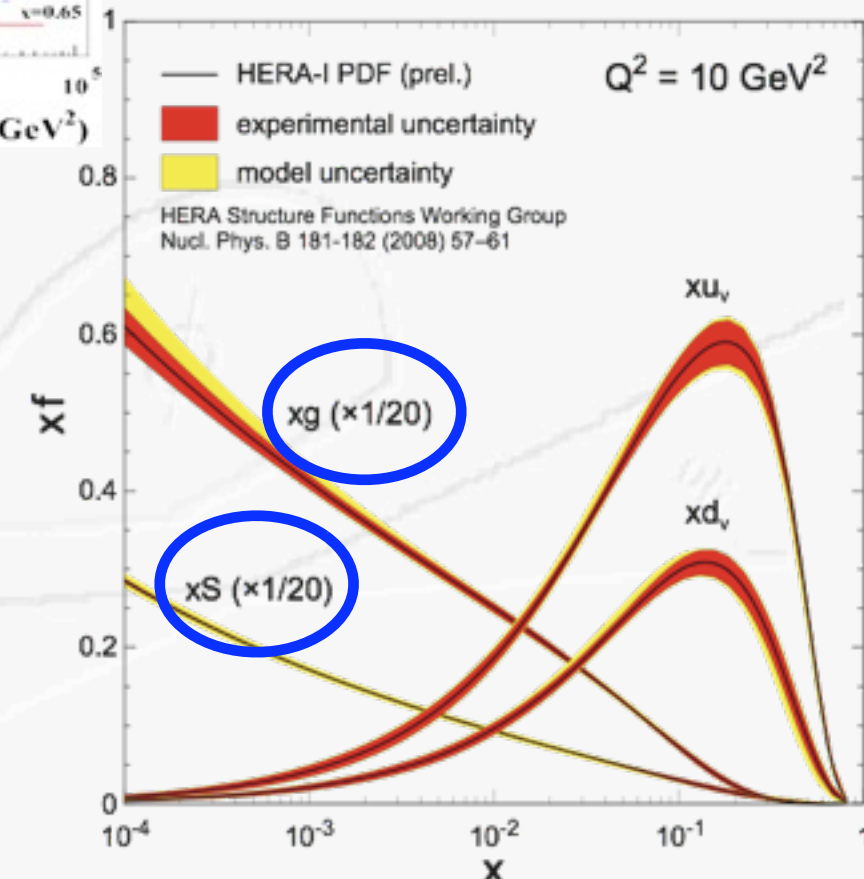
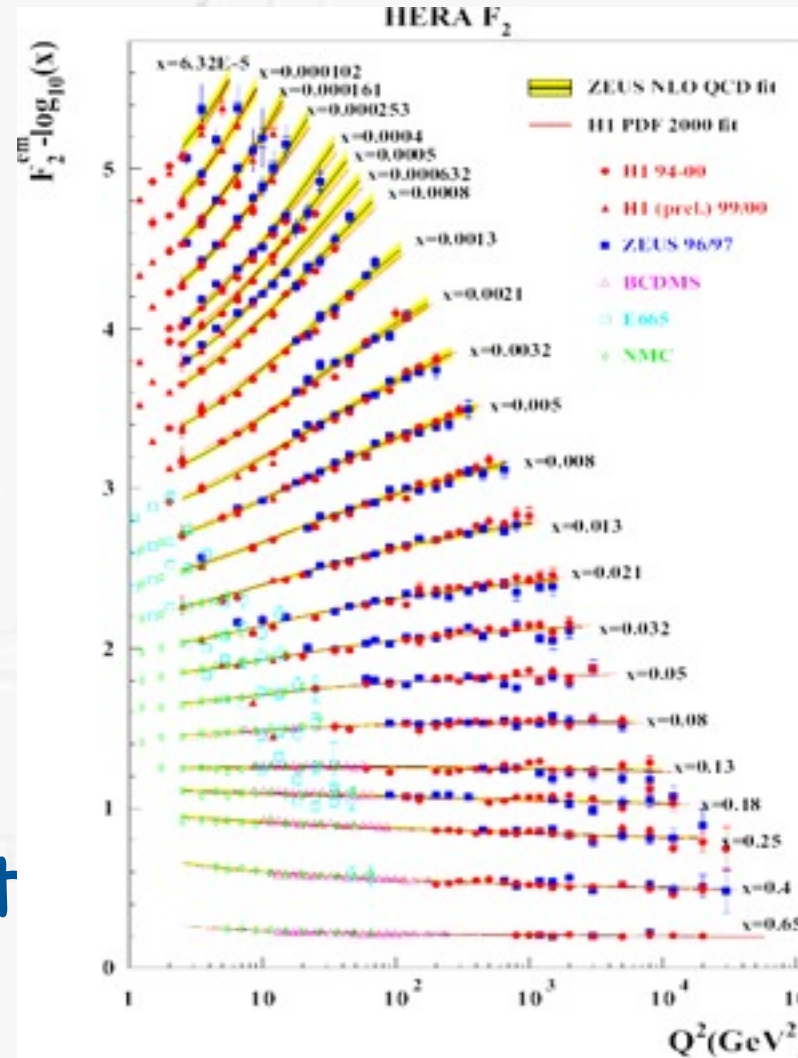
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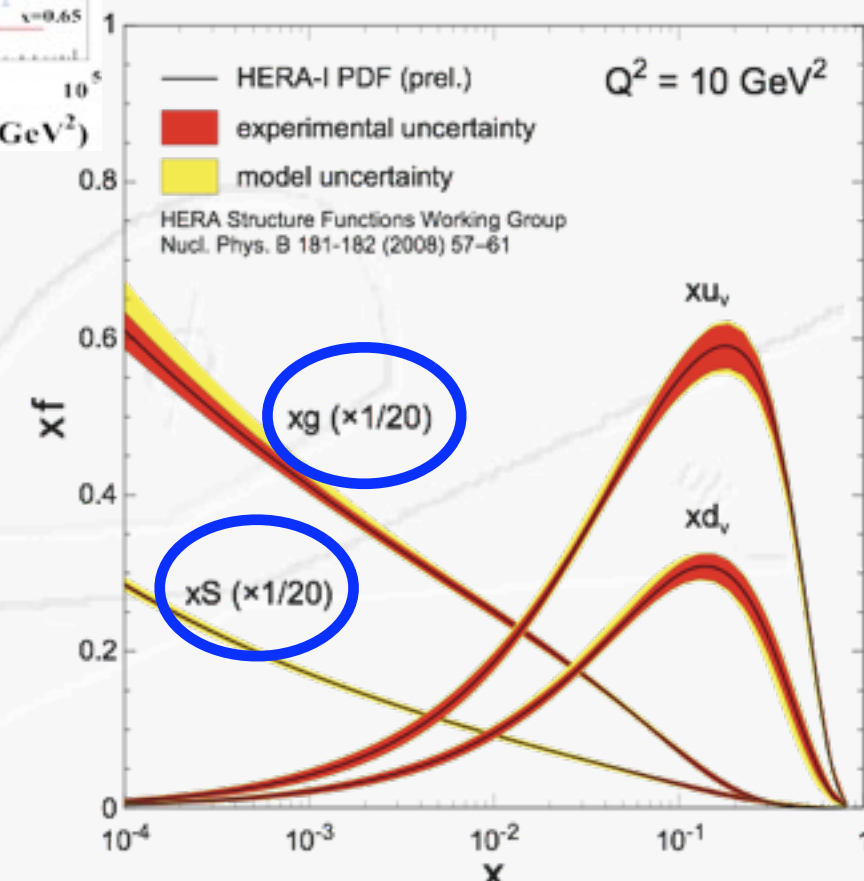
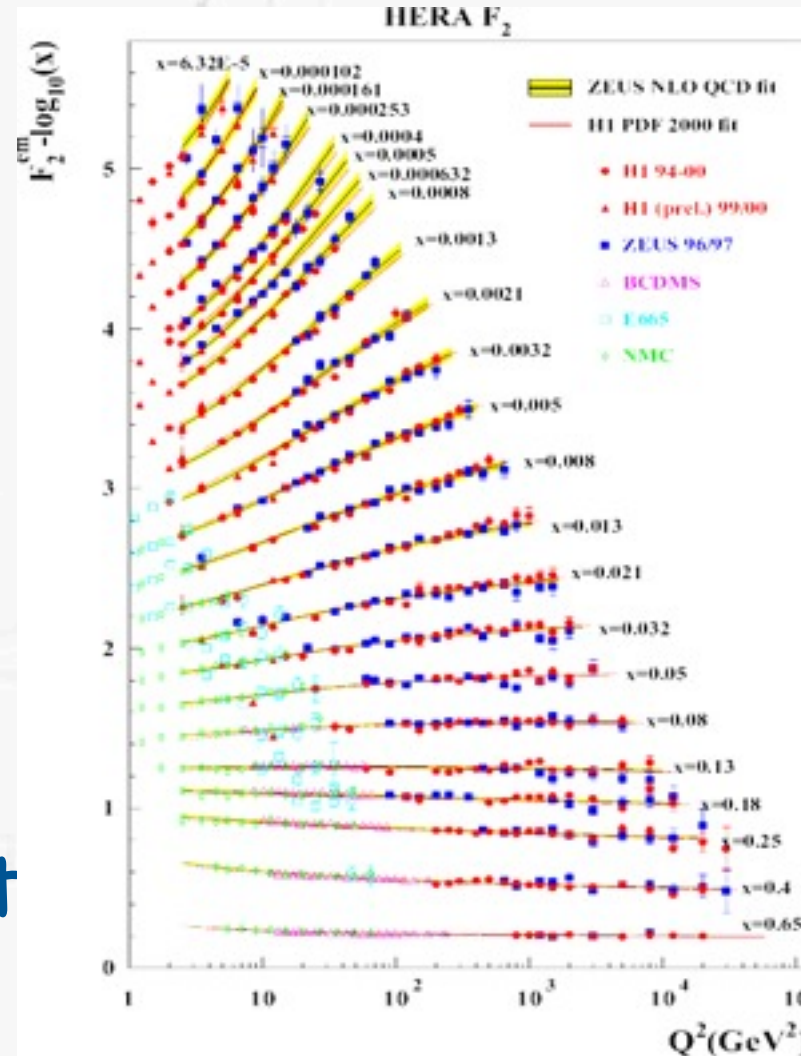
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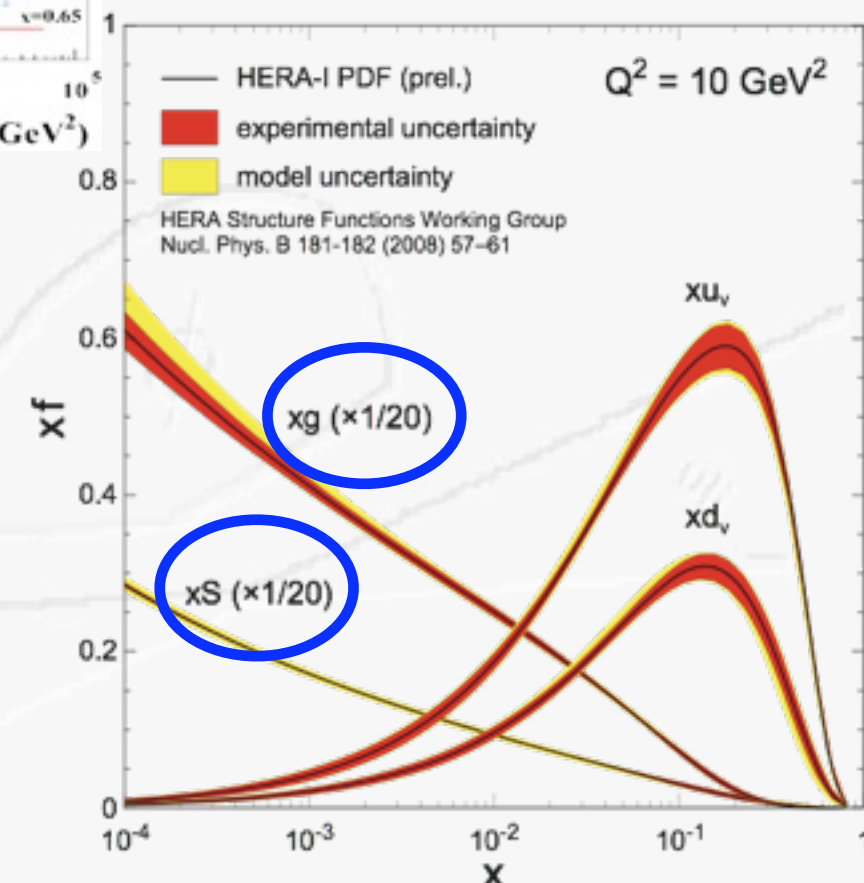
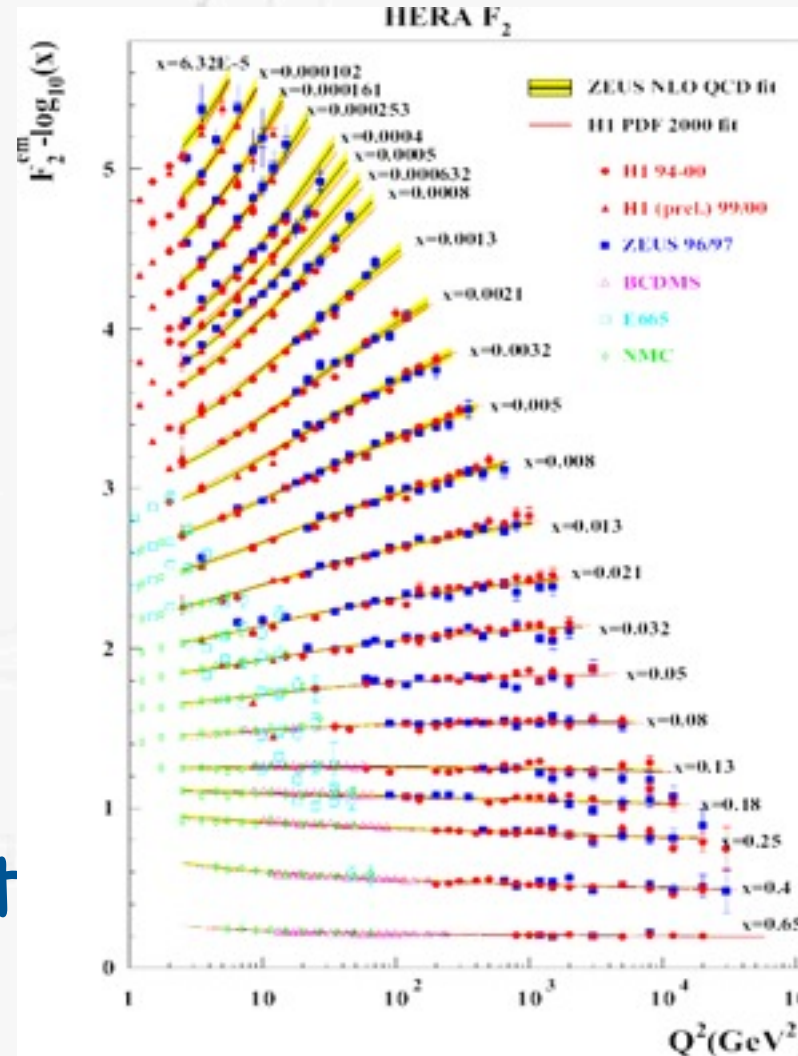
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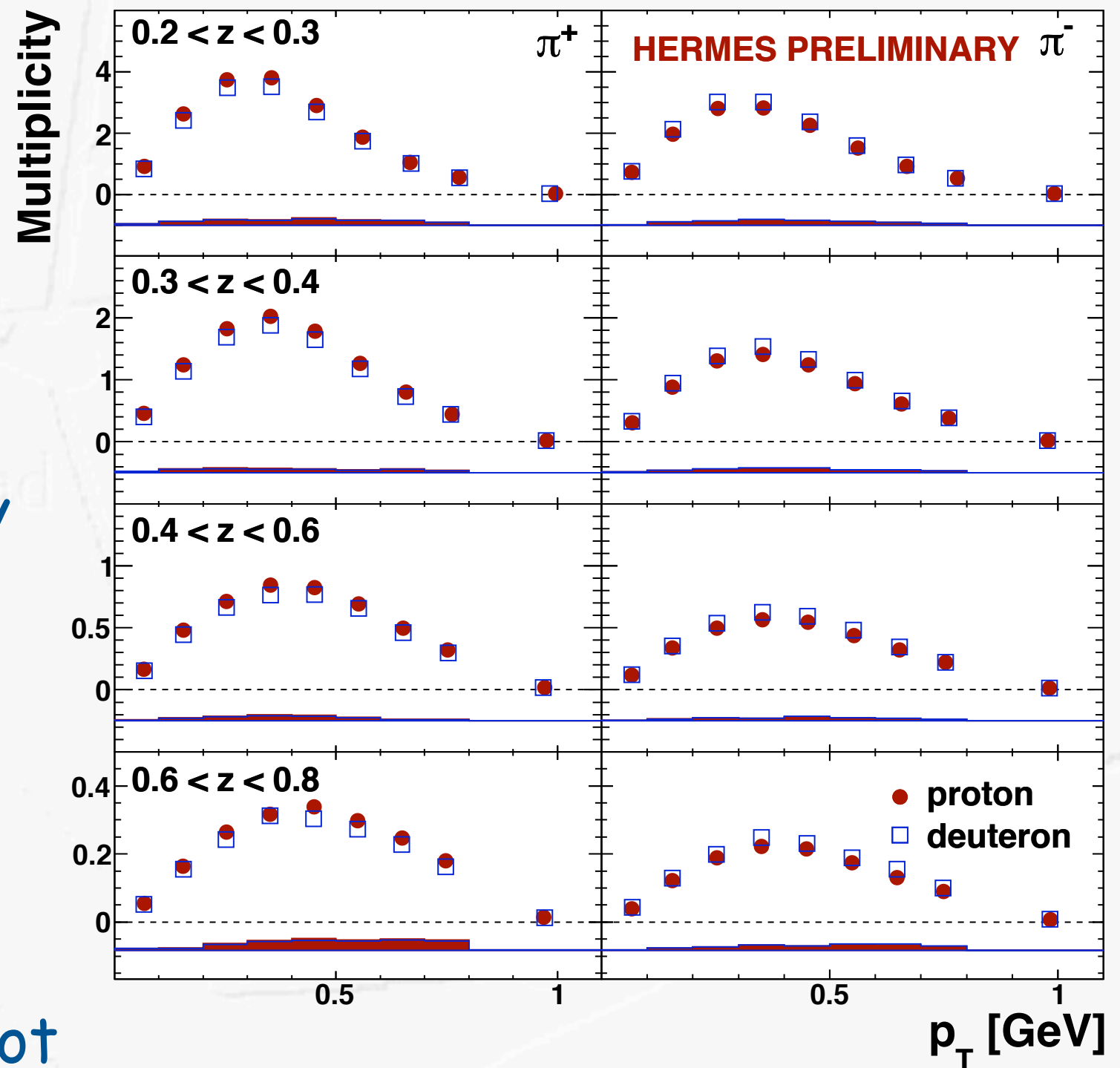
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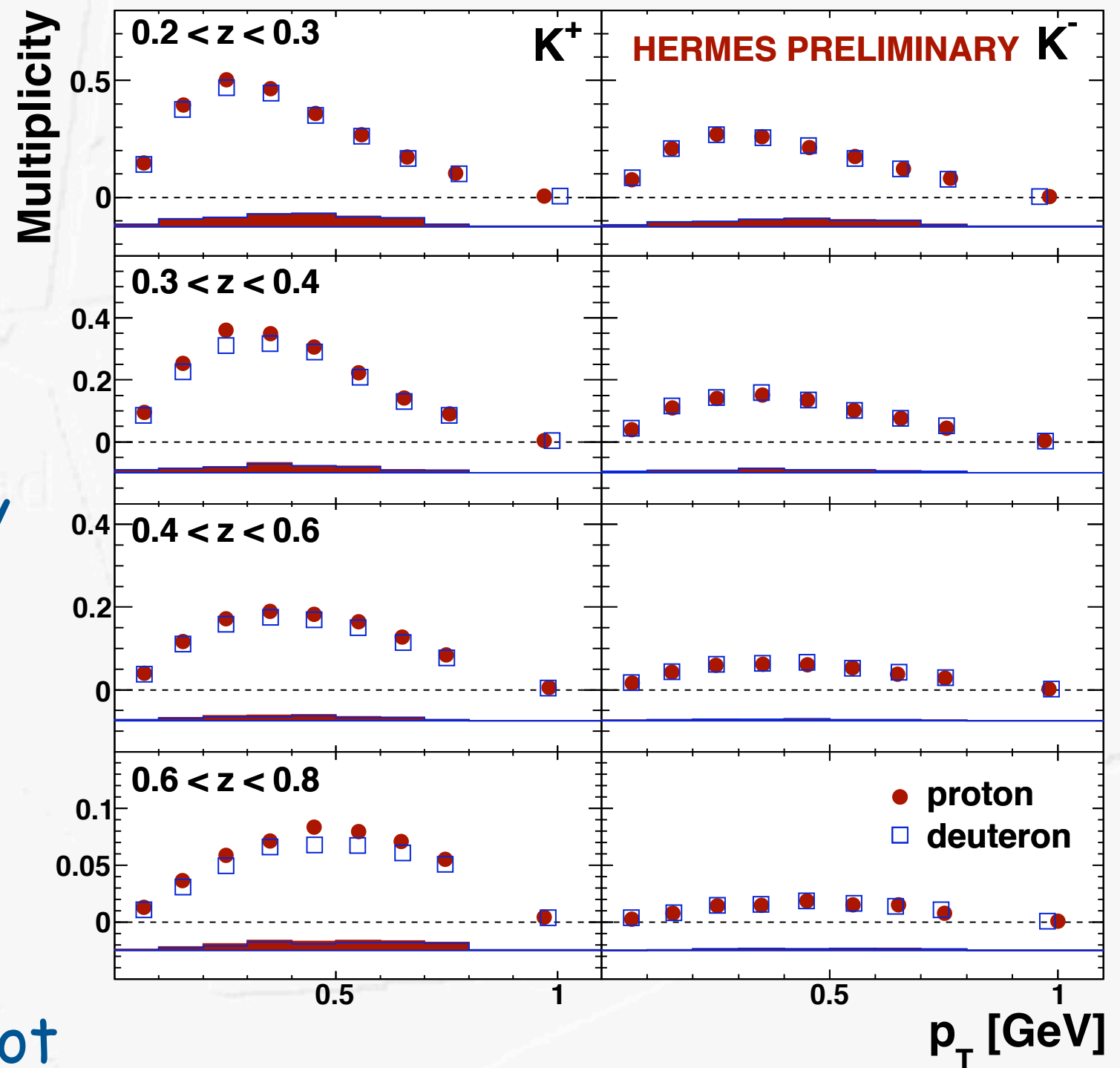


also available for kaons

Momentum density

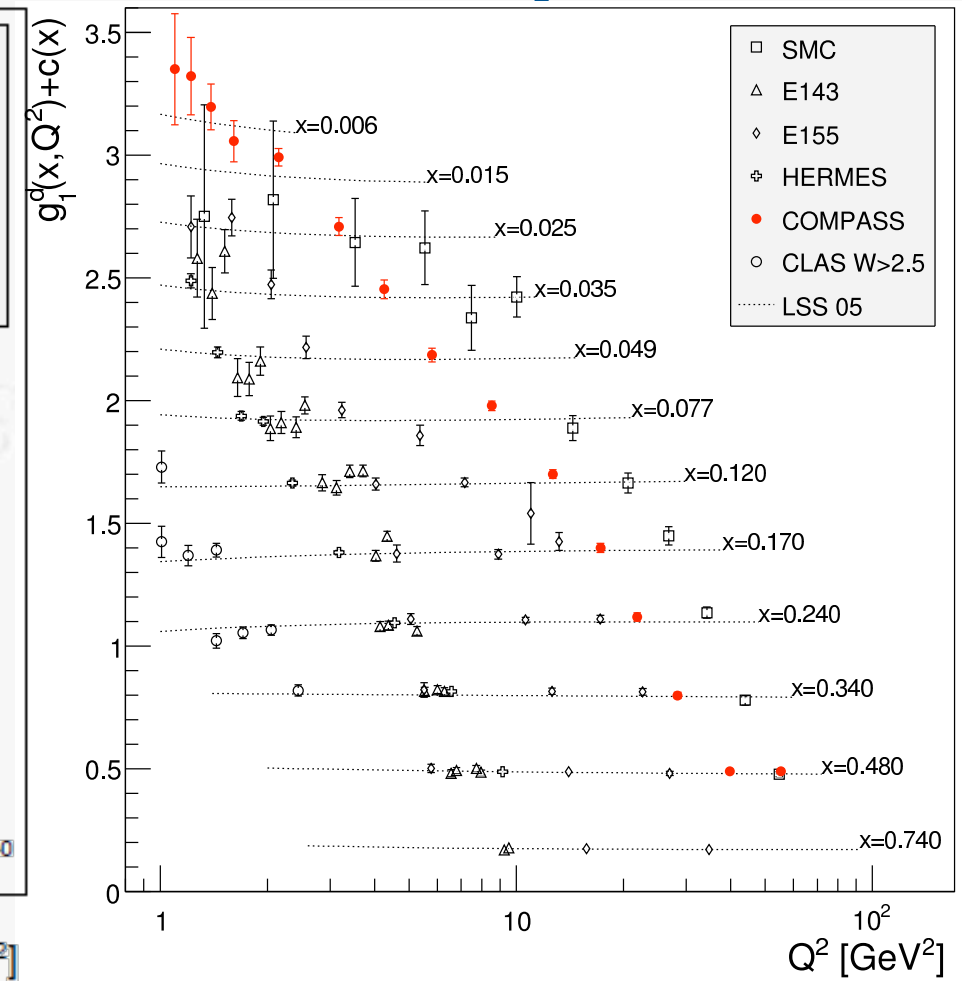
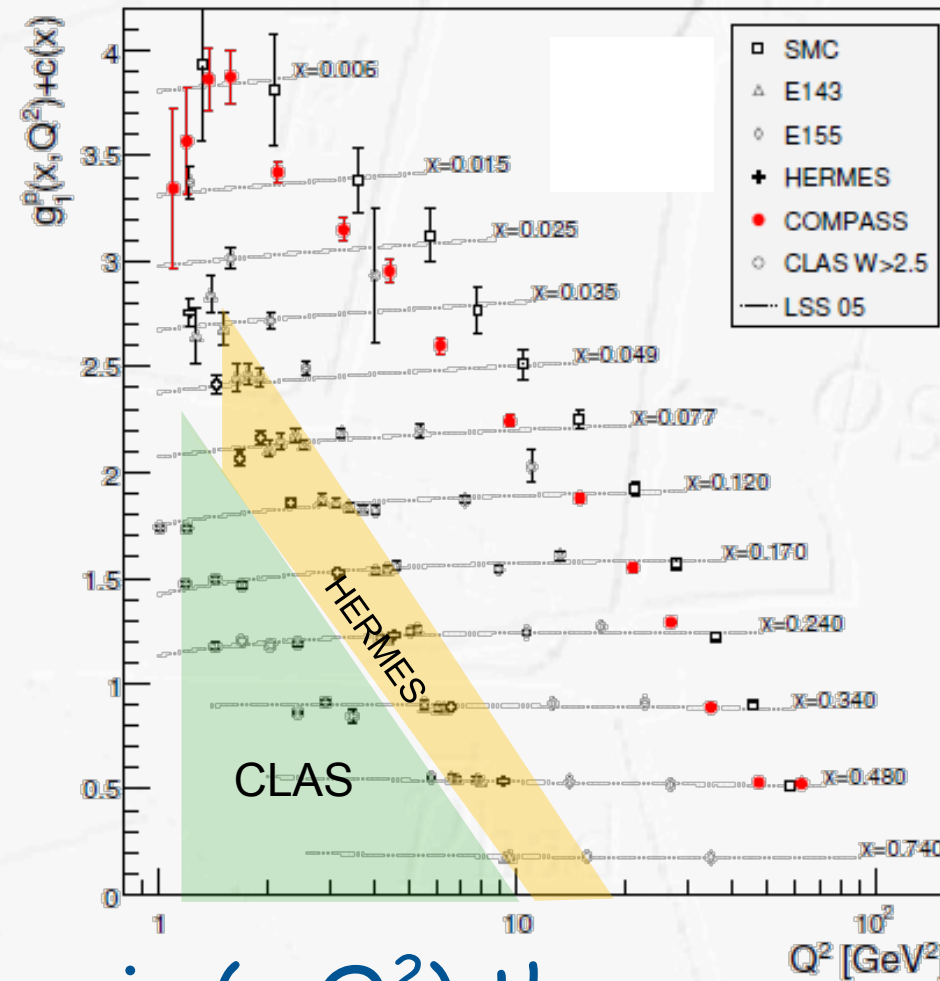
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Helicity density

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1, h_{1T}^\perp$

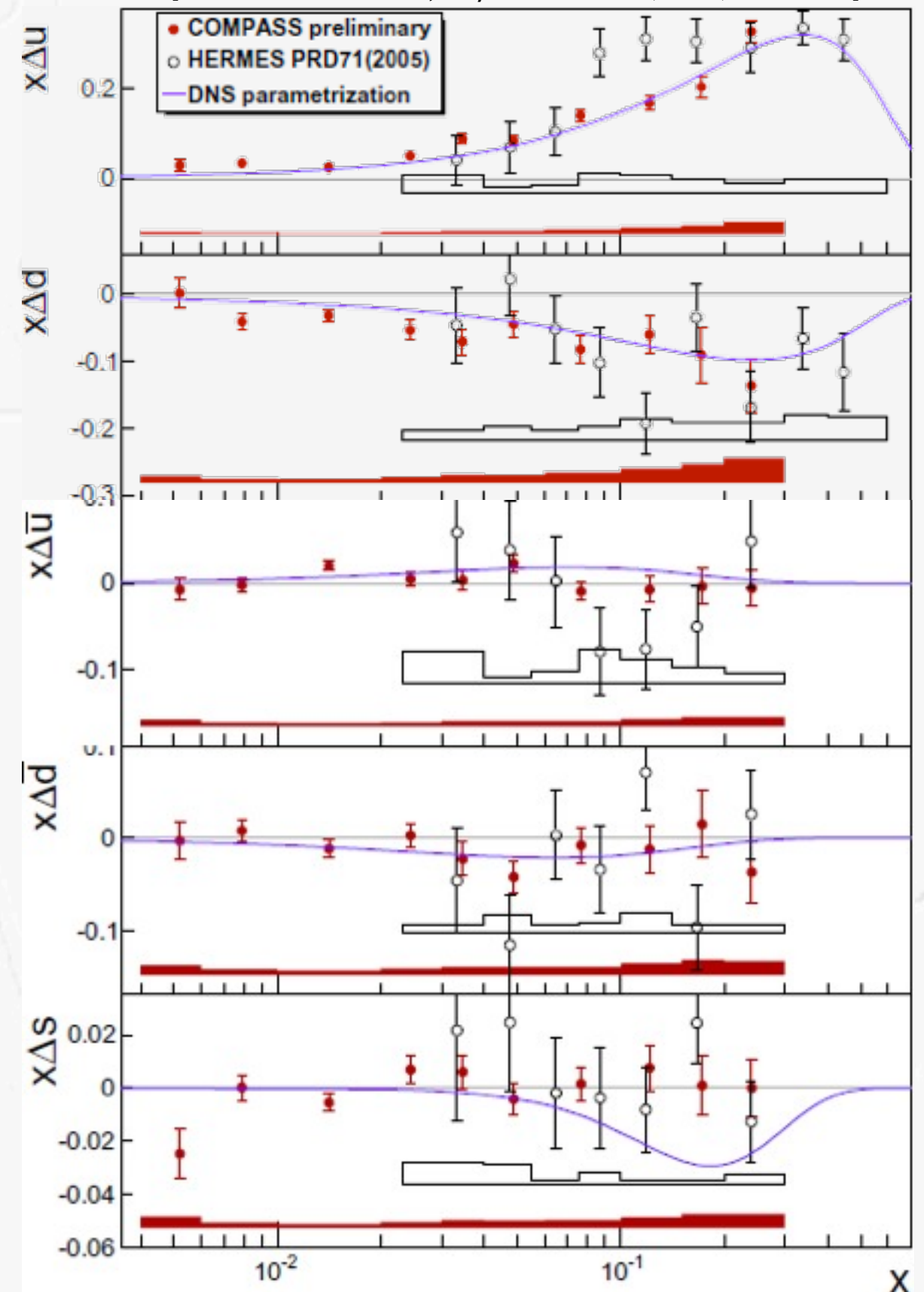


- smaller range in (x, Q^2) than for f_1
- data mainly for integrated version of g_{1L}
- need asymmetries not only binned in x but also in $P_{h\perp}$

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1, h_{1T}^\perp$

Helicity density

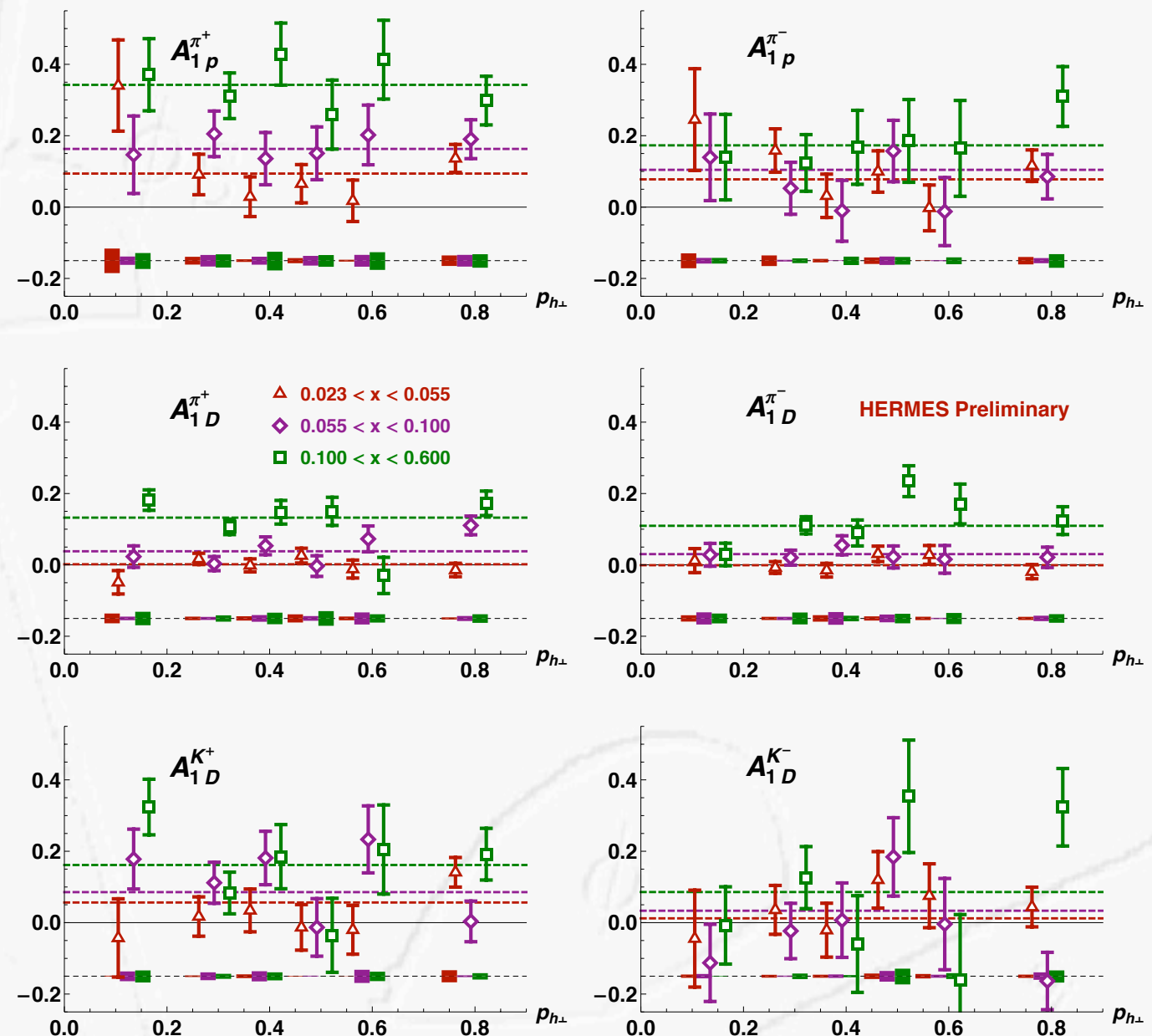
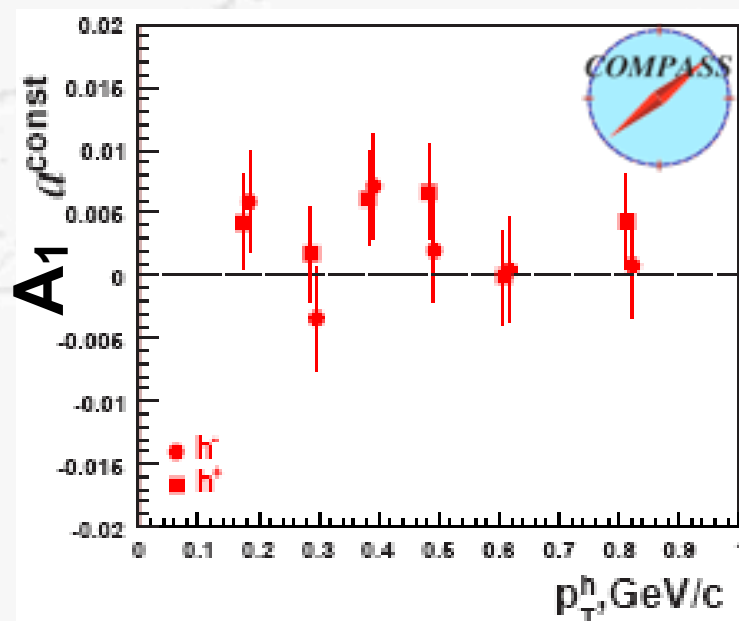
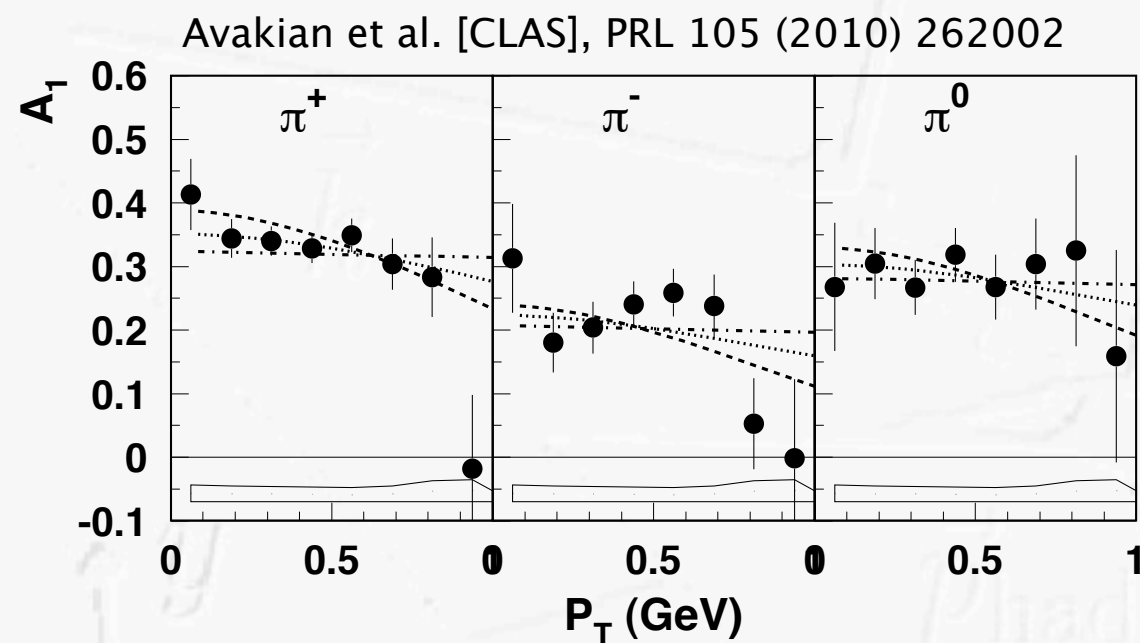
[M.G. Alekseev et al., Phys.Lett. B693 (2010) 227–235]



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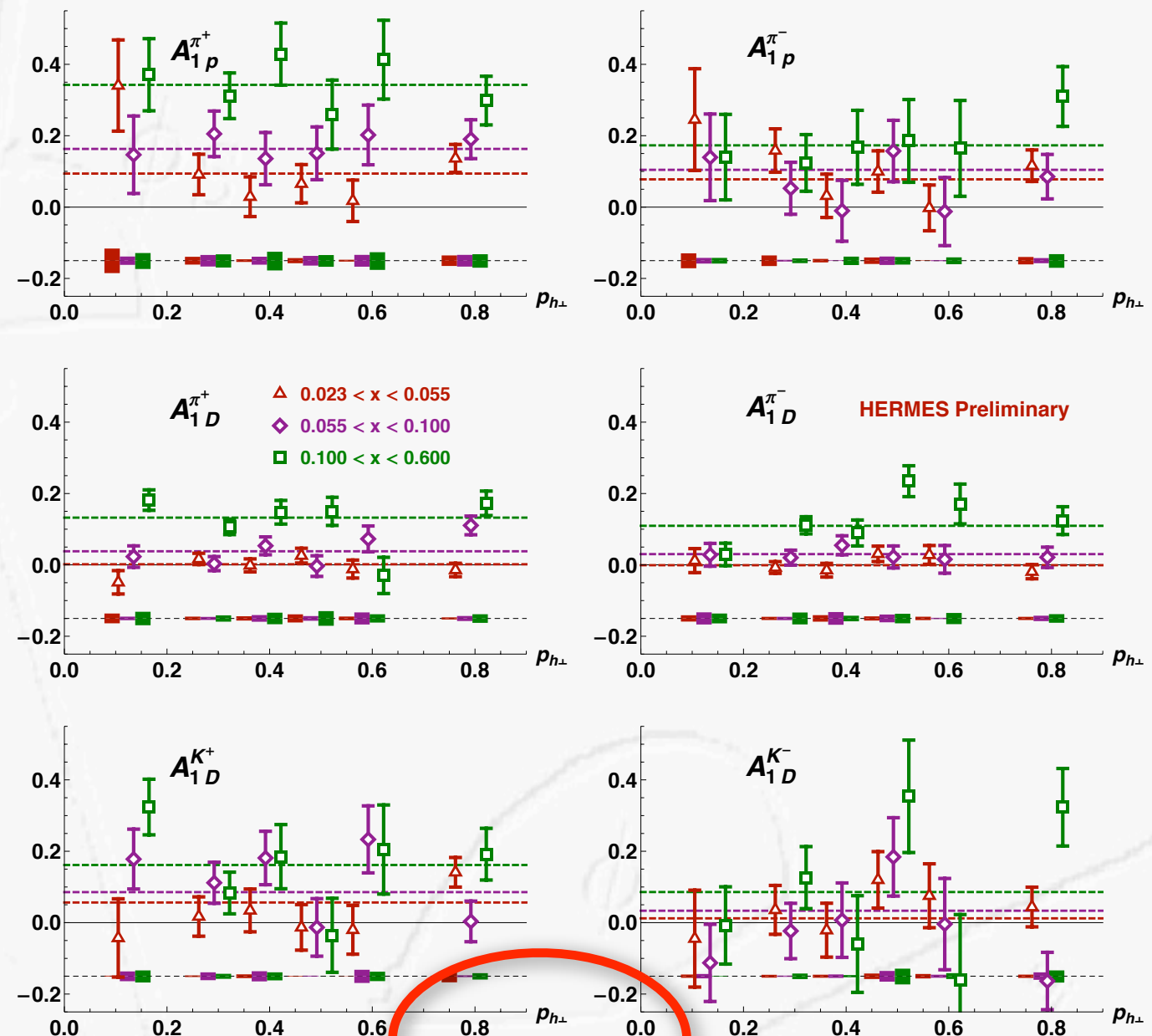
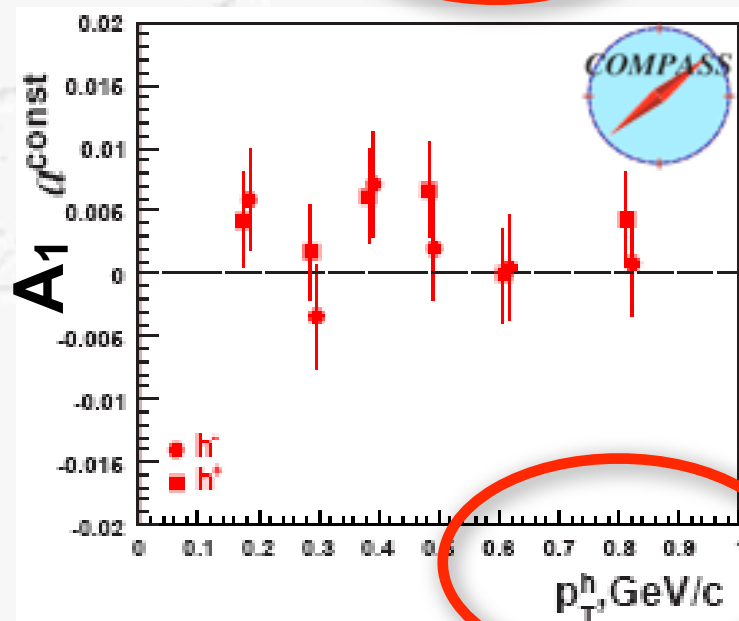
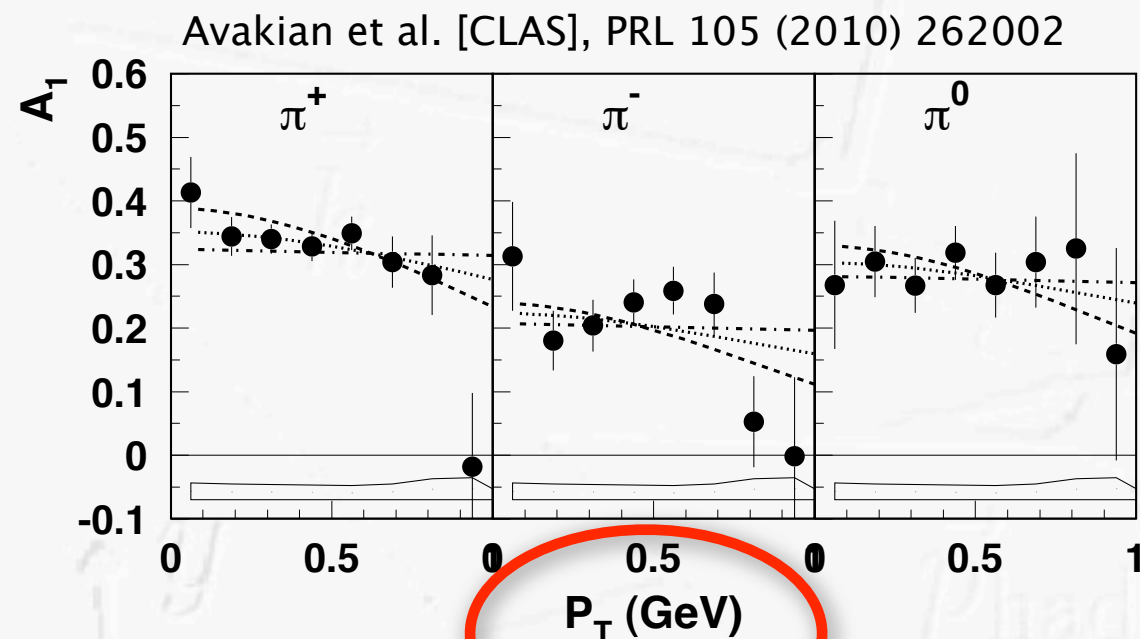
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Helicity density (unintegrated)



	U	L	T
U	f_1		$h_1^\perp$
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T	$f_{1T}^\perp$	g_{1T}	$h_1, h_{1T}^\perp$

Helicity density (unintegrated)

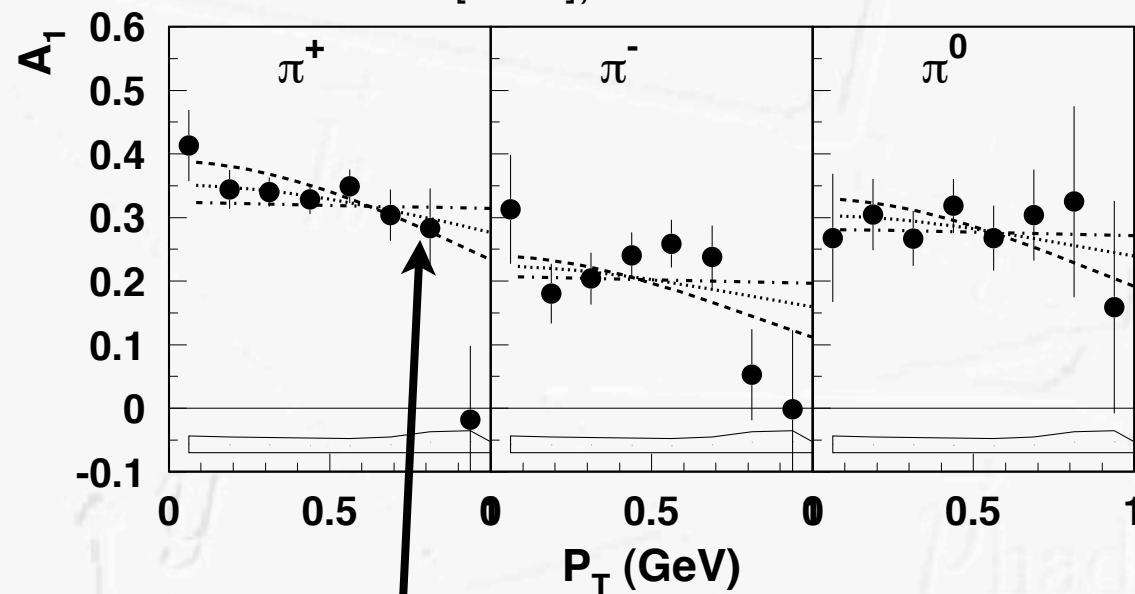


only weak if any dependence on $P_{h\perp}$ seen

Helicity density (unintegrated)

	U	L	T
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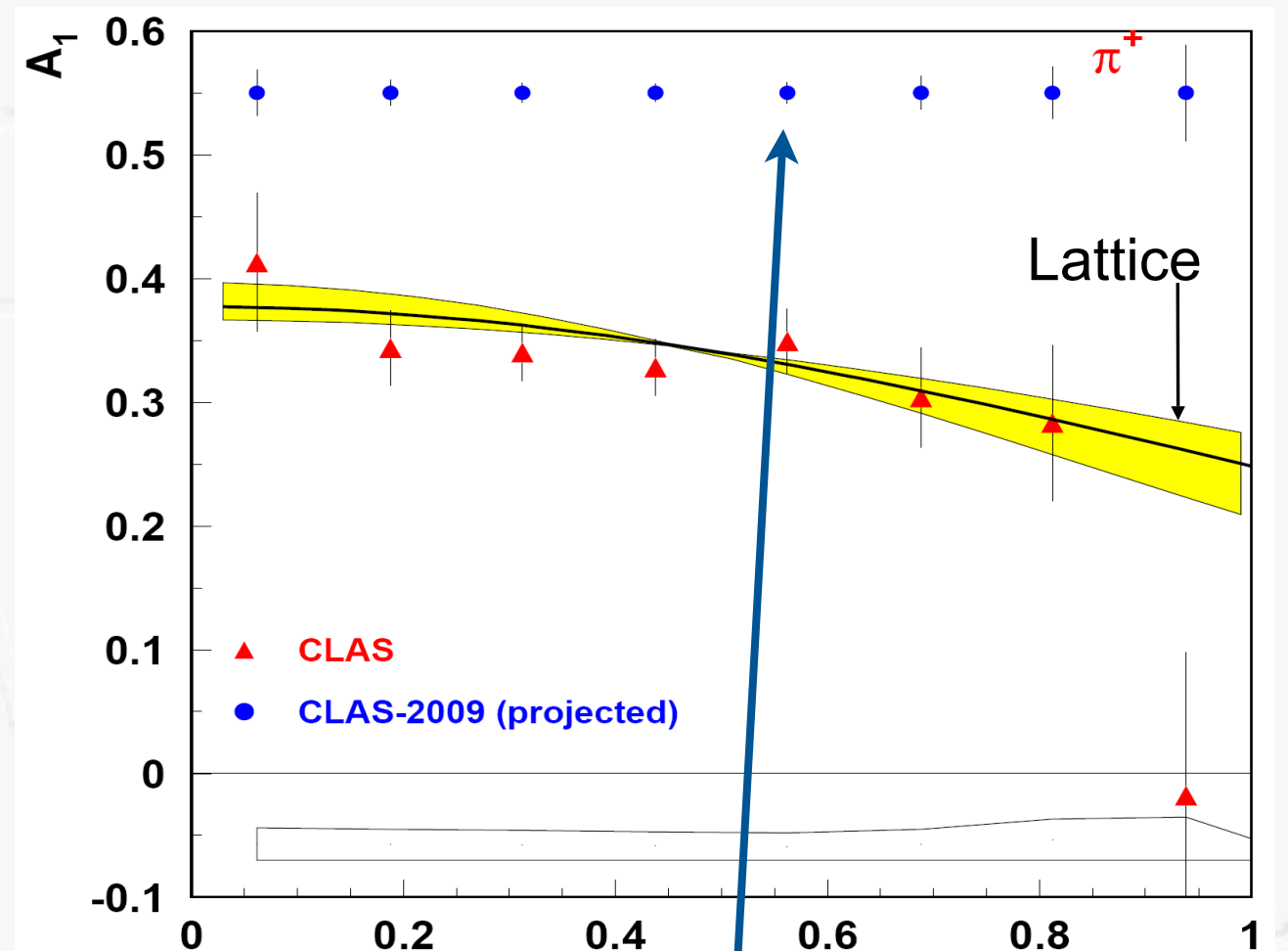
Avakian et al. [CLAS], arXiv:1003.4549



CLAS data hints at width μ_2 of g_1
that is less than the width μ_0 of f_1

$$f_1^q(x, k_T) = f_1(x) \frac{1}{\pi \mu_0^2} \exp\left(-\frac{k_T^2}{\mu_0^2}\right)$$

$$g_1^q(x, k_T) = g_1(x) \frac{1}{\pi \mu_2^2} \exp\left(-\frac{k_T^2}{\mu_2^2}\right)$$

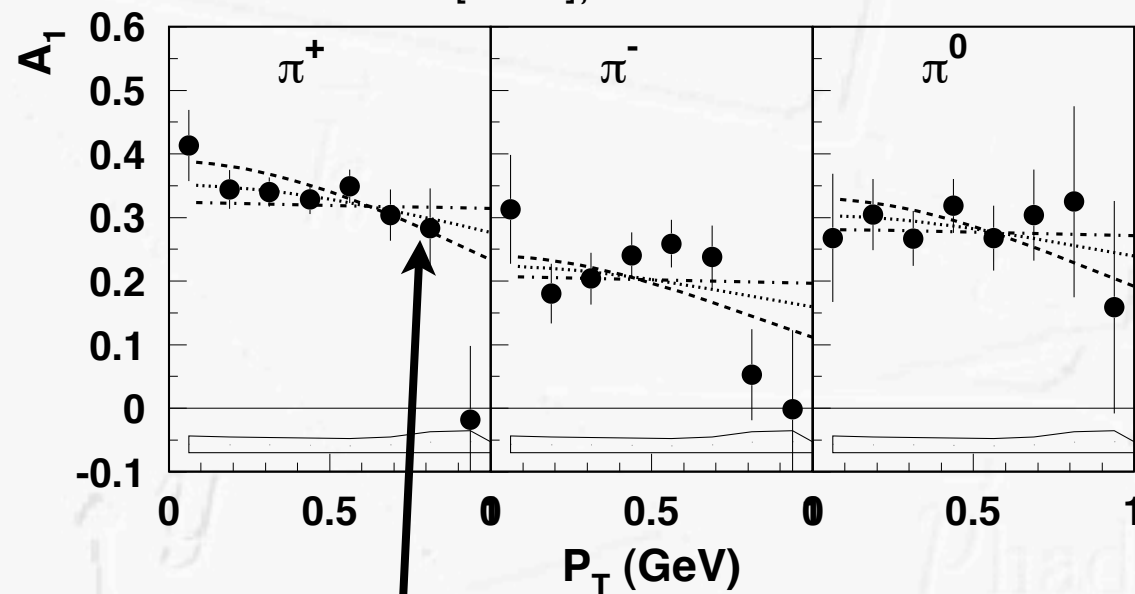


New CLAS data will allow multi-D binning
to study $P_{h\perp}$ dependence for fixed x

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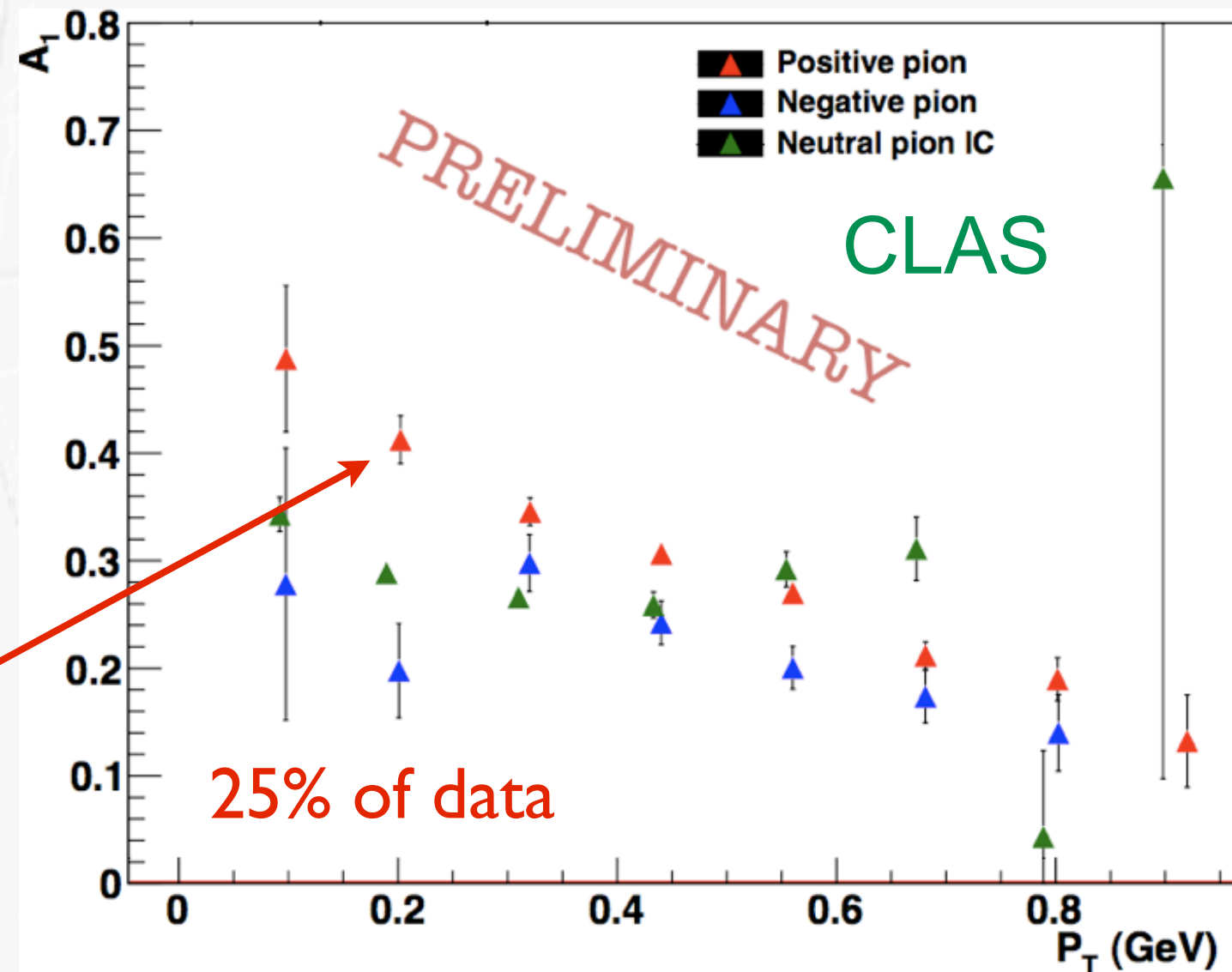
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The quest for transversity

Transversity distribution

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L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1, h_{1T}^\perp$

chiral-odd transversity involves quark helicity flip

$$f_1^q = \text{[red circle with white center]}$$

$$g_1^q = \text{[red circle with white center, right arrow]} - \text{[red circle with white center, left arrow]}$$

$$h_1^q = \text{[red circle with white center, up arrow]} - \text{[red circle with white center, down arrow]}$$

Transversity distribution

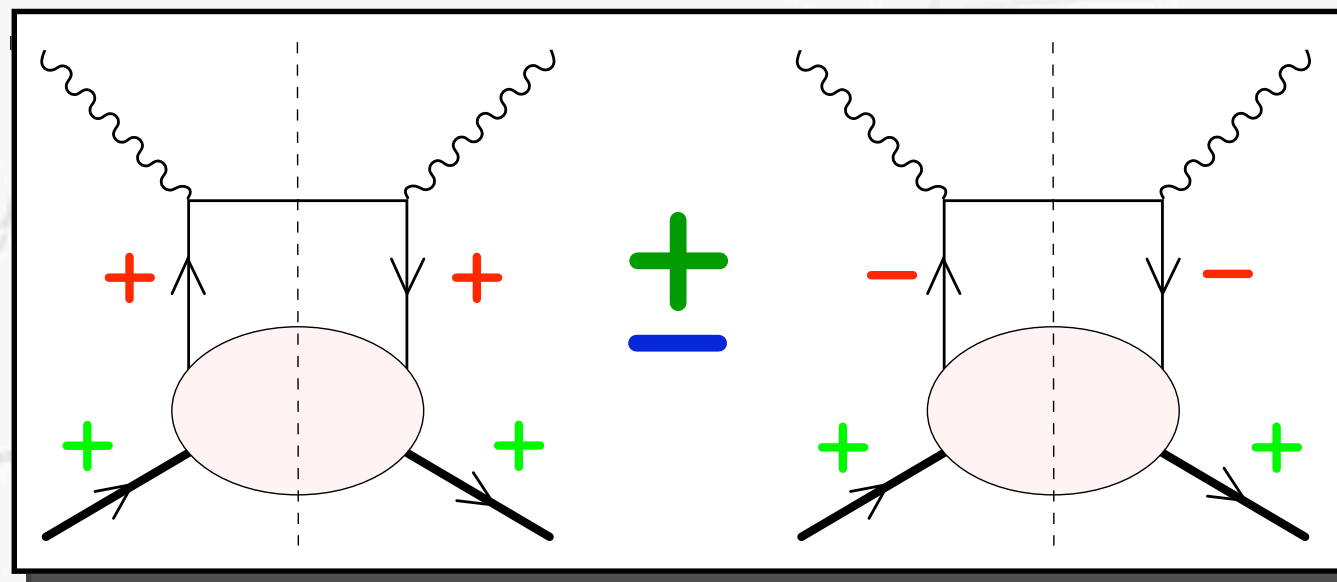
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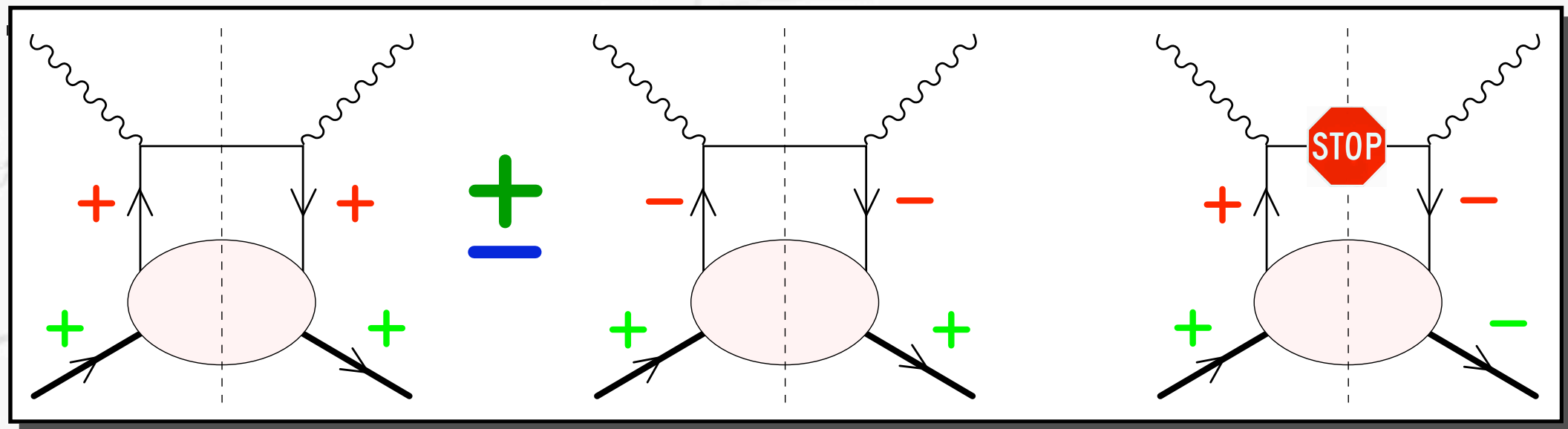
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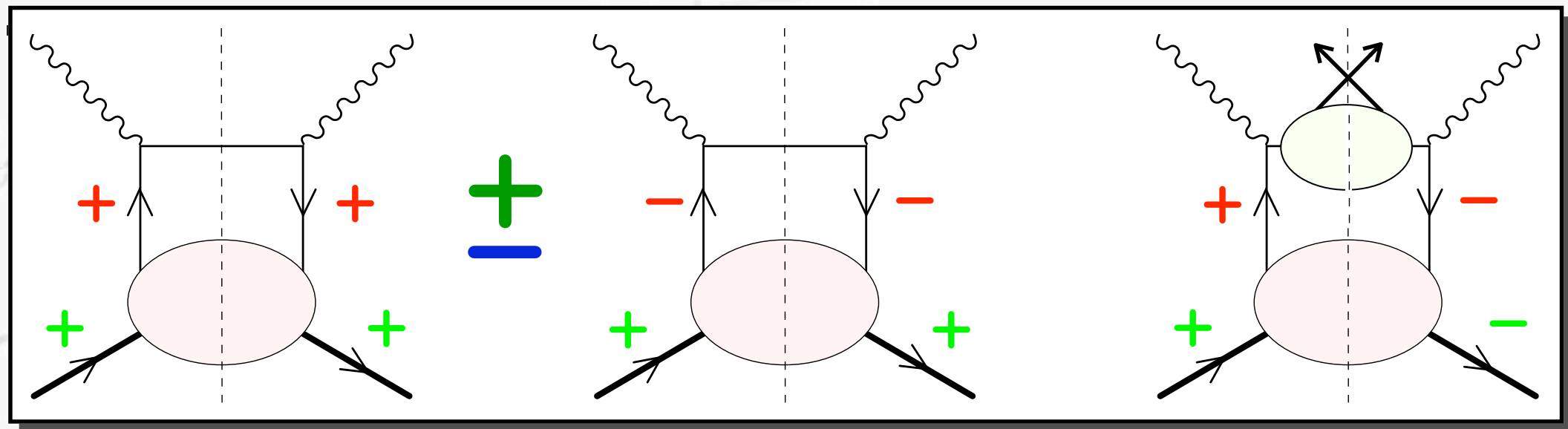
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need to couple to chiral-odd fragmentation function:

Transversity distribution

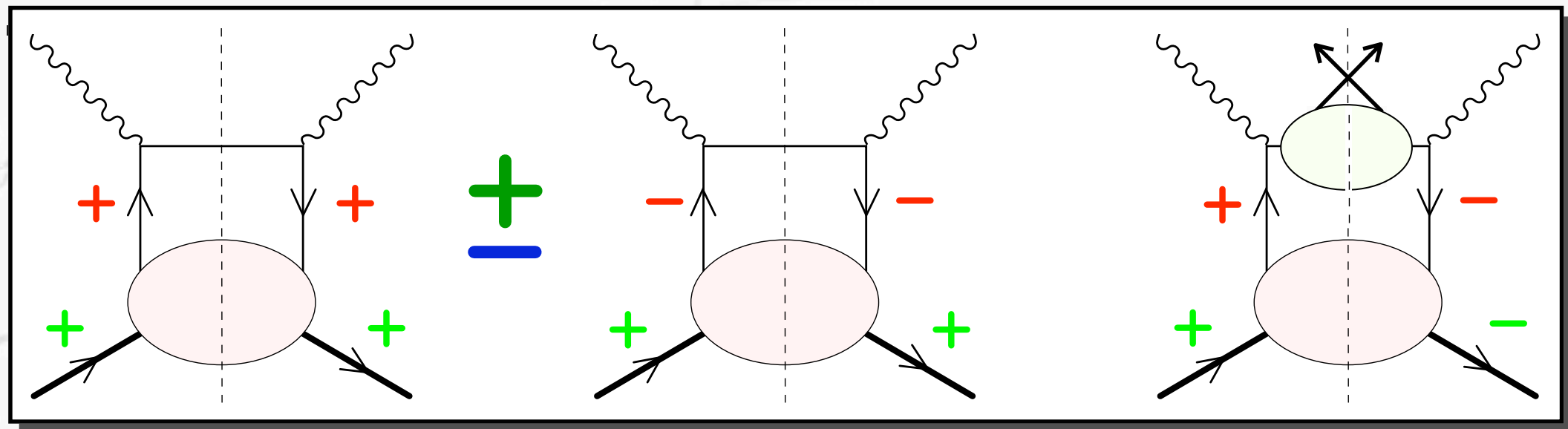
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need to couple to chiral-odd fragmentation function:

- transversity spin transfer (polarized final-state hadron)

Transversity distribution

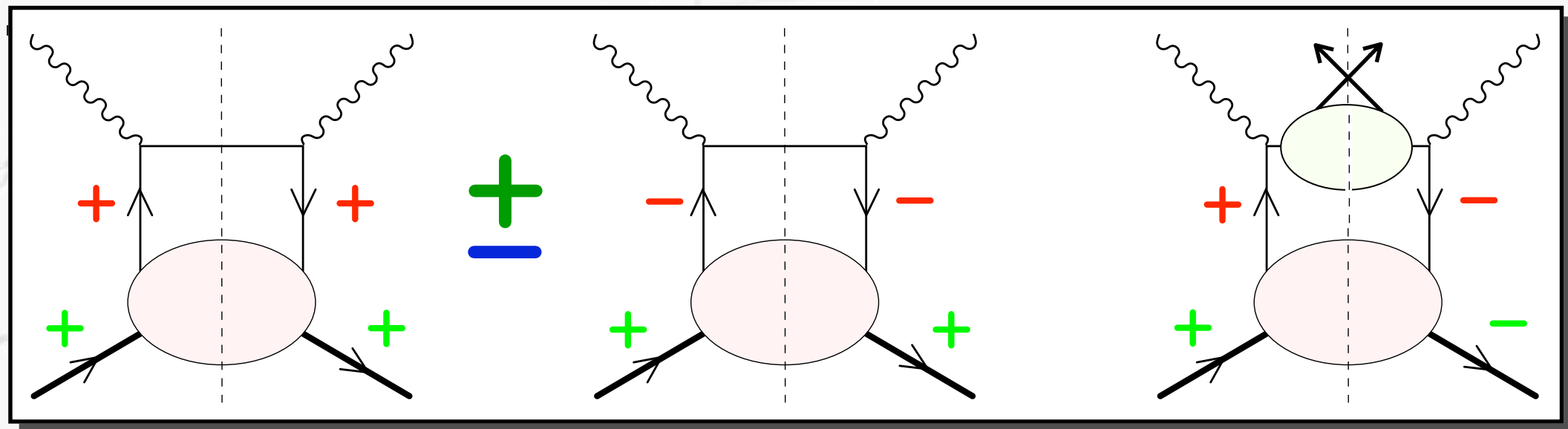
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need to couple to chiral-odd fragmentation function:

- transverse spin transfer (polarized final-state hadron)
- 2-hadron fragmentation

Transversity distribution

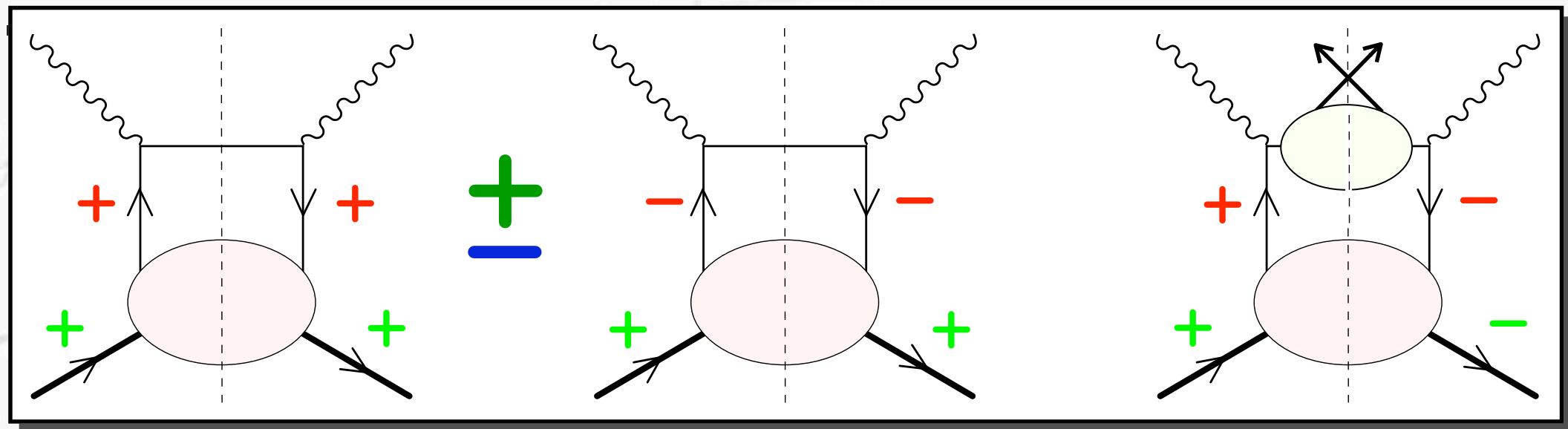
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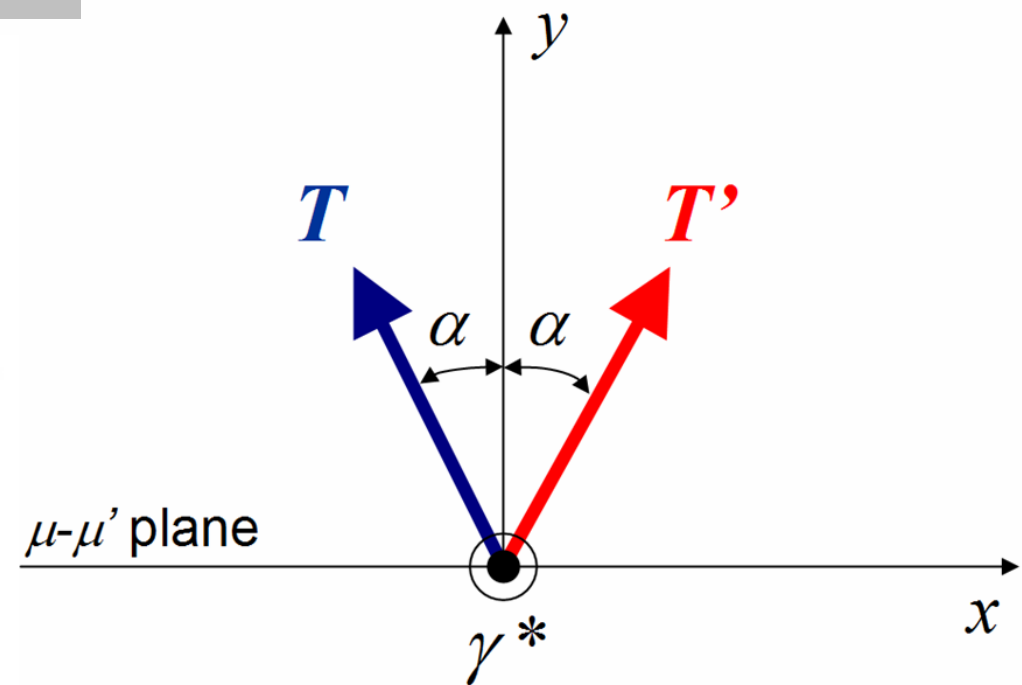
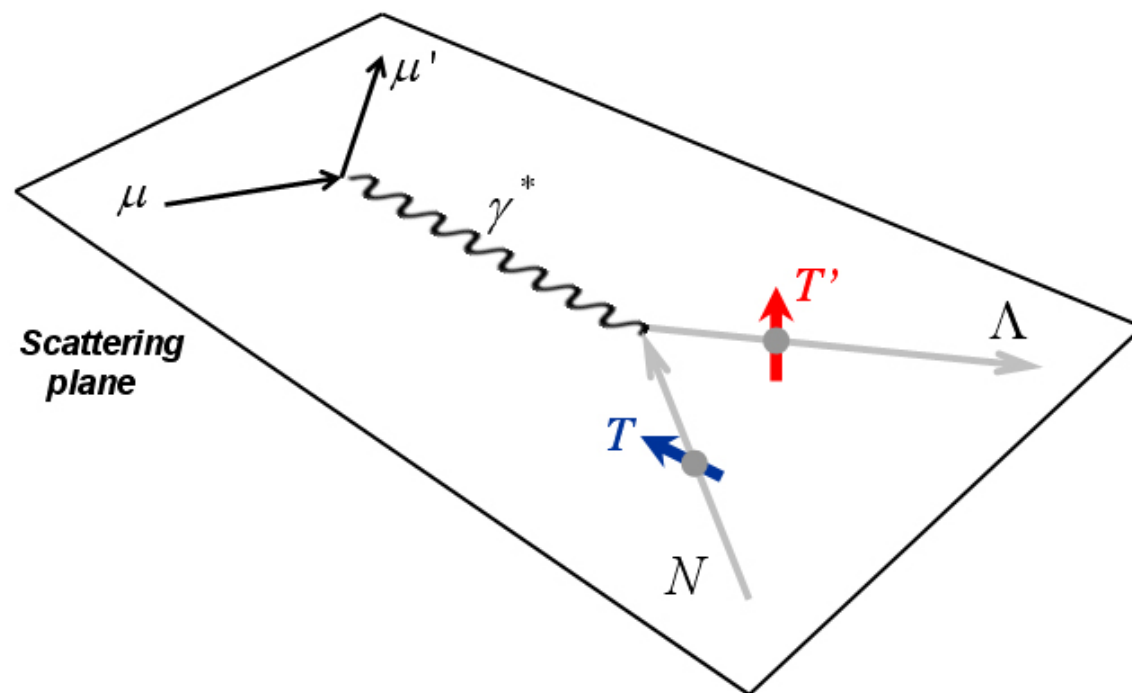
need to couple to chiral-odd fragmentation function:

- transverse spin transfer (polarized final-state hadron)
- 2-hadron fragmentation
- Collins fragmentation

Transversity distribution (transverse-spin transfer)

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Quantization axis for transverse Λ polarization



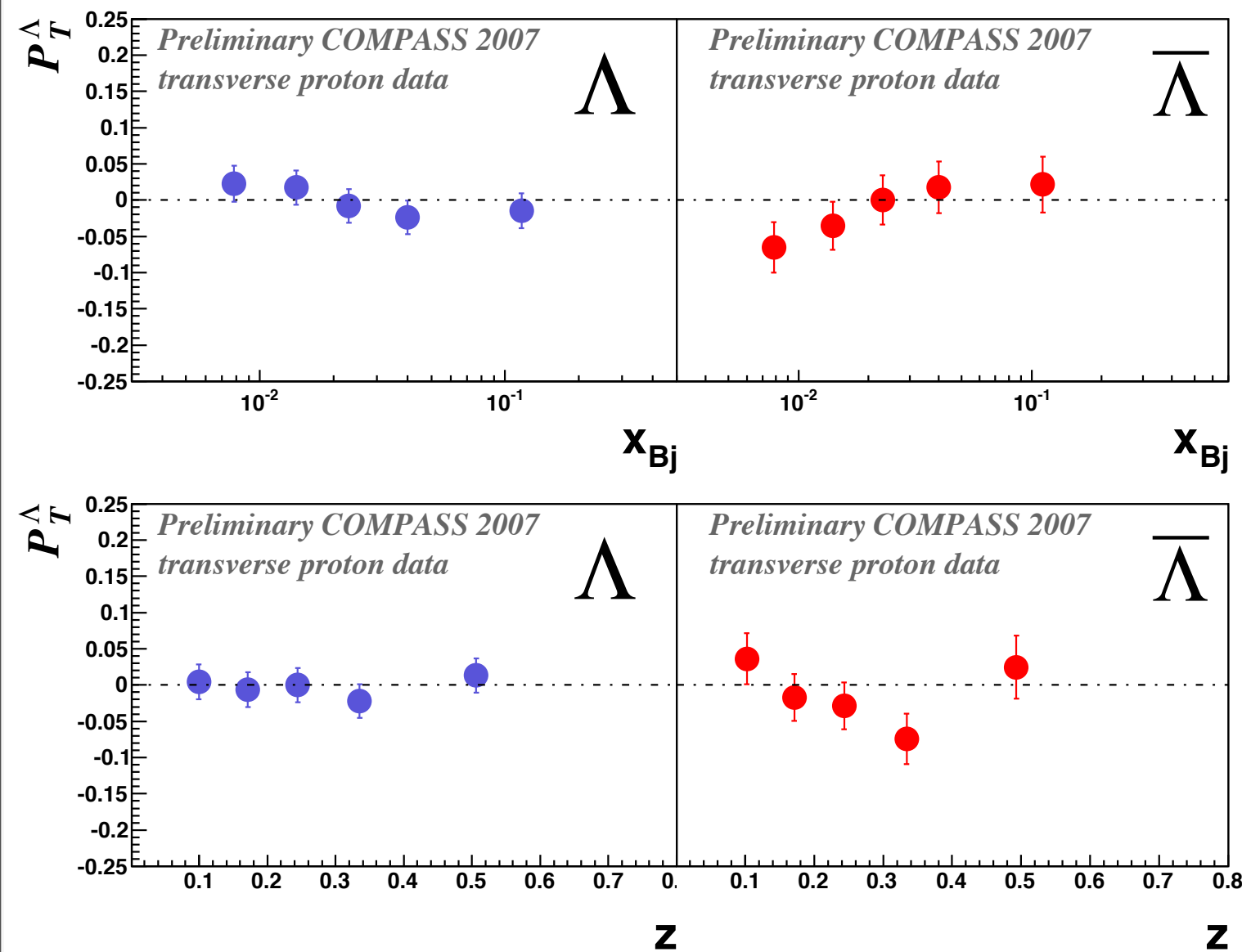
M. Anselmino & F. Murgia,
Physics Letters B 483 (2000) 74-86

T (initial quark spin) : component of target spin perpendicular to γ^*

T' (final quark spin) : symmetric of the T w.r.t. the normal to the scattering plane

Transversity distribution (transverse-spin transfer)

	U	L	T
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L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1, h_{1T}^\perp$



- compatible with zero
- low sensitivity to u & d quark polarization?
- measured at lower x where transversity is expected not to be large
- 2010 data will reduce statistical uncertainty by factor 2
- need to look at other hyperons?

Quark polarizations in hyperons

	Δu		Δd		Δs	
p	$\frac{1}{3}(\Delta\Sigma + D + 3F)$	0.79 ± 0.04	$\frac{1}{3}(\Delta\Sigma - 2D)$	-0.45 ± 0.04	$\frac{1}{3}(\Delta\Sigma + D - 3F)$	-0.16 ± 0.05
n	$\frac{1}{3}(\Delta\Sigma - 2D)$	-0.45 ± 0.04	$\frac{1}{3}(\Delta\Sigma + D + 3F)$	0.79 ± 0.04	$\frac{1}{3}(\Delta\Sigma + D - 3F)$	-0.16 ± 0.05
Σ^+	$\frac{1}{3}(\Delta\Sigma + D + 3F)$	0.79 ± 0.04	$\frac{1}{3}(\Delta\Sigma + D - 3F)$	-0.16 ± 0.05	$\frac{1}{3}(\Delta\Sigma - 2D)$	-0.45 ± 0.04
Σ^0	$\frac{1}{3}(\Delta\Sigma + D)$	0.32 ± 0.04	$\frac{1}{3}(\Delta\Sigma + D)$	0.32 ± 0.04	$\frac{1}{3}(\Delta\Sigma - 2D)$	-0.45 ± 0.04
Σ^-	$\frac{1}{3}(\Delta\Sigma + D - 3F)$	-0.16 ± 0.05	$\frac{1}{3}(\Delta\Sigma + D + 3F)$	0.79 ± 0.04	$\frac{1}{3}(\Delta\Sigma - 2D)$	-0.45 ± 0.04
Λ	$\frac{1}{3}(\Delta\Sigma - D)$	-0.20 ± 0.04	$\frac{1}{3}(\Delta\Sigma - D)$	-0.20 ± 0.04	$\frac{1}{3}(\Delta\Sigma + 2D)$	0.58 ± 0.04
Ξ^0	$\frac{1}{3}(\Delta\Sigma - 2D)$	-0.45 ± 0.04	$\frac{1}{3}(\Delta\Sigma + D - 3F)$	-0.16 ± 0.05	$\frac{1}{3}(\Delta\Sigma + D + 3F)$	0.79 ± 0.04
Ξ^-	$\frac{1}{3}(\Delta\Sigma + D - 3F)$	-0.16 ± 0.05	$\frac{1}{3}(\Delta\Sigma - 2D)$	-0.45 ± 0.04	$\frac{1}{3}(\Delta\Sigma + D + 3F)$	0.79 ± 0.04

Quark polarizations in hyperons

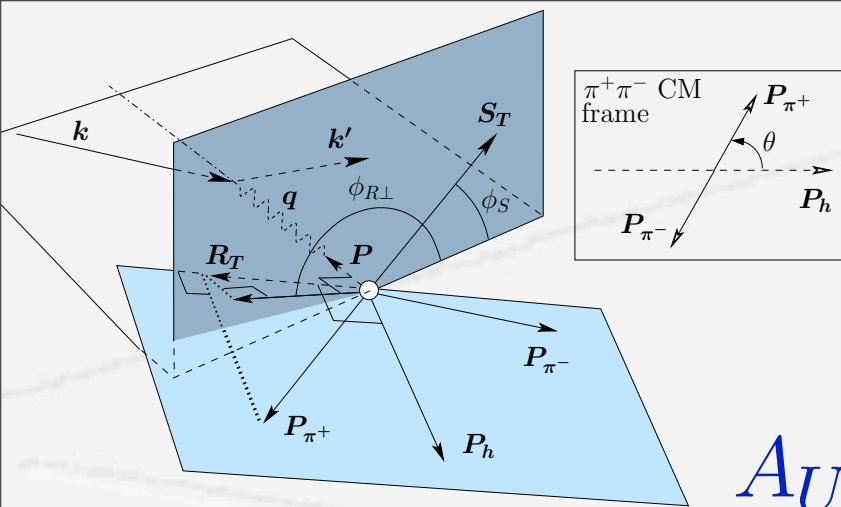
	Δu		Δd		Δs	
p	$\frac{1}{3}(\Delta\Sigma + D + 3F)$	0.79 ± 0.04	$\frac{1}{3}(\Delta\Sigma - 2D)$	-0.45 ± 0.04	$\frac{1}{3}(\Delta\Sigma + D - 3F)$	-0.16 ± 0.05
n	$\frac{1}{3}(\Delta\Sigma - 2D)$	-0.45 ± 0.04	$\frac{1}{3}(\Delta\Sigma + D + 3F)$	0.79 ± 0.04	$\frac{1}{3}(\Delta\Sigma + D - 3F)$	-0.16 ± 0.05
Σ^+	$\frac{1}{3}(\Delta\Sigma + D + 3F)$	0.79 ± 0.04	$\frac{1}{3}(\Delta\Sigma + D - 3F)$	-0.16 ± 0.05	$\frac{1}{3}(\Delta\Sigma - 2D)$	-0.45 ± 0.04
Σ^0	$\frac{1}{3}(\Delta\Sigma + D)$	0.32 ± 0.04	$\frac{1}{3}(\Delta\Sigma + D)$	0.32 ± 0.04	$\frac{1}{3}(\Delta\Sigma - 2D)$	-0.45 ± 0.04
Σ^-	$\frac{1}{3}(\Delta\Sigma + D - 3F)$	-0.16 ± 0.05	$\frac{1}{3}(\Delta\Sigma + D + 3F)$	0.79 ± 0.04	$\frac{1}{3}(\Delta\Sigma - 2D)$	-0.45 ± 0.04
Λ	$\frac{1}{3}(\Delta\Sigma - D)$	-0.20 ± 0.04	$\frac{1}{3}(\Delta\Sigma - D)$	-0.20 ± 0.04	$\frac{1}{3}(\Delta\Sigma + 2D)$	0.58 ± 0.04
Ξ^0	$\frac{1}{3}(\Delta\Sigma - 2D)$	-0.45 ± 0.04	$\frac{1}{3}(\Delta\Sigma + D - 3F)$	-0.16 ± 0.05	$\frac{1}{3}(\Delta\Sigma + D + 3F)$	0.79 ± 0.04
Ξ^-	$\frac{1}{3}(\Delta\Sigma + D - 3F)$	-0.16 ± 0.05	$\frac{1}{3}(\Delta\Sigma - 2D)$	-0.45 ± 0.04	$\frac{1}{3}(\Delta\Sigma + D + 3F)$	0.79 ± 0.04

Quark polarizations in hyperons

	Δu		Δd		Δs	
p	$\frac{1}{3}(\Delta\Sigma + D + 3F)$	0.79 ± 0.04	$\frac{1}{3}(\Delta\Sigma - 2D)$	-0.45 ± 0.04	$\frac{1}{3}(\Delta\Sigma + D - 3F)$	-0.16 ± 0.05
n	$\frac{1}{3}(\Delta\Sigma - 2D)$	-0.45 ± 0.04	$\frac{1}{3}(\Delta\Sigma + D + 3F)$	0.79 ± 0.04	$\frac{1}{3}(\Delta\Sigma + D - 3F)$	-0.16 ± 0.05
Σ^+	$\frac{1}{3}(\Delta\Sigma + D + 3F)$	0.79 ± 0.04	$\frac{1}{3}(\Delta\Sigma + D - 3F)$	-0.16 ± 0.05	$\frac{1}{3}(\Delta\Sigma - 2D)$	-0.45 ± 0.04
Σ^0	$\frac{1}{3}(\Delta\Sigma + D)$	0.32 ± 0.04	$\frac{1}{3}(\Delta\Sigma + D)$	0.32 ± 0.04	$\frac{1}{3}(\Delta\Sigma - 2D)$	-0.45 ± 0.04
Σ^-	$\frac{1}{3}(\Delta\Sigma + D - 3F)$	-0.16 ± 0.05	$\frac{1}{3}(\Delta\Sigma + D + 3F)$	0.79 ± 0.04	$\frac{1}{3}(\Delta\Sigma - 2D)$	-0.45 ± 0.04
Λ	$\frac{1}{3}(\Delta\Sigma - D)$	-0.20 ± 0.04	$\frac{1}{3}(\Delta\Sigma - D)$	-0.20 ± 0.04	$\frac{1}{3}(\Delta\Sigma + 2D)$	0.58 ± 0.04
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- better sensitivity to u and d quarks via charged Sigma's
- large analyzing power of the parity-violating decay $\Sigma^+ \rightarrow p\pi^0$
 - good probe of u-quark polarization
 - need good acceptance and neutral pion reconstruction

2-Hadron Fragmentation



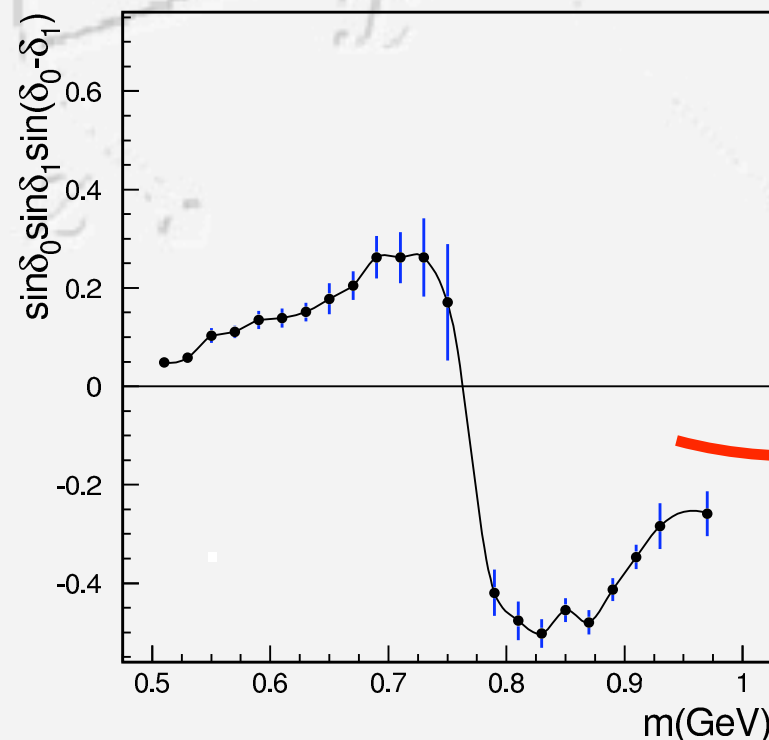
$$A_{UT} \sim \sin(\phi_{R\perp} + \phi_S) \sin \theta h_1 H_1^{\triangleleft}$$

Expansion of $H_1^{\triangleleft}$ in Legendre moments:

$$H_1^{\triangleleft}(z, \cos \theta, M_{\pi\pi}^2) = H_1^{\triangleleft,sp}(z, M_{\pi\pi}^2) + \cos \theta H_1^{\triangleleft,pp}(z, M_{\pi\pi}^2)$$

describe interference between 2 pion pairs coming from different production channels.

about $H_1^{\triangleleft,sp}$:



Jaffe et al. [[hep-ph/9709322](#)]:

$$H_1^{\triangleleft,sp}(z, M_{\pi\pi}^2) = \frac{\sin \delta_0 \sin \delta_1 \sin(\delta_0 - \delta_1)}{\delta_0 (\delta_1)} H_1^{\triangleleft,sp'}(z)$$

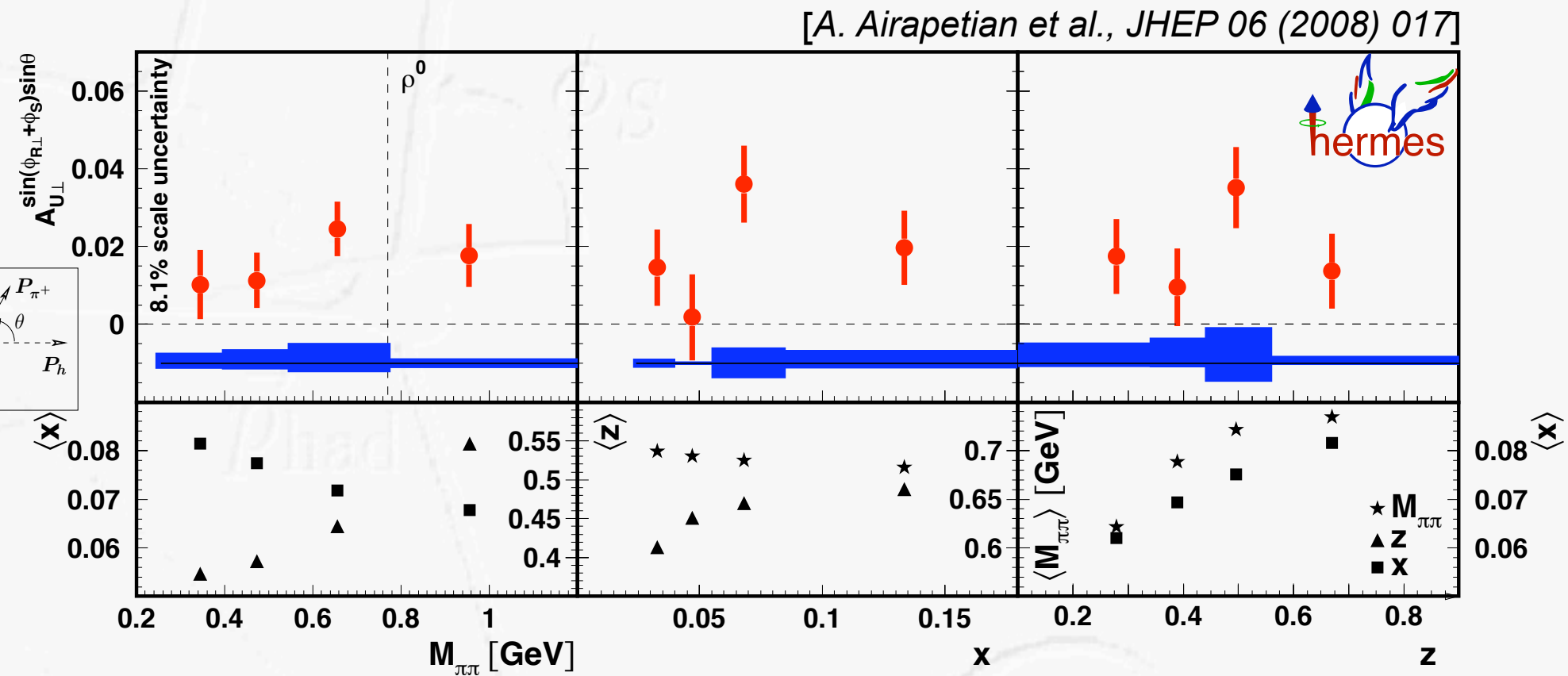
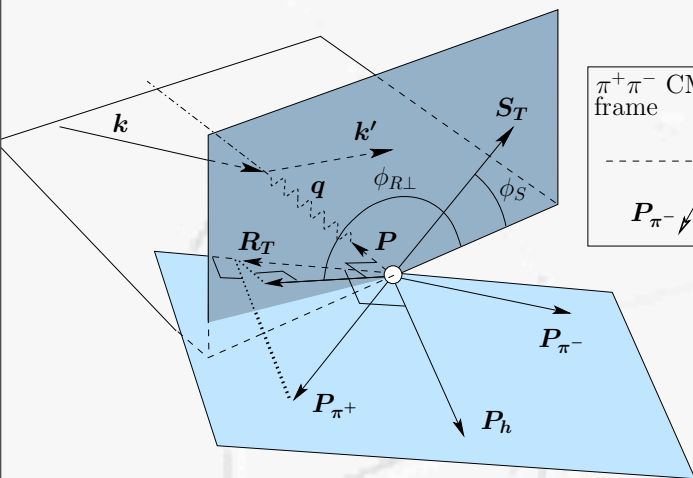
$\delta_0 (\delta_1) \rightarrow$ S(P)-wave phase shifts

$$= \mathcal{P}(M_{\pi\pi}^2) H_1^{\triangleleft,sp'}(z)$$

$\Rightarrow A_{UT}$ might depend strongly on $M_{\pi\pi}$

Transversity distribution (2-hadron fragmentation)

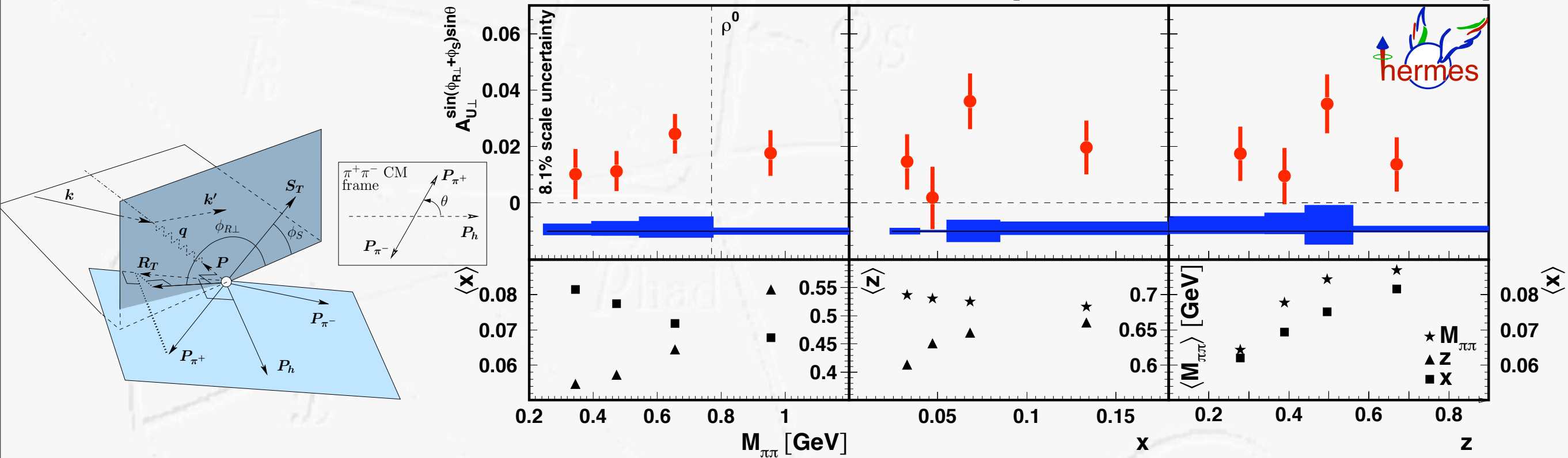
	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1, h_{1T}^\perp$



Transversity distribution (2-hadron fragmentation)

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1, h_{1T}^\perp$

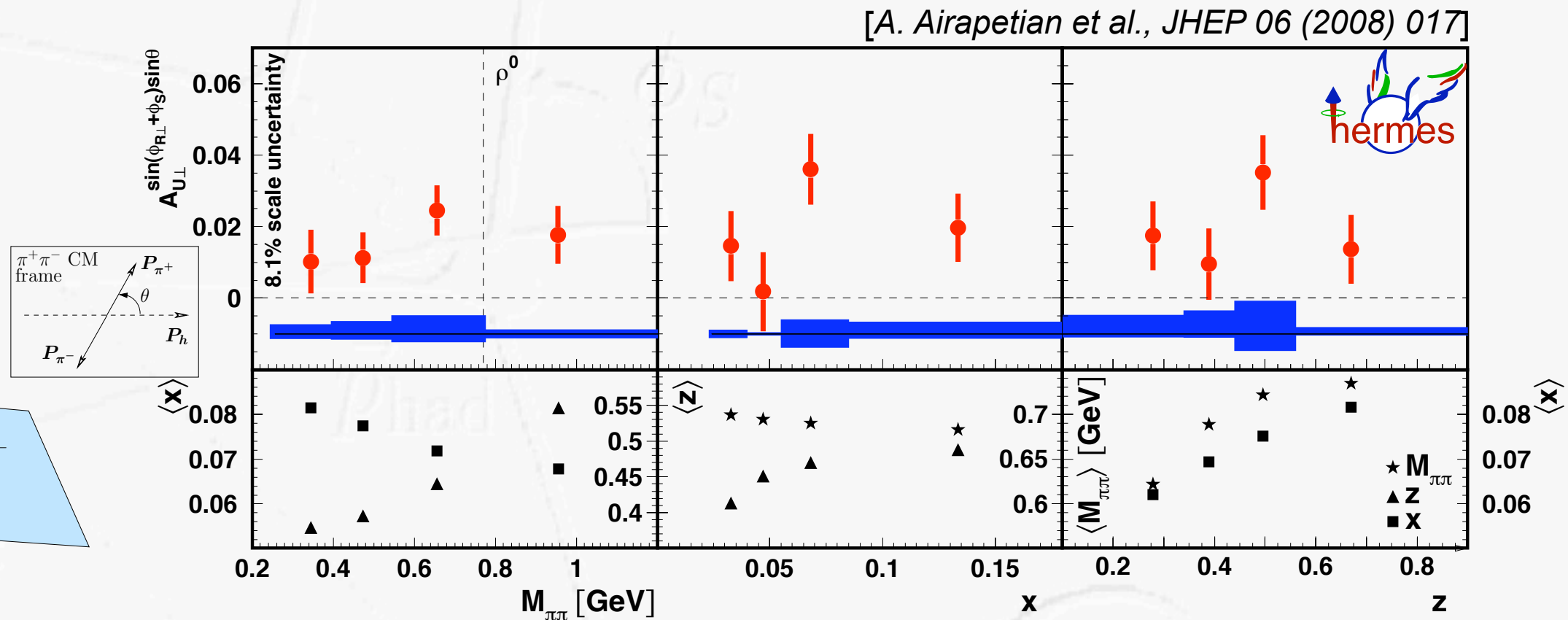
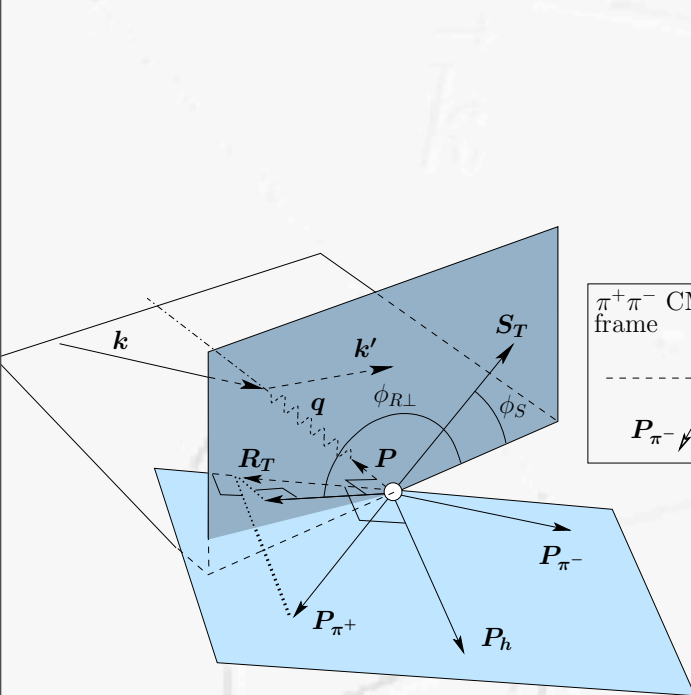
[A. Airapetian et al., JHEP 06 (2008) 017]



✓ first evidence for T-odd 2-hadron fragmentation function in semi-inclusive DIS!

Transversity distribution (2-hadron fragmentation)

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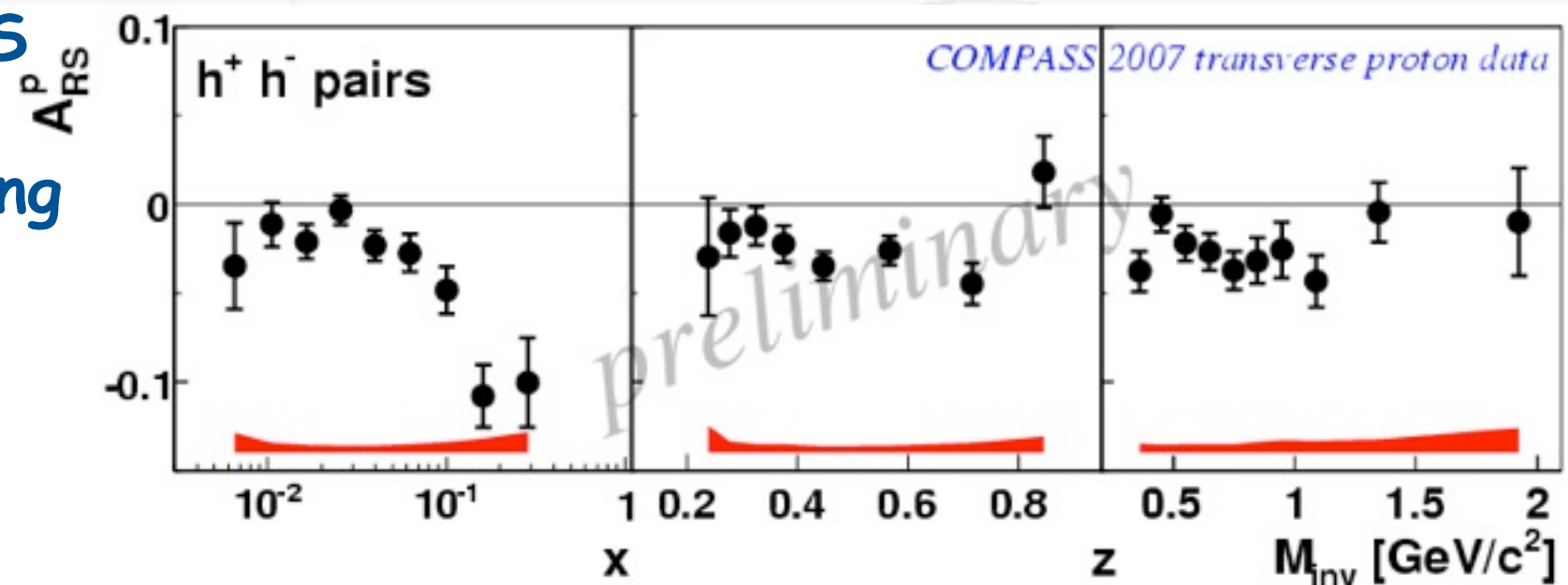
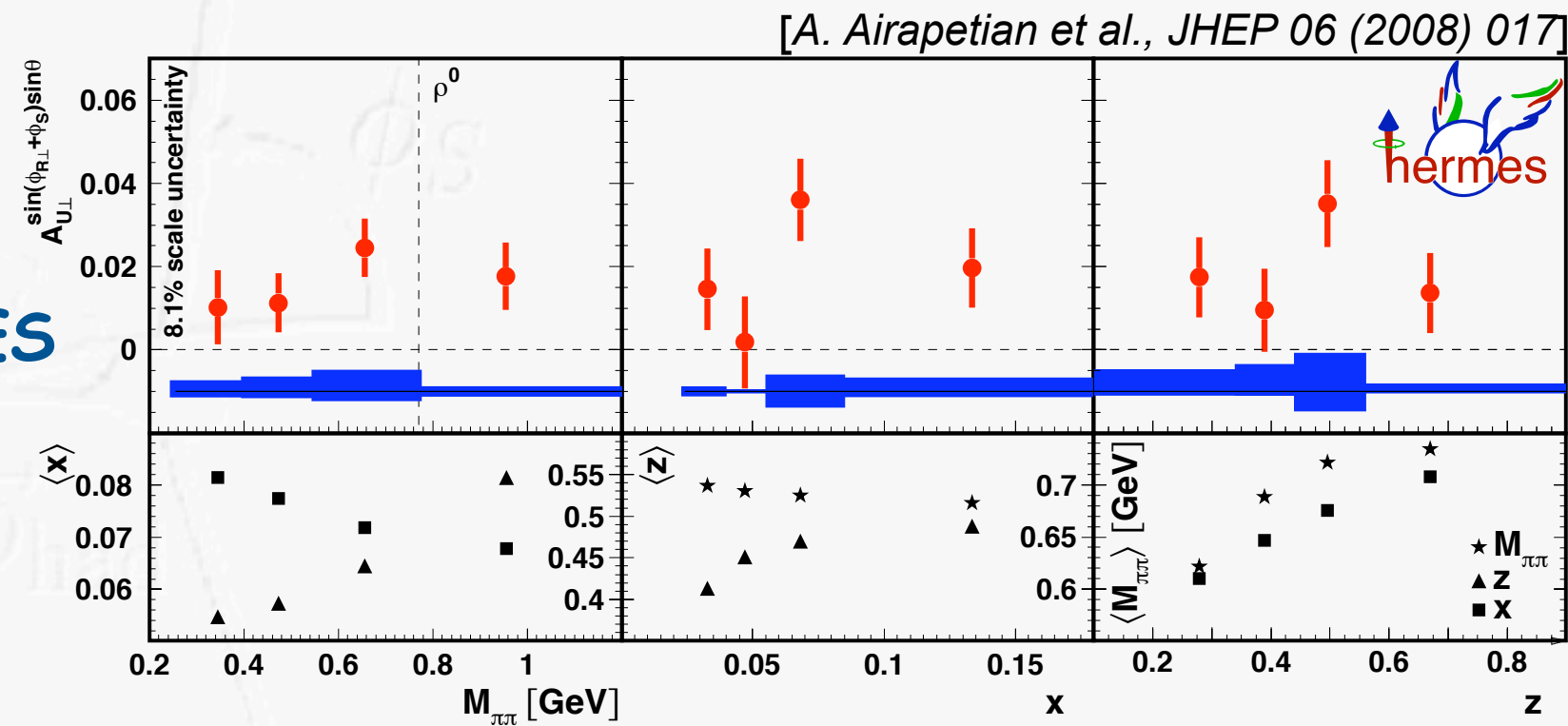


- ✓ first evidence for T-odd 2-hadron fragmentation function in semi-inclusive DIS!
- ✓ invariant-mass dependence rules out Jaffe model predicting a sign change to rho mass

Transversity distribution (2-hadron fragmentation)

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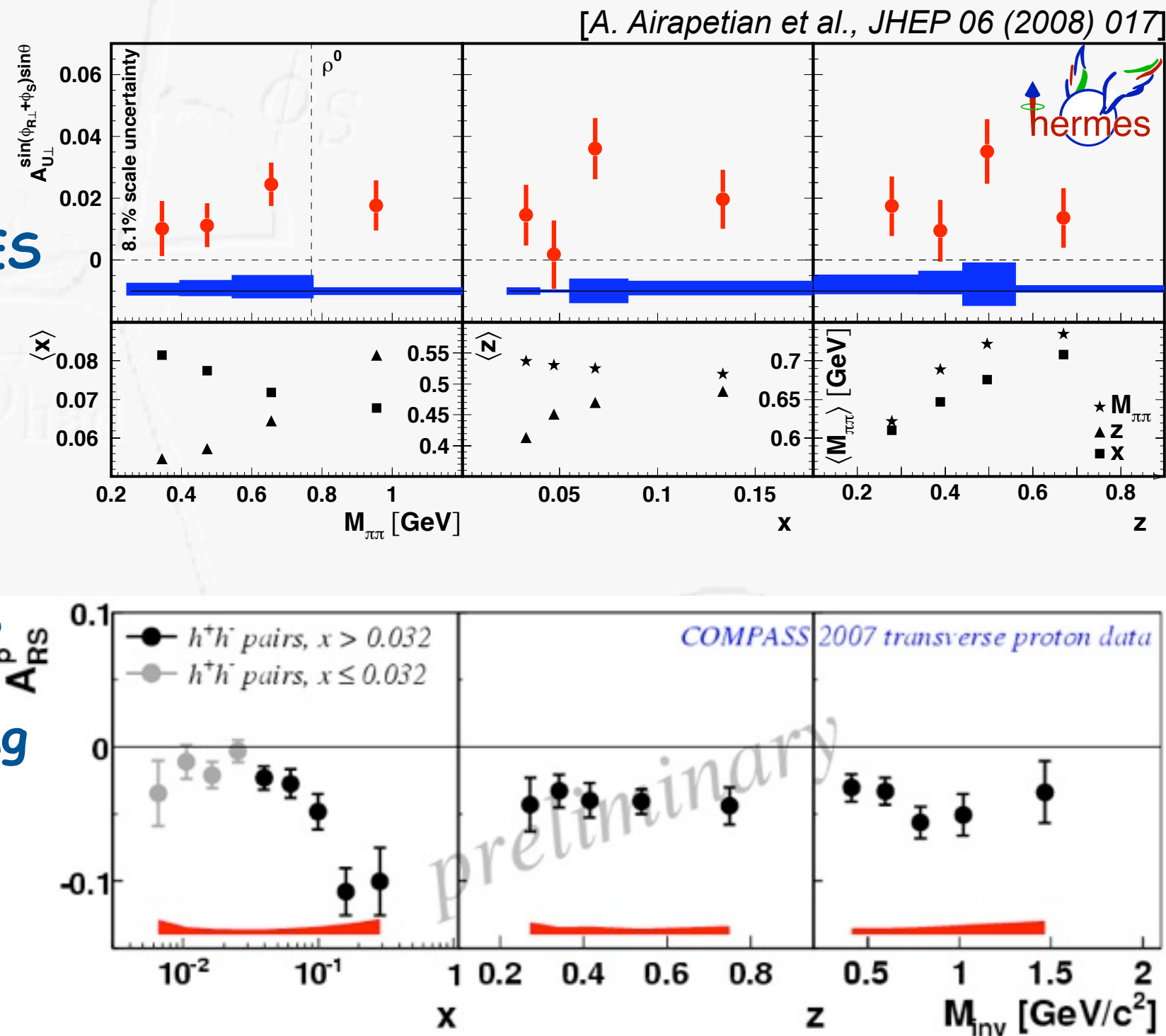
- non-zero amplitudes both from COMPASS and HERMES
- COMPASS: hadron pairs
HERMES: pion pairs
- larger amplitudes at COMPASS than at HERMES
- similar $M_{\pi\pi}$ dependence ruling out Jaffe model
- first results from e^+e^- by BELLE



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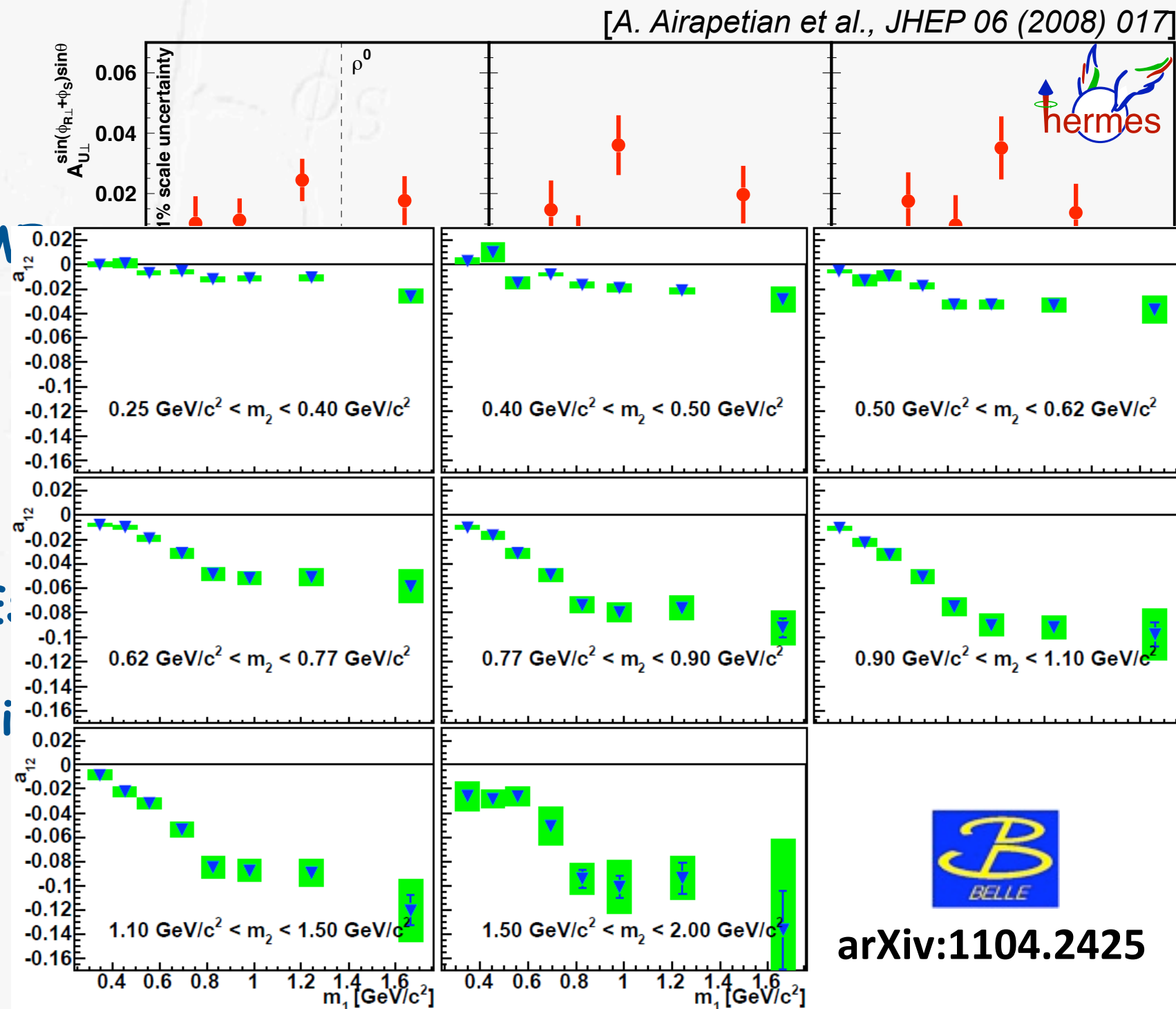
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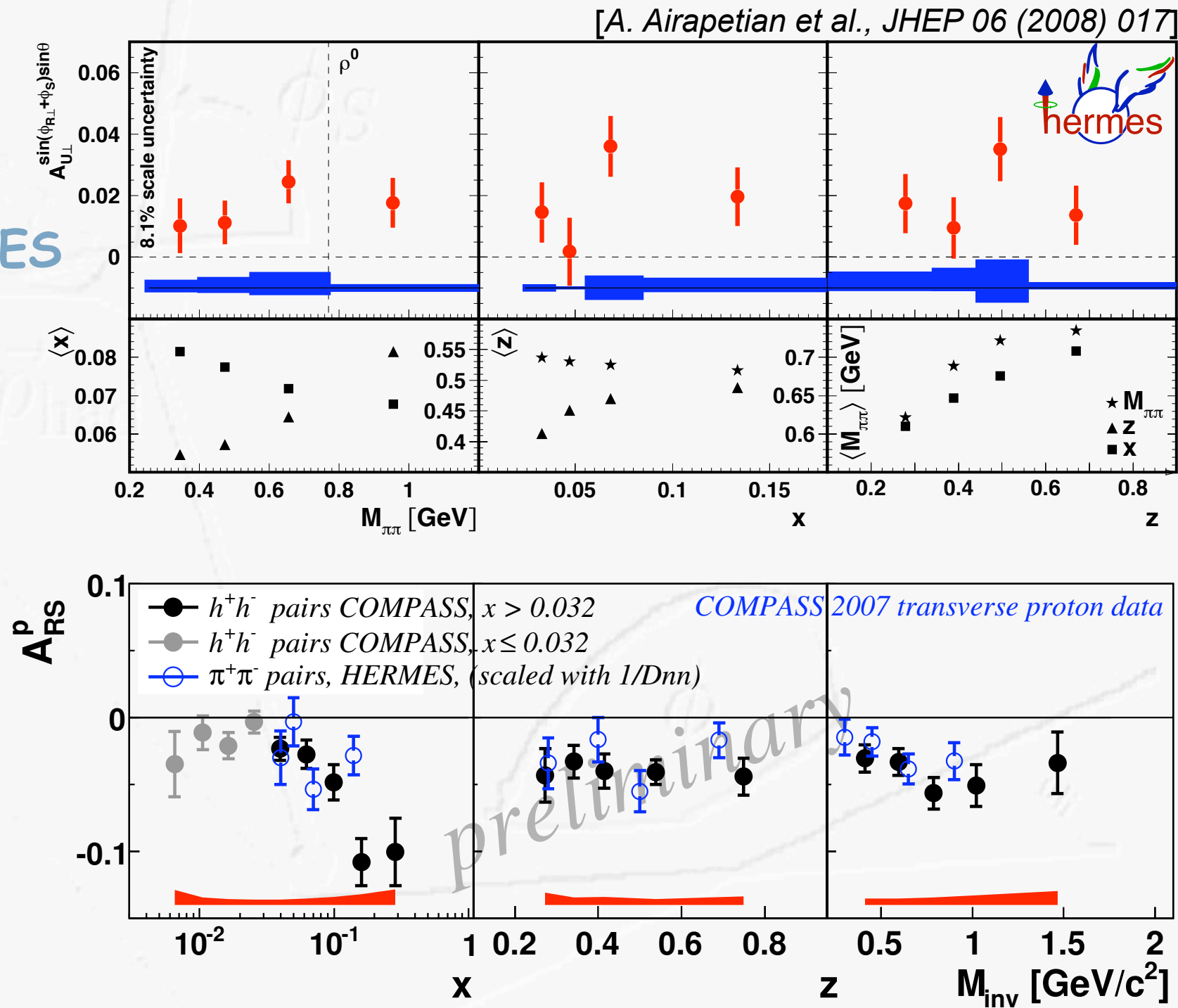
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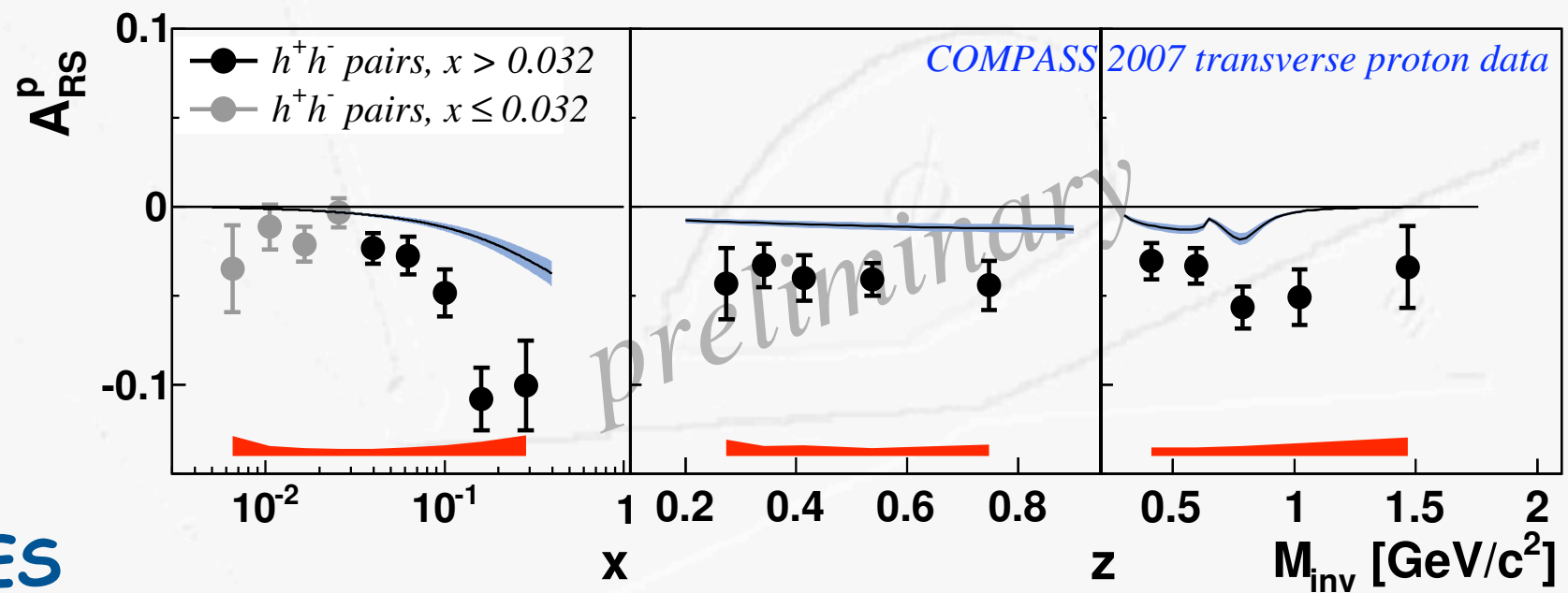
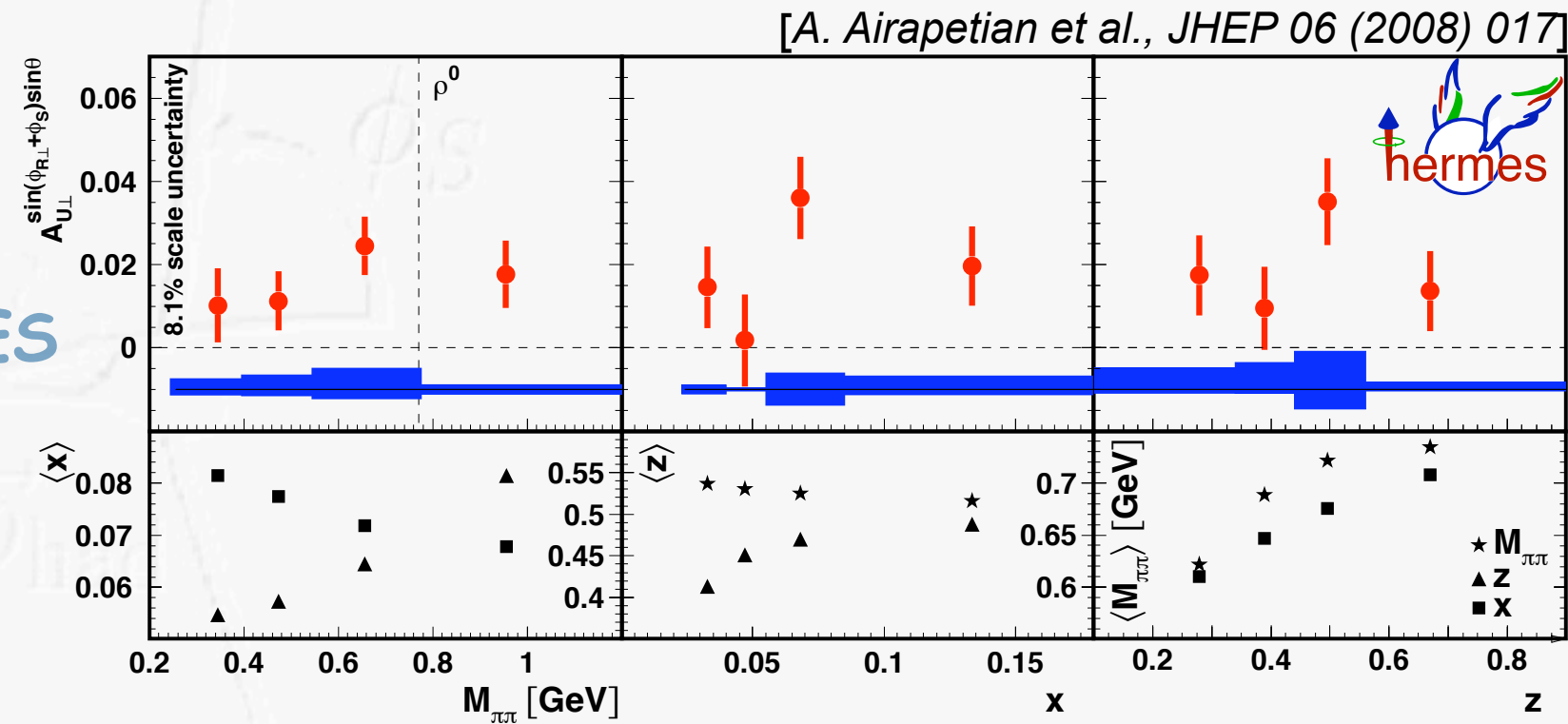
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- COMPASS divides out depolarization factor D_{nn}
- comparable when also done for HERMES data



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- non-zero amplitudes both from COMPASS and HERMES
- COMPASS: hadron pairs
HERMES: pion pairs
- COMPASS divides out depolarization factor D_{nn}
- comparable when also done for HERMES data
- bad comparison w/ model prediction based on HERMES



Transversity with TMD FFs

Ex.: Appearance of TMDs in SIDIS

Leading-Twist

Distribution Functions

$$f_1 = \text{yellow circle with blue center}$$

$$g_1 = \text{yellow circle with blue center and right arrow} - \text{yellow circle with blue center and left arrow}$$

$$h_1 = \text{yellow circle with blue center and up arrow} - \text{yellow circle with blue center and down arrow}$$

$$f_{1T}^\perp = \text{yellow circle with blue center and up arrow} - \text{yellow circle with blue center and down arrow}$$

$$h_1^\perp = \text{yellow circle with blue center and left arrow} - \text{yellow circle with blue center and right arrow}$$

$$h_{1L}^\perp = \text{yellow circle with blue center and up-right arrow} - \text{yellow circle with blue center and down-left arrow}$$

$$g_{1T} = \text{yellow circle with blue center and up arrow} - \text{yellow circle with blue center and down arrow}$$

$$h_{1T}^\perp = \text{yellow circle with blue center and left arrow} - \text{yellow circle with blue center and right arrow}$$

Fragmentation Functions

$$D_1 = \text{yellow circle with blue center}$$

$$G_1 = \text{yellow circle with blue center and right arrow} - \text{yellow circle with blue center and left arrow}$$

$$H_1 = \text{yellow circle with blue center and up arrow} - \text{yellow circle with blue center and down arrow}$$

$$D_{1T}^\perp = \text{yellow circle with blue center and up arrow} - \text{yellow circle with blue center and down arrow}$$

$$H_1^\perp = \text{yellow circle with blue center and left arrow} - \text{yellow circle with blue center and right arrow}$$

$$H_{1L}^\perp = \text{yellow circle with blue center and up-right arrow} - \text{yellow circle with blue center and down-left arrow}$$

$$G_{1T} = \text{yellow circle with blue center and up arrow} - \text{yellow circle with blue center and down arrow}$$

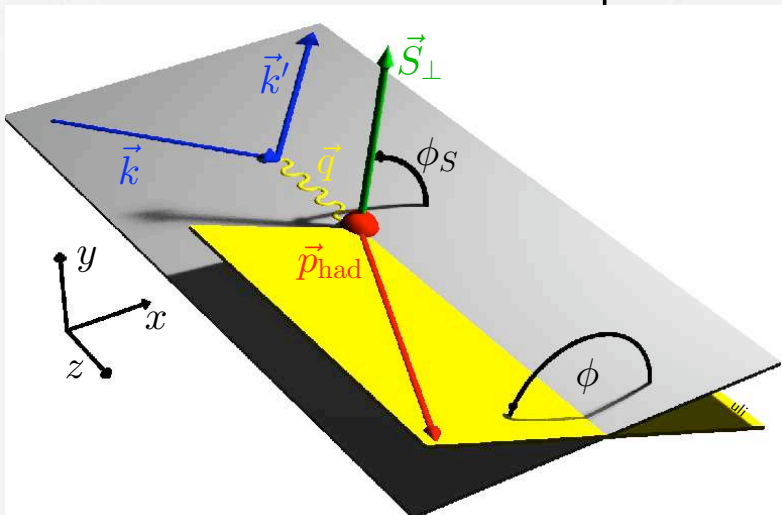
$$H_{1T}^\perp = \text{yellow circle with blue center and left arrow} - \text{yellow circle with blue center and right arrow}$$

Chiral-odd transversity h_1 must couple to chiral-odd FF
 can use k_T -unintegrated chiral-odd FF $\Rightarrow$ T-odd Collins FF
 $\Rightarrow$ leads to Single-Spin Asymmetrie (SSA)

Ex.: Appearance of TMDs in SIDIS

$$\begin{aligned}
 d\sigma = & d\sigma_{UU}^0 + \cos 2\phi d\sigma_{UU}^1 + \frac{1}{Q} \cos \phi d\sigma_{UU}^2 + \lambda_e \frac{1}{Q} \sin \phi d\sigma_{LU}^3 \\
 & + S_L \left\{ \sin 2\phi d\sigma_{UL}^4 + \frac{1}{Q} \sin \phi d\sigma_{UL}^5 + \lambda_e \left[d\sigma_{LL}^6 + \frac{1}{Q} \cos \phi d\sigma_{LL}^7 \right] \right\} \\
 & + S_T \left\{ \sin(\phi - \phi_S) d\sigma_{UT}^8 + \sin(\phi + \phi_S) d\sigma_{UT}^9 + \sin(3\phi - \phi_S) d\sigma_{UT}^{10} \frac{1}{Q} \right. \\
 & \quad \left. + \frac{1}{Q} (\sin(2\phi - \phi_S) d\sigma_{UT}^{11} + \sin \phi_S d\sigma_{UT}^{12}) \right. \\
 & \quad \left. + \lambda_e \left[\cos(\phi - \phi_S) d\sigma_{LT}^{13} + \frac{1}{Q} (\cos \phi_S d\sigma_{LT}^{14} + \cos(2\phi - \phi_S) d\sigma_{LT}^{15}) \right] \right\}
 \end{aligned}$$

σ_{XY}
 Beam Target
 Polarization



Mulders and Tangemann, Nucl. Phys. B 461 (1996) 197

Boer and Mulders, Phys. Rev. D 57 (1998) 5780

Bacchetta et al., Phys. Lett. B 595 (2004) 309

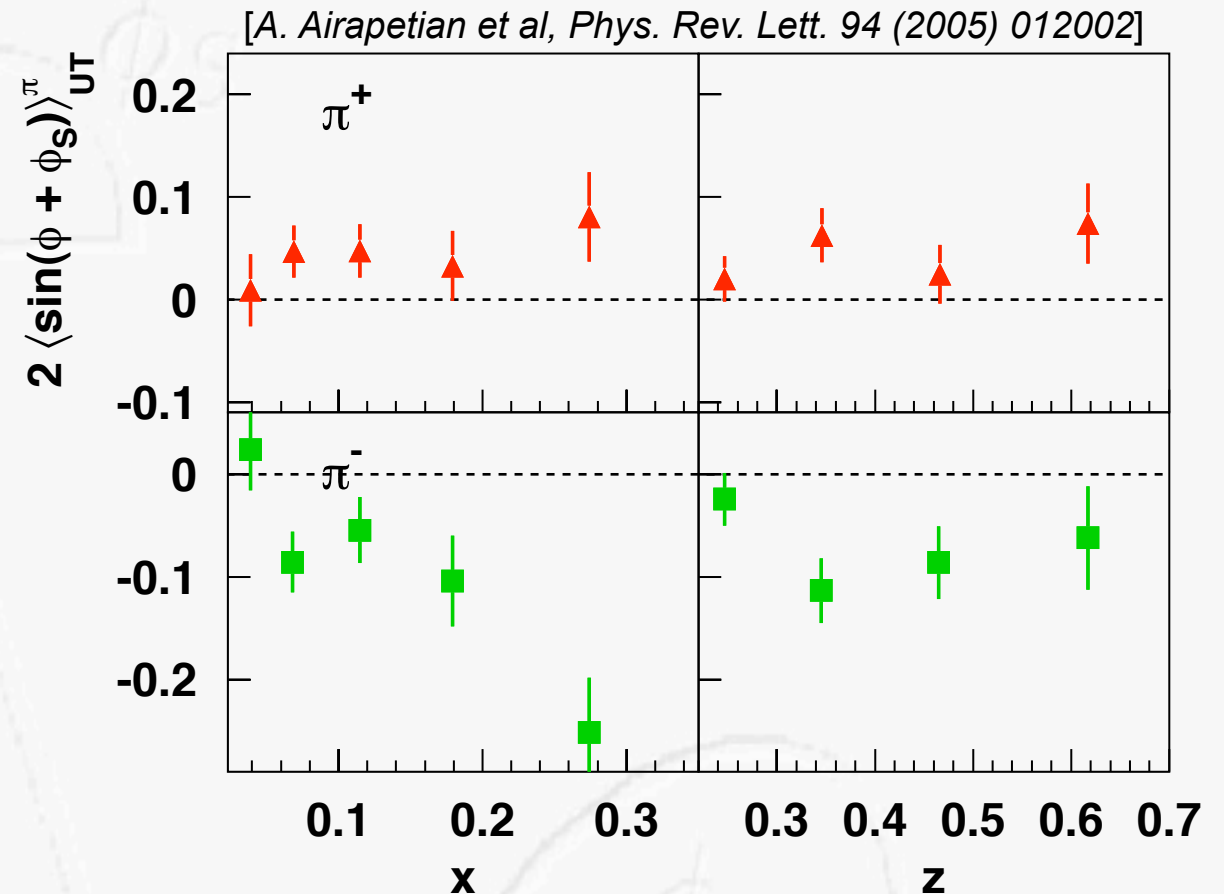
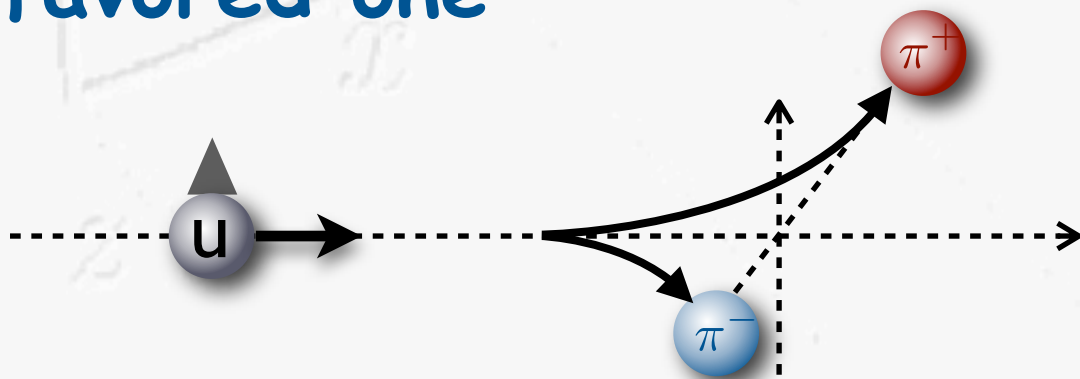
Bacchetta et al., JHEP 0702 (2007) 093

“Trento Conventions”, Phys. Rev. D 70 (2004) 117504

Transversity distribution (Collins fragmentation)

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1, h_{1T}^\perp$

- significant in size and opposite in sign for charged pions
- disfavored Collins FF large and opposite in sign to favored one



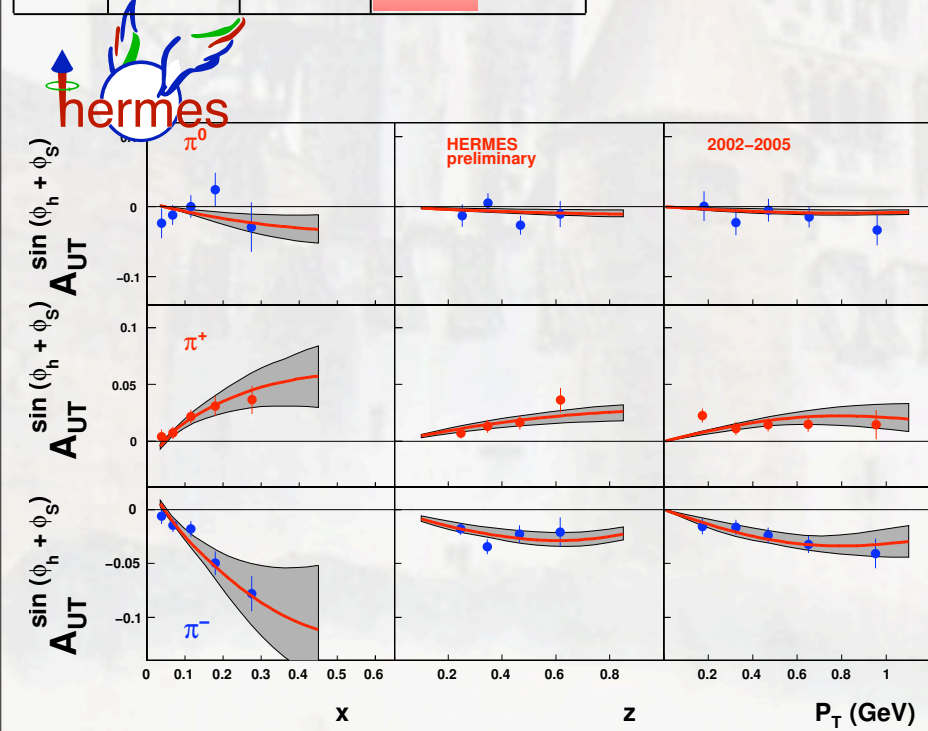
2005: First evidence from HERMES
SIDIS on proton

Non-zero transversity
Non-zero Collins function

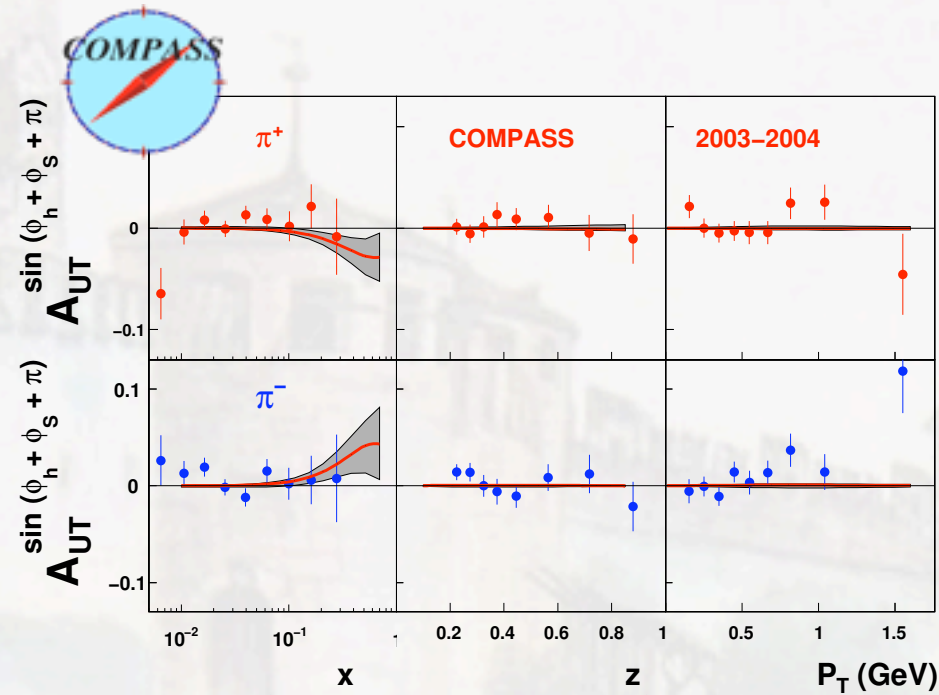
- leads to various cancellations in SSA observables

Fit of Collins amplitudes

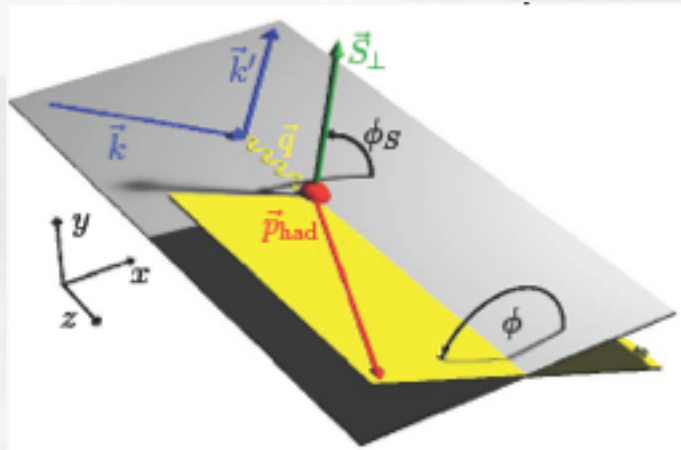
	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1, h_{1T}^\perp$



$$e^\pm p^\uparrow \rightarrow e^\pm \pi X$$

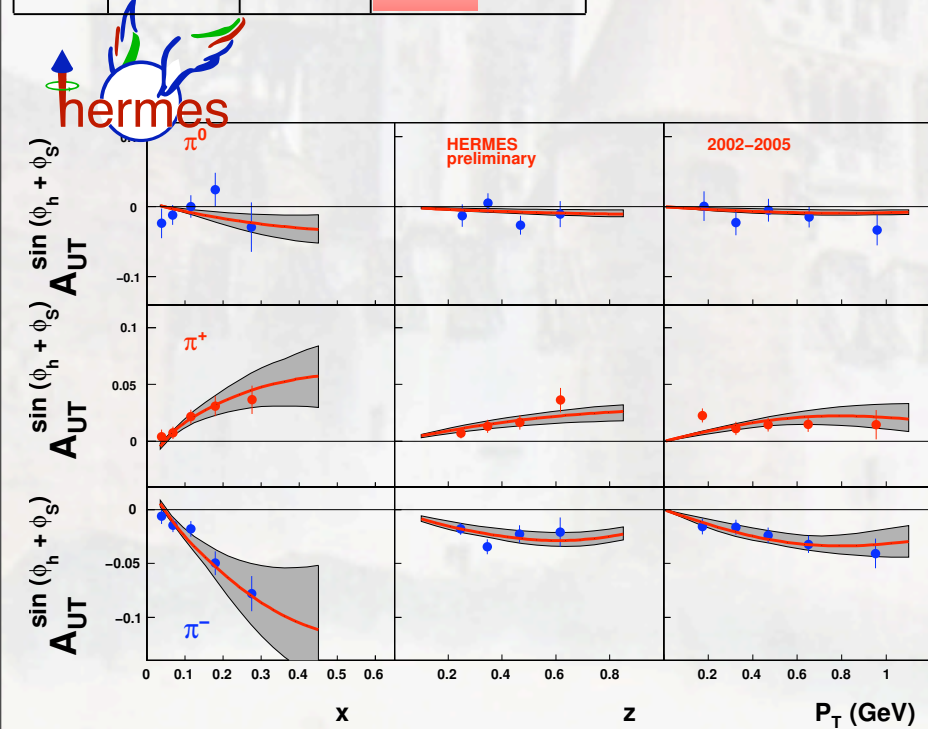


$$\mu^\pm d^\uparrow \rightarrow \mu^\pm \pi X$$

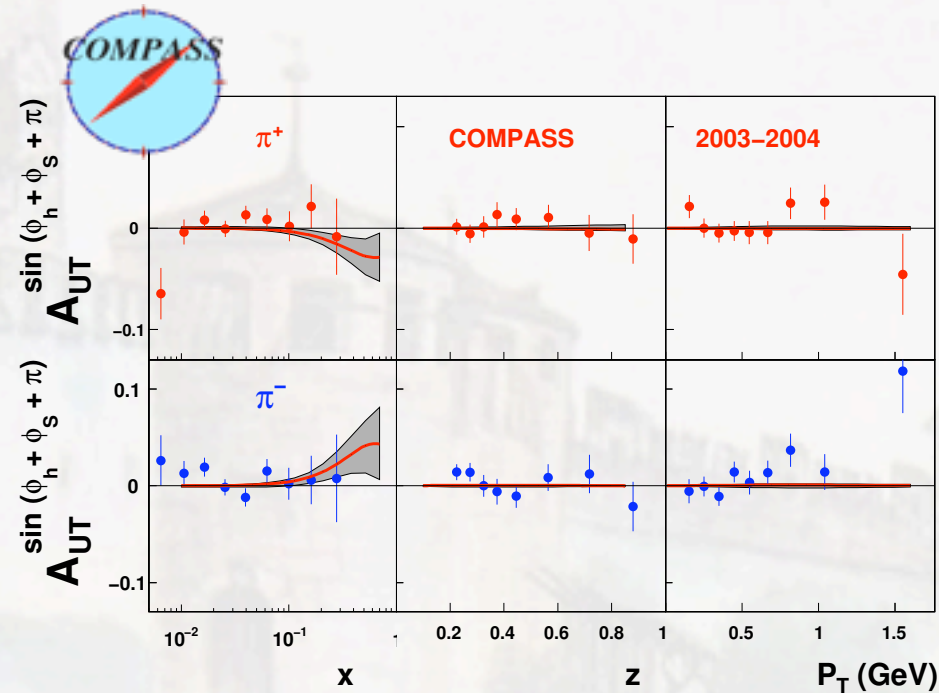
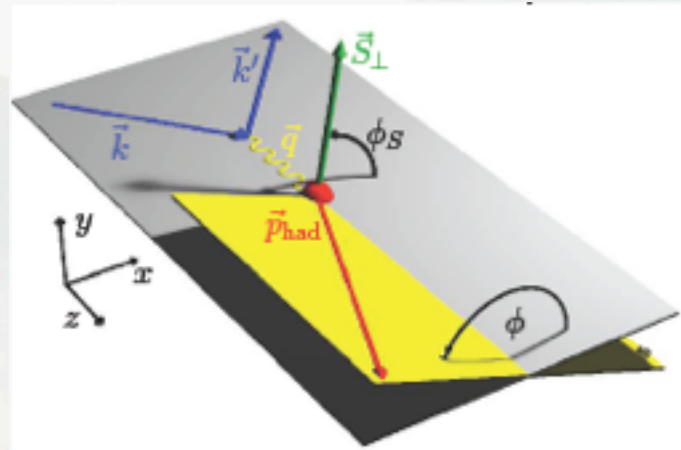


Fit of Collins amplitudes

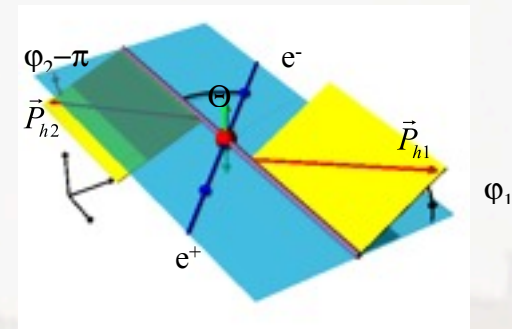
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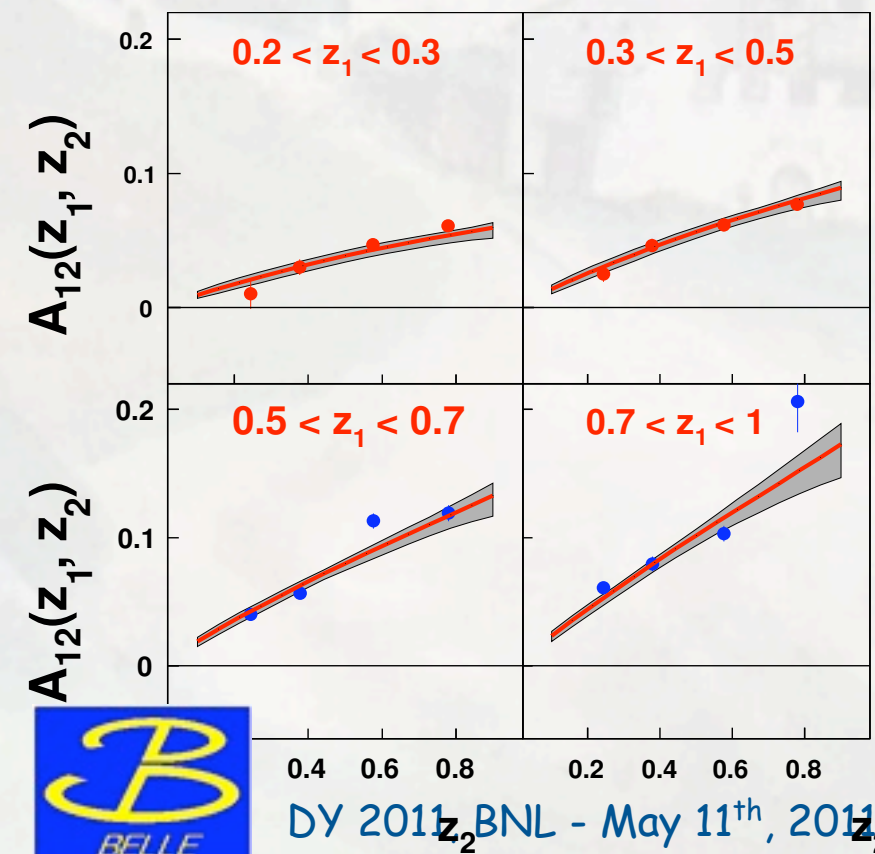
$$e^\pm p^\uparrow \rightarrow e^\pm \pi X$$



$$\mu^\pm d^\uparrow \rightarrow \mu^\pm \pi X$$

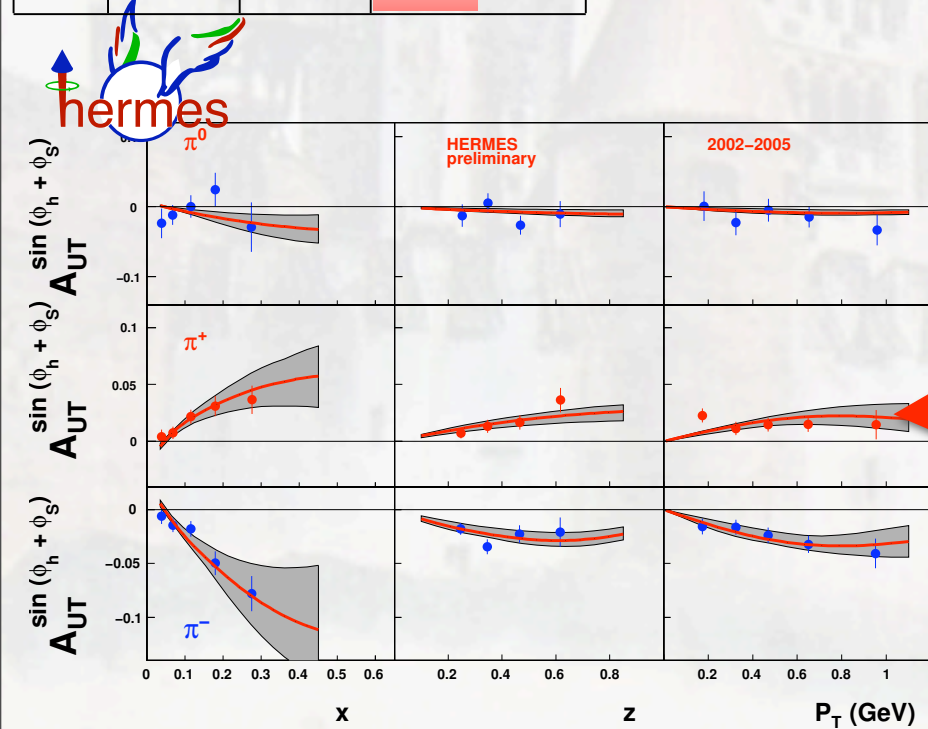


$$e^+e^- \rightarrow \pi\pi X$$

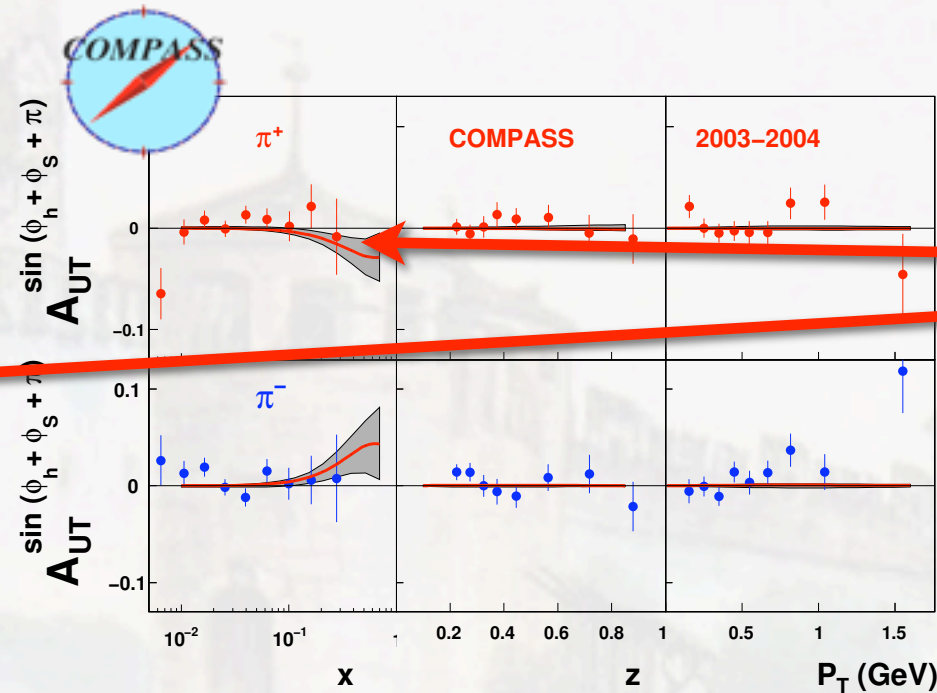
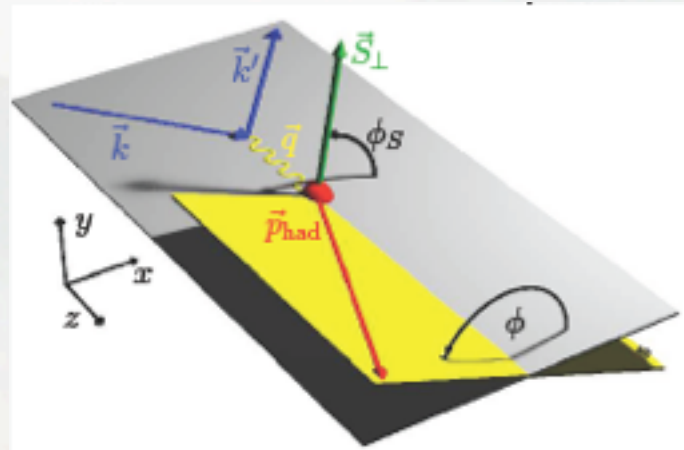


Fit of Collins amplitudes

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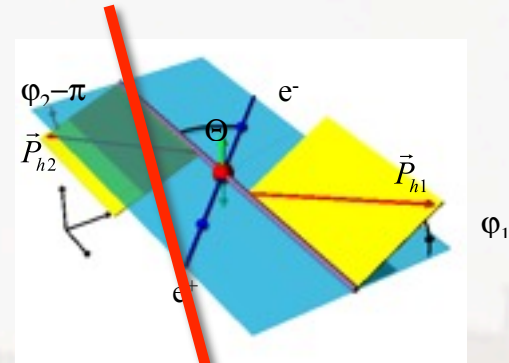


$$e^\pm p^\uparrow \rightarrow e^\pm \pi X$$

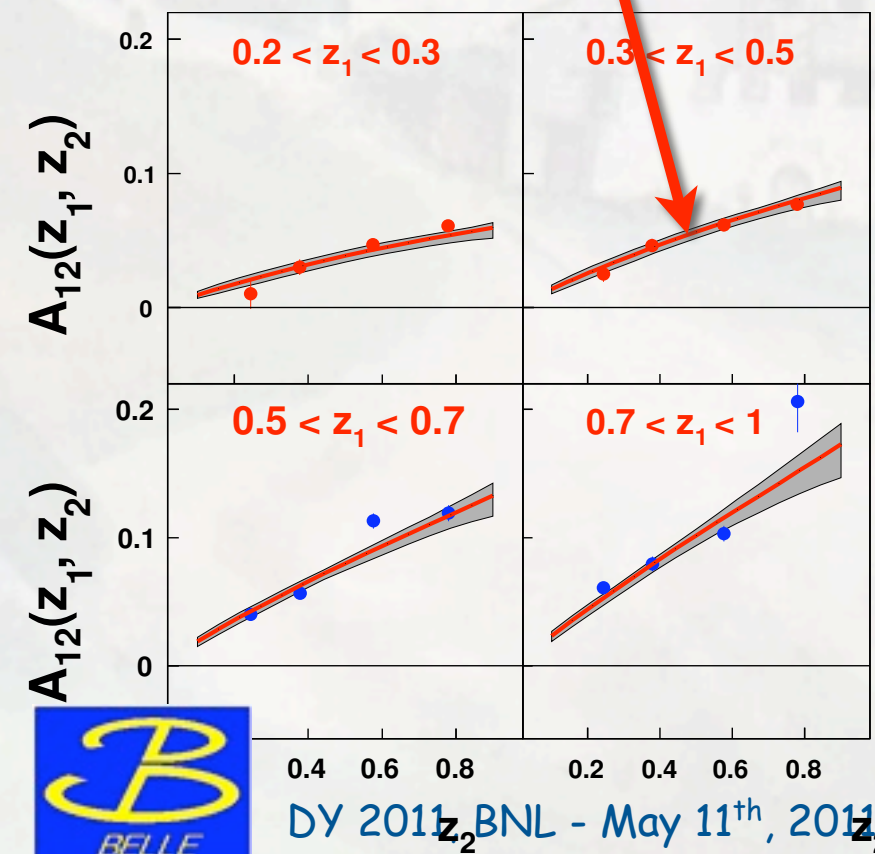


$$\mu^\pm d^\uparrow \rightarrow \mu^\pm \pi X$$

fit to data

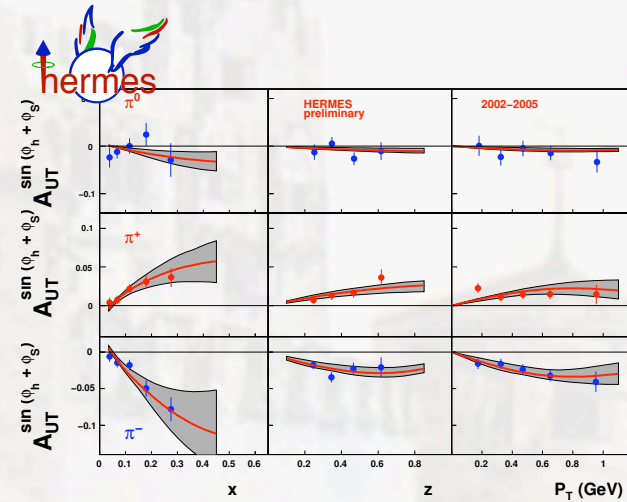


$$e^+e^- \rightarrow \pi\pi X$$

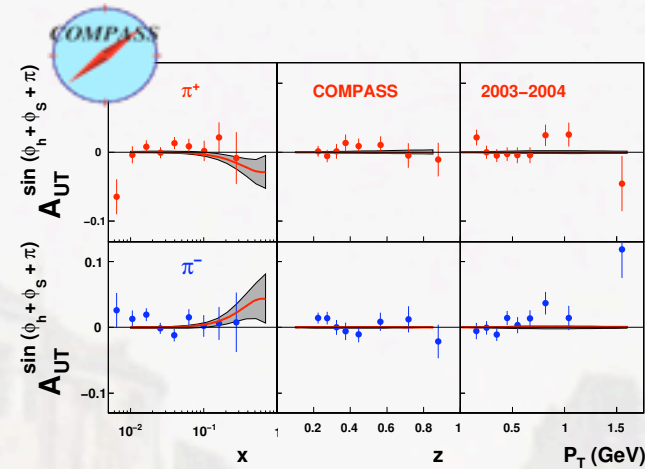


Fit of Collins amplitudes

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L		g_{1L}	$h_{1L}^\perp$
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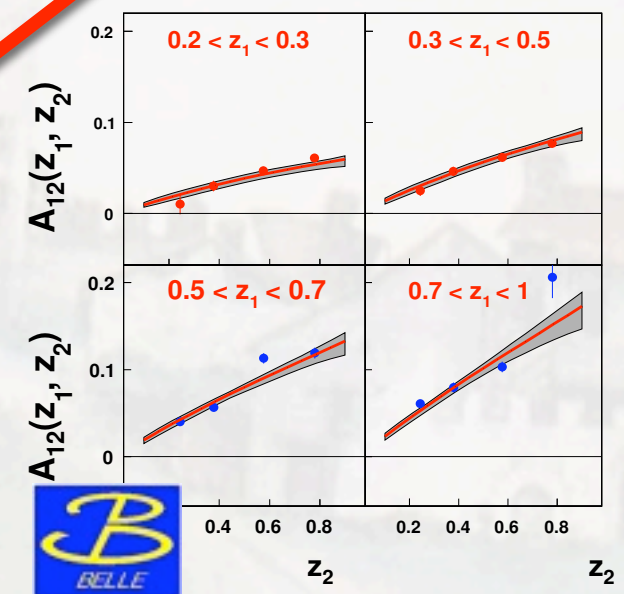
$$e^\pm p \uparrow \rightarrow e^\pm \pi X$$



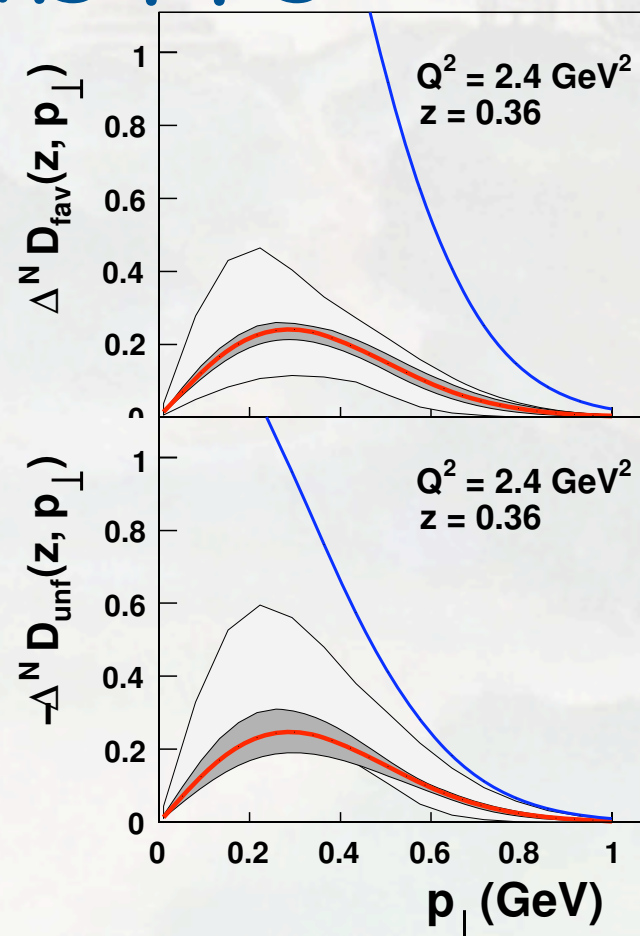
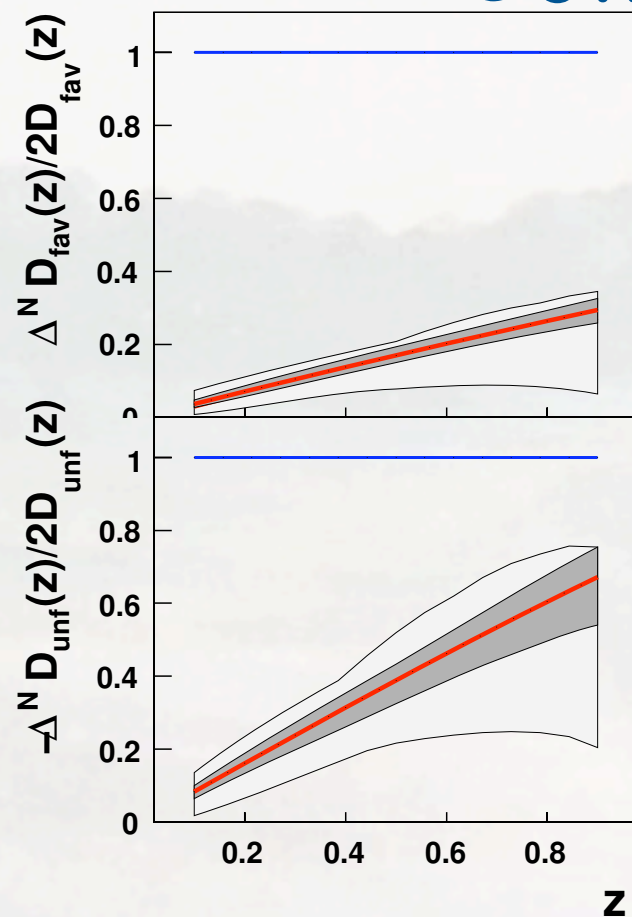
$$\mu^\pm d \uparrow \rightarrow \mu^\pm \pi X$$

fit to data

$$e^+ e^- \rightarrow \pi\pi X$$



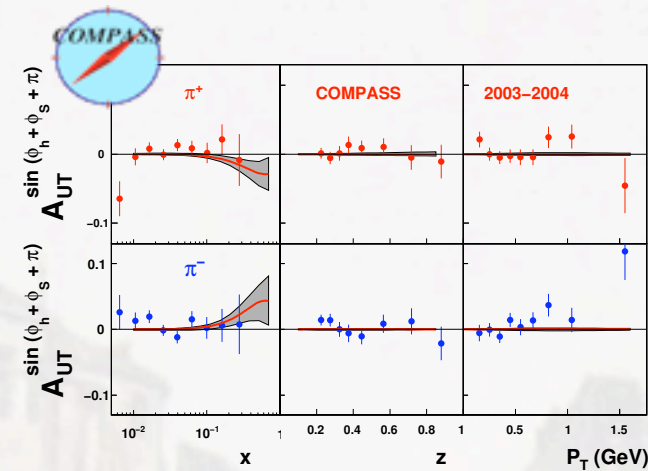
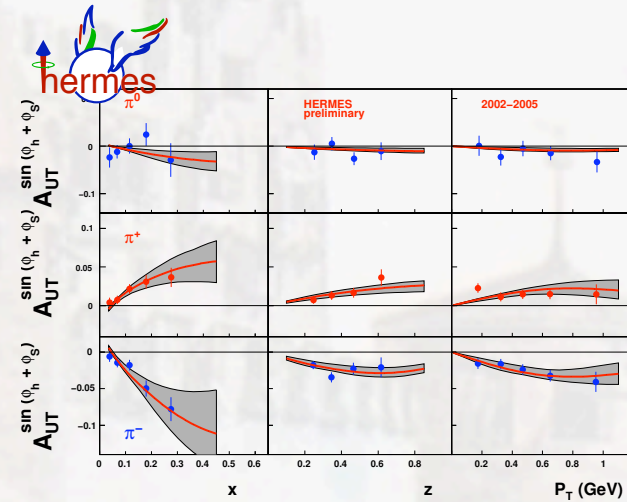
Collins FFs



[Anselmino et al., Nucl.Phys.Proc.Suppl.191 (2009) 98]

Fit of Collins amplitudes

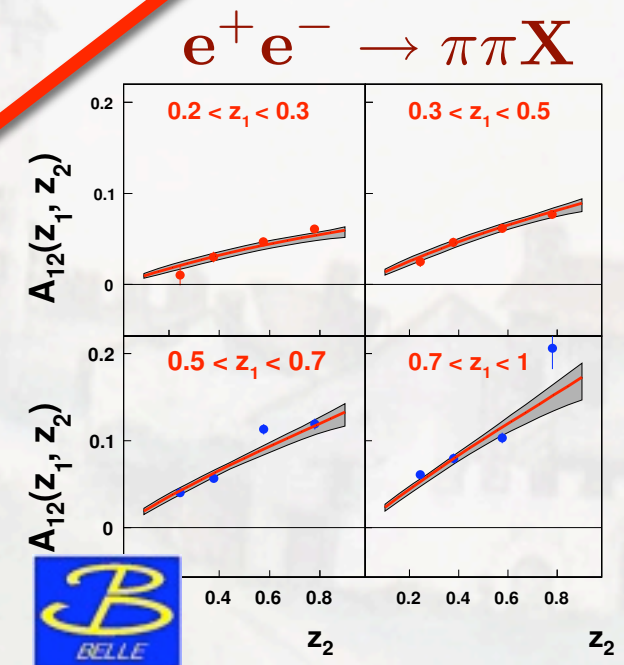
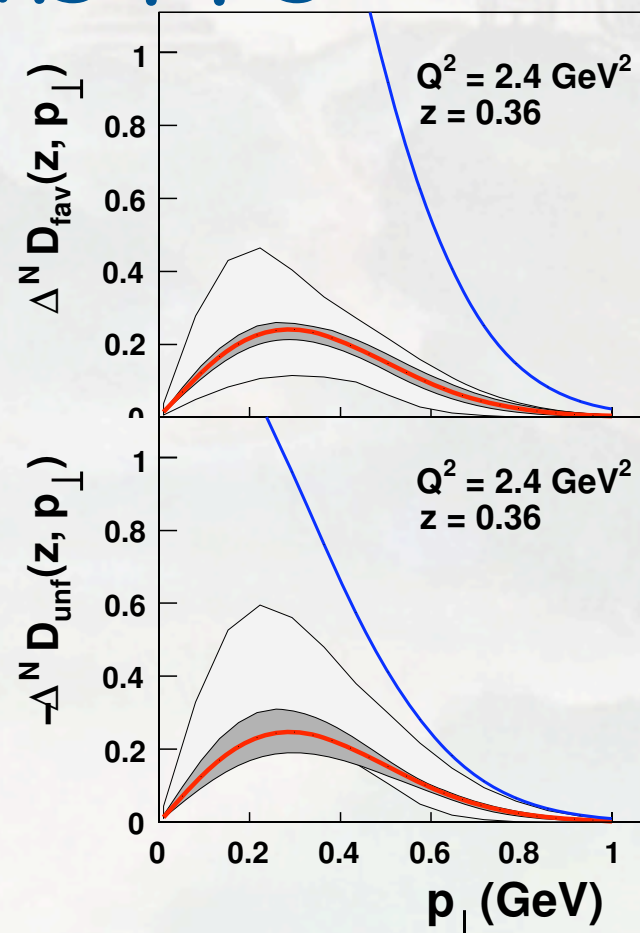
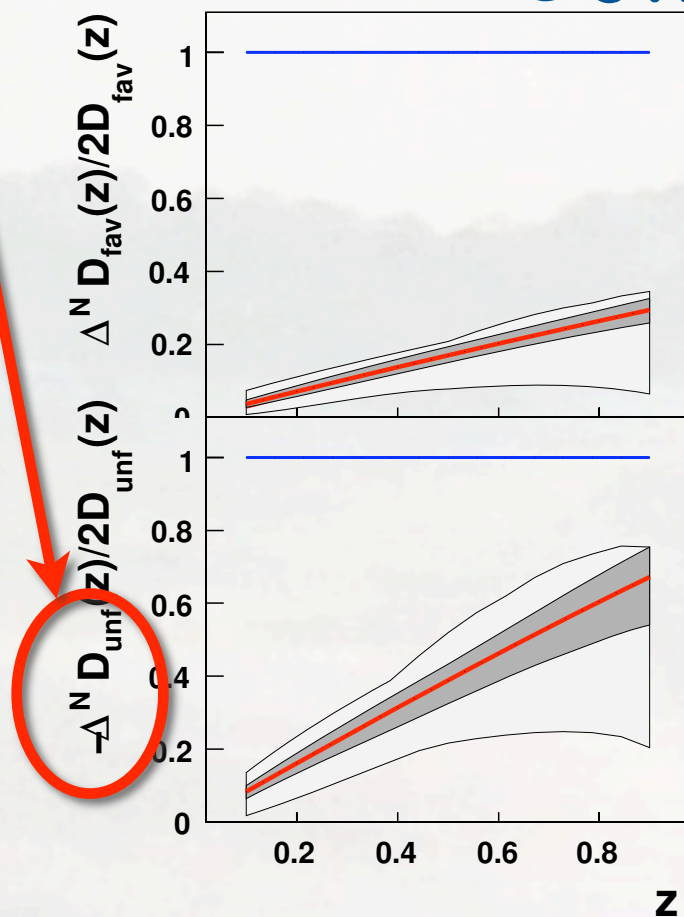
	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1, h_{1T}^\perp$



fit to data

opposite sign

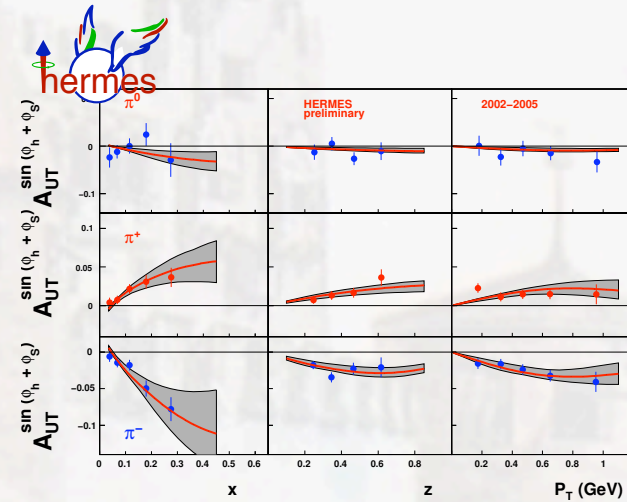
Collins FFs



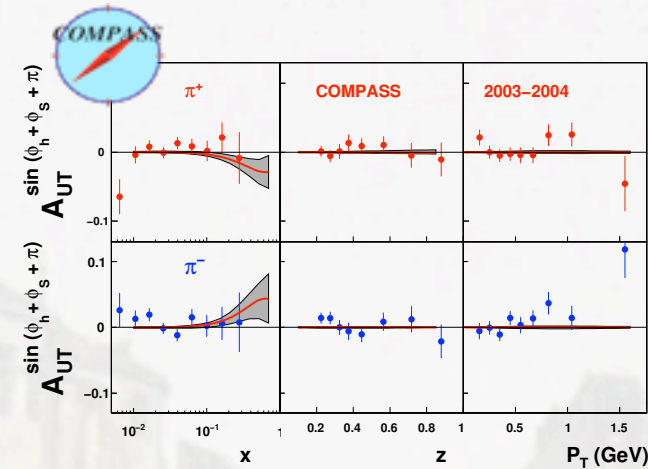
[Anselmino et al., Nucl.Phys.Proc.Suppl.191 (2009) 98]

Fit of Collins amplitudes

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1, h_{1T}^\perp$



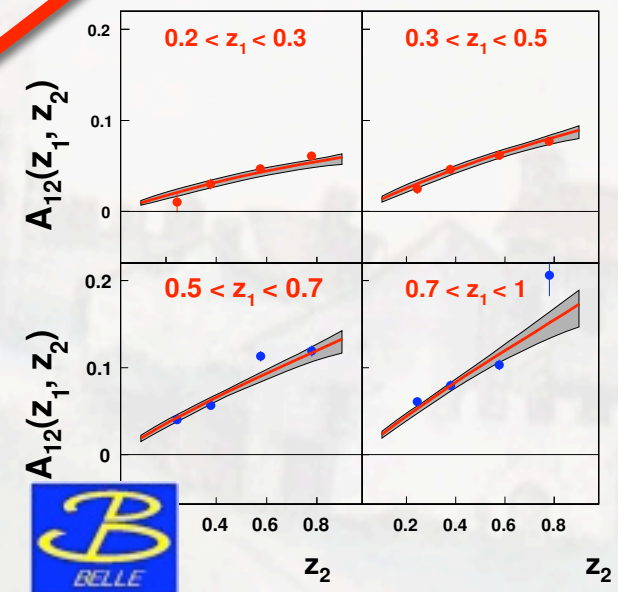
$$e^\pm p^\uparrow \rightarrow e^\pm \pi X$$



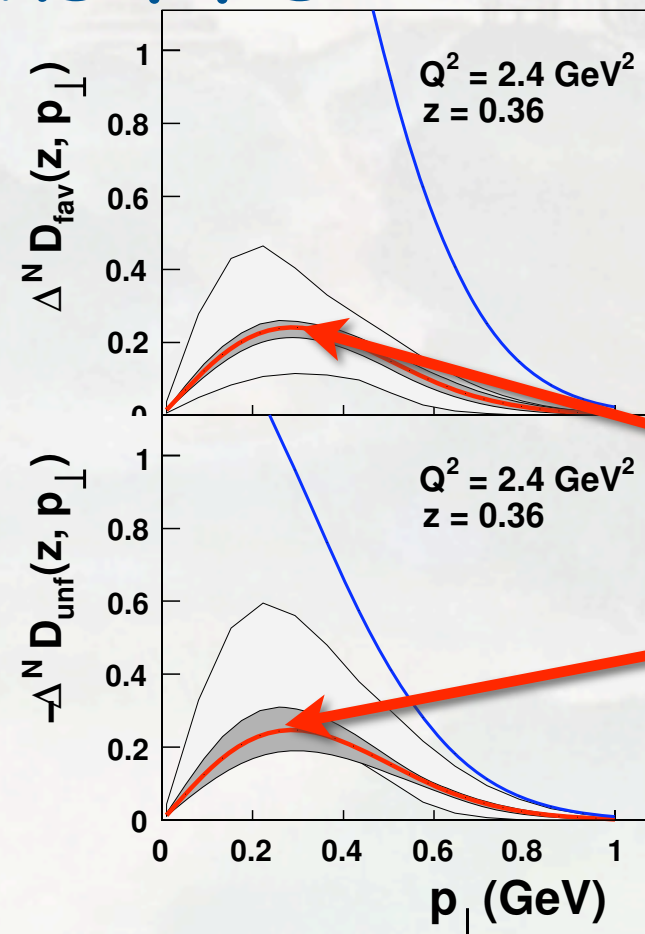
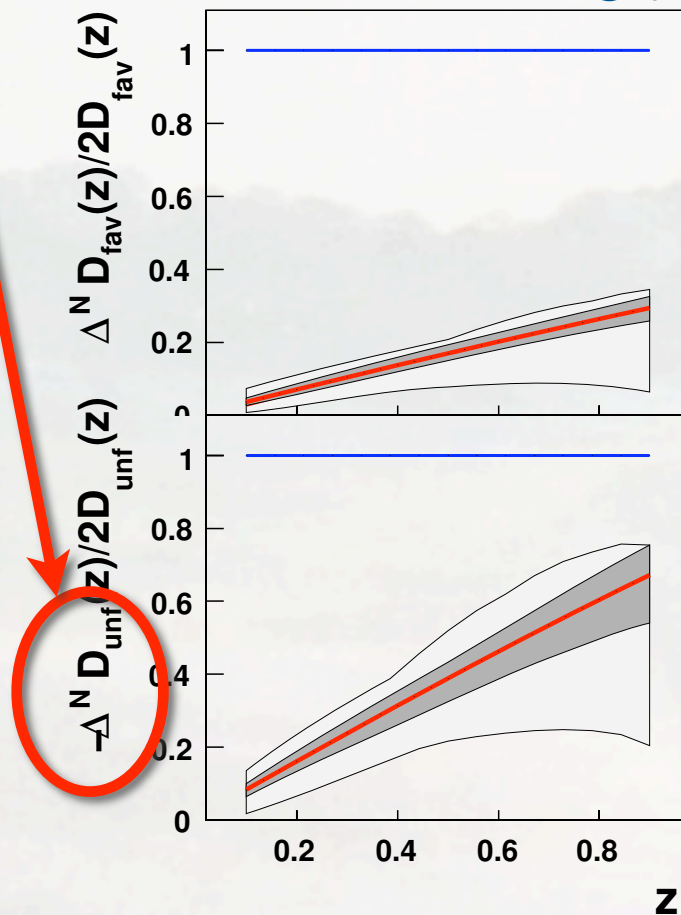
$$\mu^\pm d^\uparrow \rightarrow \mu^\pm \pi X$$

fit to data

$$e^+e^- \rightarrow \pi\pi X$$



Collins FFs



similar magnitude

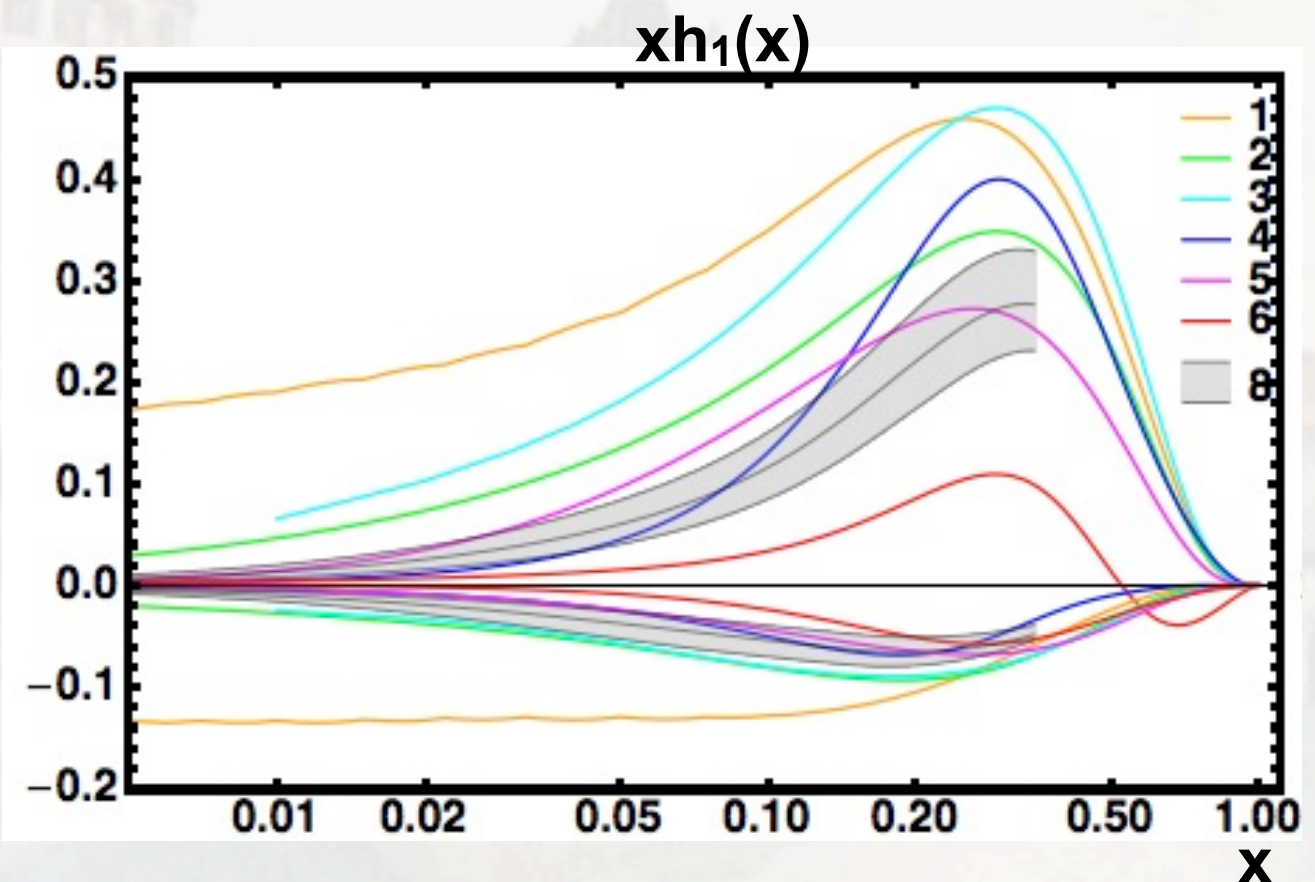
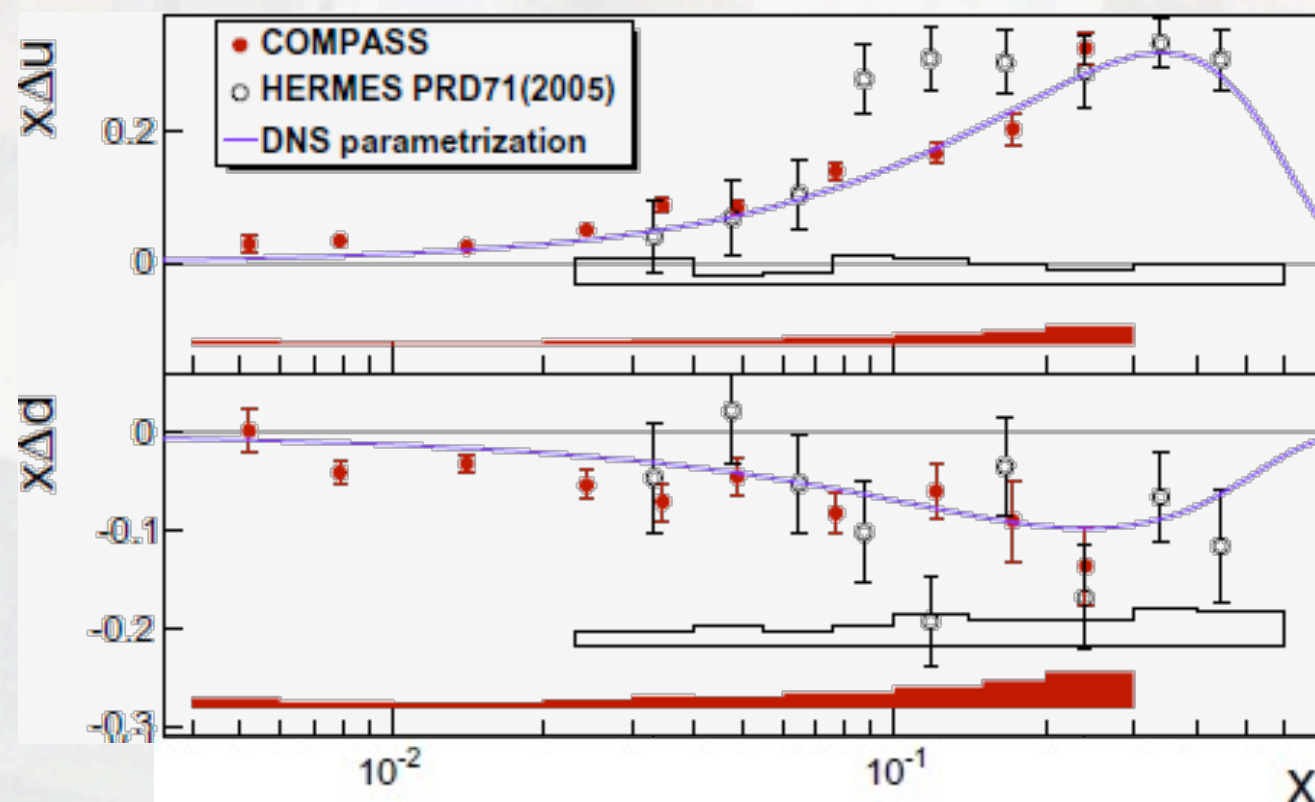
[Anselmino et al., Nucl.Phys.Proc.Suppl.191 (2009) 98]

Transversity: models and fits

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1, h_{1T}^\perp$

- [1] Soffer et al. PRD 65 (02)
- [2] Korotkov et al. EPJC 18 (01)
- [3] Schweitzer et al., PRD 64 (01)
- [4] Wakamatsu, PLB 509 (01)

- [5] Pasquini et al., PRD 72 (05)
- [6] Bacchetta, Conti, Radici, PRD 78 (08)
- [7] Anselmino et al., PRD 75 (07)
- [8] Anselmino et al., arXiv:0807.0173

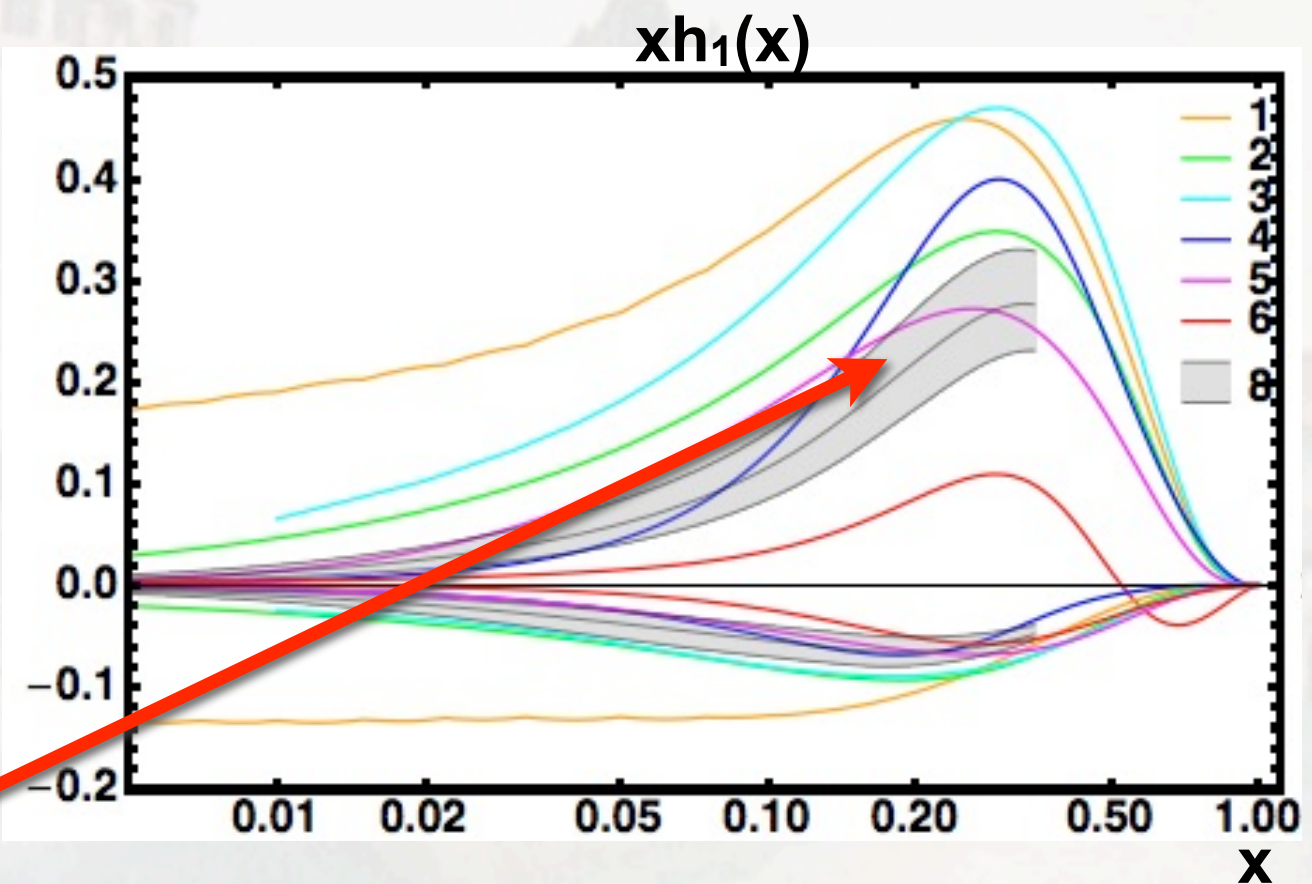
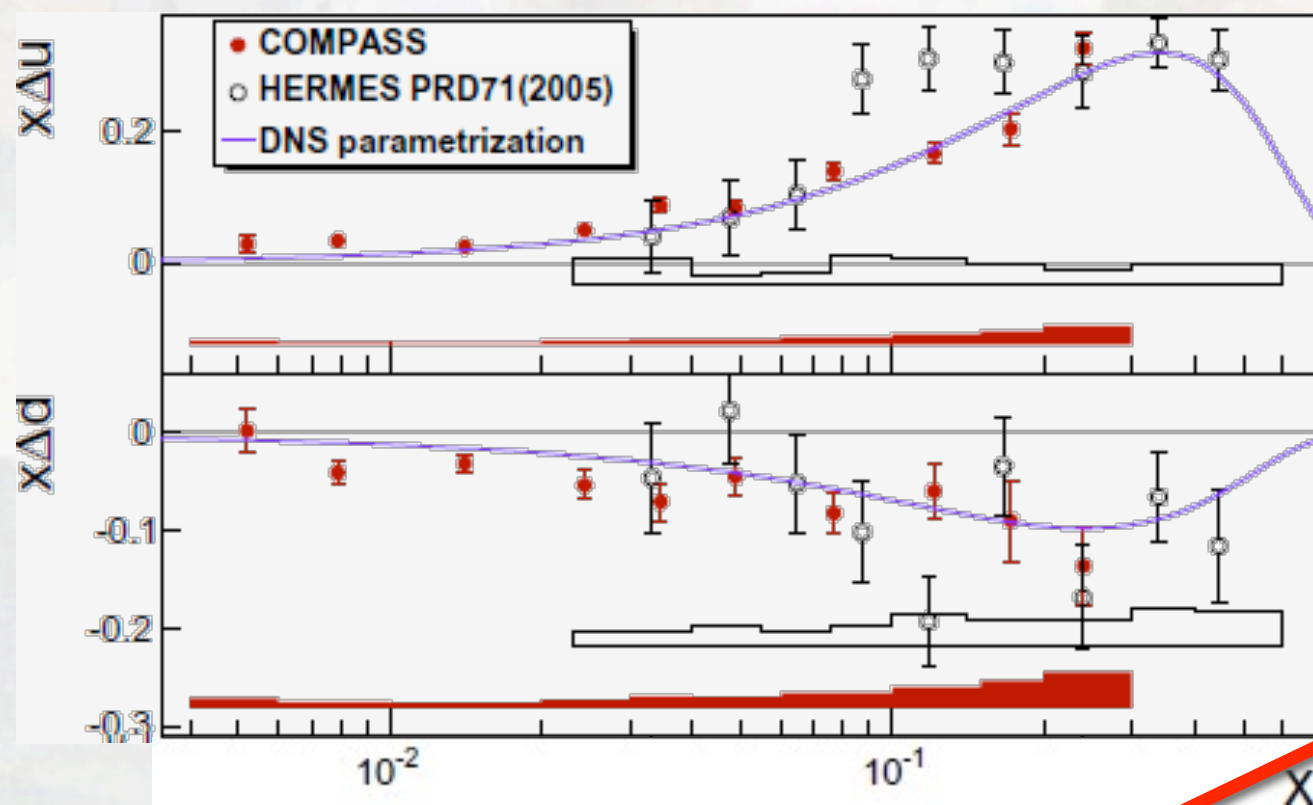


Transversity: models and fits

	U	L	T
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L		g_{1L}	$h_{1L}^\perp$
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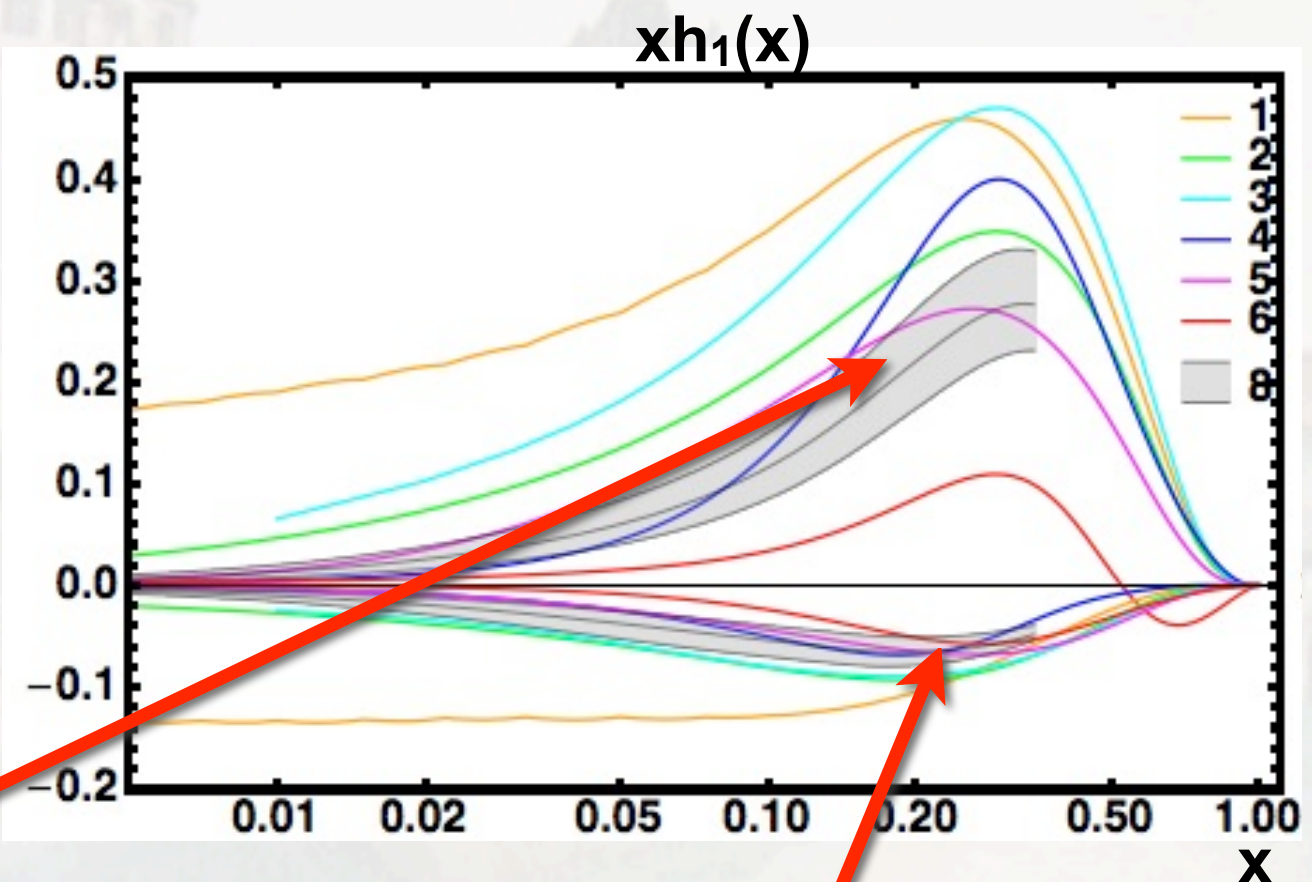
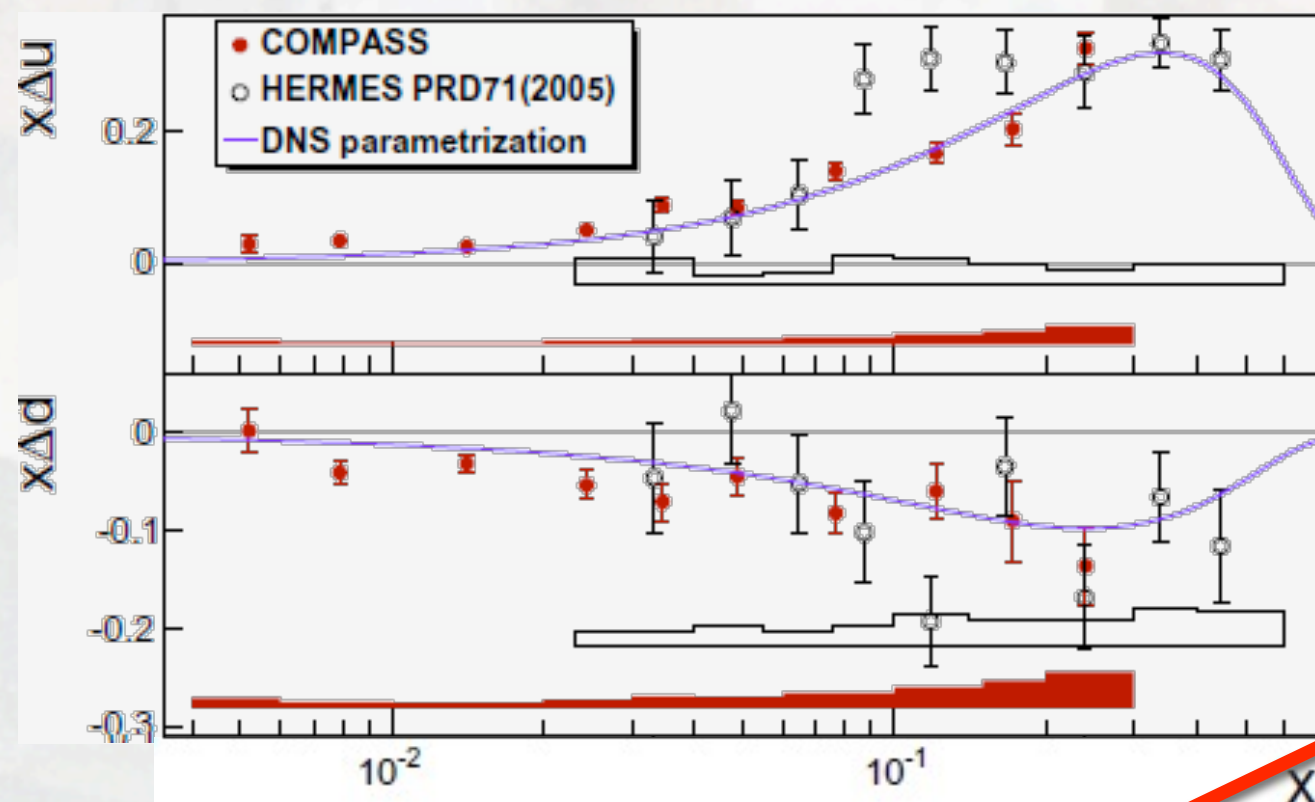
☑ u quark transversity along nucleon spin

Transversity: models and fits

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
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- [1] Soffer et al. PRD 65 (02)
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- [4] Wakamatsu, PLB 509 (01)

- [5] Pasquini et al., PRD 72 (05)
- [6] Bacchetta, Conti, Radici, PRD 78 (08)
- [7] Anselmino et al., PRD 75 (07)
- [8] Anselmino et al., arXiv:0807.0173



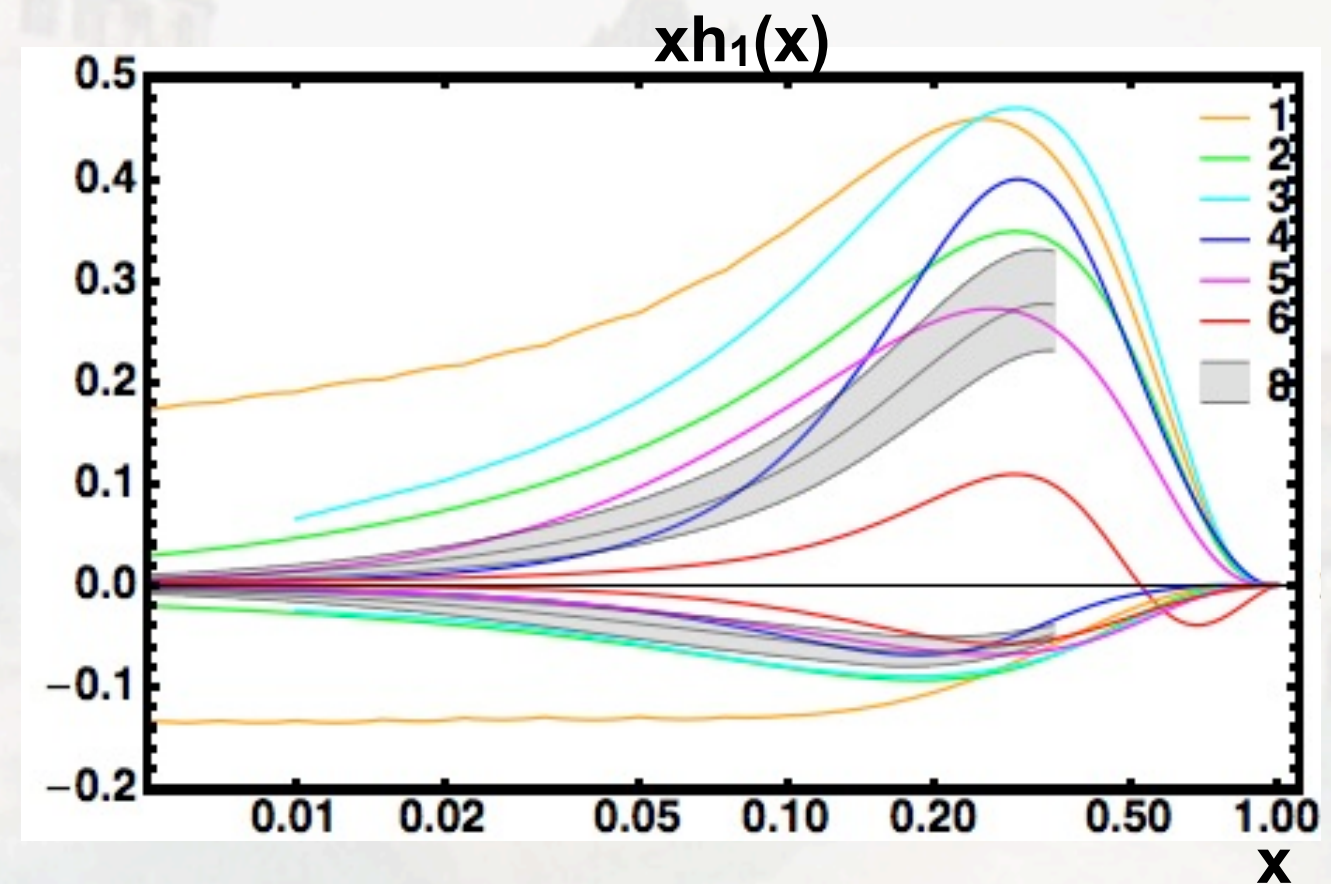
- ☑ u quark transversity along nucleon spin
- ☑ d quark transversity anti-parallel to nucleon spin

Transversity: models and fits

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1, h_{1T}^\perp$

- [1] Soffer et al. PRD 65 (02)
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- [6] Bacchetta, Conti, Radici, PRD 78 (08)
- [7] Anselmino et al., PRD 75 (07)
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tensor charge:

$$\delta q \equiv \int_0^1 dx [h_1^q(x) - h_1^{\bar{q}}(x)]$$

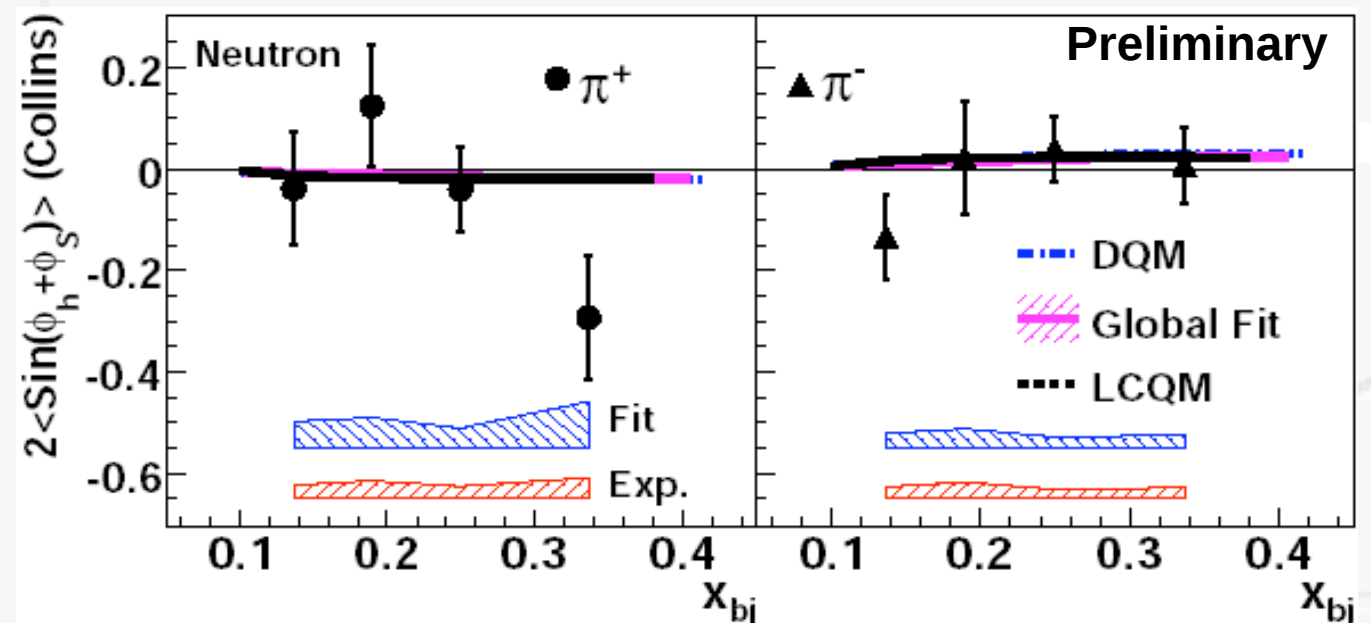
Transversity distribution (Collins fragmentation)

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L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1, h_{1T}^\perp$

- wealth of new results available and/or analyses ongoing

- JLab
- COMPASS
- HERMES
- BELLE
- BaBar

³He Target Single-Spin Asymmetry in SIDIS: JLab E06-010



Transversity distribution (Collins fragmentation)

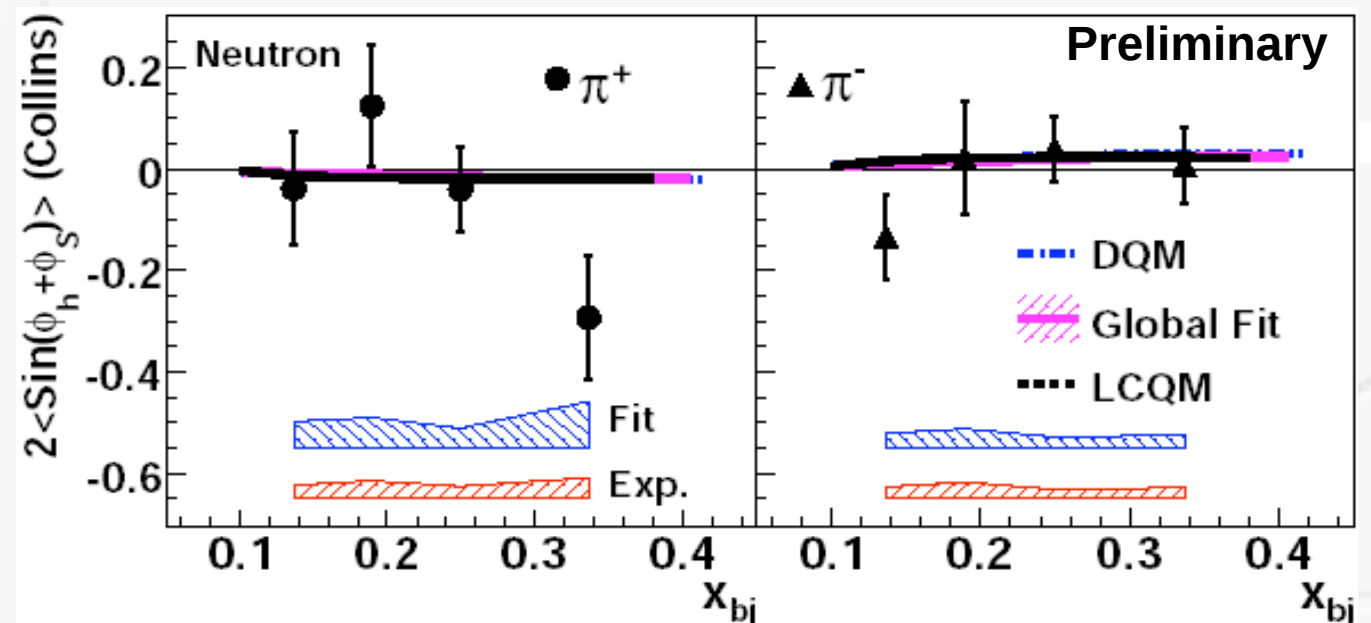
	U	L	T
U	f_1		$h_1^\perp$
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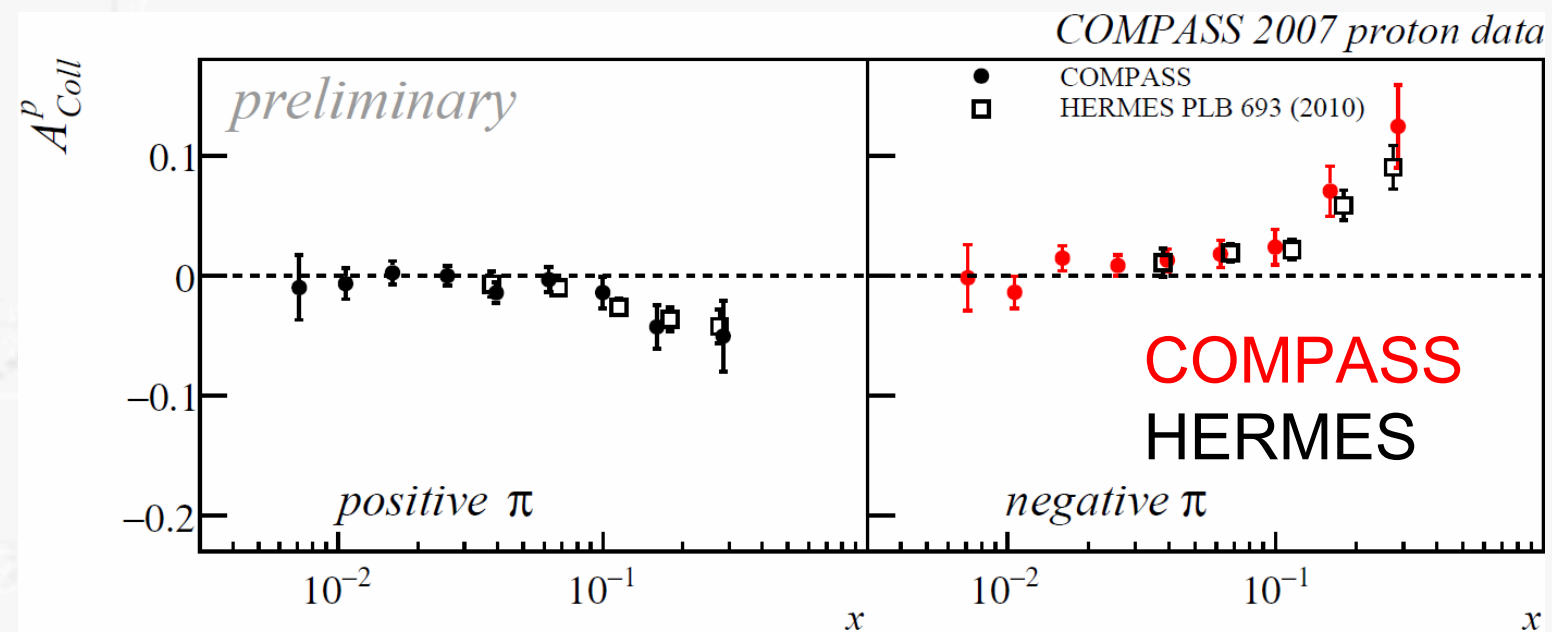
- JLab
- COMPASS
- HERMES
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^3He Target Single-Spin Asymmetry in SIDIS: JLab E06-010



Collins amplitudes COMPASS & HERMES

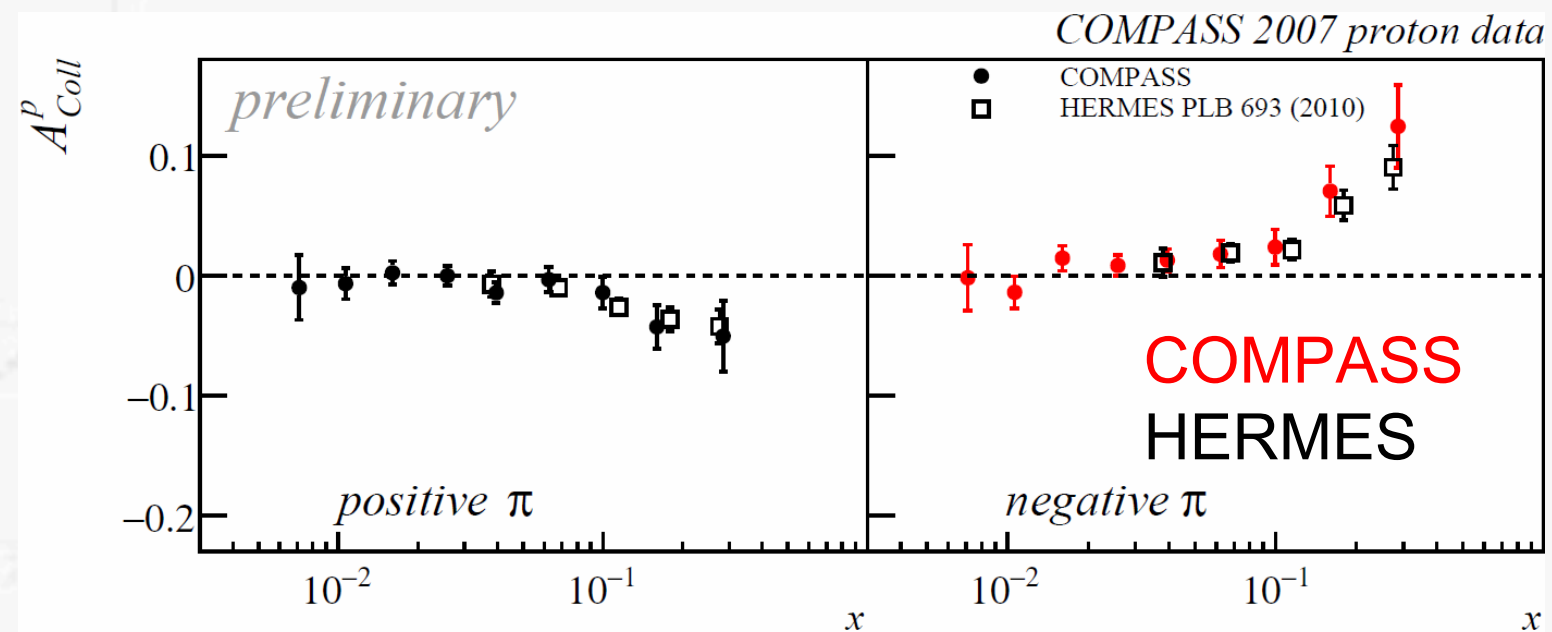
	U	L	T
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Collins amplitudes COMPASS & HERMES

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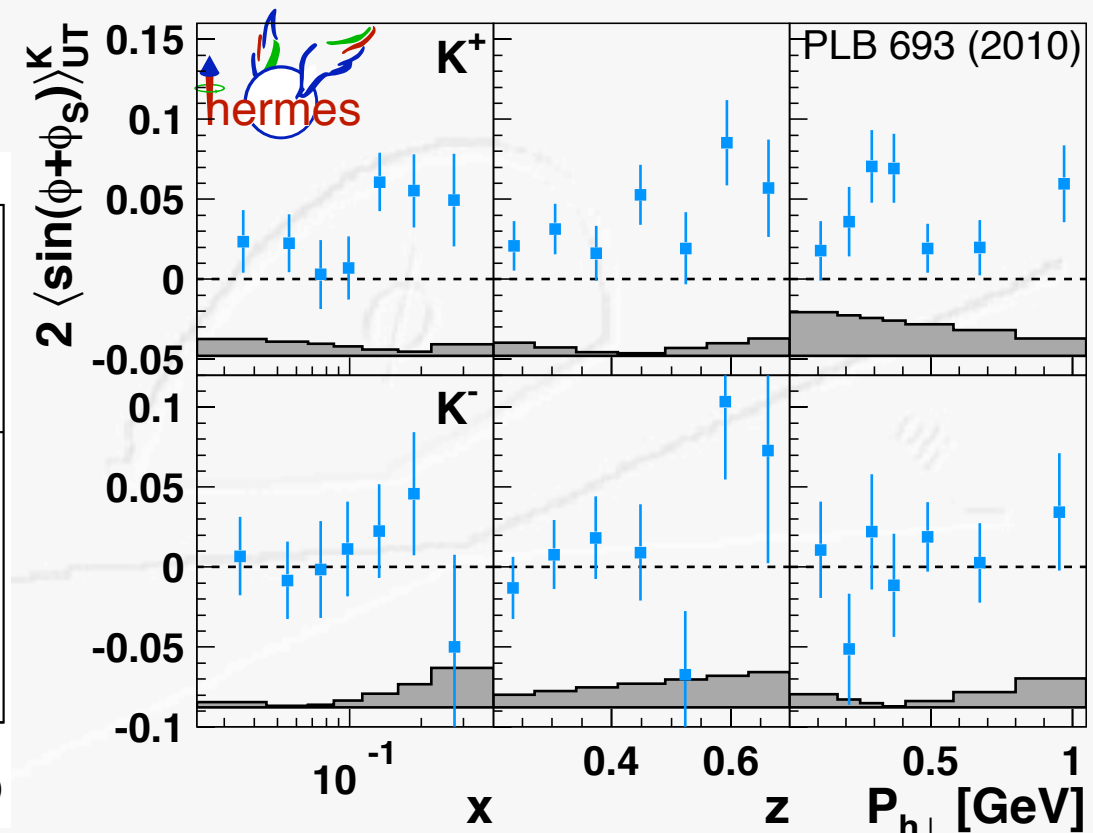
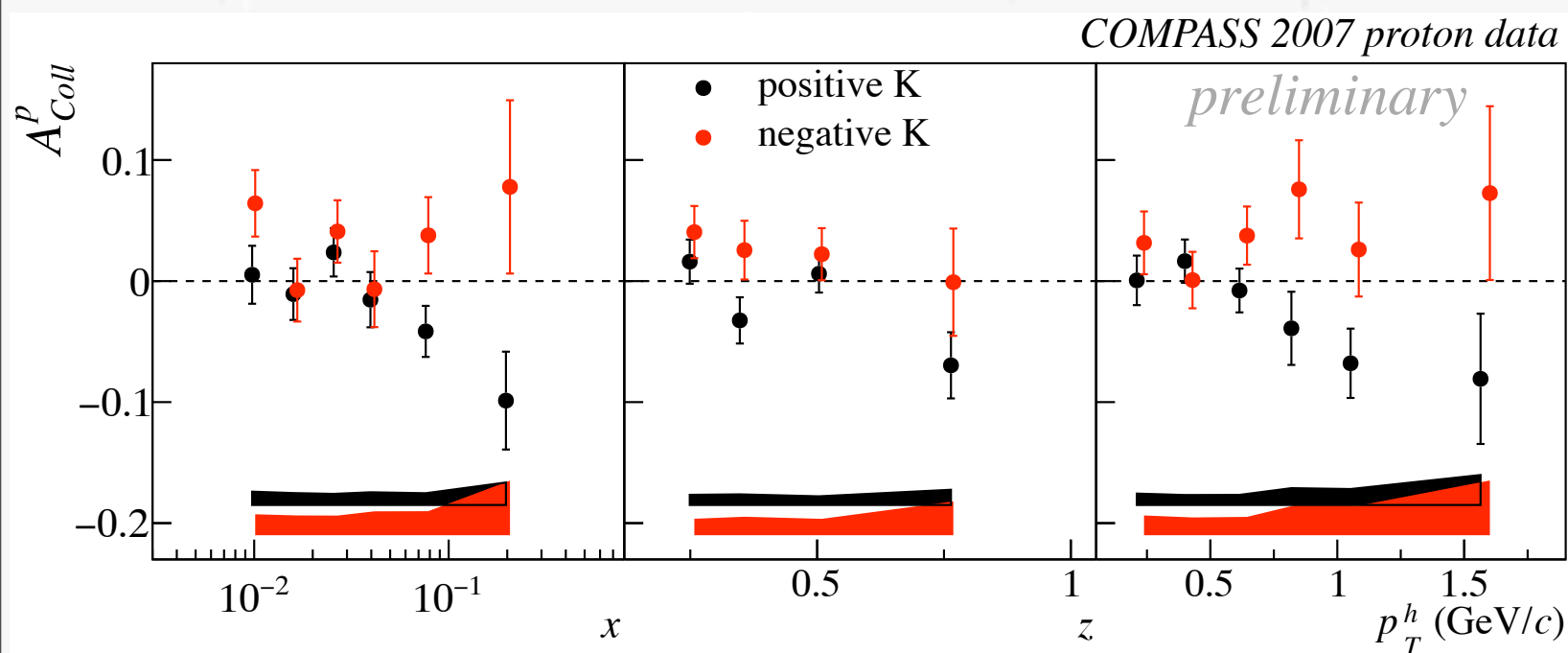
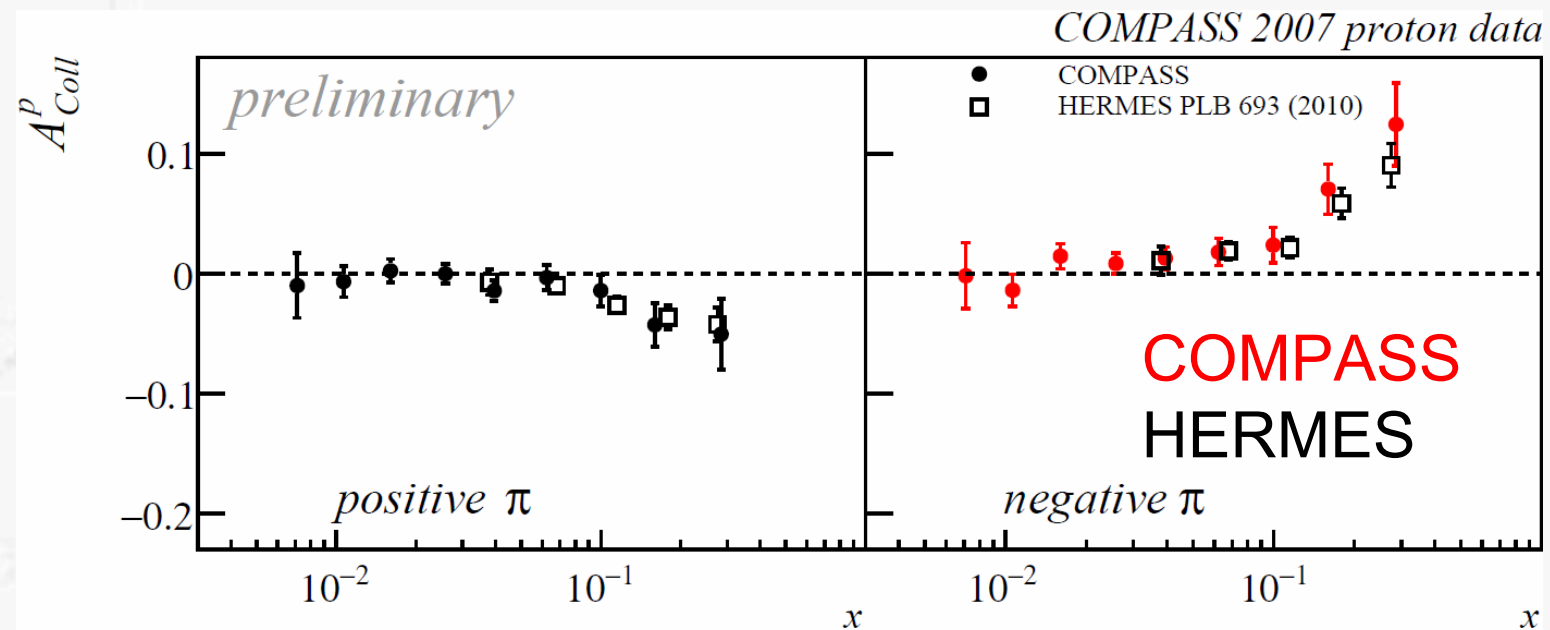
- similar behavior for pions



Collins amplitudes COMPASS & HERMES

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U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1, h_{1T}^\perp$

- similar behavior for pions
- similar behavior for K^+
- different trend for K^-
- opposite sign conventions!



Transversity's friends

Pretzelosity

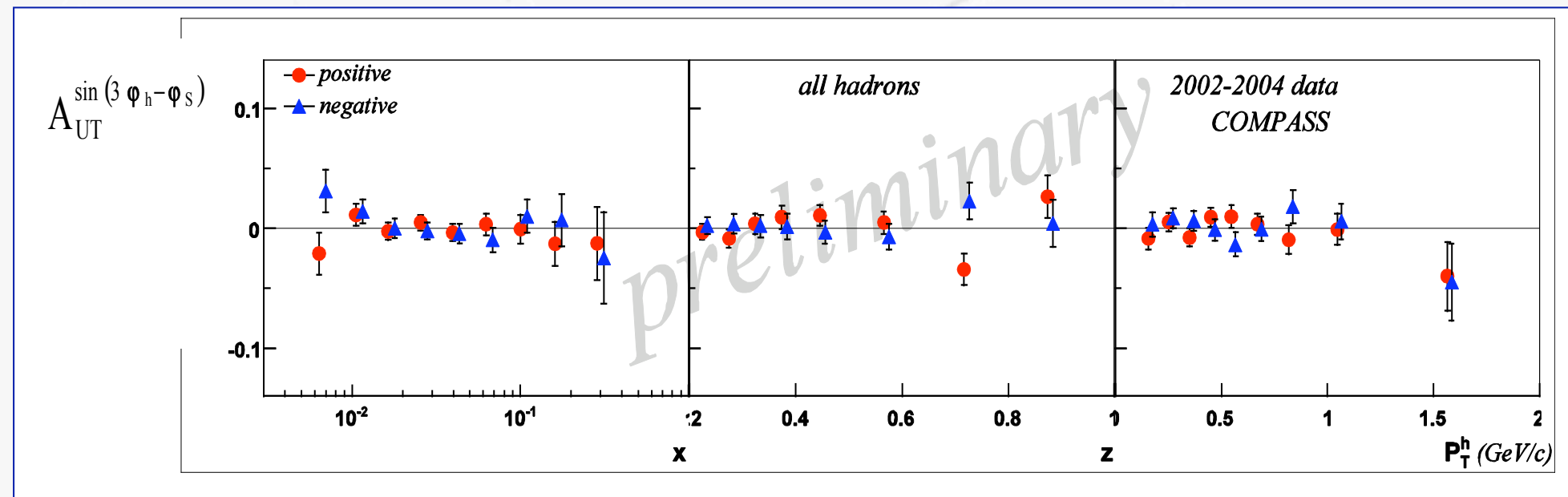
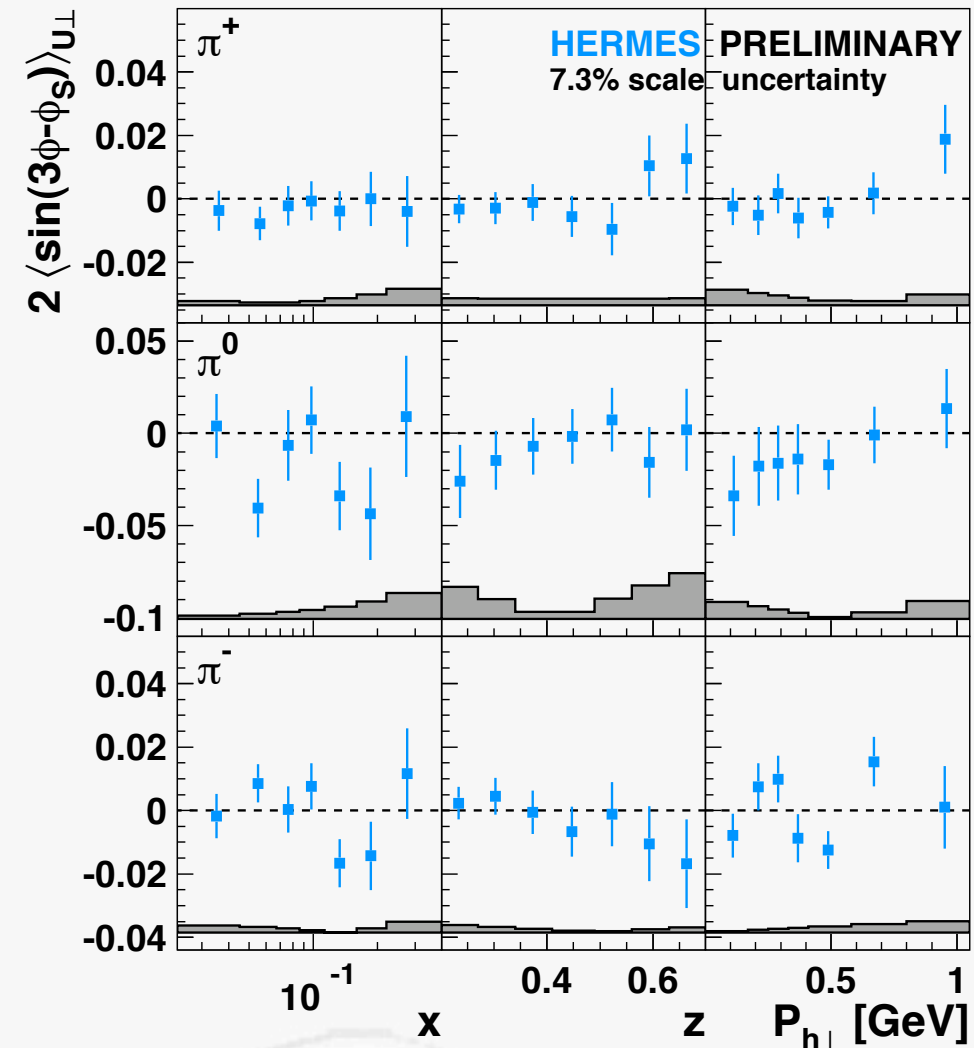
	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1, h_{1T}^\perp$

- chiral-odd $\Rightarrow$ needs Collins FF (or similar)
- leads to $\sin(3\phi - \phi_s)$ modulation in A_{UT}
- proton and deuteron data consistent with zero
- cancelations? pretzelosity=zero?
or just the additional suppression by two powers of $P_{h\perp}$

Pretzelosity

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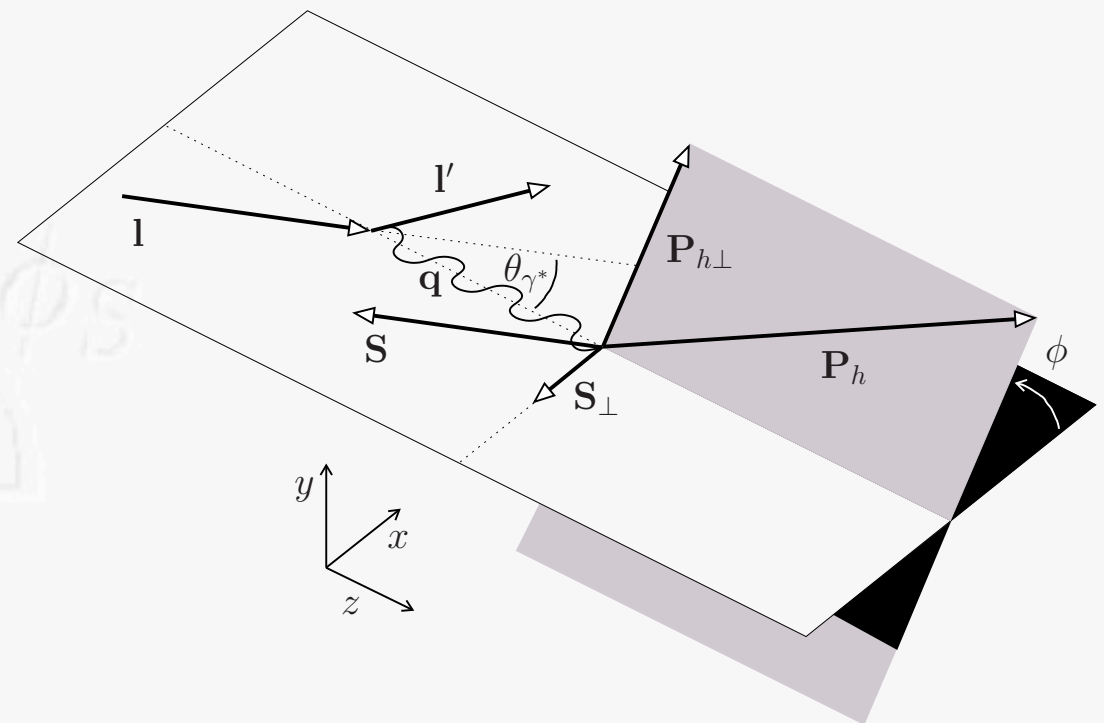


Pretzelosity

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1, h_{1T}^\perp$

- could also use longitudinally polarized targets:

$$\sin(\mathbf{3}\phi - \phi_S) \rightsquigarrow -\sin(\mathbf{3}\phi)$$

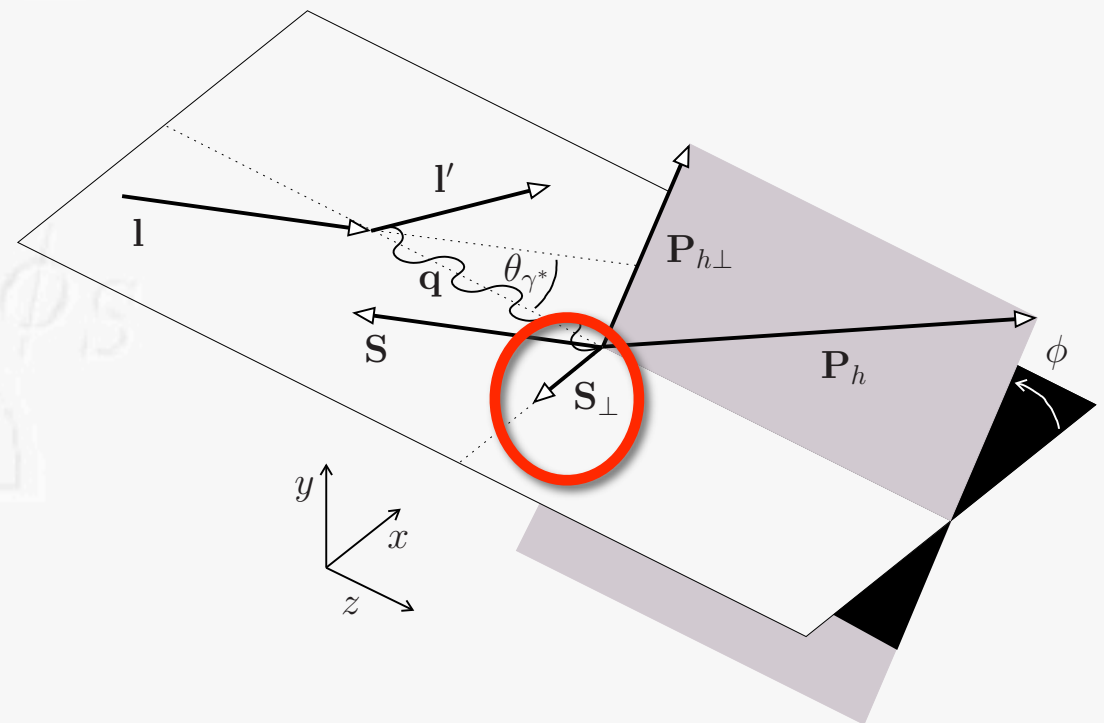


Pretzelosity

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1, h_{1T}^\perp$

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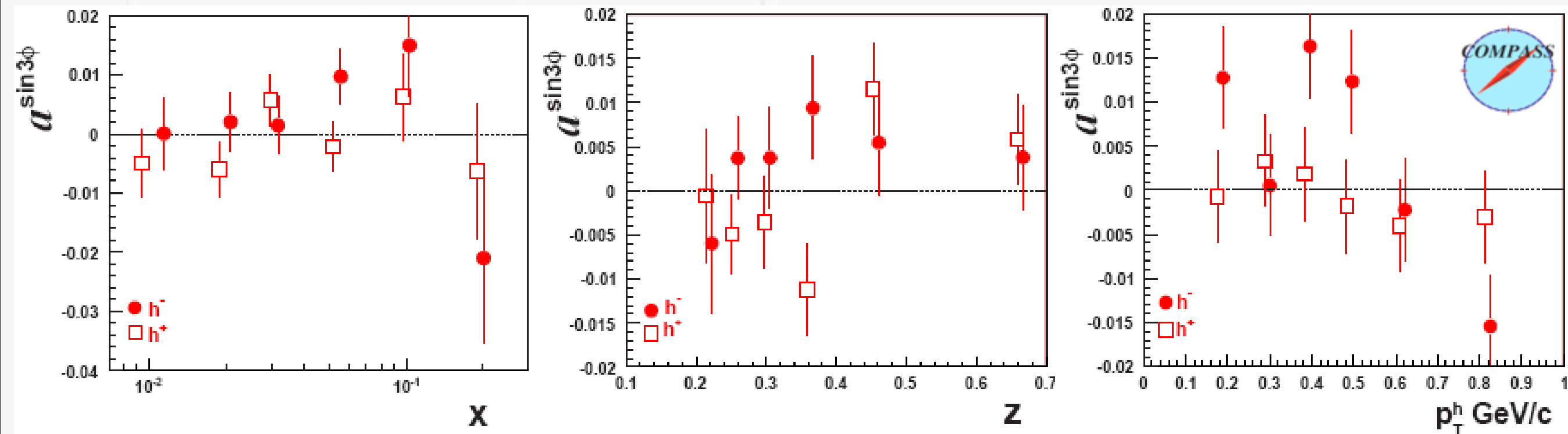
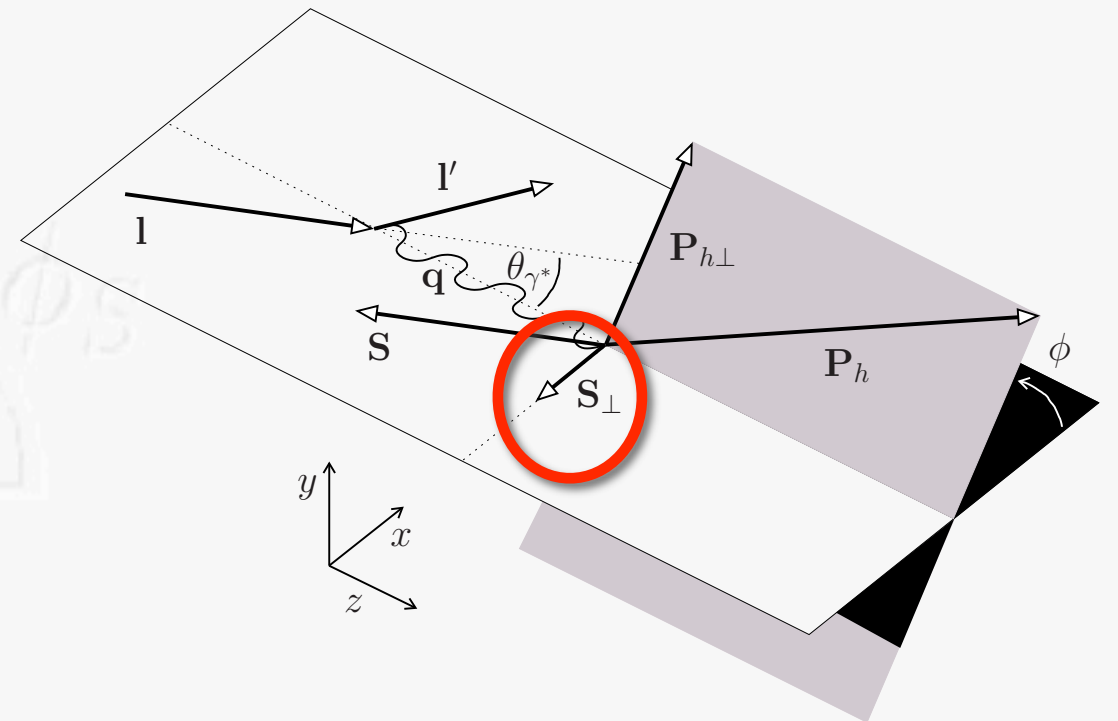


Pretzelosity

	U	L	T
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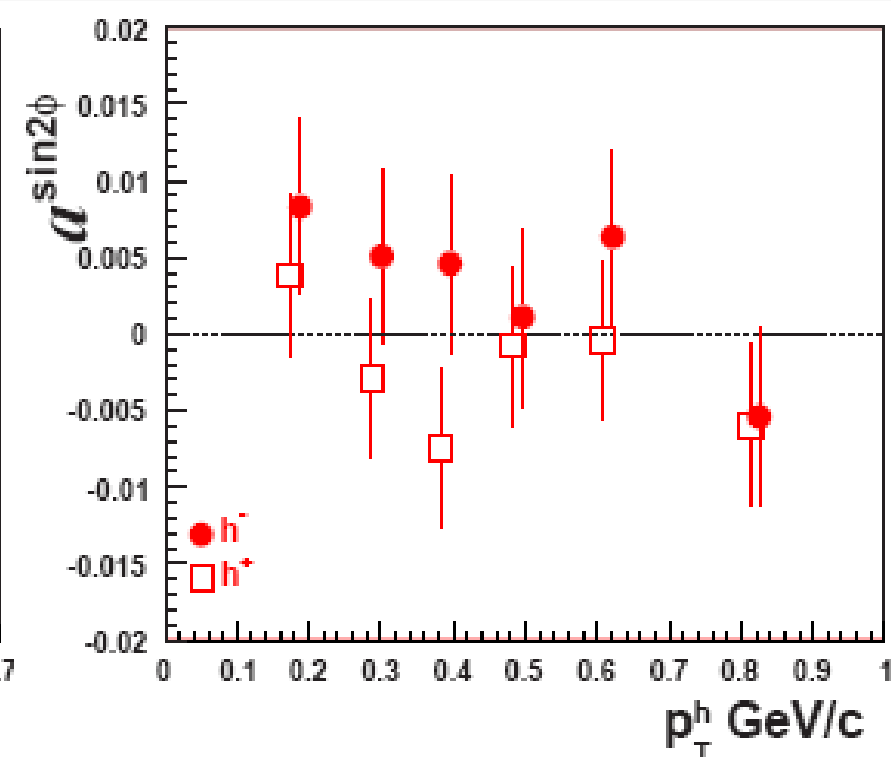
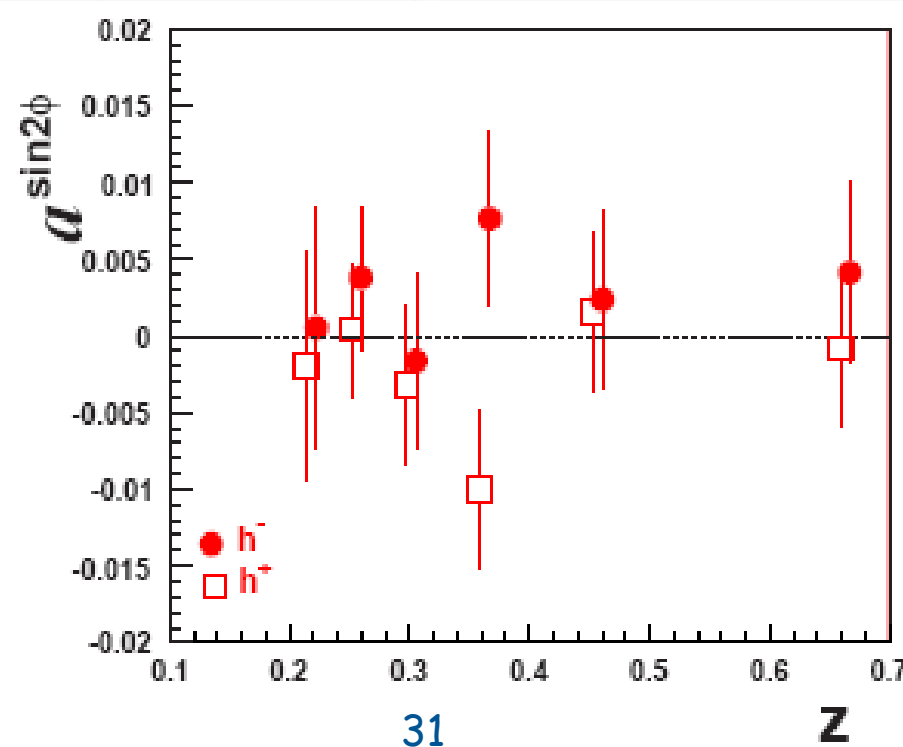
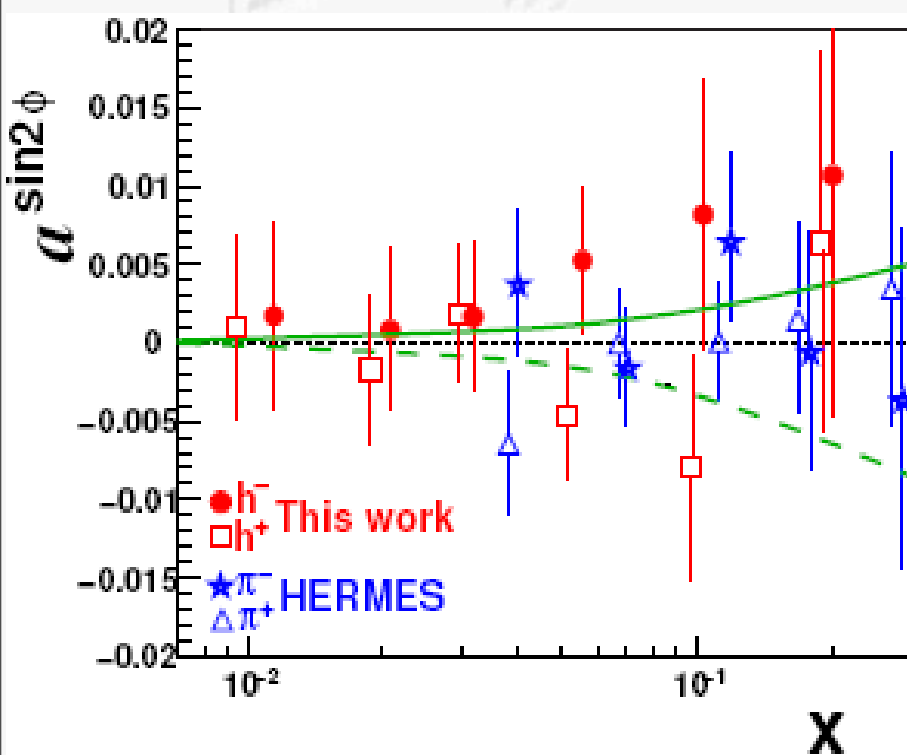
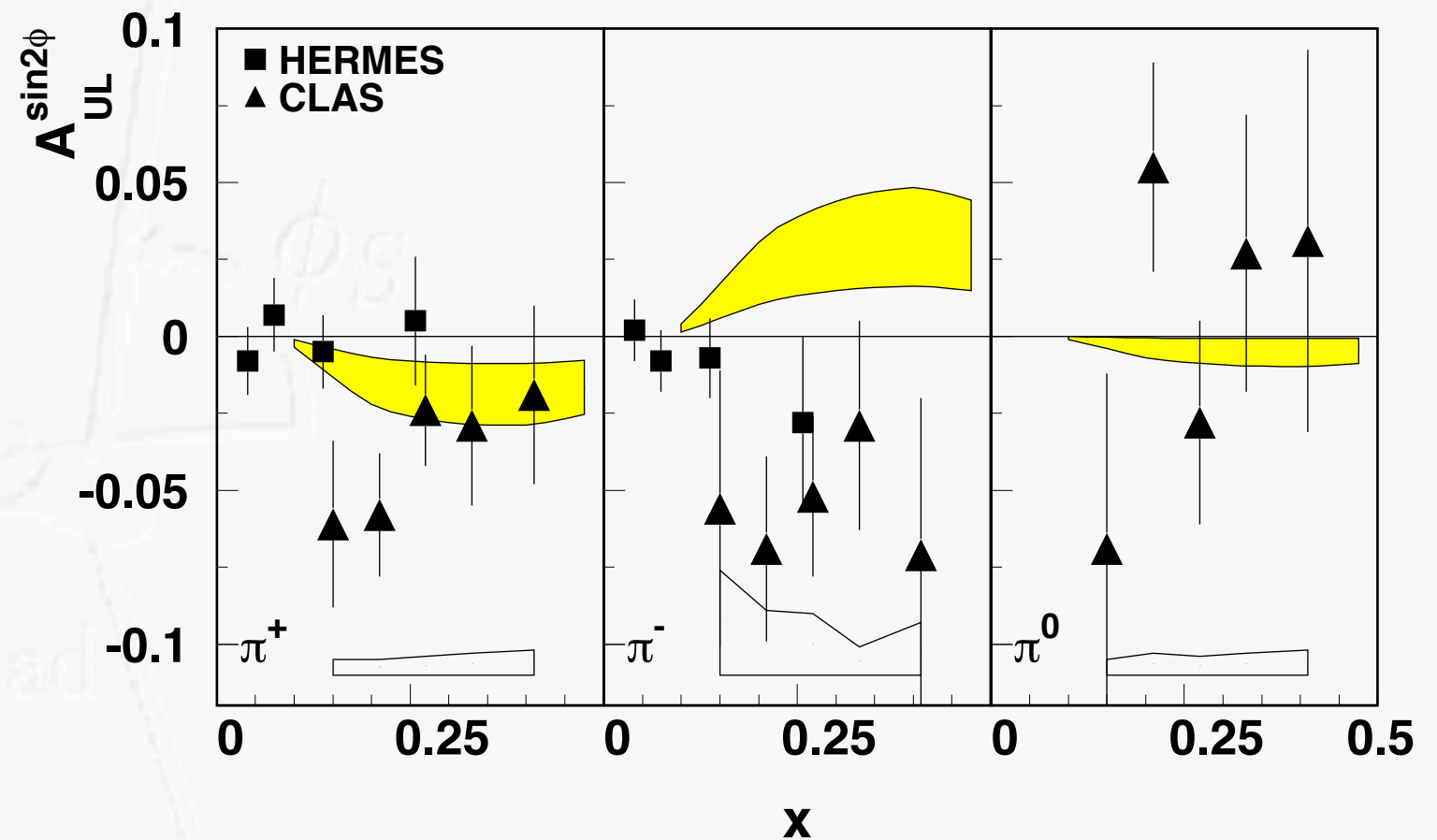


	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1, h_{1T}^\perp$



Worm-Gear I

- again chiral-odd
- evidence from CLAS (violating isospin symmetry?)
- consistent with zero at COMPASS and HERMES

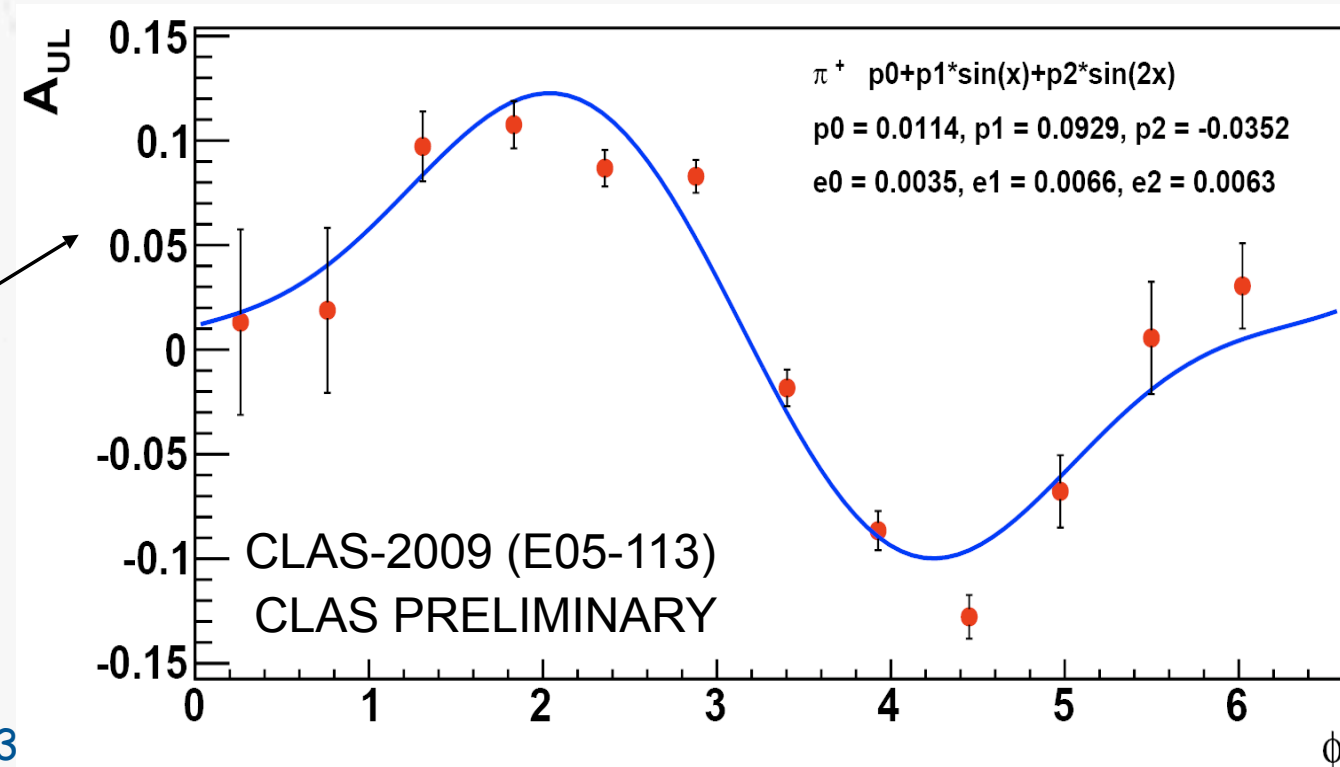
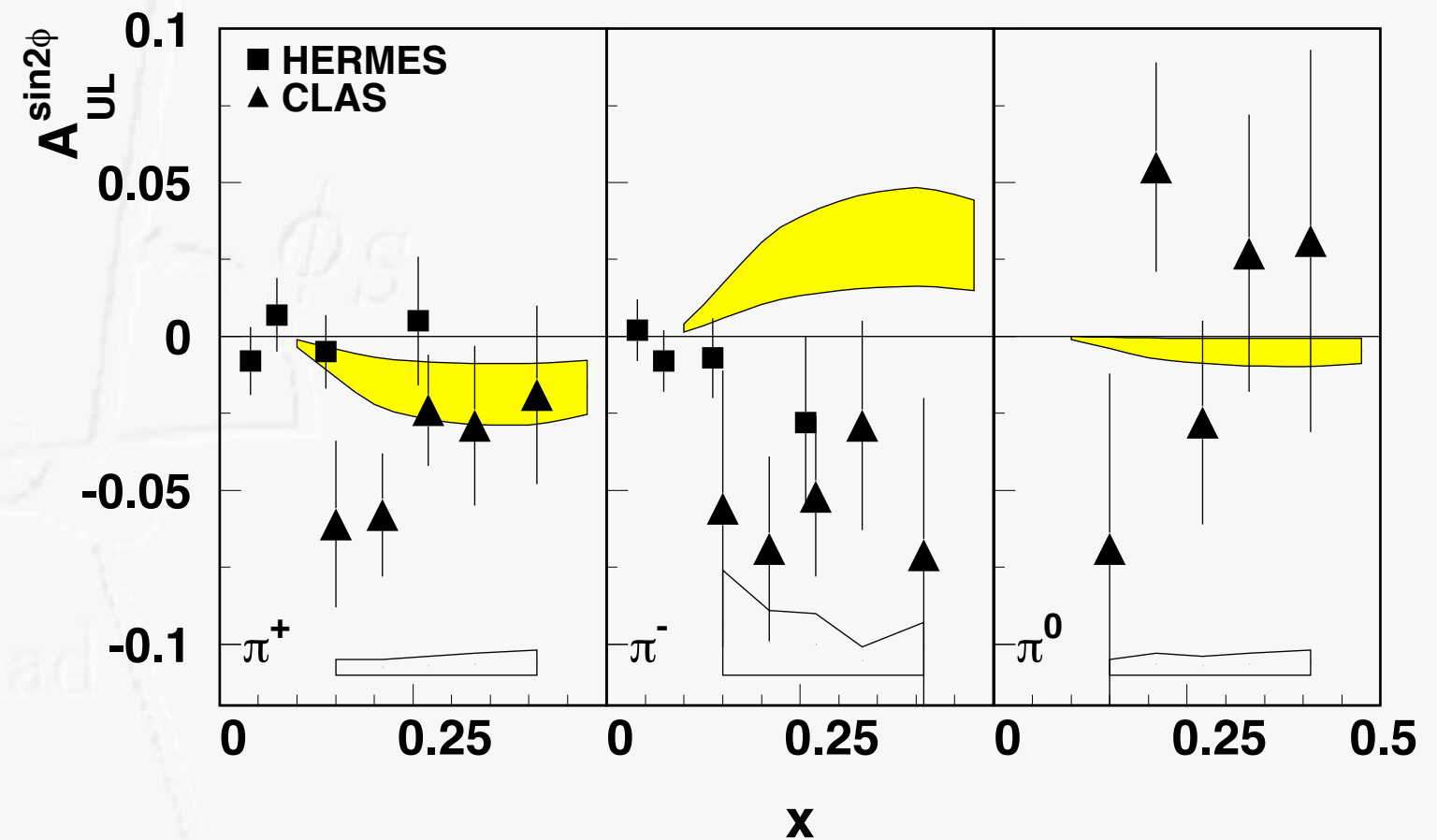


	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1, h_{1T}^\perp$



Worm-Gear I

- again chiral-odd
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- new data from CLAS



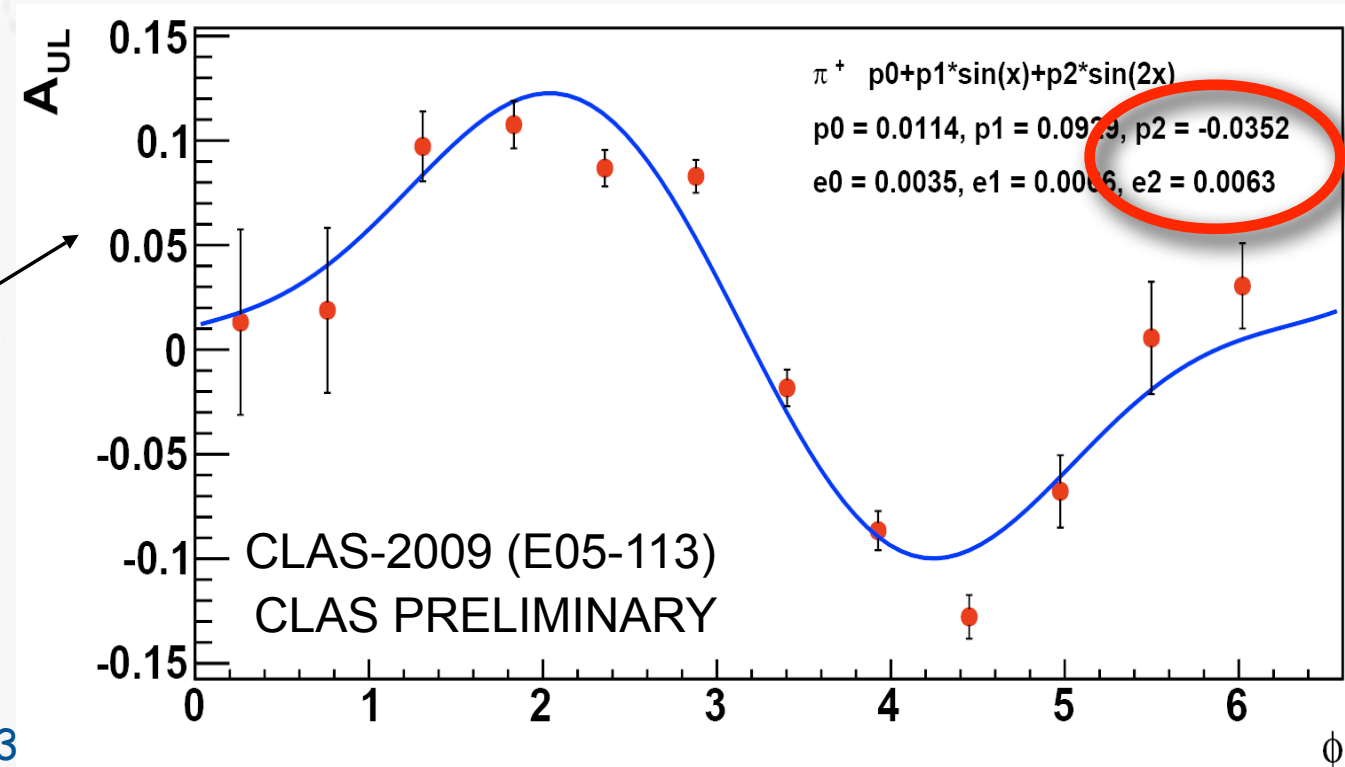
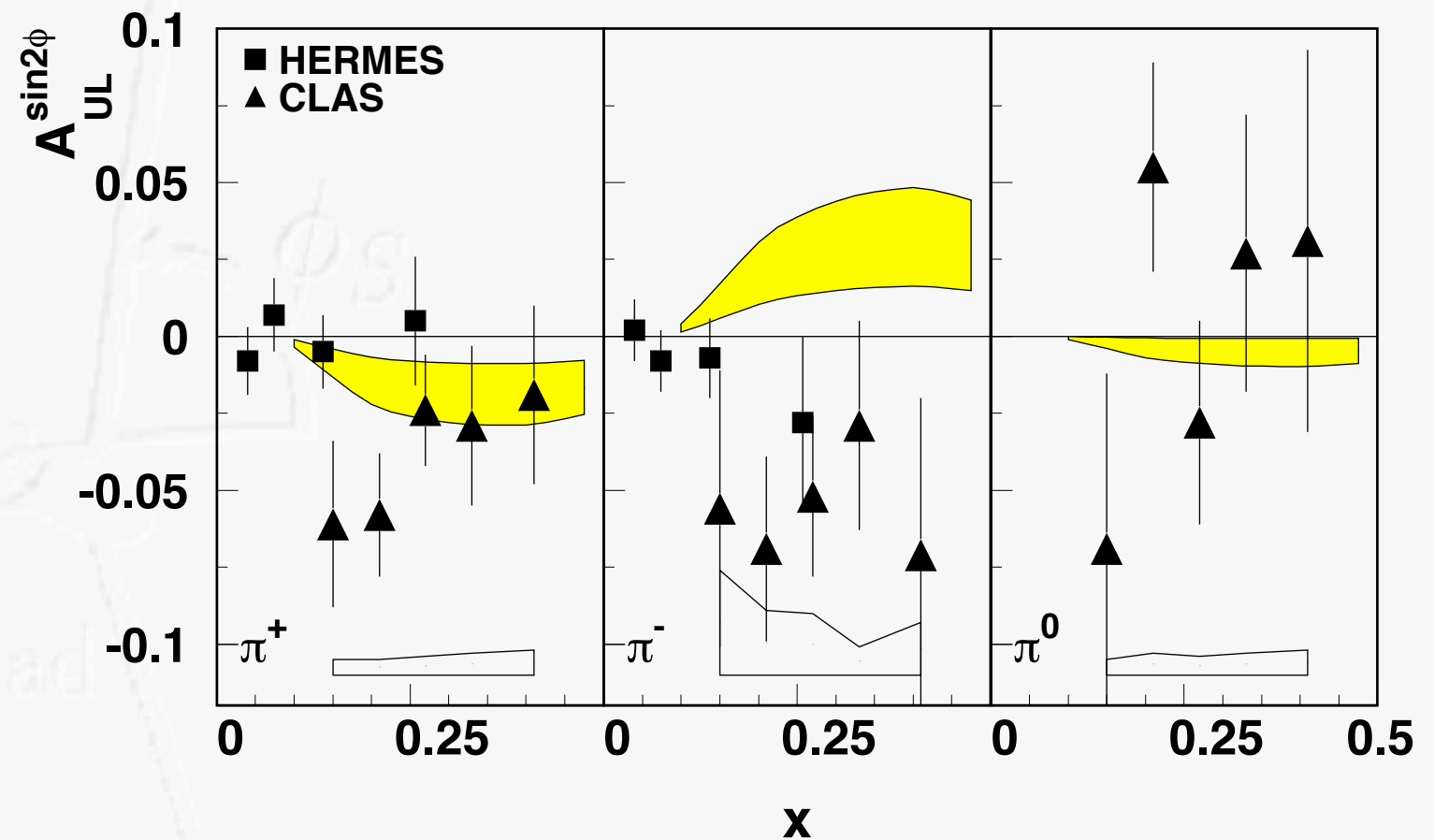
~10% of E05-113 data

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L		g_{1L}	$h_{1L}^\perp$
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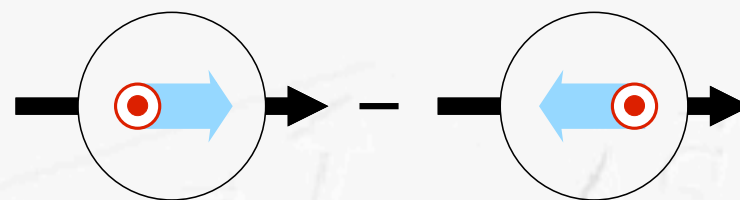
Worm-Gear I

- again chiral-odd
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- consistent with zero at COMPASS and HERMES
- new data from CLAS



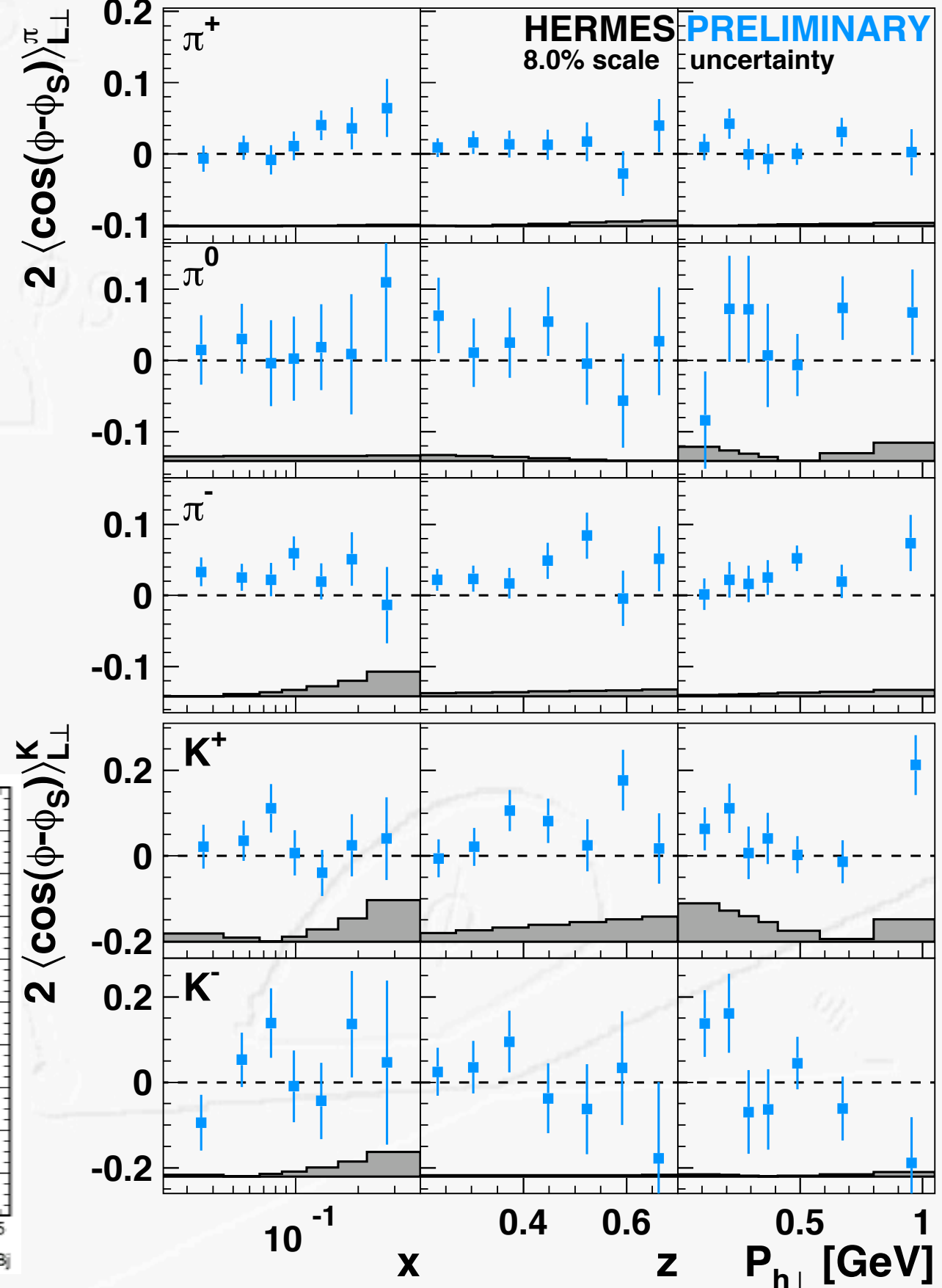
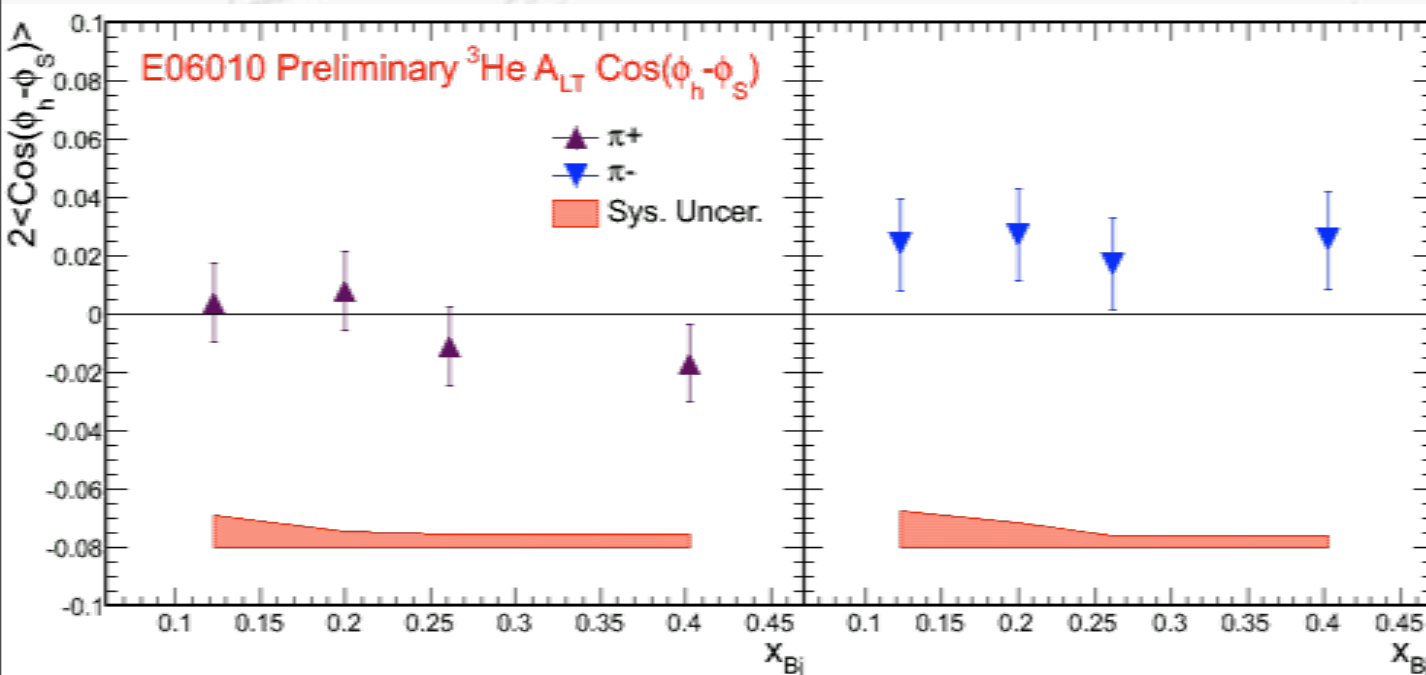
~10% of E05-113 data

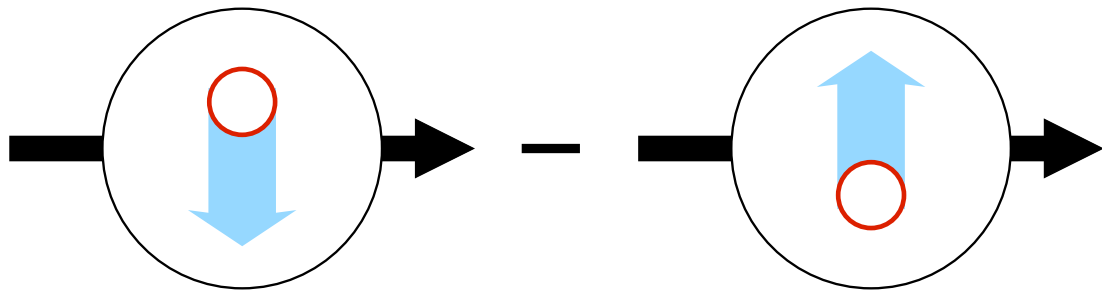
	U	L	T
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L		g_{1L}	$h_{1L}^\perp$
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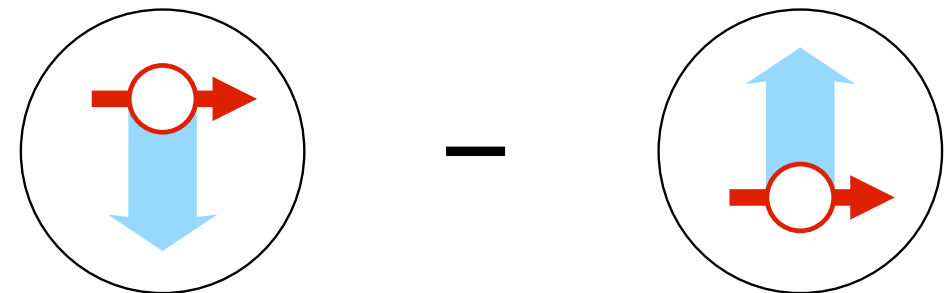
Worm-Gear II

- chiral even
- first direct evidence for worm-gear g_{1T} on
- ^3He target at JLab
- H target at HERMES





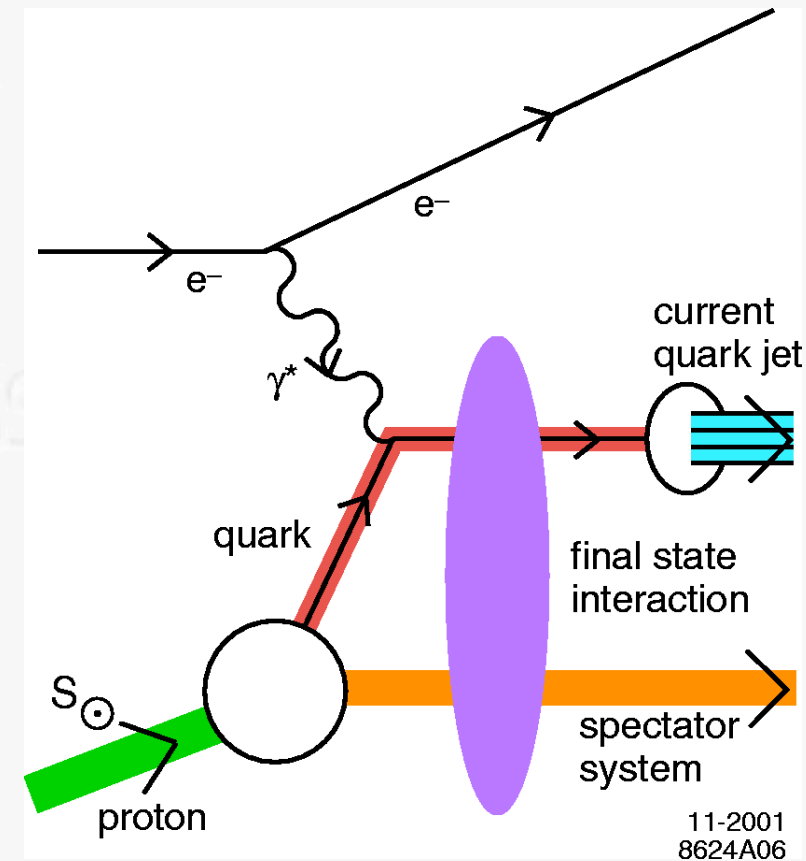
“gauge-link physics”
naively T-odd distributions



	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1, h_{1T}^\perp$



Sivers function



[S. Brodsky et al., Phys. Lett. B530, 99 (2002)]

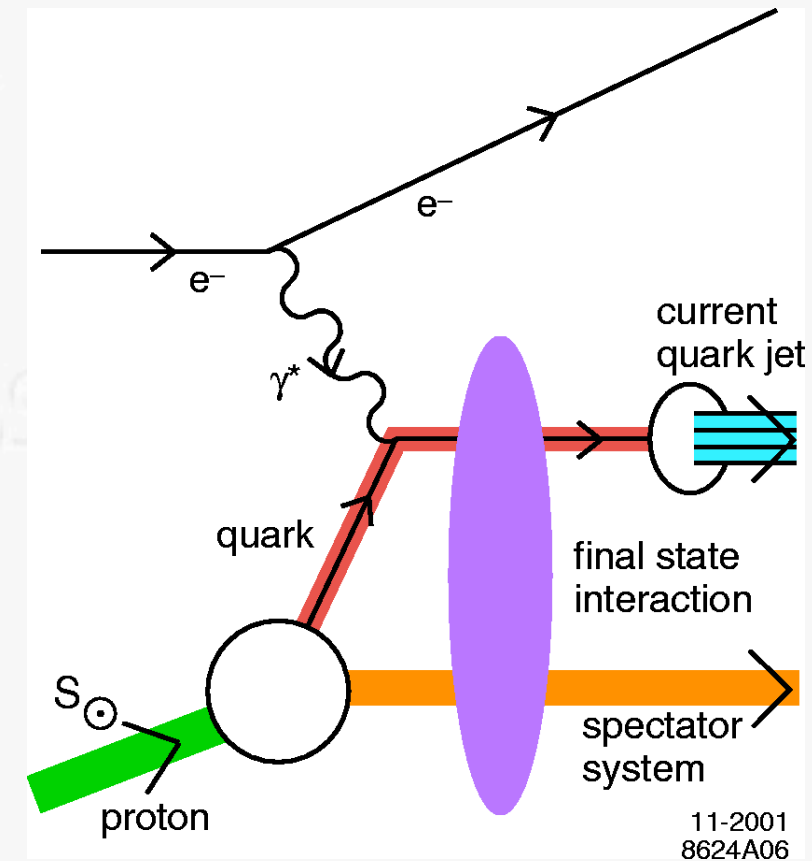
- naive T-odd
- requires FSI via non-perturbative gluon (gauge link) exchange(s)
- leads to opposite sign in DIS and DY (firm QCD prediction!)
- relation to GPD E + FSI yields opposite signs of Sivers fct. for up and down quarks

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1, h_{1T}^\perp$

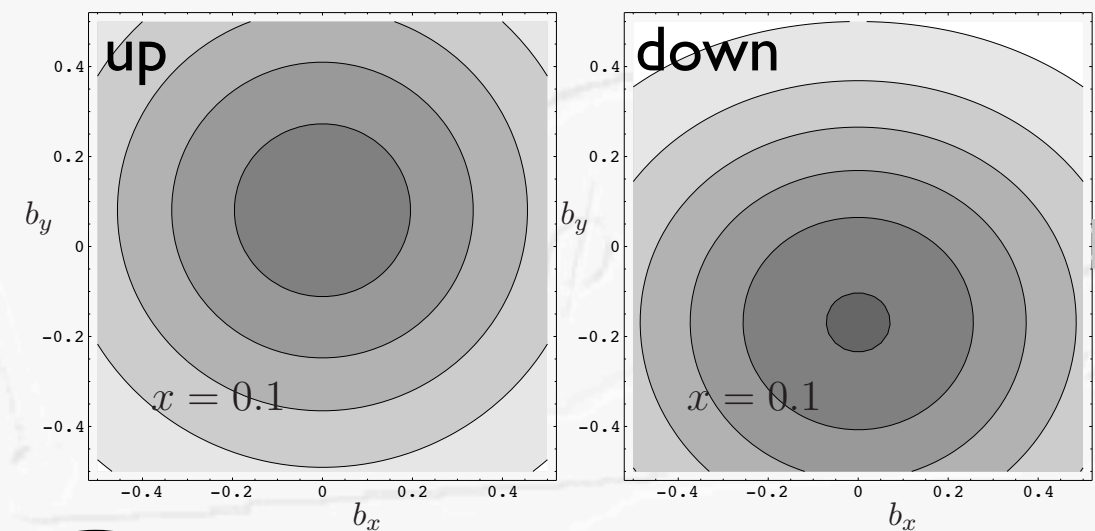


Sivers function

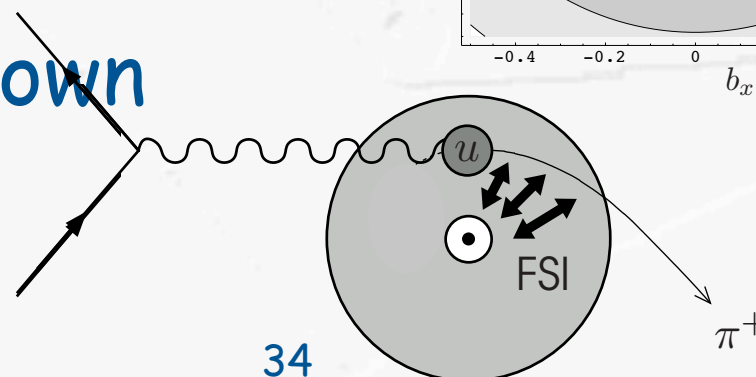
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[S. Brodsky et al., Phys. Lett. B530, 99 (2002)]



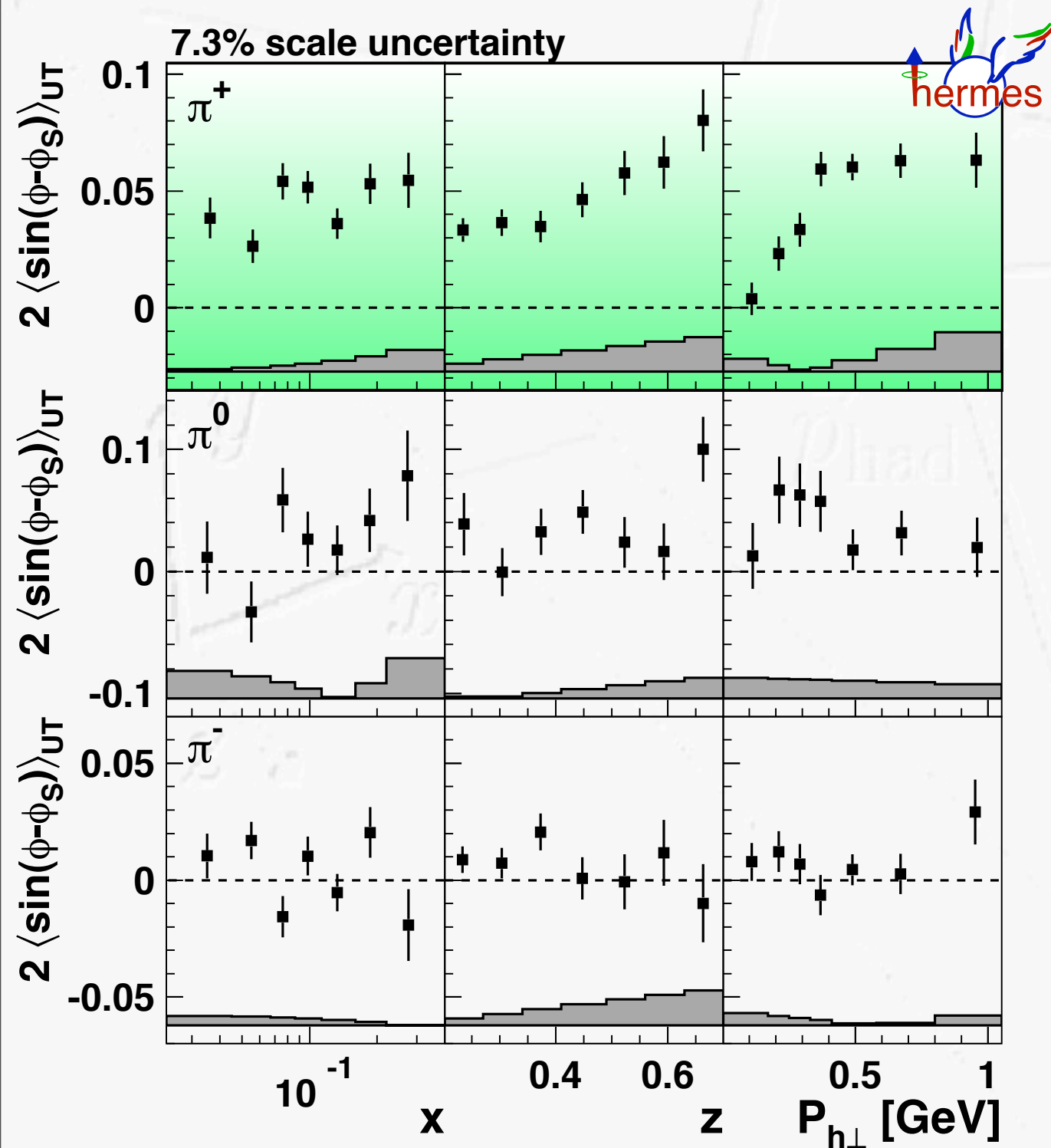
[M. Burkardt, Phys. Rev. D66 (2002) 114005]



Sivers amplitudes for pions

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1, h_{1T}^\perp$

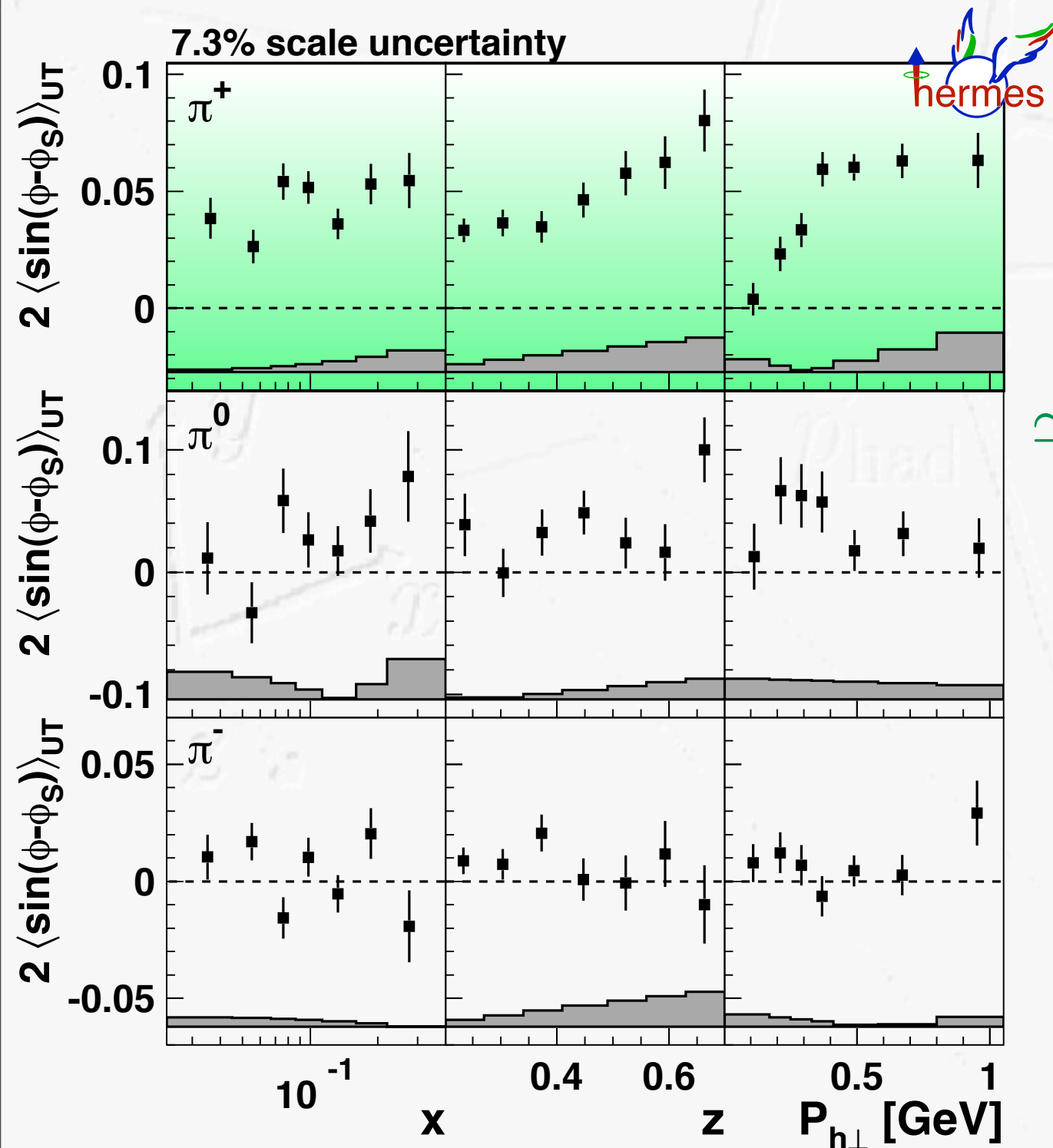
$$2\langle \sin(\phi - \phi_S) \rangle_{UT} = - \frac{\sum_q e_q^2 f_{1T}^{\perp,q}(x, p_T^2) \otimes_{\mathcal{W}} D_1^q(z, k_T^2)}{\sum_q e_q^2 f_1^q(x, p_T^2) \otimes D_1^q(z, k_T^2)}$$



Sivers amplitudes for pions

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
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π^+ dominated by u-quark scattering:

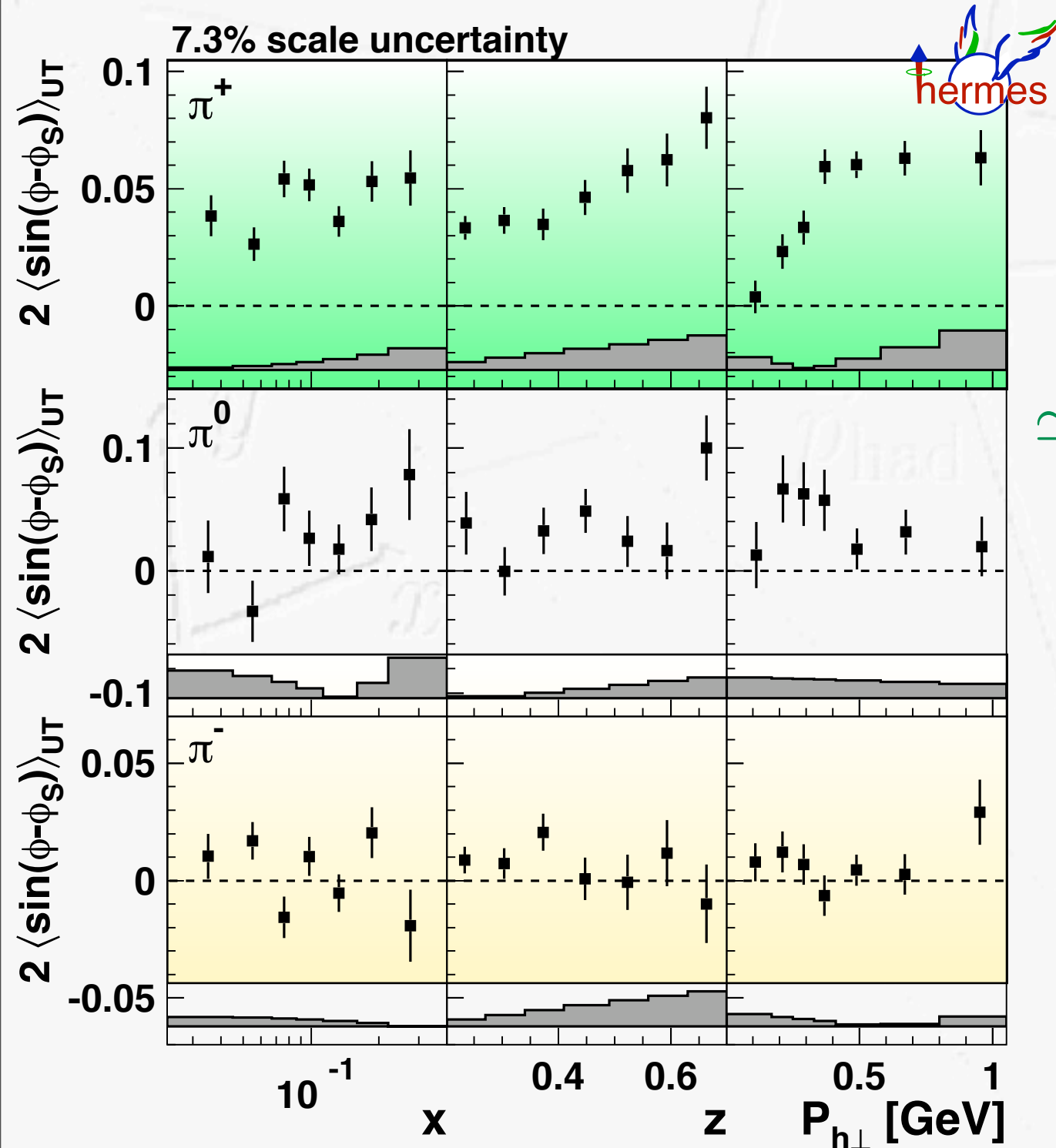
$$\simeq - \frac{f_{1T}^{\perp,u}(x, p_T^2) \otimes_{\mathcal{W}} D_1^{u \rightarrow \pi^+}(z, k_T^2)}{f_1^u(x, p_T^2) \otimes D_1^{u \rightarrow \pi^+}(z, k_T^2)}$$

➡ u-quark Sivers DF < 0

Sivers amplitudes for pions

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1, h_{1T}^\perp$

$$2\langle \sin(\phi - \phi_S) \rangle_{UT} = - \frac{\sum_q e_q^2 f_{1T}^{\perp,q}(x, p_T^2) \otimes_{\mathcal{W}} D_1^q(z, k_T^2)}{\sum_q e_q^2 f_1^q(x, p_T^2) \otimes D_1^q(z, k_T^2)}$$



π^+ dominated by u-quark scattering:

$$\simeq - \frac{f_{1T}^{\perp,u}(x, p_T^2) \otimes_{\mathcal{W}} D_1^{u \rightarrow \pi^+}(z, k_T^2)}{f_1^u(x, p_T^2) \otimes D_1^{u \rightarrow \pi^+}(z, k_T^2)}$$

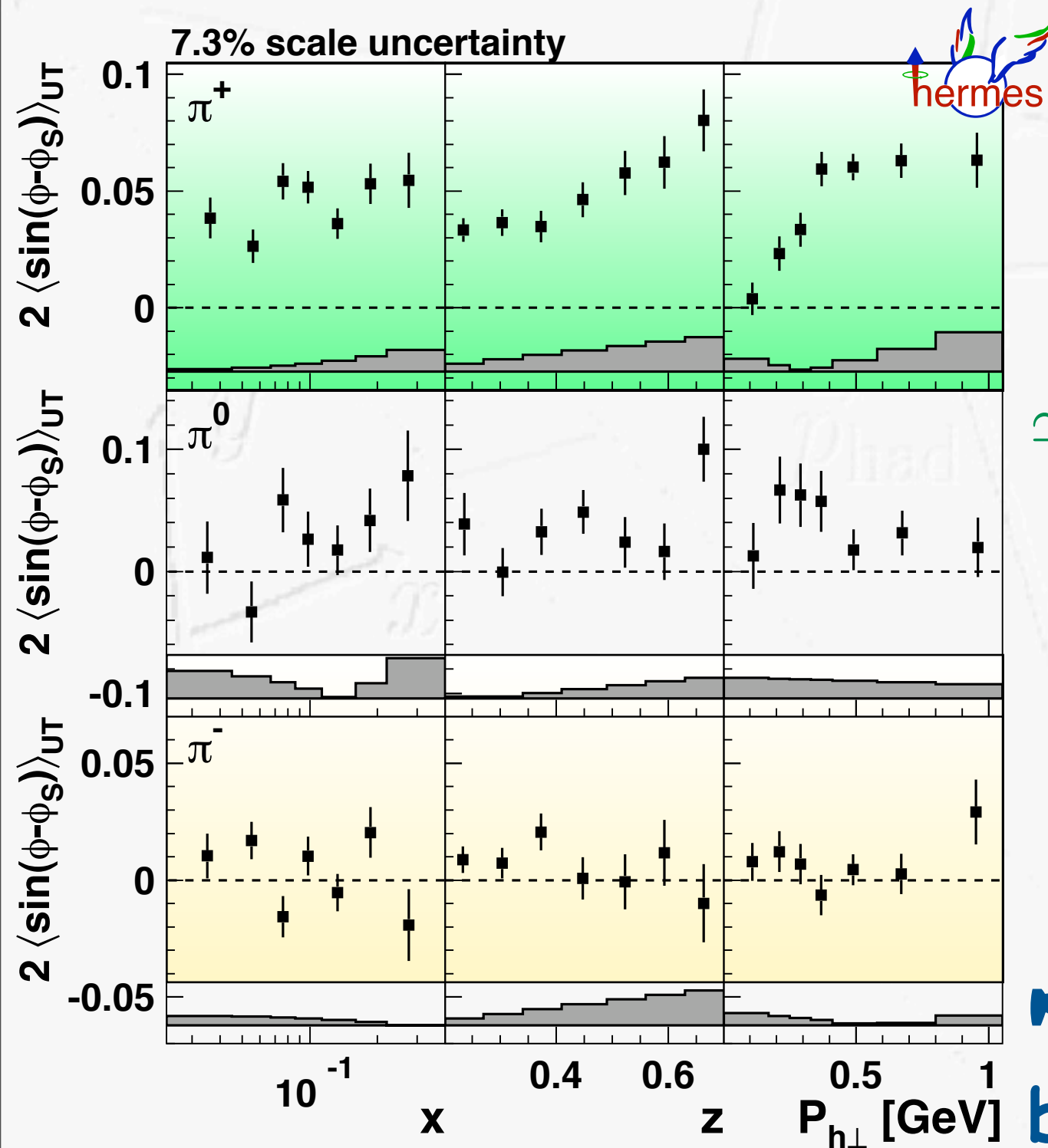
➡ u-quark Sivers DF < 0

➡ d-quark Sivers DF > 0
(cancellation for π^-)

Sivers amplitudes for pions

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1, h_{1T}^\perp$

$$2\langle \sin(\phi - \phi_S) \rangle_{UT} = - \frac{\sum_q e_q^2 f_{1T}^{\perp,q}(x, p_T^2) \otimes_{\mathcal{W}} D_1^q(z, k_T^2)}{\sum_q e_q^2 f_1^q(x, p_T^2) \otimes D_1^q(z, k_T^2)}$$



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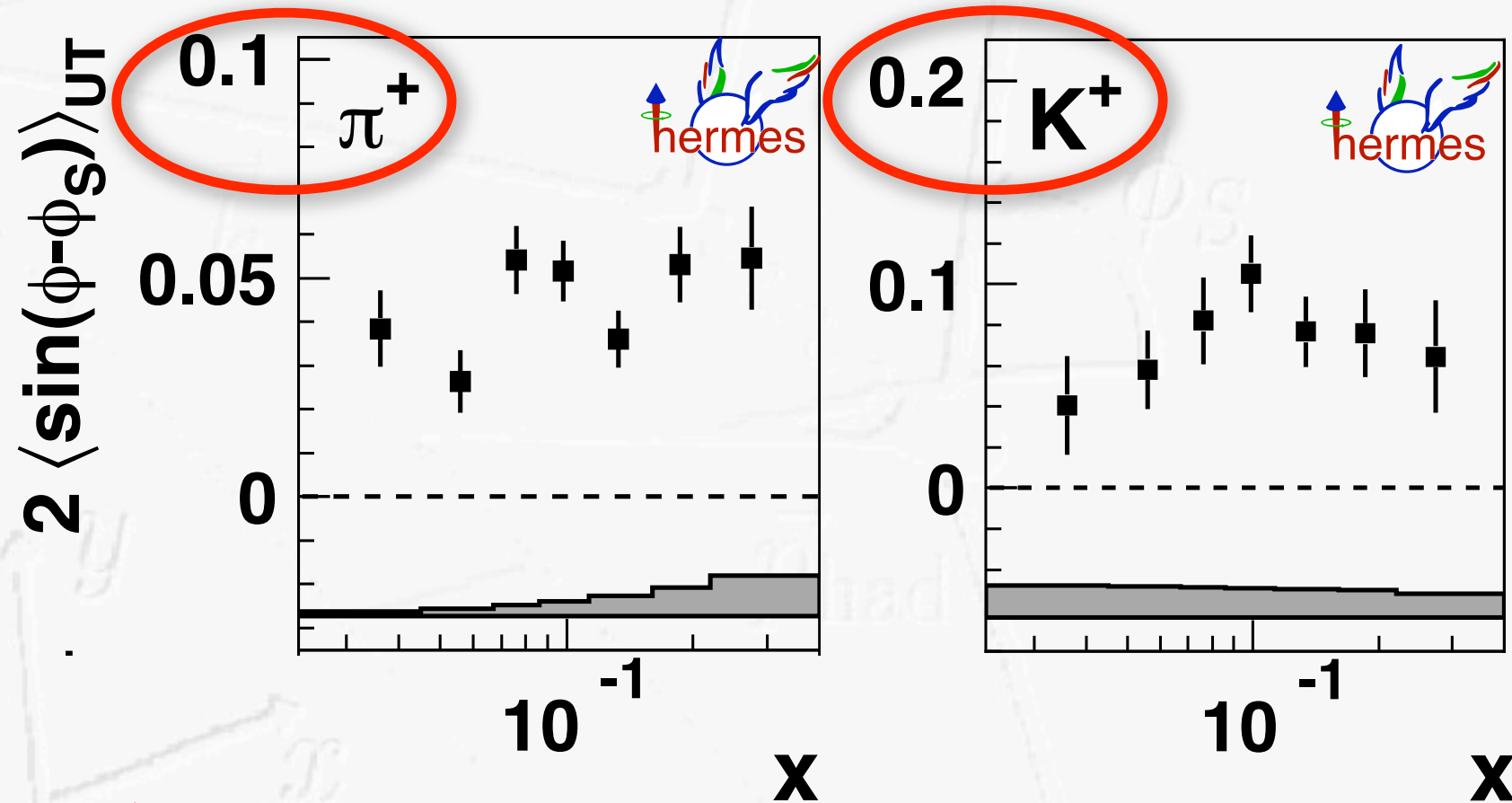
➡ u-quark Sivers DF < 0

➡ d-quark Sivers DF > 0
(cancellation for π^-)

➡ u-d cancellation supported by COMPASS D data

Sivers amplitudes pions vs. kaons

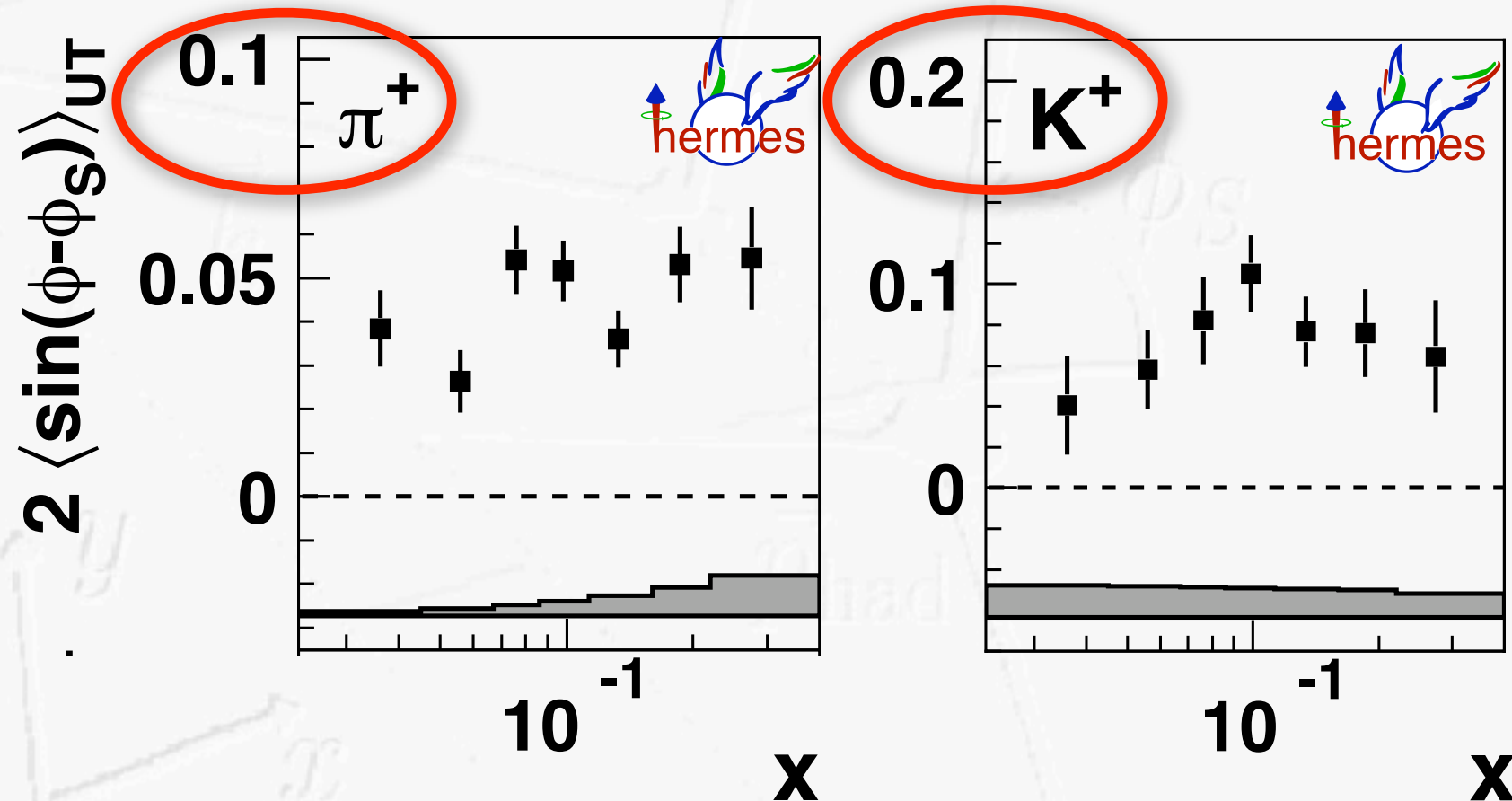
	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1, h_{1T}^\perp$



π^+ / K^+ production dominated by scattering off u-quarks: $\simeq - \frac{f_{1T}^{\perp,u}(\mathbf{x}, p_T^2) \otimes_{\mathcal{W}} D_1^{u \rightarrow \pi^+ / K^+}(z, k_T^2)}{f_1^u(\mathbf{x}, p_T^2) \otimes D_1^{u \rightarrow \pi^+ / K^+}(z, k_T^2)}$

Sivers amplitudes pions vs. kaons

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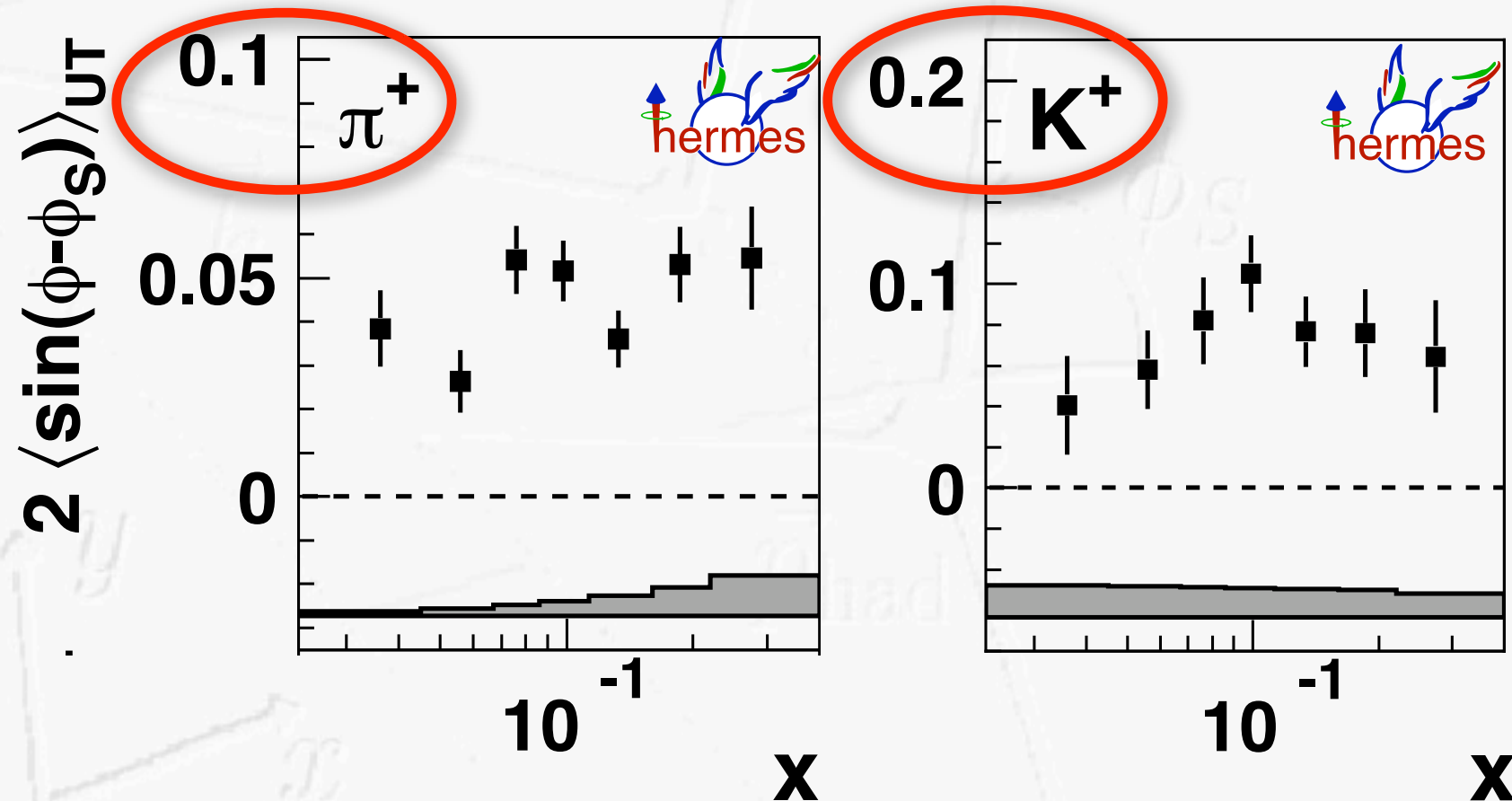


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□ $K^+ = |u\bar{s}\rangle$ & $\pi^+ = |u\bar{d}\rangle \Rightarrow$ non-trivial role of sea quarks?

Sivers amplitudes pions vs. kaons

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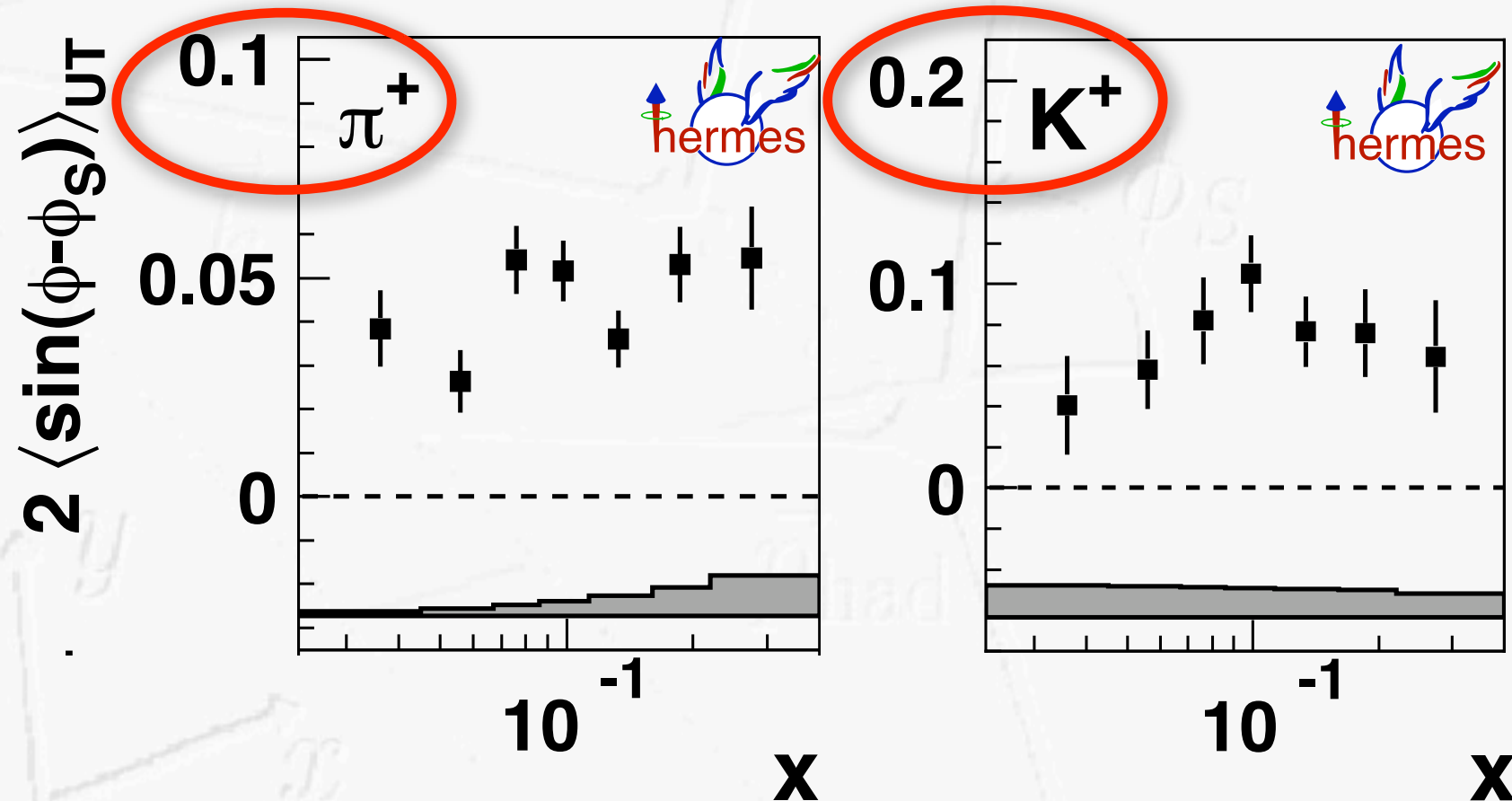


π^+ / K^+ production dominated by scattering off u-quarks: $\simeq - \frac{f_{1T}^{\perp,u}(\mathbf{x}, p_T^2) \otimes_{\mathcal{W}} D_1^{u \rightarrow \pi^+ / K^+}(z, k_T^2)}{f_1^u(\mathbf{x}, p_T^2) \otimes D_1^{u \rightarrow \pi^+ / K^+}(z, k_T^2)}$

- ☐ $K^+ = |u\bar{s}\rangle$ & $\pi^+ = |u\bar{d}\rangle \Rightarrow$ non-trivial role of sea quarks?
- ☐ convolution integrals depend on k_T dependence of fragmentation functions

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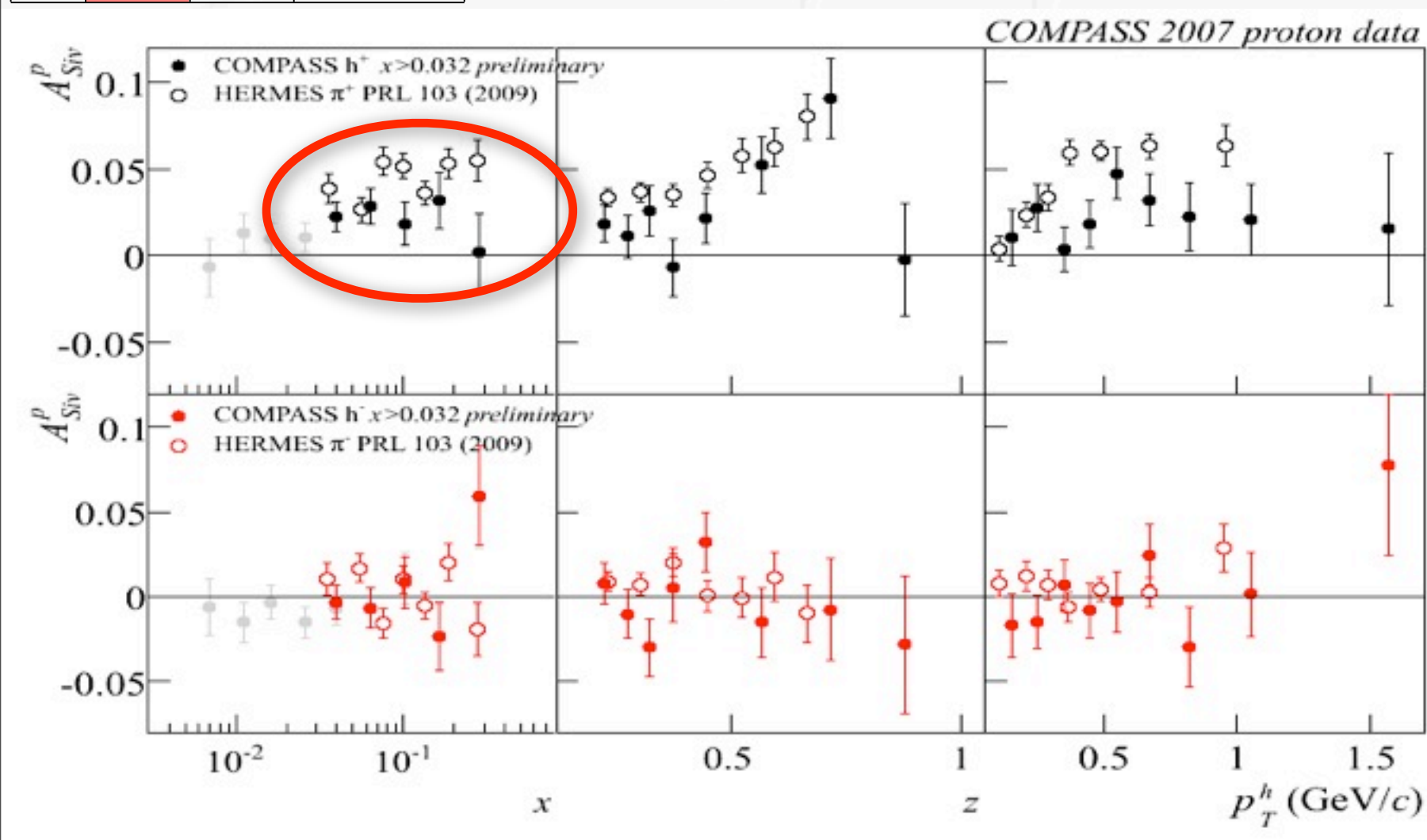


π^+ / K^+ production dominated by scattering off u-quarks: $\simeq - \frac{f_{1T}^{\perp,u}(\mathbf{x}, p_T^2) \otimes_{\mathcal{W}} D_1^{u \rightarrow \pi^+ / K^+}(z, k_T^2)}{f_1^u(\mathbf{x}, p_T^2) \otimes D_1^{u \rightarrow \pi^+ / K^+}(z, k_T^2)}$

- ☐ $K^+ = |u\bar{s}\rangle$ & $\pi^+ = |u\bar{d}\rangle \Rightarrow$ non-trivial role of sea quarks?
- ☐ convolution integrals depend on k_T dependence of fragmentation functions
- ☐ possible difference in dependences on the kinematics integrated over

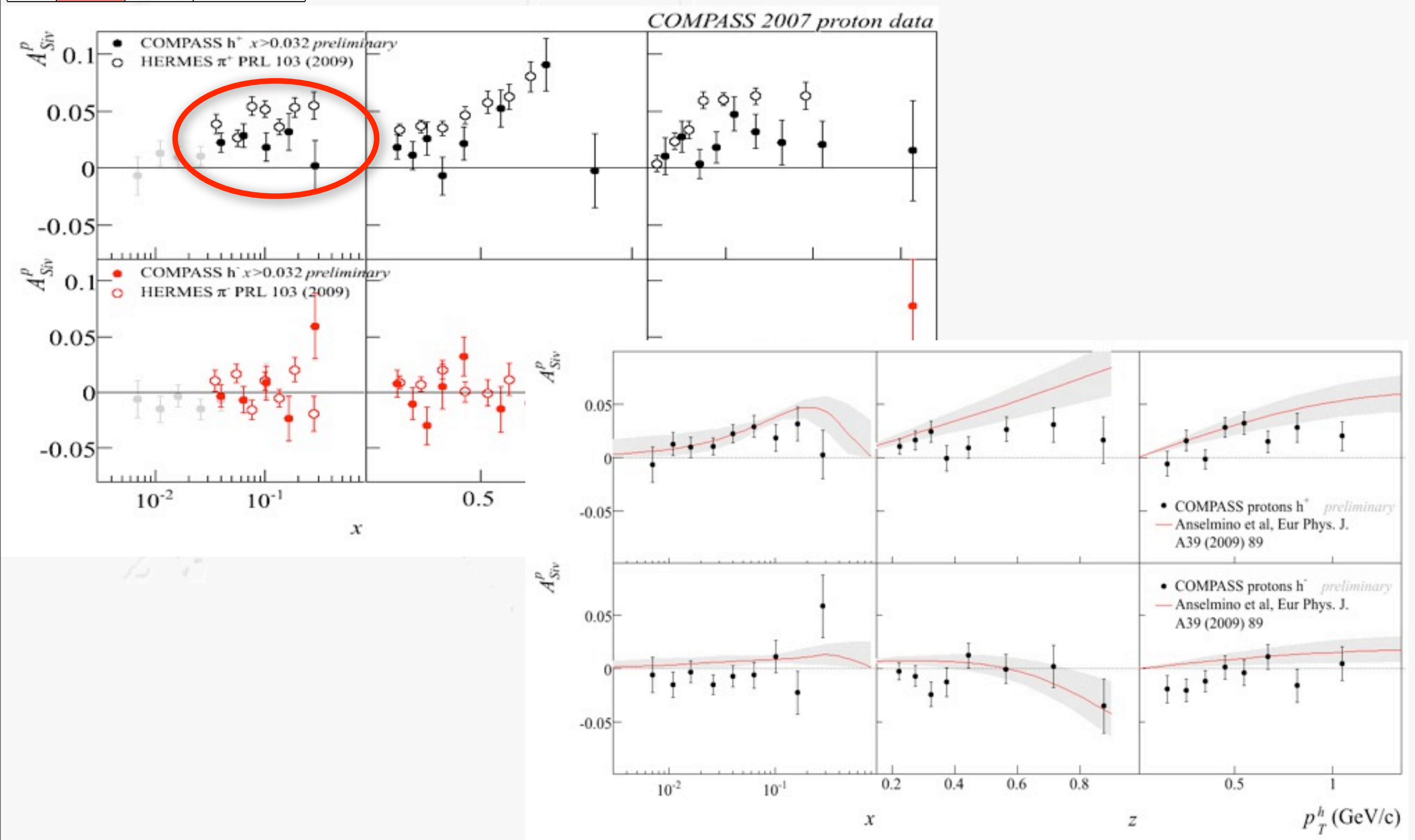
Sivers amplitudes COMPASS & HERMES

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
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Sivers amplitudes COMPASS & HERMES

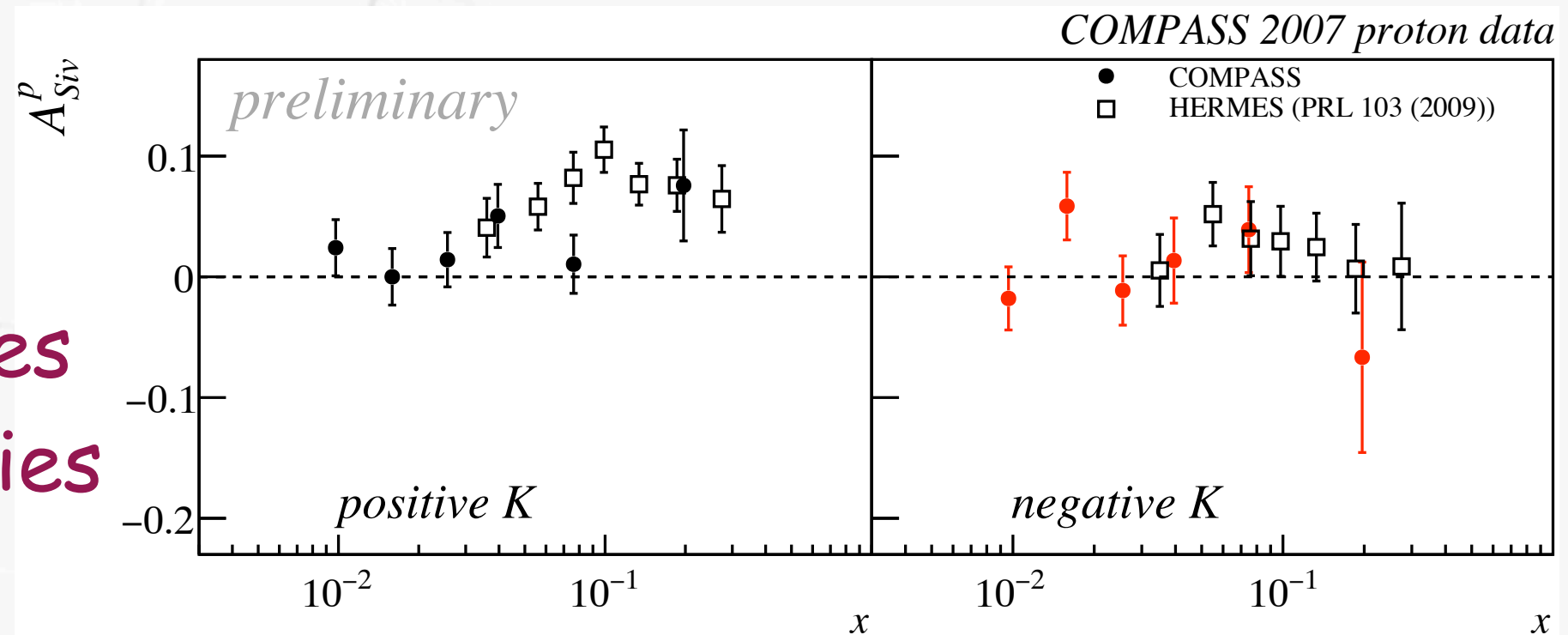
	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1, h_{1T}^\perp$



Sivers amplitudes

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1, h_{1T}^\perp$

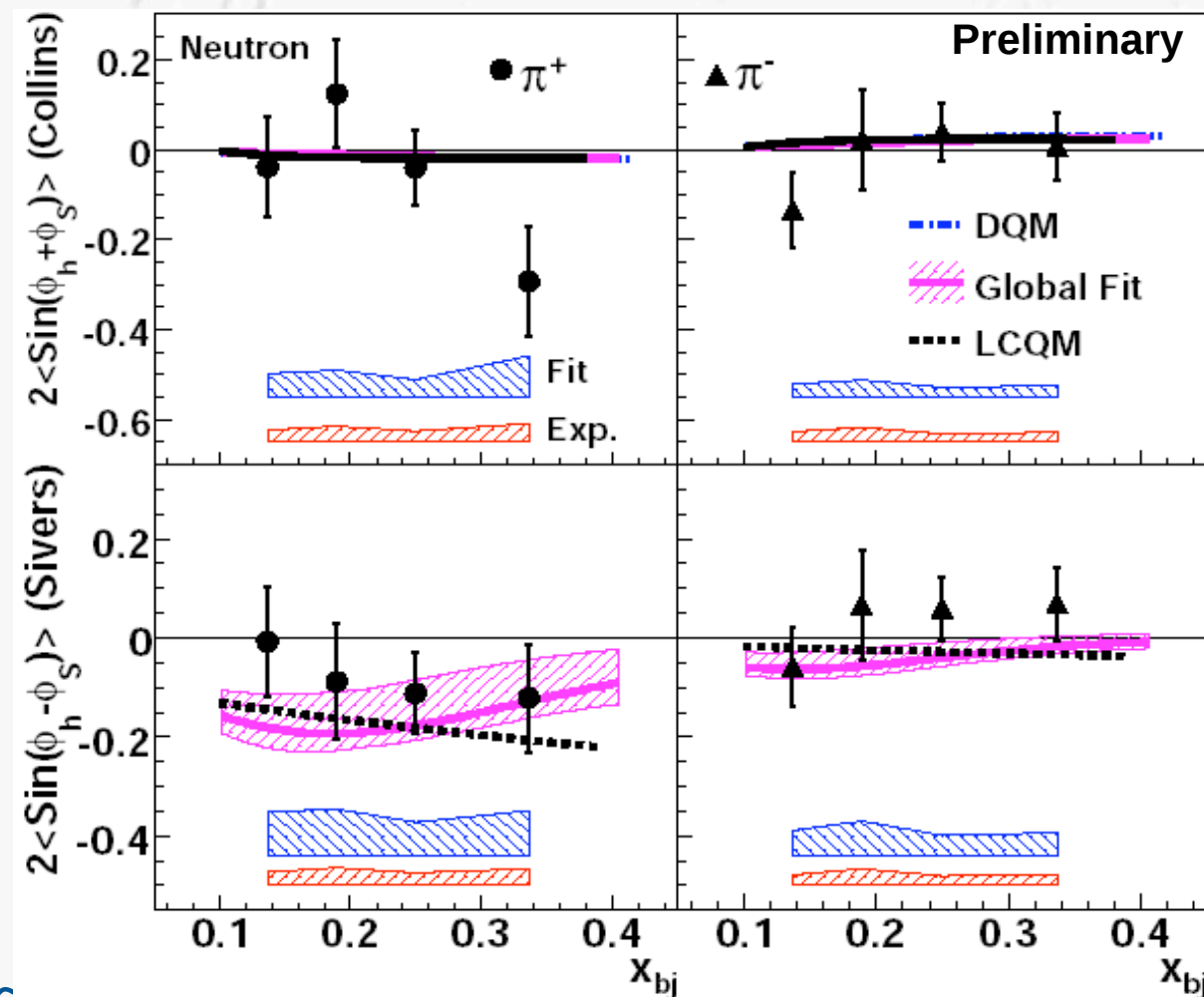
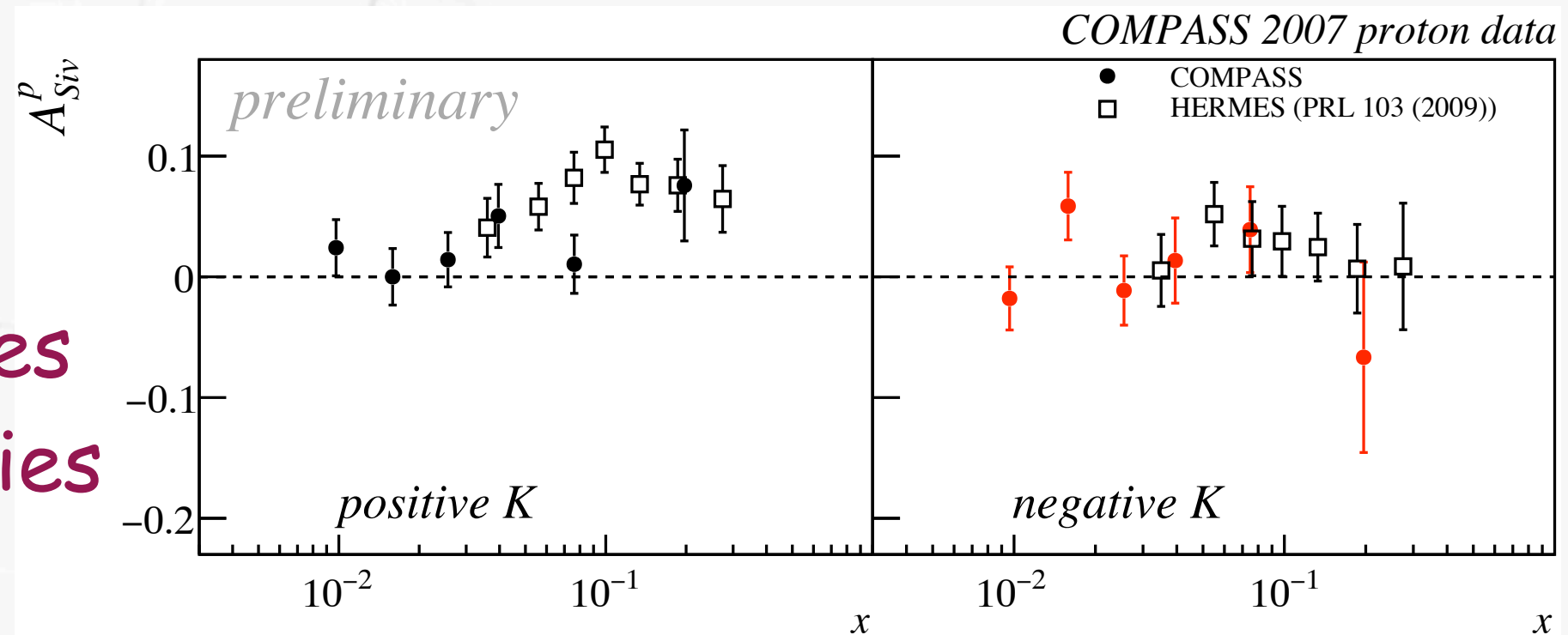
also COMPASS sees large K^+ asymmetries



Sivers amplitudes

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
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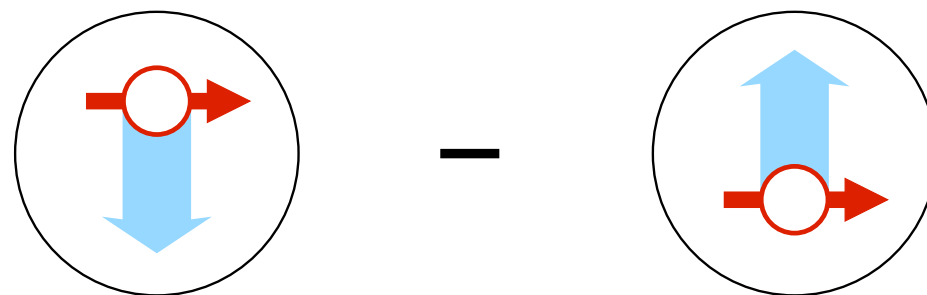
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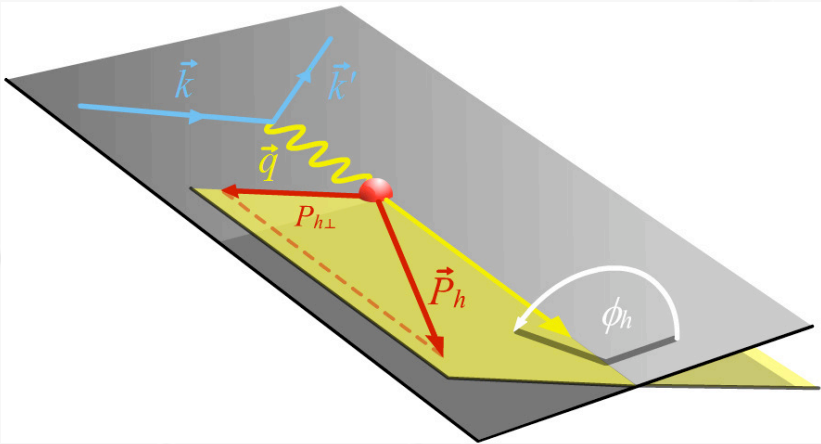
data on ^3He target from JLab available for pions

Boer-Mulders

the other naively T-odd distribution



Modulations in spin-independent SIDIS cross section



$$\frac{d^5\sigma}{dx dy dz d\phi_h dP_{h\perp}^2} = \frac{\alpha^2}{xyQ^2} \left(1 + \frac{\gamma^2}{2x}\right) \{A(y) F_{UU,T} + B(y) F_{UU,L} + C(y) \cos \phi_h F_{UU}^{\cos \phi_h} + B(y) \cos 2\phi_h F_{UU}^{\cos 2\phi_h}\}$$

leading twist

$$F_{UU}^{\cos 2\phi_h} \propto C \left[-\frac{2(\hat{P}_{h\perp} \cdot \vec{k}_T)(\hat{P}_{h\perp} \cdot \vec{p}_T) - \vec{k}_T \cdot \vec{p}_T}{MM_h} h_1^\perp H_1^\perp \right]$$

BOER-MULDERS EFFECT

next to leading twist

$$F_{UU}^{\cos \phi_h} \propto \frac{2M}{Q} C \left[-\frac{\hat{P}_{h\perp} \cdot \vec{p}_T}{M_h} x h_1^\perp H_1^\perp - \frac{\hat{P}_{h\perp} \cdot \vec{k}_T}{M} x f_1 D_1 + \dots \right]$$

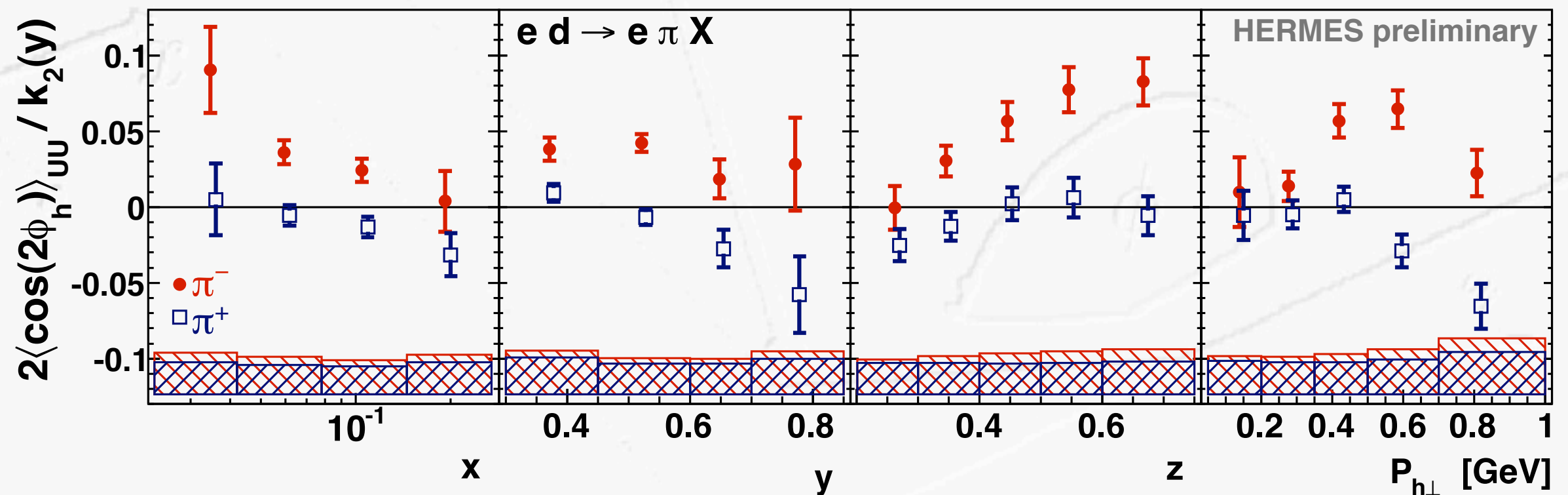
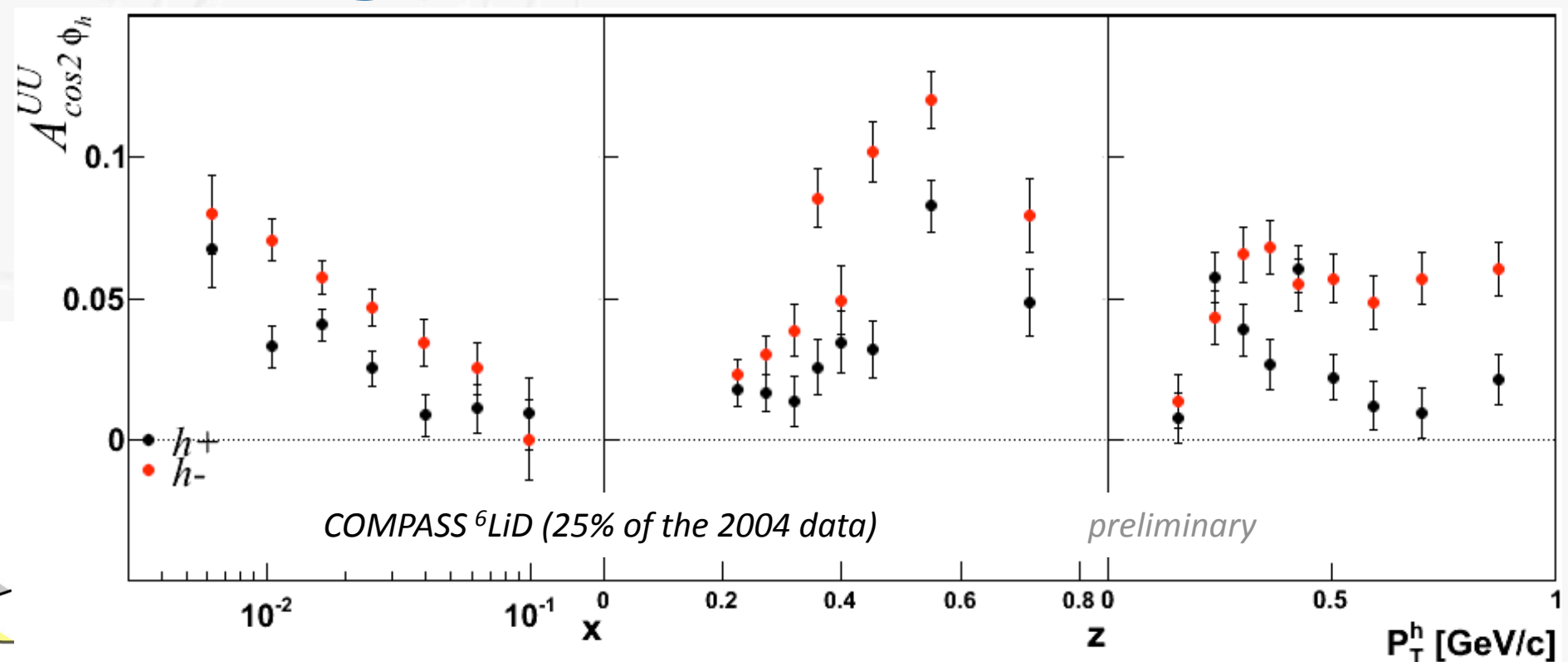
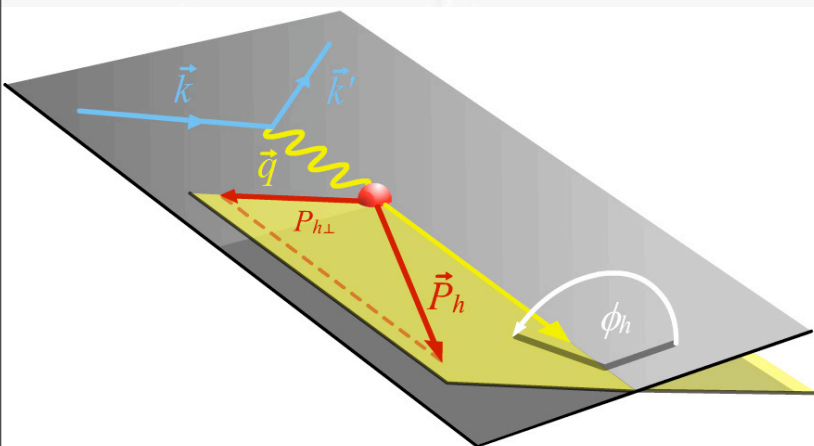
CAHN EFFECT

Interaction dependent terms neglected

(Implicit sum over quark flavours)

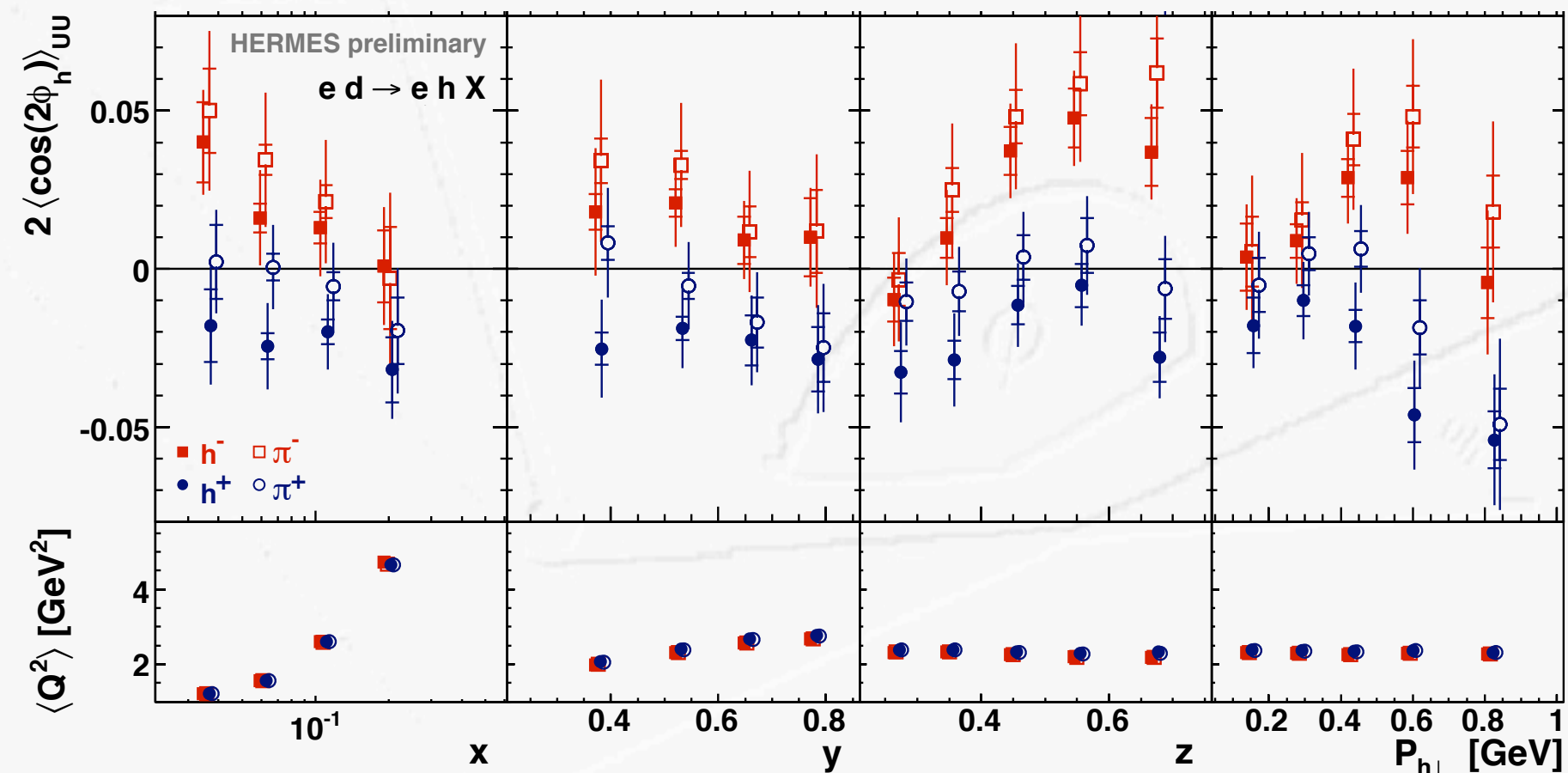
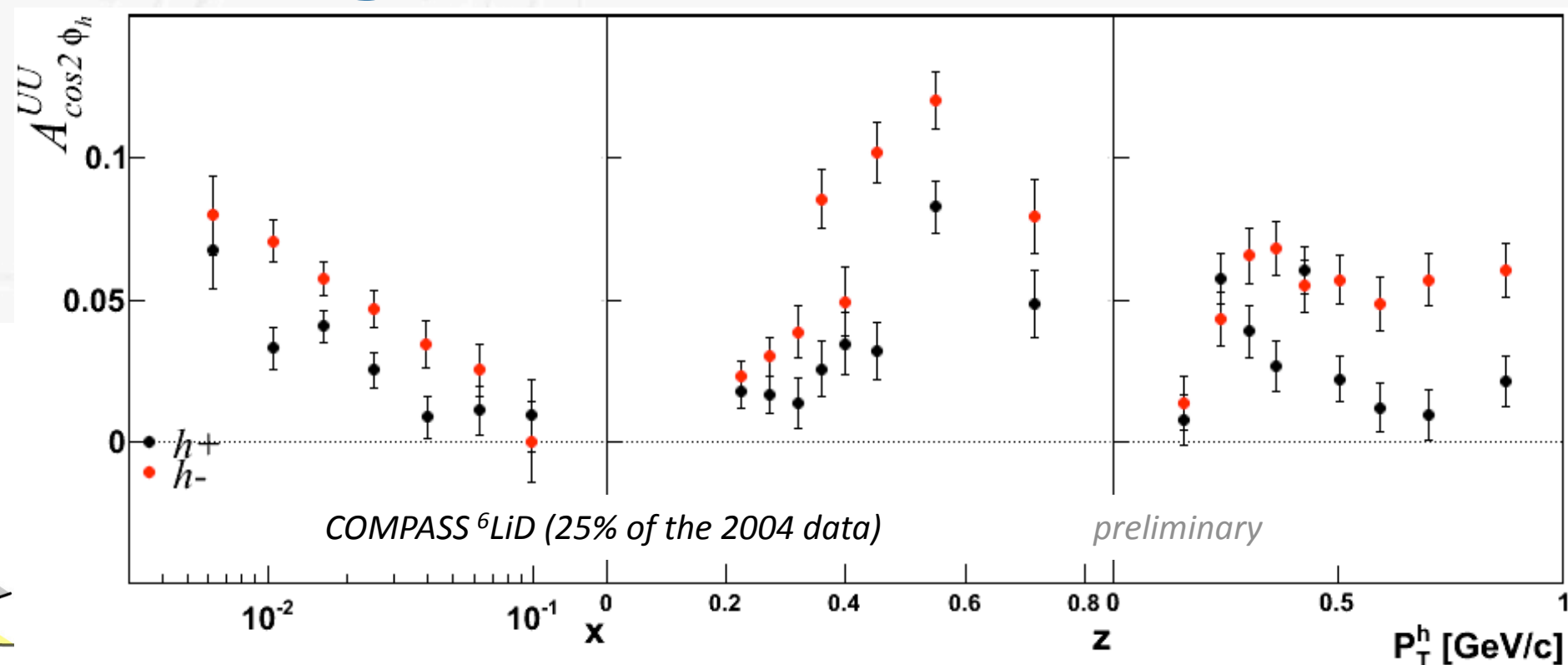
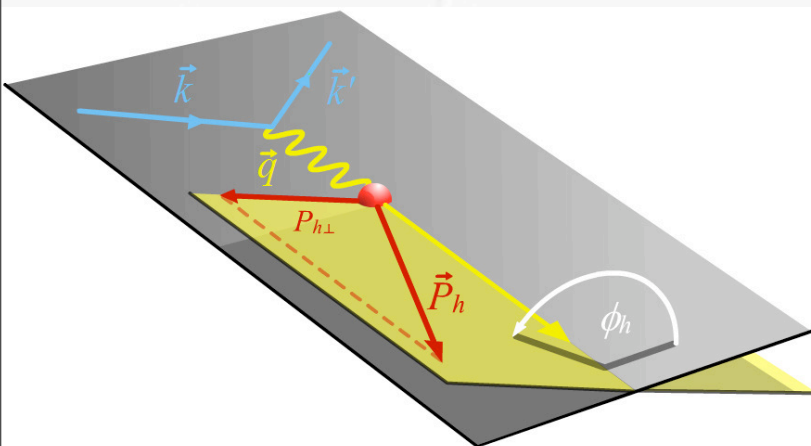
Signs of Boer-Mulders

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1, h_{1T}^\perp$



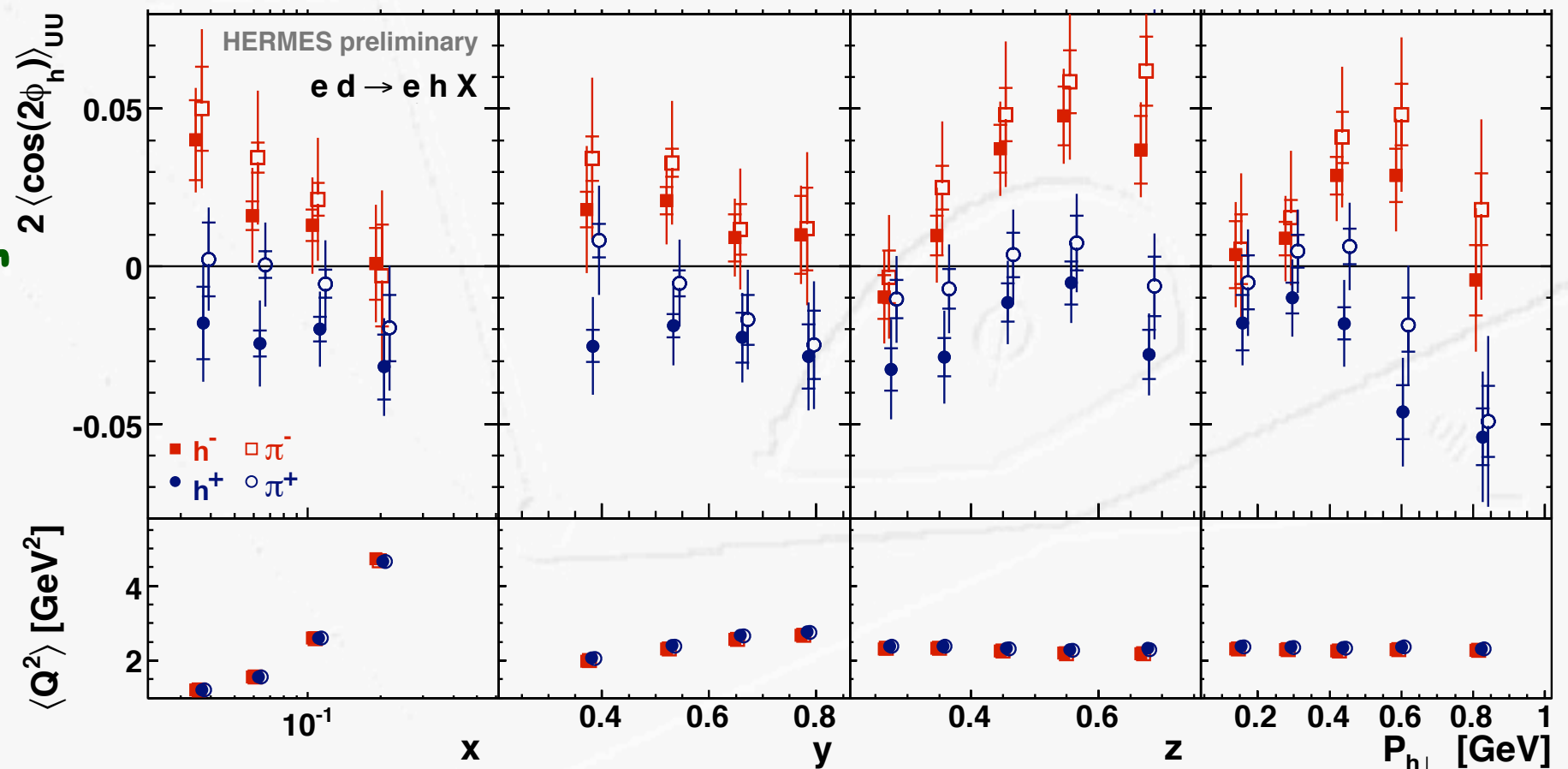
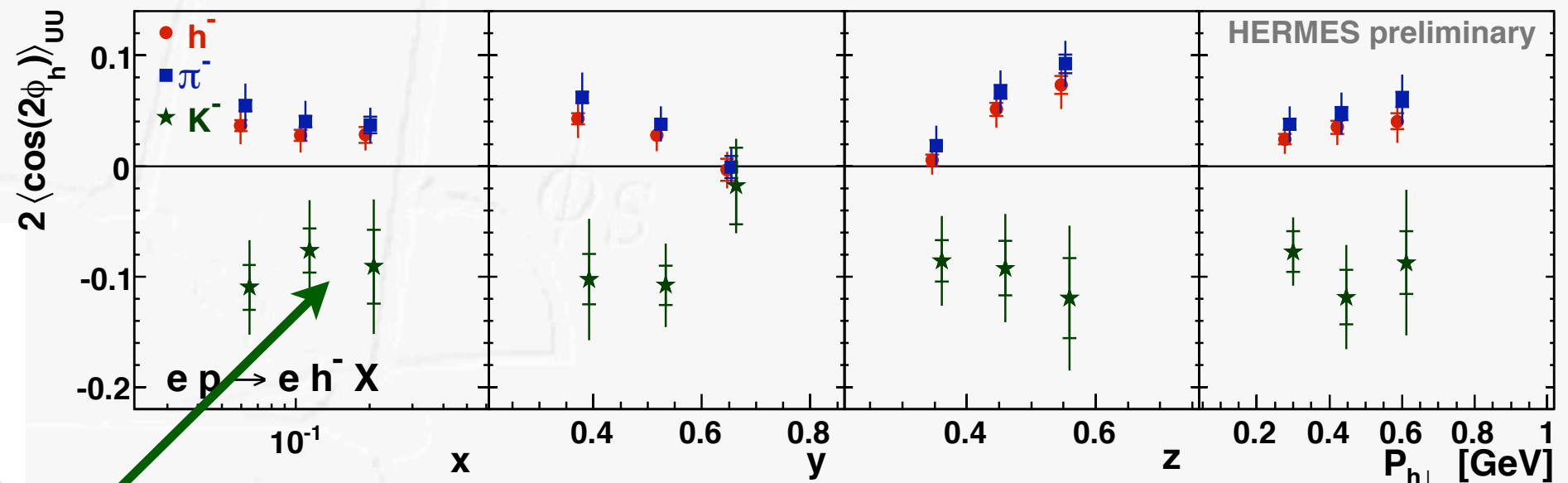
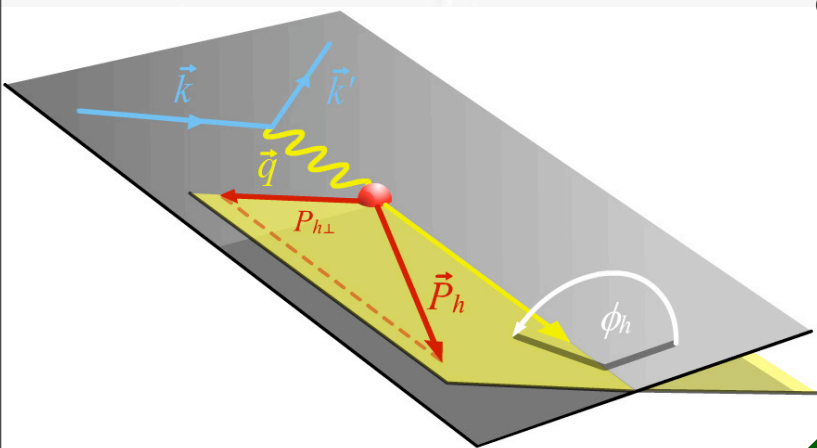
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Signs of Boer-Mulders

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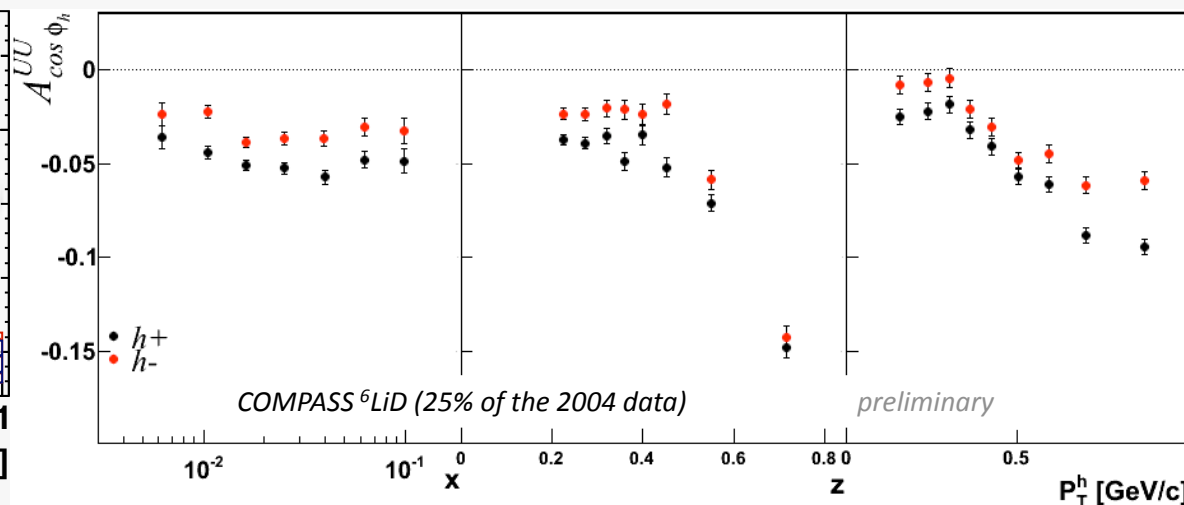
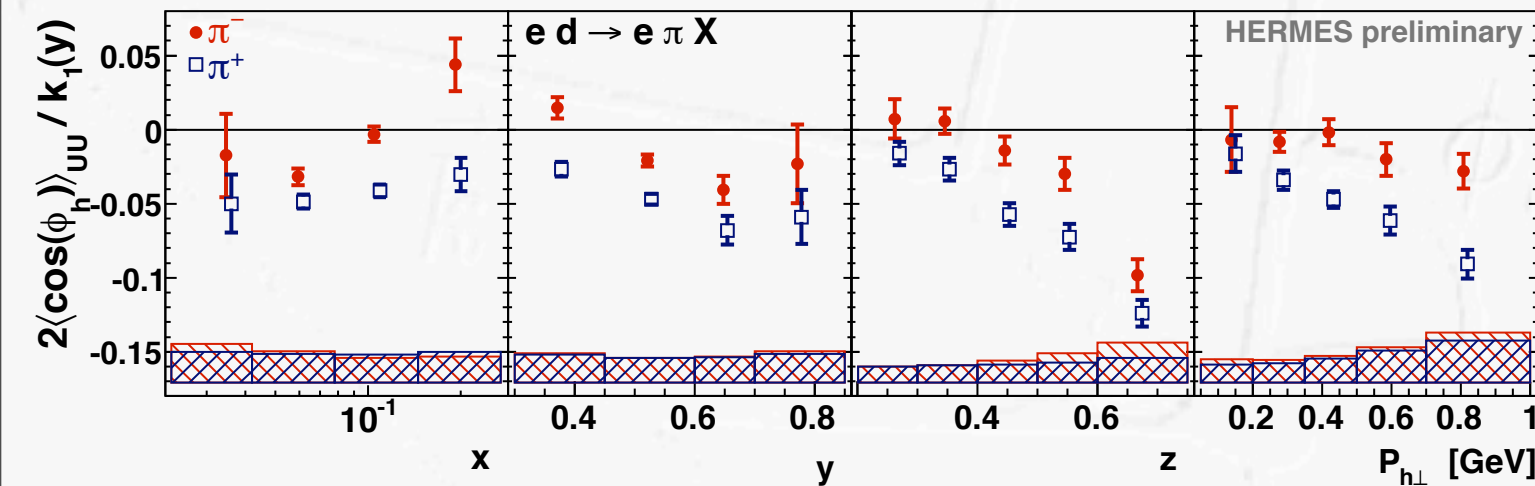
intriguing behavior
for kaons

Cahn effect?

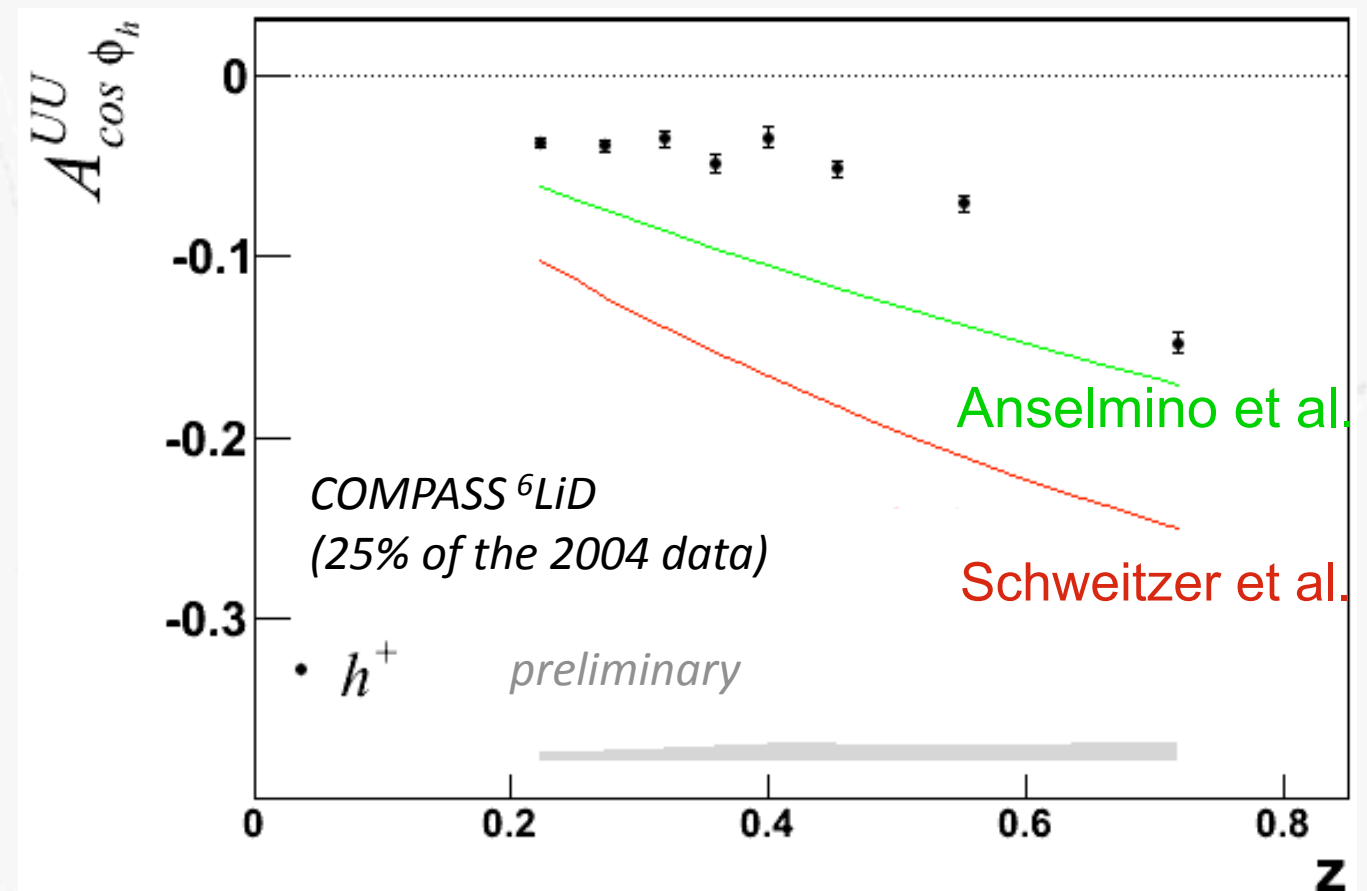
next to leading twist

$$F_{UU}^{\cos \phi_h} \propto \frac{2M}{Q} C \left[\underbrace{-\frac{\hat{P}_{h\perp} \cdot \vec{p}_T}{M_h} x h_1^\perp H_1^\perp}_{\text{BOER-MULDERS EFFECT}} - \underbrace{\frac{\hat{P}_{h\perp} \cdot \vec{k}_T}{M} x f_1 D_1}_{\text{CAHN EFFECT}} + \dots \right]$$

Interaction dependent terms neglected



- no dependence on hadron charge expected
- prediction off from data

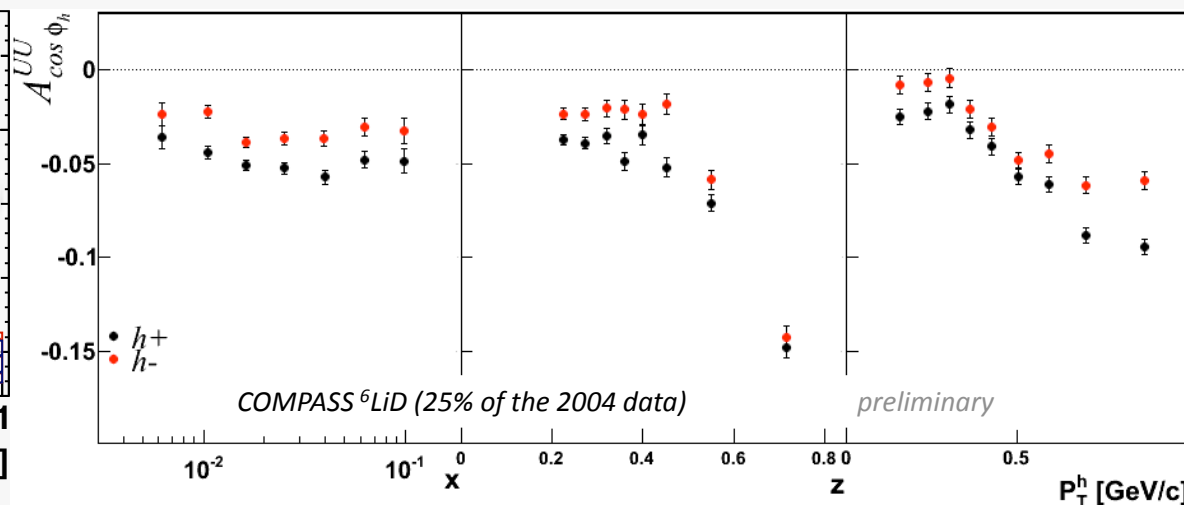
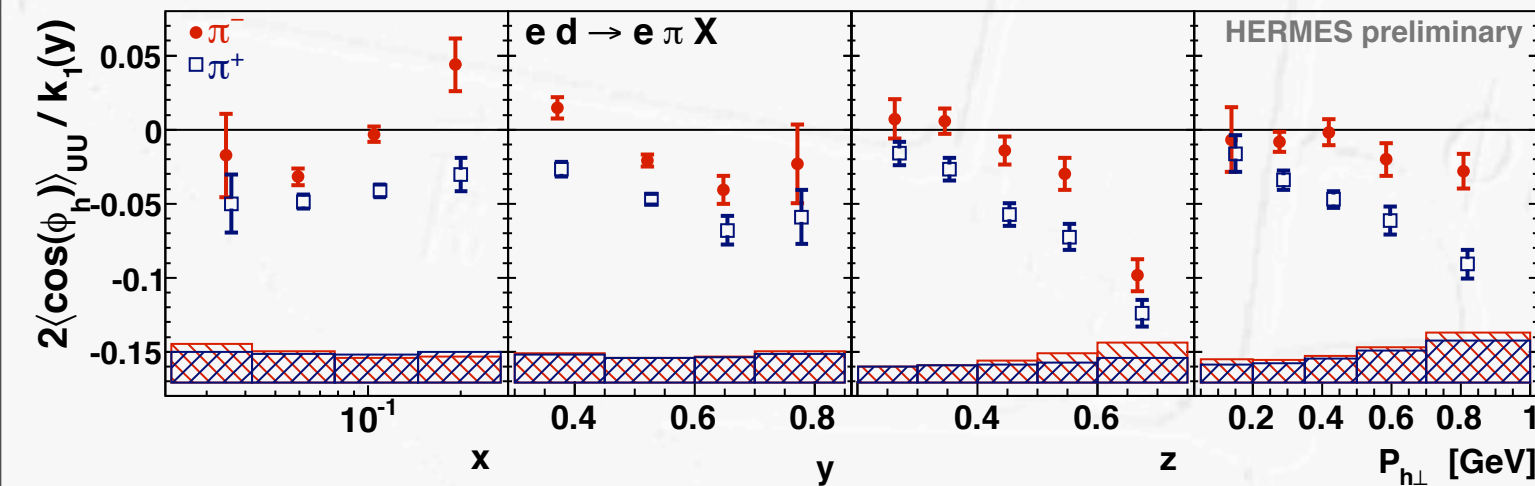


Cahn effect?

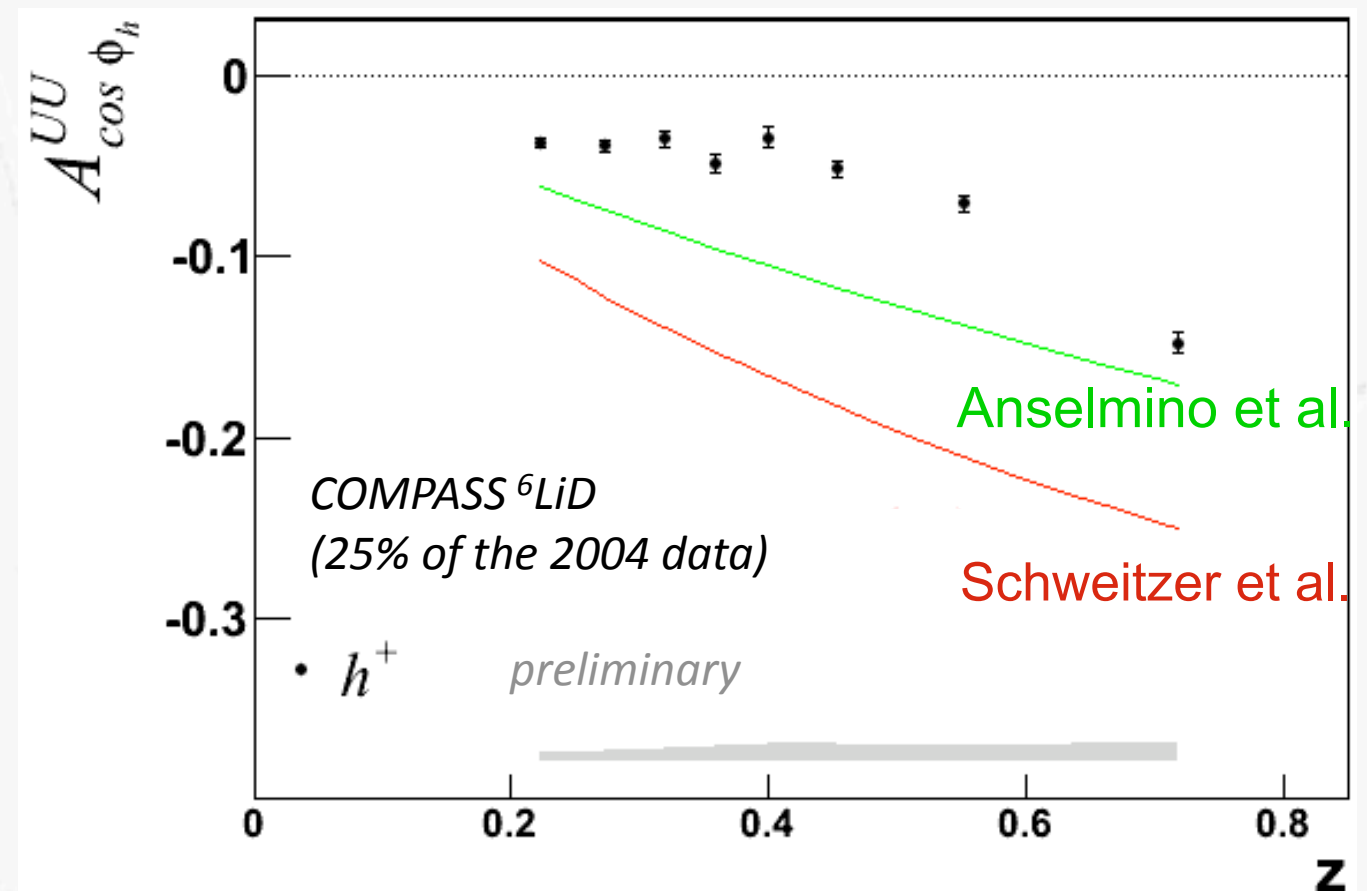
next to leading twist

$$F_{UU}^{\cos\phi_h} \propto \frac{2M}{Q} C \left[\underbrace{-\frac{\hat{P}_{h\perp} \cdot \vec{p}_T}{M_h} x h_1^\perp H_1^\perp}_{\text{BOER-MULDERS EFFECT}} - \underbrace{\frac{\hat{P}_{h\perp} \cdot \vec{k}_T}{M} x f_1 D_1}_{\text{CAHN EFFECT}} + \dots \right]$$

Interaction dependent terms neglected

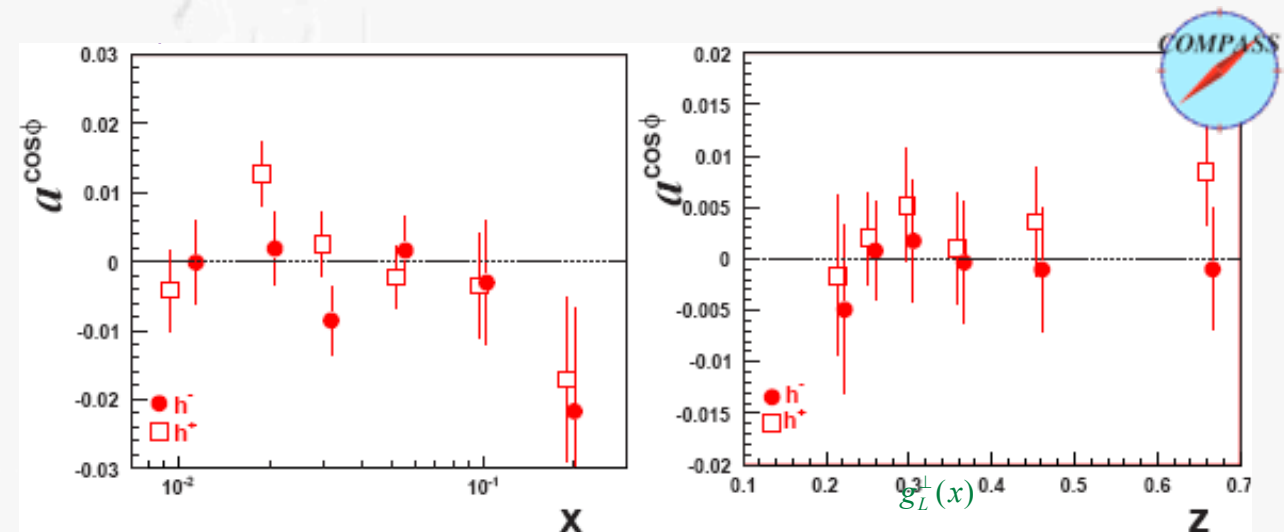


- no dependence on hadron charge expected
- prediction off from data
- ➔ sign of Boer-Mulders in $\cos\phi$ modulation or "real" twist-3?



Other twist-3 effects

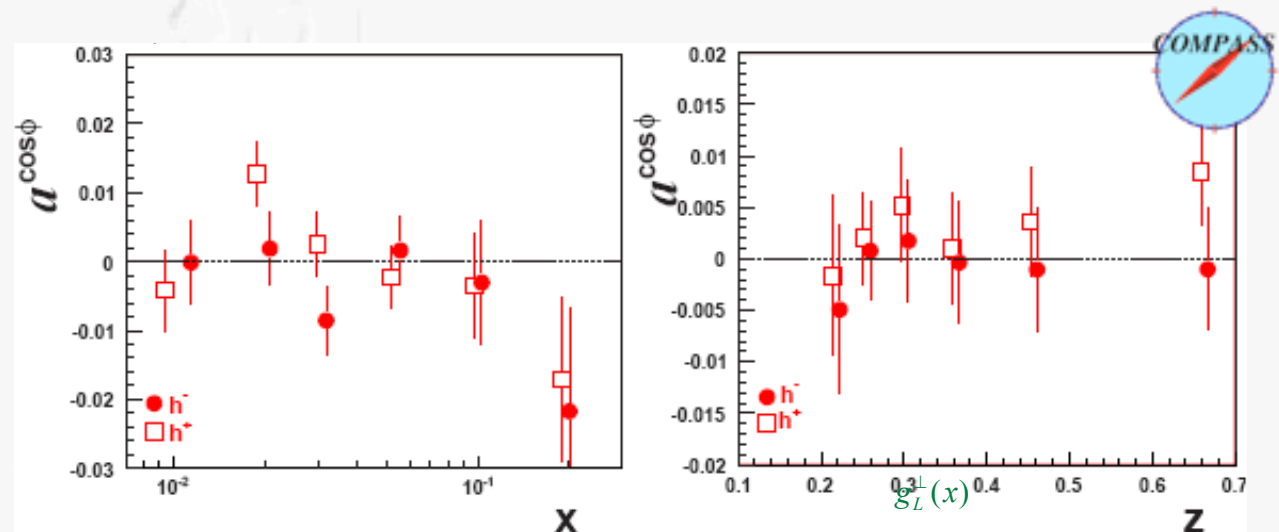
$A_{LL}^{\cos \phi}$



$$= \frac{2M}{Q} \mathcal{C} \left[\frac{\hat{\mathbf{h}} \cdot \mathbf{k}_T}{M_h} \left(x e_L H_1^\perp - \frac{M_h}{M} g_{1L} \frac{\tilde{D}^\perp}{z} \right) - \frac{\hat{\mathbf{h}} \cdot \mathbf{p}_T}{M} \left(x g_L^\perp D_1 + \frac{M_h}{M} h_{1L}^\perp \frac{\tilde{E}}{z} \right) \right]$$

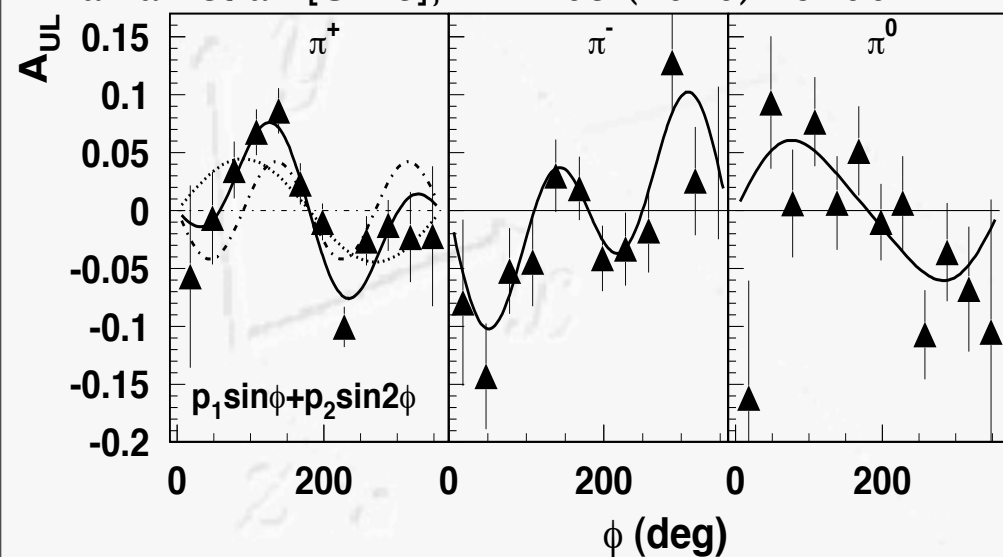
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$$A_{LL}^{\cos \phi}$$



$$= \frac{2M}{Q} \mathcal{C} \left[\frac{\hat{h} \cdot \mathbf{k}_T}{M_h} \left(x e_L H_1^\perp - \frac{M_h}{M} g_{1L} \frac{\tilde{D}^\perp}{z} \right) - \frac{\hat{h} \cdot \mathbf{p}_T}{M} \left(x g_L^\perp D_1 + \frac{M_h}{M} h_{1L}^\perp \frac{\tilde{E}}{z} \right) \right]$$

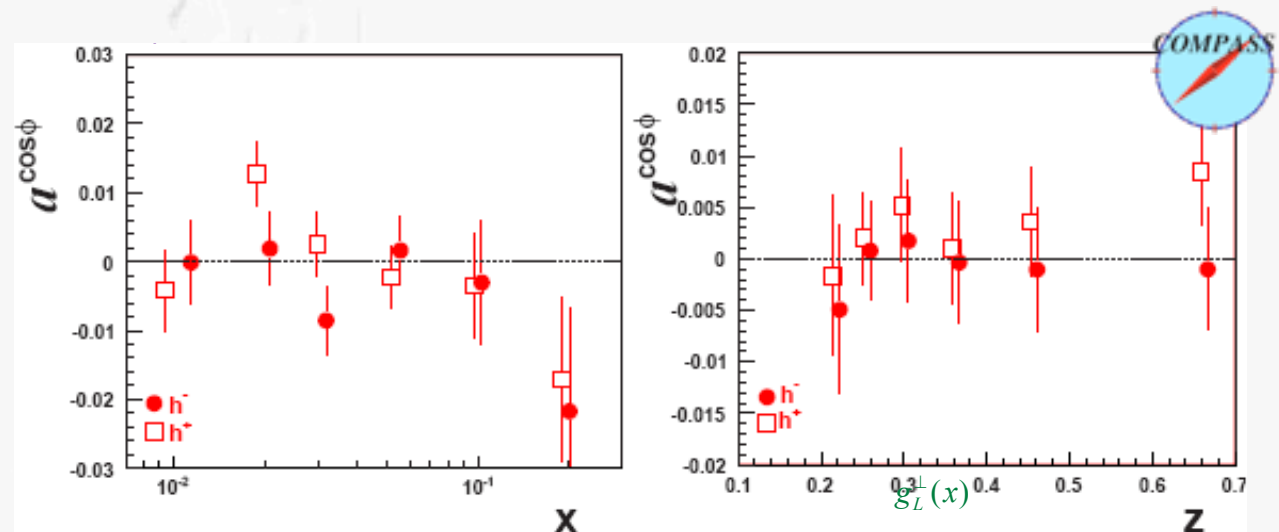
Avakian et al. [CLAS], PRL 105 (2010) 262002



$$= \frac{2M}{Q} \mathcal{C} \left[-\frac{\hat{h} \cdot \mathbf{k}_T}{M_h} \left(x h_L H_1^\perp + \frac{M_h}{M} g_{1L} \frac{\tilde{G}^\perp}{z} \right) + \frac{\hat{h} \cdot \mathbf{p}_T}{M} \left(x f_L^\perp D_1 - \frac{M_h}{M} h_{1L}^\perp \frac{\tilde{H}}{z} \right) \right]$$

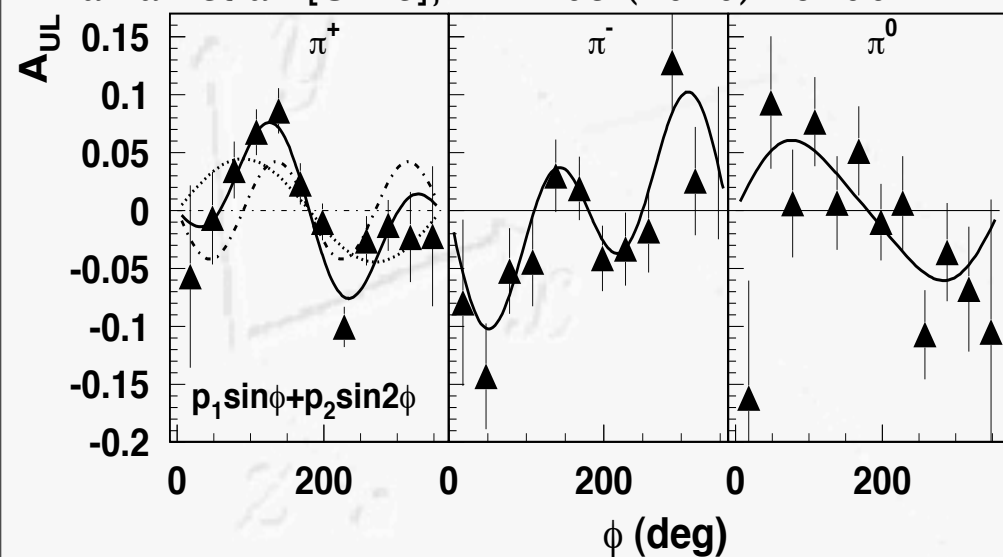
Other twist-3 effects

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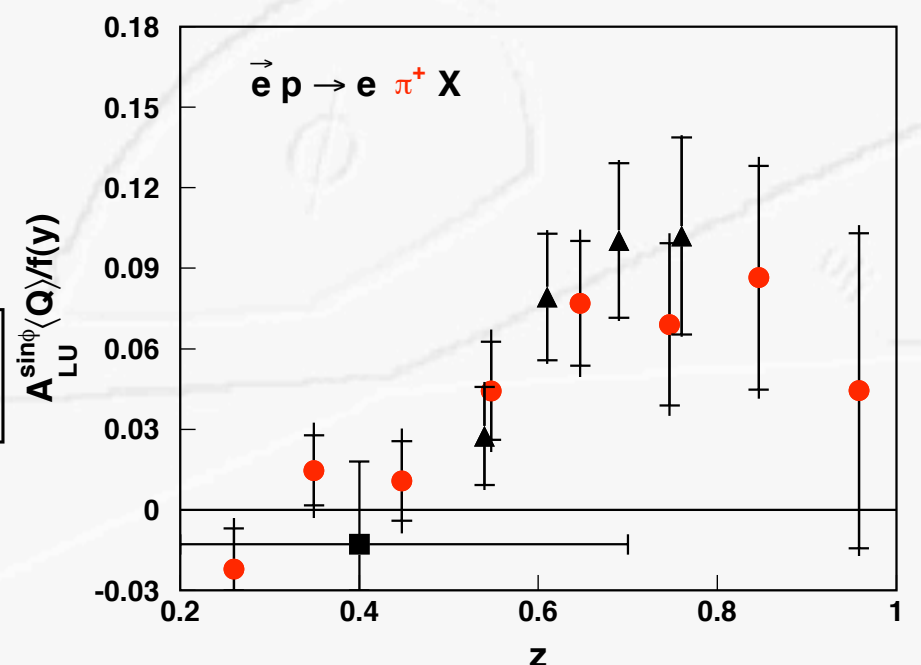
$$= \frac{2M}{Q} \mathcal{C} \left[\frac{\hat{h} \cdot \mathbf{k}_T}{M_h} \left(x e_L H_1^\perp - \frac{M_h}{M} g_{1L} \frac{\tilde{D}^\perp}{z} \right) - \frac{\hat{h} \cdot \mathbf{p}_T}{M} \left(x g_L^\perp D_1 + \frac{M_h}{M} h_{1L}^\perp \frac{\tilde{E}}{z} \right) \right]$$

Avakian et al. [CLAS], PRL 105 (2010) 262002



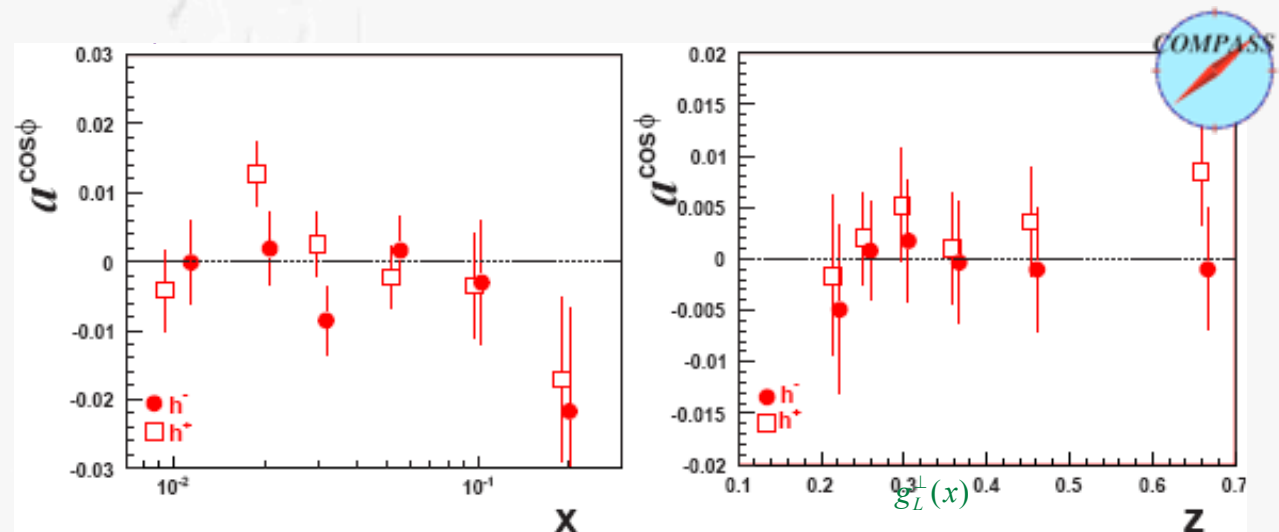
$$= \frac{2M}{Q} \mathcal{C} \left[-\frac{\hat{h} \cdot \mathbf{k}_T}{M_h} \left(x h_L H_1^\perp + \frac{M_h}{M} g_{1L} \frac{\tilde{G}^\perp}{z} \right) + \frac{\hat{h} \cdot \mathbf{p}_T}{M} \left(x f_L^\perp D_1 - \frac{M_h}{M} h_{1L}^\perp \frac{\tilde{H}}{z} \right) \right]$$

$$\frac{2M}{Q} \mathcal{C} \left[-\frac{\hat{h} \cdot \mathbf{k}_T}{M_h} \left(x e H_1^\perp + \frac{M_h}{M} f_1 \frac{\tilde{G}^\perp}{z} \right) + \frac{\hat{h} \cdot \mathbf{p}_T}{M} \left(x g^\perp D_1 + \frac{M_h}{M} h_1^\perp \frac{\tilde{E}}{z} \right) \right]$$



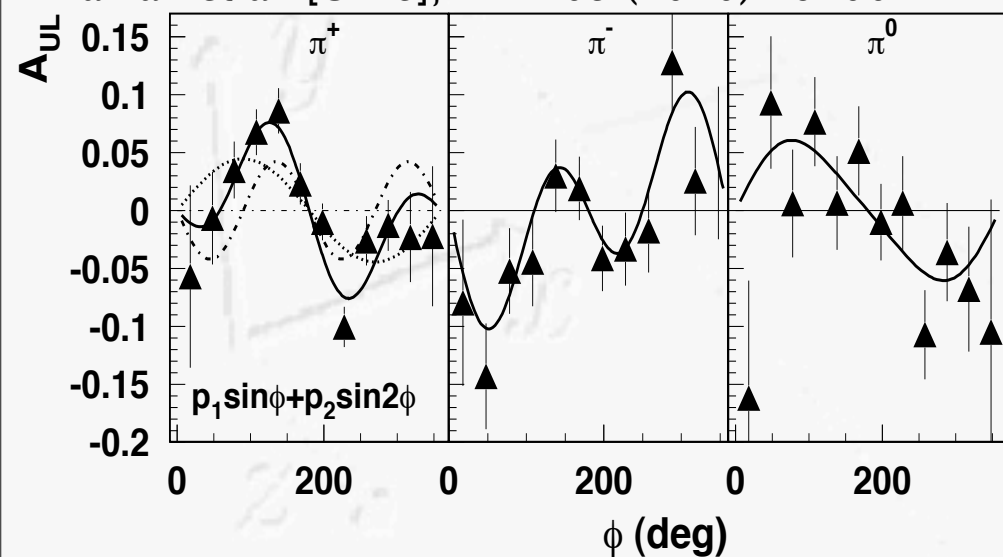
Other twist-3 effects

$$A_{LL}^{\cos \phi}$$



$$= \frac{2M}{Q} \mathcal{C} \left[\frac{\hat{h} \cdot \mathbf{k}_T}{M_h} \left(x e_L H_1^\perp - \frac{M_h}{M} g_{1L} \frac{\tilde{D}^\perp}{z} \right) - \frac{\hat{h} \cdot \mathbf{p}_T}{M} \left(x g_L^\perp D_1 + \frac{M_h}{M} h_{1L}^\perp \frac{\tilde{E}}{z} \right) \right]$$

Avakian et al. [CLAS], PRL 105 (2010) 262002

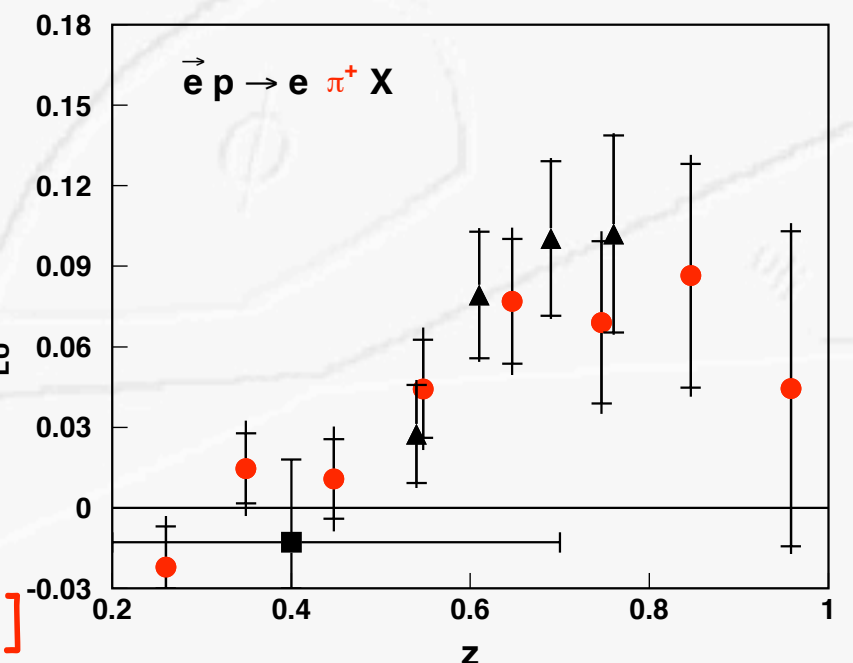


$$= \frac{2M}{Q} \mathcal{C} \left[-\frac{\hat{h} \cdot \mathbf{k}_T}{M_h} \left(x h_L H_1^\perp + \frac{M_h}{M} g_{1L} \frac{\tilde{G}^\perp}{z} \right) + \frac{\hat{h} \cdot \mathbf{p}_T}{M} \left(x f_L^\perp D_1 - \frac{M_h}{M} h_{1L}^\perp \frac{\tilde{H}}{z} \right) \right]$$

$$\frac{2M}{Q} \mathcal{C} \left[-\frac{\hat{h} \cdot \mathbf{k}_T}{M_h} \left(x e_L H_1^\perp + \frac{M_h}{M} f_1 \frac{\tilde{G}^\perp}{z} \right) + \frac{\hat{h} \cdot \mathbf{p}_T}{M} \left(x g_L^\perp D_1 + \frac{M_h}{M} h_{1L}^\perp \frac{\tilde{E}}{z} \right) \right] A_{LU}^{\sin \phi(Q)/f(y)}$$

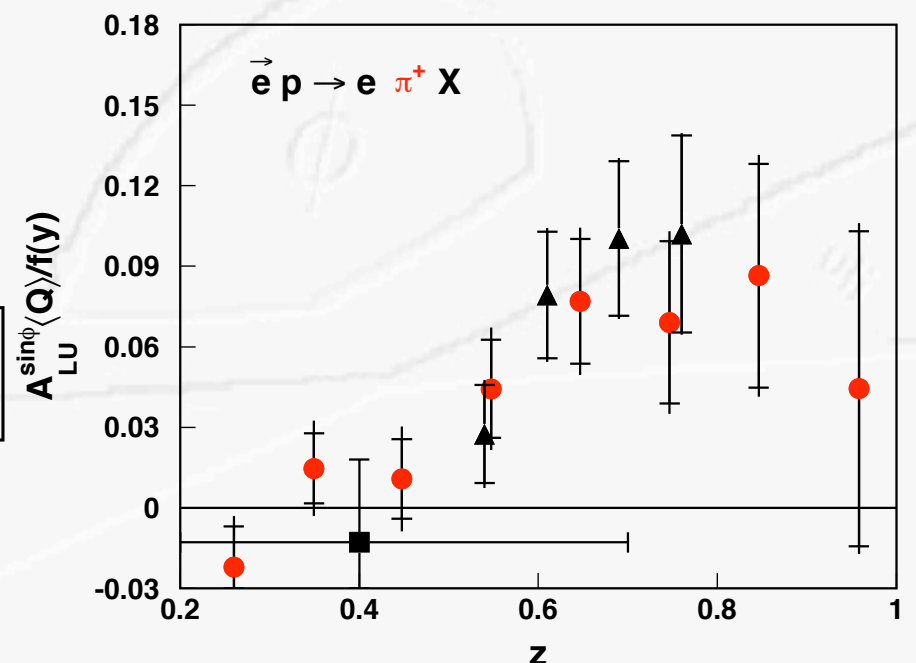
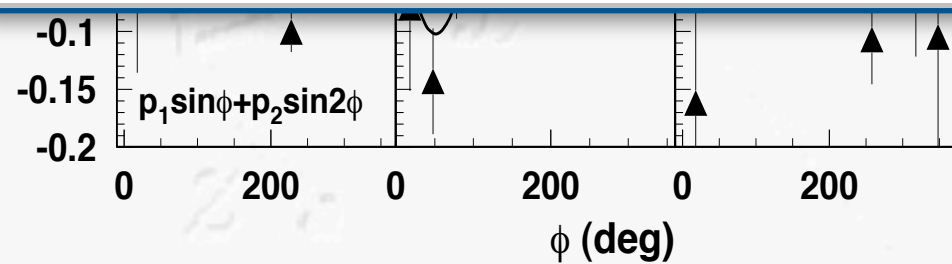
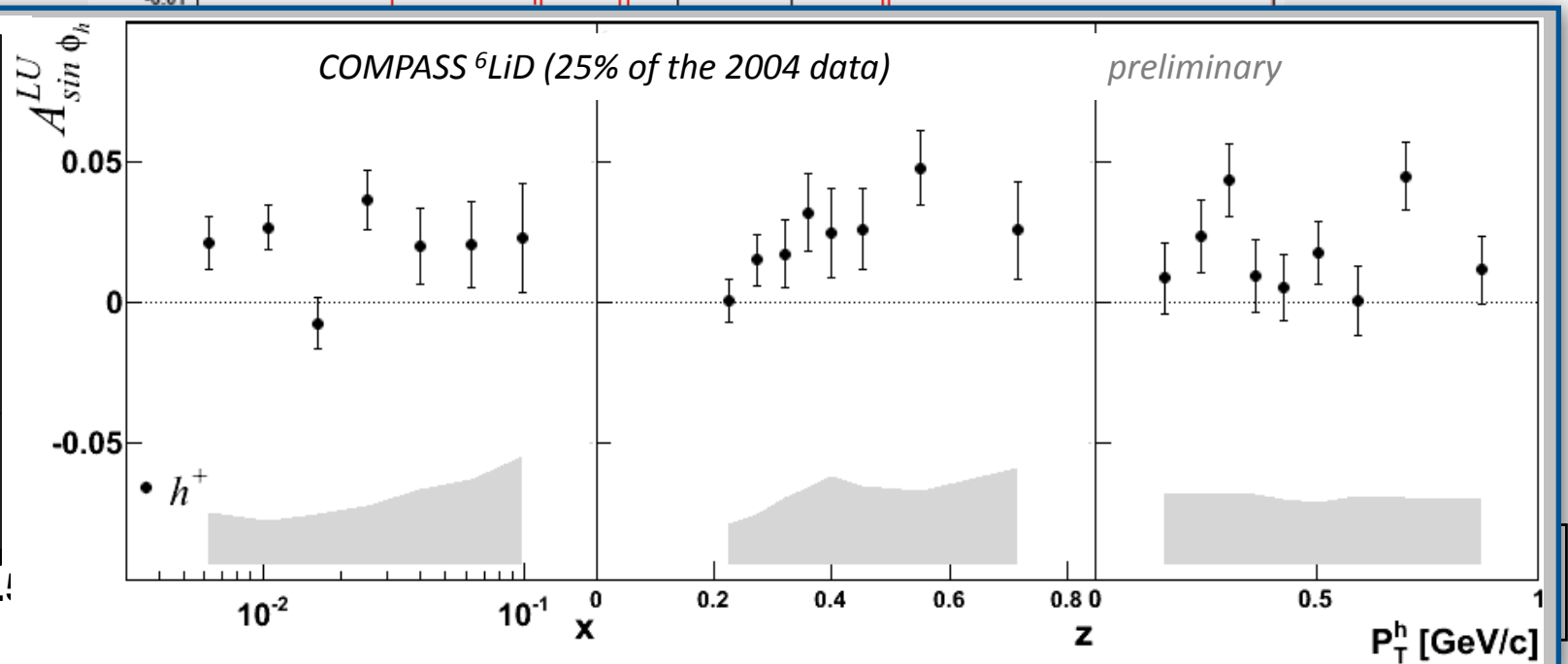
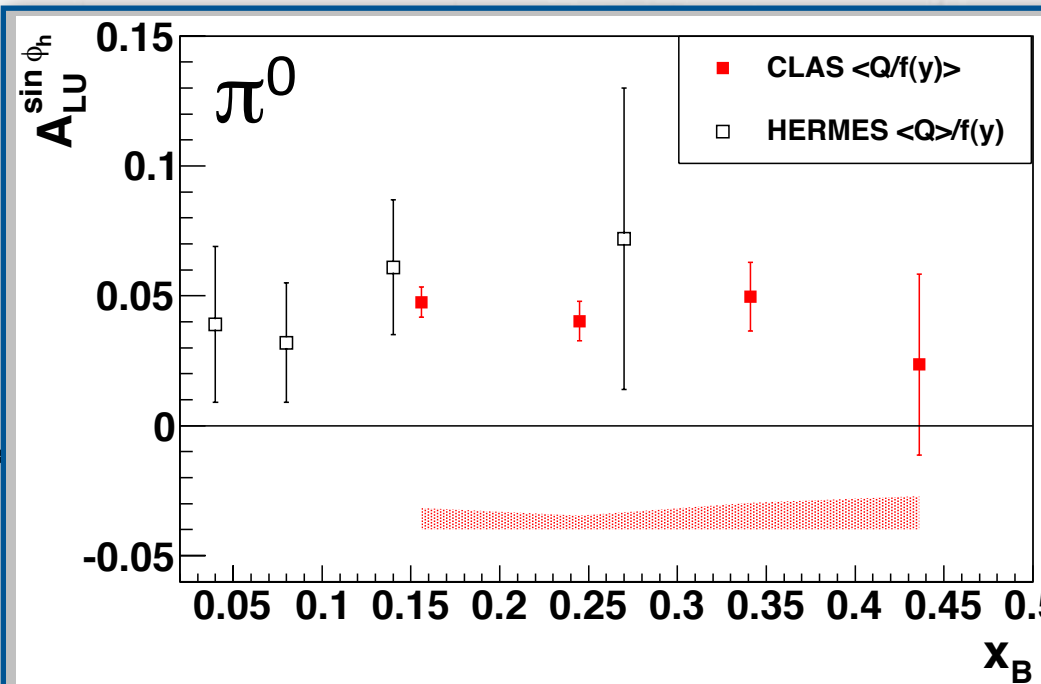
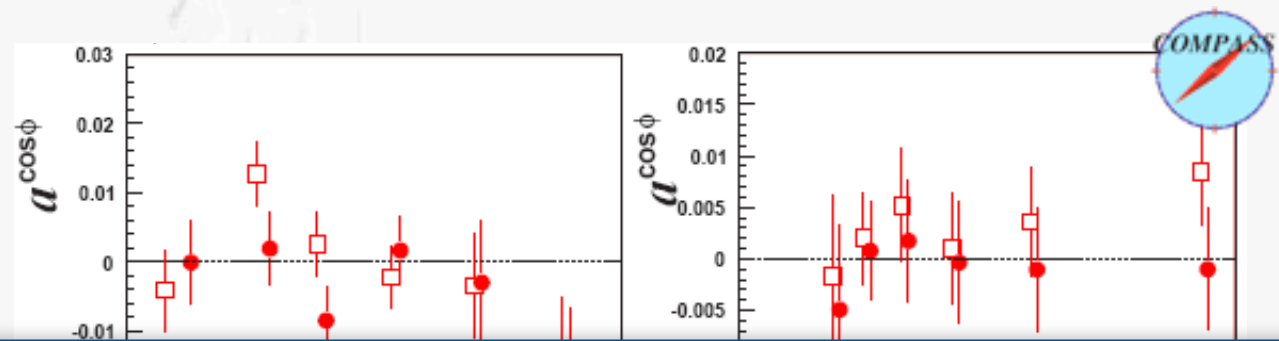
transverse force on transversely pol. quarks [M. Burkardt]

Gunar Schnell



Other twist-3 effects

$A_{LL}^{\cos \phi}$



$$\frac{2M}{Q} c \left[-\frac{\hat{h} \cdot \mathbf{k}_T}{M_h} \left(x e H_1^\perp + \frac{M_h}{M} f_1 \frac{\tilde{G}^\perp}{z} \right) + \frac{\hat{h} \cdot \mathbf{p}_T}{M} \left(x g^\perp D_1 + \frac{M_h}{M} h_1^\perp \frac{\tilde{E}}{z} \right) \right] A_{LU}^{\sin \phi} \langle Q \rangle / f(y)$$

Any help in disentangling contributions?

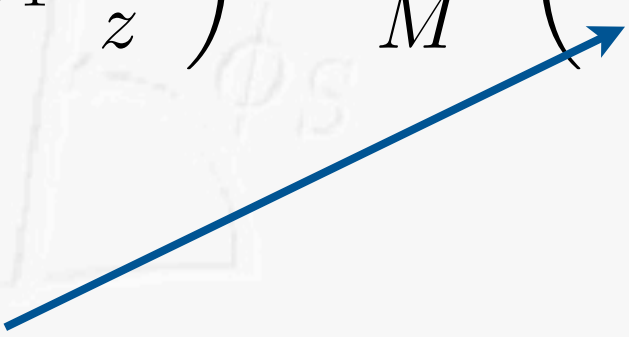
$$\frac{2M}{Q} c \left[-\frac{\hat{\mathbf{h}} \cdot \mathbf{k}_T}{M_h} \left(x e H_1^\perp + \frac{M_h}{M} f_1 \frac{\tilde{G}^\perp}{z} \right) + \frac{\hat{\mathbf{h}} \cdot \mathbf{p}_T}{M} \left(x g^\perp D_1 + \frac{M_h}{M} h_1^\perp \frac{\tilde{E}}{z} \right) \right]$$

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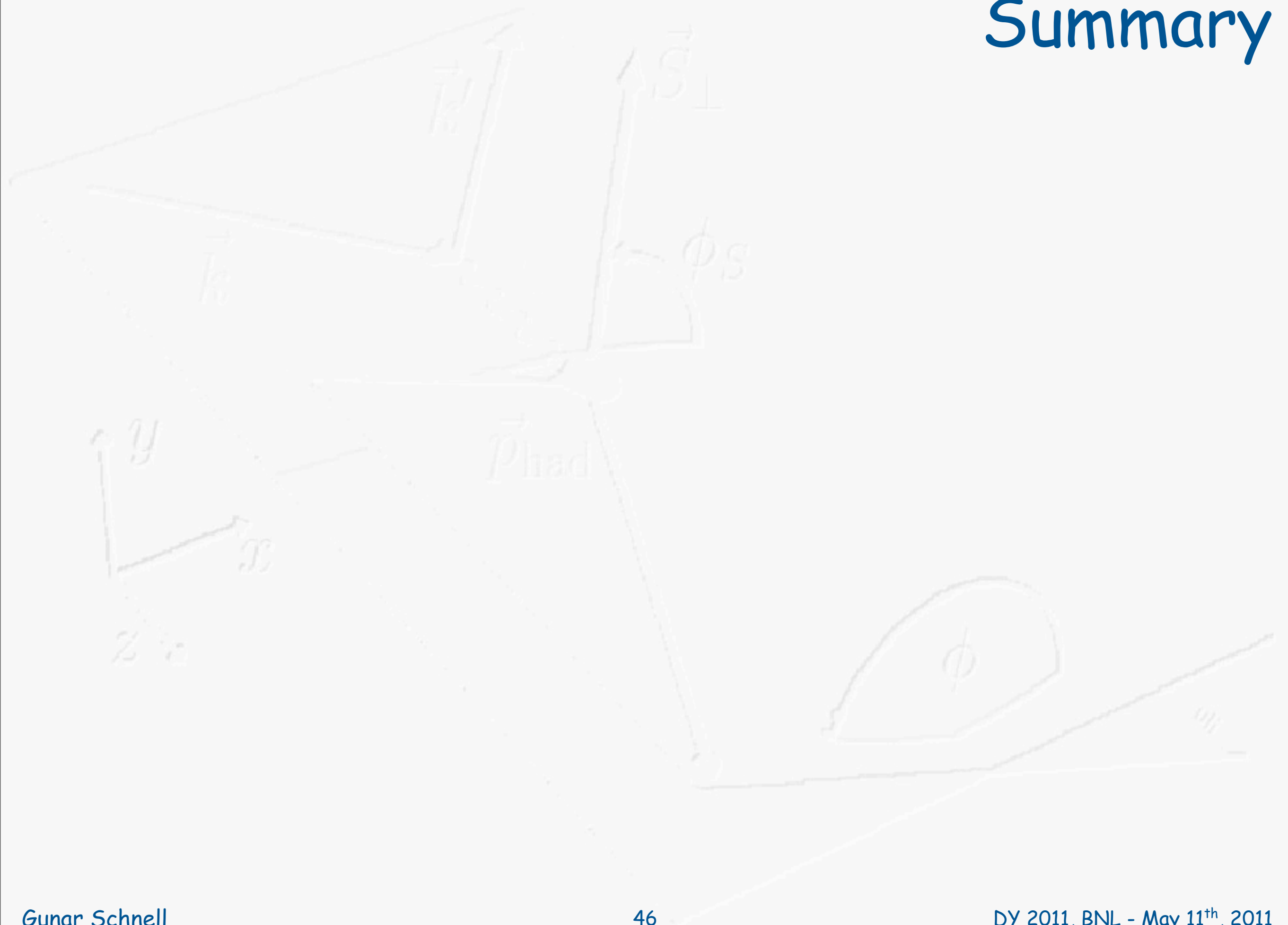
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- 2-hadron fragmentation:

$$\sigma_{LU} \propto \sin \phi_{R\perp} \left[x e(x) H_1^\triangleleft(z, \zeta, M_h^2) + \frac{1}{z} f_1(x) \tilde{G}^\triangleleft(z, \zeta, M_h^2) \right]$$

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- great opportunities for measurements in Drell-Yan especially for naive-T-odd TMDs