Orbital Angular Momentum

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- Deeply virtual Compton scattering (DVCS)

$\hookrightarrow$ Generalized parton distributions (GPDs)

- Probabilistic interpretation of GPDs as Fourier transforms of impact parameter dependent PDFs
  - $H(x, 0, -\mathbf{\Delta}_\perp^2) \longrightarrow q(x, \mathbf{b}_\perp)$
  - $\tilde{H}(x, 0, -\mathbf{\Delta}_\perp^2) \longrightarrow \Delta q(x, \mathbf{b}_\perp)$
  - $E(x, 0, -\mathbf{\Delta}_\perp^2) \longrightarrow$ $\perp$ deformation of PDFs when the target is transversely polarized
- Chromodynamik lensing and $\perp$ single-spin asymmetries (SSA)

$$\left.\begin{array}{c}\text{transverse distortion of PDFs}\\ +\text{ final state interactions}\end{array}\right\} \Rightarrow \perp \text{SSA in } \gamma N \longrightarrow \pi + X$$

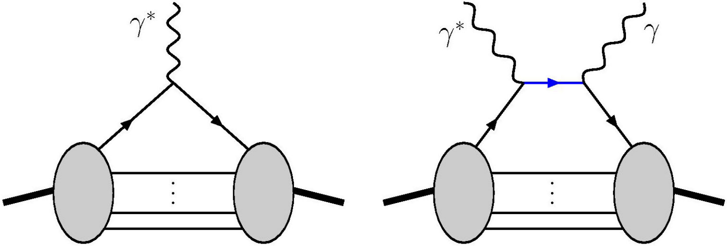
- Summary

- virtual Compton scattering: $\gamma^* p \longrightarrow \gamma p$ (actually: $e^- p \longrightarrow e^- \gamma p$)
- 'deeply': $-q_\gamma^2 \gg M_p^2, |t| \longrightarrow$ Compton amplitude dominated by (coherent superposition of) Compton scattering off single quarks

↪ only difference between form factor (a) and DVCS amplitude (b) is replacement of photon vertex by two photon vertices connected by quark (energy denominator depends on quark momentum fraction x)

↪ DVCS amplitude provides access to momentum-decomposition of form factor = Generalized Parton Distribution (GPDs).

$$\int dx H_q(x,\xi,t) = F_1^q(t) \qquad \int dx E_q(x,\xi,t) = F_2^q(t)$$



- form factors: $\stackrel{FT}{\longleftrightarrow} \rho(\vec{r})$
- $GPDs(x, \vec{\Delta})$: form factor for quarks with momentum fraction x

$\hookrightarrow$ suitable FT of $GPDs$ should provide spatial distribution of quarks with momentum fraction x

- careful: cannot measure longitudinal momentum (x) and longitudinal position simultaneously (Heisenberg)

$\hookrightarrow$ consider purely transverse momentum transfer

Impact Parameter Dependent Quark Distributions

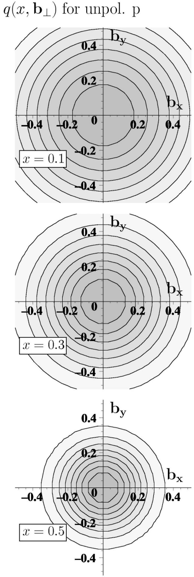
$$q(x, \mathbf{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} GPD(x, -\mathbf{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp \cdot \mathbf{\Delta}_\perp}$$

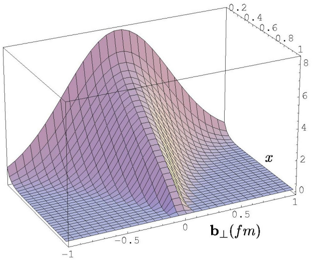
$q(x, \mathbf{b}_\perp)$ = parton distribution as a function of the separation $\mathbf{b}_\perp$ from the transverse center of momentum $\mathbf{R}_\perp \equiv \sum_{i\in q,g} \mathbf{r}_{\perp,i} x_i$
MB, Phys. Rev. D62, 071503 (2000)

- No relativistic corrections (Galilean subgroup!)

$\hookrightarrow$ corollary: interpretation of 2d-FT of $F_1(Q^2)$ as charge density in transverse plane also free from relativistic corrections

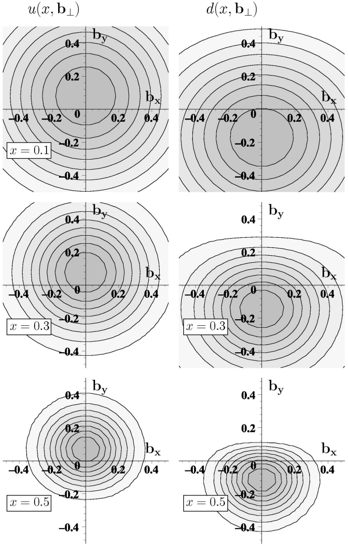
Impact parameter dependent quark distributions

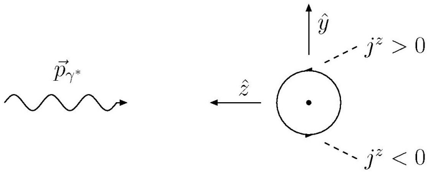




- x = momentum fraction of the quark
- $\vec{b} = \perp$ distance of quark from $\perp$ center of momentum
- small x: large 'meson cloud'
- larger x: compact 'valence core'
- $x \to 1$: active quark becomes center of momentum

$\hookrightarrow$ $\vec{b}_\perp \to 0$ (narrow distribution)





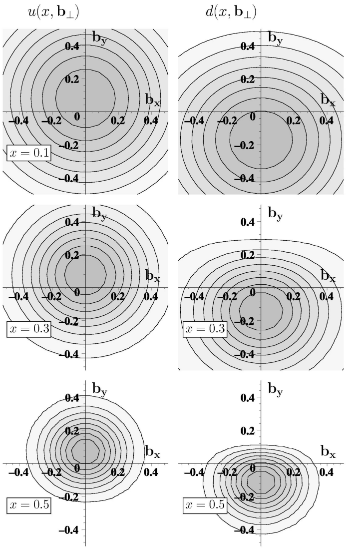
proton polarized in $+\hat{x}$ direction

no axial symmetry!

$$q(x, \mathbf{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H_q(x, 0, -\Delta_\perp^2) e^{-i\mathbf{b}_\perp \cdot \Delta_\perp}$$

$$- \frac{1}{2M} \frac{\partial}{\partial b_y} \int \frac{d^2\Delta_\perp}{(2\pi)^2} E_q(x, 0, -\Delta_\perp^2) e^{-i\mathbf{b}_\perp \cdot \Delta_\perp}$$

Physics: relevant density in DIS is $j^+ \equiv j^0 + j^3$ and left-right asymmetry from j^3



proton polarized in $+\hat{x}$ direction

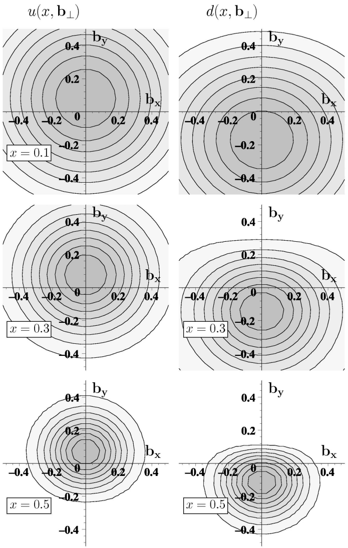
$$q(x,\mathbf{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H_q(x,0,-\Delta_\perp^2) e^{-i\mathbf{b}_\perp\cdot\Delta_\perp}$$

$$-\frac{1}{2M}\frac{\partial}{\partial b_y}\int \frac{d^2\Delta_\perp}{(2\pi)^2} E_q(x,0,-\Delta_\perp^2) e^{-i\mathbf{b}_\perp\cdot\Delta_\perp}$$

sign & magnitude of the average shift

model-independently related to p/n anomalous magnetic moments:

$$\langle b_y^q \rangle \equiv \int dx \int d^2 b_\perp q(x,\mathbf{b}_\perp) b_y$$
$$= \frac{1}{2M}\int dx E_q(x,0,0) = \frac{\kappa_q}{2M}$$



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$\kappa^p = 1.913 = \frac{2}{3}\kappa_u^p - \frac{1}{3}\kappa_d^p + \ldots$

- u-quarks: $\kappa_u^p = 2\kappa_p + \kappa_n = 1.673$

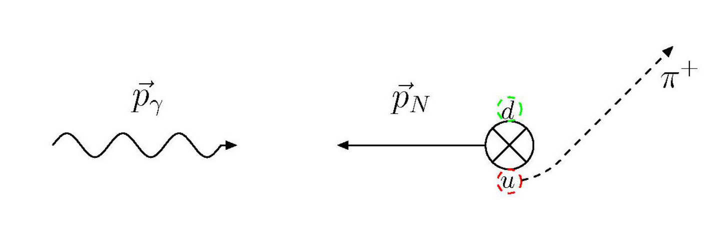
$\hookrightarrow$ shift in $+\hat{y}$ direction

- d-quarks: $\kappa_u^p = 2\kappa_n + \kappa_p = -2.033$

$\hookrightarrow$ shift in $-\hat{y}$ direction

- $\langle b_y^q \rangle = \mathcal{O}\left(\pm 0.2 fm\right)$!!!!

example: $\gamma p \to \pi X$

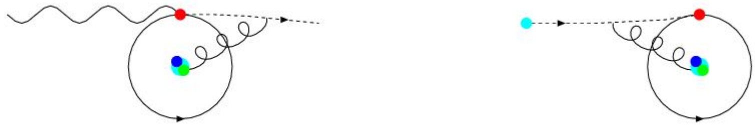


- u, d distributions in $\perp$ polarized proton have left-right asymmetry in $\perp$ position space (T-even!); sign 'determined' by κ_u & κ_d
- attractive FSI deflects active quark towards the CoM

$\hookrightarrow$ FSI translates position space distortion (before the quark is knocked out) in $+\hat{y}$-direction into momentum asymmetry that favors $-\hat{y}$ direction $\to$ 'chromodynamic lensing'

$\Rightarrow$ $\kappa_p\ , \kappa_n \quad \longleftrightarrow \quad$ sign of SSA!!!!!!!! (MB,2004)

- confirmed by HERMES p data; consistent with vanishing isoscalar Sivers (COMPASS) compare FSI for 'red' q that is being knocked out of nucleon with ISI for 'anti-red' $\bar{q}$ that is about to annihilate with a 'red' target q



a) b)

FSI in SIDIS

- knocked-out q 'red'

$\hookrightarrow$ spectators 'anti-red'

$\hookrightarrow$ interaction between knocked-out quark and spectators attractive

ISI in DY

- incoming $\bar{q}$ 'anti-red'

$\hookrightarrow$ struck target q 'red'

$\hookrightarrow$ spectators also 'anti-red'

$\hookrightarrow$ interaction between incoming $\bar{q}$ and spectators repulsive

test of $f_{1T}^{\perp}(x, \mathbf{k}-\perp)_{DY} = -f_{1T}^{\perp}(x, \mathbf{k}-\perp)_{SIDIS}$ critical test of TMD factorization approach (J.Qiu)

- treat FSI to lowest order in g

$\hookrightarrow$

$$\left\langle k_q^i \right\rangle = -\frac{g}{4p^+} \int \frac{d^2\mathbf{b}_\perp}{2\pi} \frac{b^i}{|\mathbf{b}_\perp|^2} \left\langle p,s \left| \bar{q}(0)\gamma^+ \frac{\lambda_a}{2} q(0)\rho_a(\mathbf{b}_\perp) \right| p,s \right\rangle$$

with $\rho_a(\mathbf{b}_\perp) = \int dr^- \rho_a(r^-, \mathbf{b}_\perp)$ summed over all quarks and gluons

$\hookrightarrow$ SSA related to dipole moment of density-density correlations

- GPDs (N polarized in $+\hat{x}$ direction): $u \longrightarrow +\hat{y}$ and $d \longrightarrow -\hat{y}$

$\hookrightarrow$ expect density density correlation to show same asymmetry $\langle b^y \bar{u}(0)\gamma^+ \frac{\lambda_a}{2} u(0)\rho_a(\mathbf{b}_\perp)\rangle > 0$

$\hookrightarrow$ sign of SSA opposite to sign of distortion in position space

Total (Spin+Orbital) Quark Angular Momentum

$$J_q^x = L_q^x + S_q^x = \int d^3r \left[y T_q^{0z}(\vec{r}) - z T_q^{0y}(\vec{r}) \right]$$

- $T_q^{\mu\nu}(\vec{r})$ energy momentum tensor $(T_q^{\mu\nu}(\vec{r}) = T_q^{\nu\mu}(\vec{r}))$
- $T_q^{0i}(\vec{r})$ momentum density $[P_q^i = \int d^3r T_q^{0i}(\vec{r})\ ]$
- think: $(\vec{r} \times \vec{p})^x = yp^z - zp^y$

relate to impact parameter dependent quark distributions $q(x, \mathbf{r}_\perp)$:

Consider spherically symmetric wave packet with nucleon polarized in $+\hat{x}$ direction

- eigenstate under rotations about x-axis

$\hookrightarrow$ both terms in J_q^x equal:
$J_q^x = 2\int d^3r\, y T_q^{0z}(\vec{r}) = \int d^3r\, y \left[T_q^{0z}(\vec{r}) + T_q^{z0}(\vec{r}) \right]$

- $\int d^3r\, y T_q^{00}(\vec{r}) = 0 = \int d^3r\, y T_q^{zz}(\vec{r})$

$$\Rightarrow \quad J_q^x = \int d^3r\, y T_q^{++}(\vec{r}) \quad \text{with} \quad T^{++} \equiv T^{00} + T^{0z} + T^{z0} + T^{zz}$$

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- $\int dx\, xq(x, \mathbf{r}_\perp) = \frac{1}{2m_N}\int dz T^{++}(\vec{r})$
  (note: here x is momentum fraction and not r^x)

$\hookrightarrow$ $J_q^x = m_N \int dx\, xr^y q(x, \mathbf{r}_\perp)$

- before applying this result to $\perp$ shifted PDFs, need to consider 'overall $\perp$ shift' of CoM for $\perp$ polarized target...

spherically symmetric wave packet has center of momentum off-center:

- relativistic effect $\longrightarrow$ use Dirac wave packet for nucleon

$$\psi = \begin{pmatrix} f(r) \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+M_N} f(r) \end{pmatrix} \chi \qquad \text{with} \qquad \chi = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$\int d^3r f^2(r) = 1$, take limit of large 'radius' for wave packet

- evaluate $T_q^{0z} = \frac{i}{2}\bar{q}\left(\gamma^0\partial^z + \gamma^z\partial^0\right) q$ in this state
- $\psi^\dagger \partial_z \psi$ even under $y \to -y$, i.e. no contribution to $\langle y T_q^{0z} \rangle$
- use $i\psi^\dagger\gamma^0\gamma^z\partial^0\psi = E\psi^\dagger\gamma^0\gamma^z\psi$

$$\begin{aligned}
\langle T^{0z} y \rangle &= E\int d^3r \psi^\dagger \gamma^0\gamma^z \psi y = E\int d^3r \psi^\dagger \begin{pmatrix} 0 & \sigma^z \\ \sigma^z & 0 \end{pmatrix} \psi y \\
&= \frac{2E}{E+M_N}\int d^3r \chi^\dagger \sigma^z\sigma^y \chi f(r)(-i)\partial^y f(r) y \\
&= \frac{E}{E+M_N}\int d^3r f^2(r) \overset{R\to\infty}{\longrightarrow} \frac{1}{2}
\end{aligned}$$

$\hookrightarrow$ p pol. in $+\hat{x}$ direction has CoM shifted by $\frac{1}{2M_N}$ in $+\hat{y}$ direction!

Angular Momentum carried by Quarks

spherically symmetric wave packet has center of momentum off-center:

- relativistic effect $\longrightarrow$ use Dirac wave packet for nucleon

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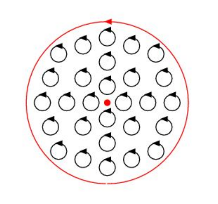
$$\langle T^{0z} y \rangle \xrightarrow{R\to\infty} \frac{1}{2}$$

$\hookrightarrow$ p pol. in $+\hat{x}$ direction has CoM shifted by $\frac{1}{2M_N}$ in $+\hat{y}$ direction!

origin of 'shift' of CoM

- nucleon polization: $\odot$
- counterclockwise momentum density from lower component
- $p \sim \frac{1}{R}$, but $y \sim R$

$\hookrightarrow$ $\langle T^{++} y \rangle = \mathcal{O}(1)$



relate to impact parameter dependent quark distributions $q(x, \mathbf{b}_\perp)$:

- Thus $J_q^x = m_N \int dx\, x r^y q(x, \mathbf{r}_\perp)$ with $b^y = r^y - \frac{1}{2m_N}$, where $q(x, \mathbf{r}_\perp)$ is distribution relative to CoM of whole nucleon
- recall: $q(x, \mathbf{b}_\perp)$ for nucleon polarized in $+\hat{x}$ direction

$$\begin{aligned} q(x, \mathbf{b}_\perp) &= \int \frac{d^2\Delta_\perp}{(2\pi)^2} H_q(x, 0, -\Delta_\perp^2) e^{-i\mathbf{b}_\perp \cdot \Delta_\perp} \\ &\quad - \frac{1}{2M_N} \frac{\partial}{\partial b_y} \int \frac{d^2\Delta_\perp}{(2\pi)^2} E_q(x, 0, -\Delta_\perp^2) e^{-i\mathbf{b}_\perp \cdot \Delta_\perp} \end{aligned}$$

$$\begin{aligned} \Rightarrow J_q^x &= M_N \int dx\, x r^y q(x, \mathbf{r}_\perp) = \int dx\, x \left(m_N b^y + \frac{1}{2} \right) q(x, \mathbf{r}_\perp) \\ &= \frac{1}{2} \int dx\, x \left[H(x, 0, 0) + E(x, 0, 0) \right] \end{aligned}$$

relate to impact parameter dependent quark distributions $q(x, \mathbf{b}_\perp)$:

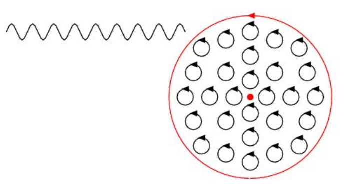
- Thus $J_q^x = m_N \int dx\, x r^y q(x, \mathbf{r}_\perp)$ with $b^y = r^y - \frac{1}{2m_N}$, where $q(x, \mathbf{r}_\perp)$ is distribution relative to CoM of whole nucleon
- recall: $q(x, \mathbf{b}_\perp)$ for nucleon polarized in $+\hat{x}$ direction

$$q(x, \mathbf{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H_q(x, 0, -\Delta_\perp^2) e^{-i\mathbf{b}_\perp \cdot \Delta_\perp} - \frac{1}{2M_N} \frac{\partial}{\partial b_y} \int \frac{d^2\Delta_\perp}{(2\pi)^2} E_q(x, 0, -\Delta_\perp^2) e^{-i\mathbf{b}_\perp \cdot \Delta_\perp}$$

$$\Rightarrow J_q^x = M_N \int dx\, x r^y q(x, \mathbf{r}_\perp) = \int dx\, x \left(m_N b^y + \frac{1}{2} \right) q(x, \mathbf{r}_\perp) = \frac{1}{2} \int dx\, x \left[H(x, 0, 0) + E(x, 0, 0) \right]$$

- X.Ji (1996): rotational invariance $\Rightarrow$ apply to all components of $\vec{J}$
- partonic interpretation exists only for $\perp$ components!

q with polarization $\odot$

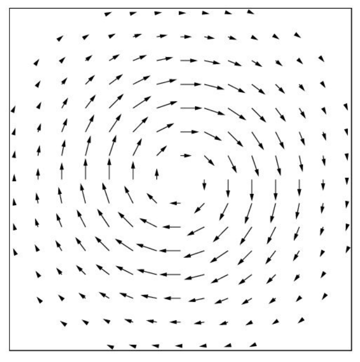


$\hookrightarrow$ counterclockwise current from lower component

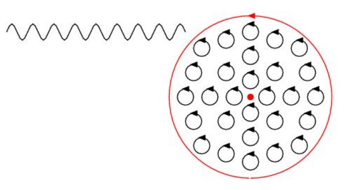
$\hookrightarrow$ q distribution shifted to top

unpolarized target

- all q polns. equally likely



q with polarization $\odot$

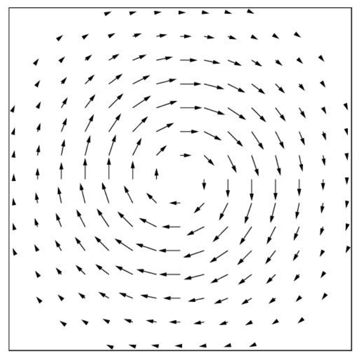


$\hookrightarrow$ counterclockwise current from lower component

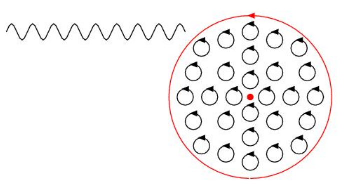
$\hookrightarrow$ q distribution shifted to top

unpolarized target

- q with pol. $\uparrow$ shifted to left



q with polarization $\odot$

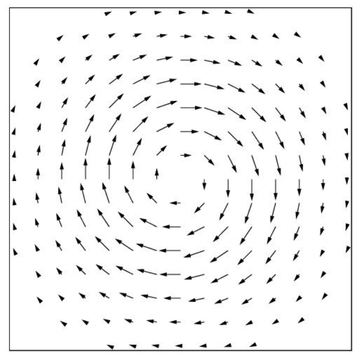


$\hookrightarrow$ counterclockwise current from lower component

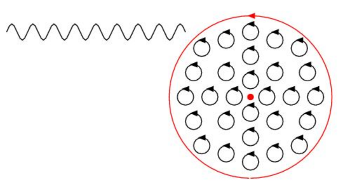
$\hookrightarrow$ q distribution shifted to top

unpolarized target

- q with pol. $\downarrow$ shifted to right



q with polarization $\odot$

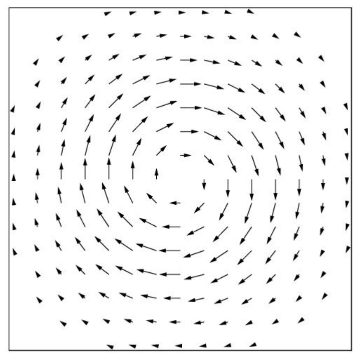


$\hookrightarrow$ counterclockwise current from lower component

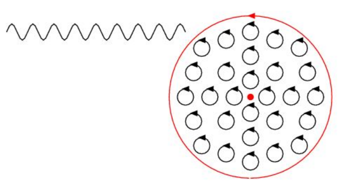
$\hookrightarrow$ q distribution shifted to top

unpolarized target

- q with pol. $\rightarrow$ shifted to top



q with polarization $\odot$

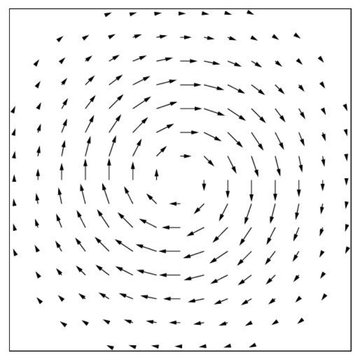


$\hookrightarrow$ counterclockwise current from lower component

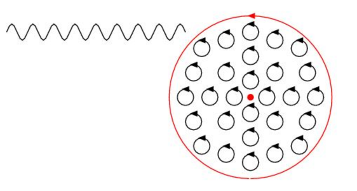
$\hookrightarrow$ q distribution shifted to top

unpolarized target

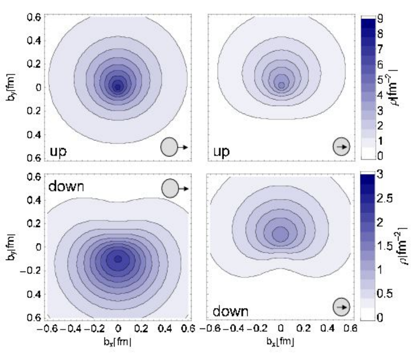
- q with pol. $\leftarrow$ shifted to bottom



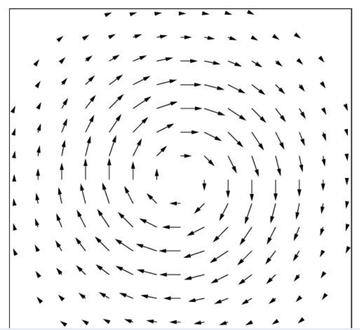
q with polarization $\odot$



lattice calculations (QCDSF)



unpolarized target



- transversity distribution in unpol. target described by chirally odd GPD $\bar{E}_T$
- $\bar{E}_T > 0$ for both u & d quarks
- connection $h_1^\perp(x, \mathbf{k}_\perp) \leftrightarrow \bar{E}_T$ similar to $f_{1T}^\perp(x, \mathbf{k}_\perp) \leftrightarrow E$.

$\hookrightarrow$ $h_1^\perp(x, \mathbf{k}_\perp) < 0$ for $u/p, d/p, u/\pi, \bar{d}/\pi, ..$

- Deeply Virtual Compton Scattering (DVCS) $\longrightarrow$ GPDs

$\hookrightarrow$ impact parameter dependent PDFs $q(x, \mathbf{b}_\perp)$

- $E^q(x, 0, -\mathbf{\Delta}_\perp^2) \leftrightarrow \kappa_{q/p}$ (contribution from quark flavor q to anom magnetic moment)
- $E^q(x, 0, -\mathbf{\Delta}_\perp^2) \longrightarrow \perp$ deformation of PDFs for $\perp$ polarized target
- $\perp$ deformation $\leftrightarrow$ (sign of) SSA (Sivers; Boer-Mulders)
- parton interpretation for Ji-relation
- $\perp$ deformation $\leftrightarrow$ (sign of) quark-gluon correlations
  ($\int dx\, x^2 \bar{g}_2(x)$, $\int dx\, x^2 \bar{e}(x)$)