Generalized TMDs and Wigner Distributions

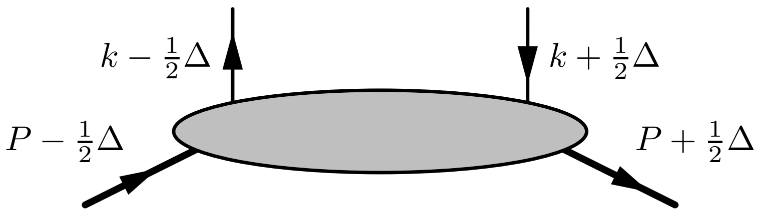
(A. Metz, Temple University, Philadelphia)

- Parameterization of GTMDs
- Nontrivial relation between GPD E and TMD $f_{1T}^{\perp}$
  (Burkardt, 2002, ... / Burkardt, Hwang, 2003)
- Overview of model-dependent nontrivial GPD-TMD relations
- GTMDs and nontrivial GPD-TMD relations
- Further developments/applications of GTMDs
- Summary

In collaboration with K. Goeke, S. Meißner, M. Schlegel
(hep-ph/0703176 ; arXiv:0805.3165 ; arXiv:0906.5323)

Definition of GPDs and TMDs

- GPDs
  - Appear in QCD-description of hard exclusive reactions (DVCS, HEMP)
  - Kinematics



$$P = \frac{p + p'}{2} \qquad \Delta = p' - p$$

  - GPD-correlator

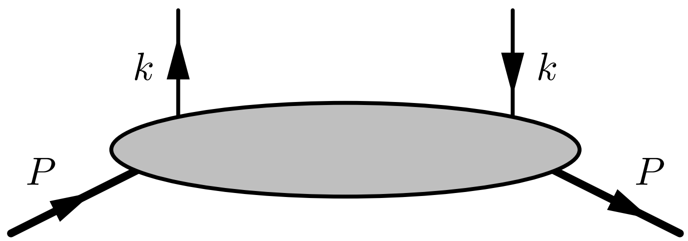
$$\begin{aligned} F^{[\gamma^+]} &= \frac{1}{2} \int \frac{dz^-}{2\pi}\, e^{ik\cdot z} \, \langle p' \,|\, \bar{\psi}\Big(-\frac{z}{2}\Big)\, \gamma^+ \, \mathcal{W}_{GPD}\, \psi\Big(\frac{z}{2}\Big)\, |\, p\rangle \Big|_{z^+ = z_T = 0} \\ &= \frac{1}{2P^+}\, \bar{u}(p') \left(\gamma^+ \, H^q(x,\xi,t) + \frac{i\sigma^{+\mu}\Delta_\mu}{2M}\, E^q(x,\xi,t) \right) u(p) \end{aligned}$$

$$x = \frac{k^+}{P^+} \qquad \xi = -\frac{\Delta^+}{2P^+} \qquad t = \Delta^2$$

  - Leading twist for

$$\bar{\psi}\,\gamma^+\,\psi \qquad \bar{\psi}\,\gamma^+\gamma_5\,\psi \qquad \bar{\psi}\,i\sigma^{j+}\gamma_5\,\psi$$

- TMDs
  - Appear in QCD-description of hard semi-inclusive reactions (SIDIS, DY)
  - Kinematics



  - TMD-correlator

$$\Phi^{[\gamma^+]} = \frac{1}{2}\int \frac{dz^-}{2\pi}\,\frac{d^2\vec{z}_T}{(2\pi)^2}\, e^{ik\cdot z}\,\langle p\,|\,\bar{\psi}\Big(-\frac{z}{2}\Big)\,\gamma^+\,\mathcal{W}_{TMD}\,\psi\Big(\frac{z}{2}\Big)\,|\,p\rangle\Big|_{z^+=0}$$

$$= f_1^q(x,\vec{k}_T^{\,2}) - \frac{\epsilon_T^{ij}k_T^i S_T^j}{M}\, f_{1T}^{\perp q}(x,\vec{k}_T^{\,2})$$

$\longrightarrow$ Sivers function $f_{1T}^{\perp}$ describes strength of (dipole) distortion of TMD correlator

  - Leading twist for

$$\bar{\psi}\,\gamma^+\,\psi \qquad \bar{\psi}\,\gamma^+\gamma_5\,\psi \qquad \bar{\psi}\,i\sigma^{j+}\gamma_5\,\psi$$

- Leading twist parton distributions of the nucleon

	Quarks				Gluons			
PDFs	f_1^q	g_1^q	h_1^q		f_1^g	g_1^g		
TMDs	f_1^q	$f_{1T}^{\perp q}$	g_{1L}^q	g_{1T}^q	f_1^g	$f_{1T}^{\perp g}$	g_{1L}^g	g_{1T}^g
	h_{1T}^q	$h_{1L}^{\perp q}$	$h_{1T}^{\perp q}$	$h_1^{\perp q}$	h_{1T}^g	$h_{1L}^{\perp g}$	$h_{1T}^{\perp g}$	$h_1^{\perp g}$
GPDs	H^q	E^q	$\tilde{H}^q$	$\tilde{E}^q$	H^g	E^g	$\tilde{H}^g$	$\tilde{E}^g$
	H_T^q	E_T^q	$\tilde{H}_T^q$	$\tilde{E}_T^q$	H_T^g	E_T^g	$\tilde{H}_T^g$	$\tilde{E}_T^g$

  - Trivial relations:

$$H^q(x,0,0) = f_1^q(x) = \int d^2\vec{k}_T \, f_1^q(x, \vec{k}_T^{\,2}) \quad \text{etc.}$$

  - Nontrivial relations: 3 for quarks, 4 for gluons

Parameterization of GTMDs

- GTMD-correlator

$$W^{[\Gamma]} = \frac{1}{2}\int \frac{dz^-}{2\pi}\,\frac{d^2\vec{z}_T}{(2\pi)^2}\, e^{ik\cdot z}\,\langle p' \,|\, \bar{\psi}\Big(-\frac{z}{2}\Big)\,\Gamma\,\mathcal{W}_{GTMD}\,\psi\Big(\frac{z}{2}\Big)\,|\, p\rangle\,\Big|_{z^+=0}$$

$\longrightarrow$ $W^{[\Gamma]}$ appears, e.g., in handbag diagram of DVCS

- Projection onto GPDs and TMDs

$$\begin{aligned} F^{[\Gamma]} &= \frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ik\cdot z}\,\langle p' \,|\, \bar{\psi}\Big(-\frac{z}{2}\Big)\,\Gamma\,\mathcal{W}_{GPD}\,\psi\Big(\frac{z}{2}\Big)\,|\, p\rangle\,\Big|_{z^+=z_T=0} \\ &= \int d^2\vec{k}_T\, W^{[\Gamma]} \end{aligned}$$

$$\begin{aligned} \Phi^{[\Gamma]} &= \frac{1}{2}\int \frac{dz^-}{2\pi}\,\frac{d^2\vec{z}_T}{(2\pi)^2}\, e^{ik\cdot z}\,\langle p \,|\, \bar{\psi}\Big(-\frac{z}{2}\Big)\,\Gamma\,\mathcal{W}_{TMD}\,\psi\Big(\frac{z}{2}\Big)\,|\, p\rangle\,\Big|_{z^+=0} \\ &= W^{[\Gamma]}\Big|_{\Delta=0} \end{aligned}$$

$\longrightarrow$ GPDs and TMDs appear as certain limits of GTMDs (mother distributions)

- Parameterization of GTMD-correlator (Meißner, Metz, Schlegel, 2009)
  - Use constraints from hermiticity and parity
  - Eliminate redundant terms by means of Gordon identities, etc.

$$\det \begin{pmatrix} g^{\alpha\mu} & g^{\beta\mu} & g^{\gamma\mu} & g^{\delta\mu} & g^{\varepsilon\mu} \\ g^{\alpha\nu} & g^{\beta\nu} & g^{\gamma\nu} & g^{\delta\nu} & g^{\varepsilon\nu} \\ g^{\alpha\rho} & g^{\beta\rho} & g^{\gamma\rho} & g^{\delta\rho} & g^{\varepsilon\rho} \\ g^{\alpha\sigma} & g^{\beta\sigma} & g^{\gamma\sigma} & g^{\delta\sigma} & g^{\varepsilon\sigma} \\ g^{\alpha\tau} & g^{\beta\tau} & g^{\gamma\tau} & g^{\delta\tau} & g^{\varepsilon\tau} \end{pmatrix} = 0$$

  - Example

$$W^{[\gamma^+]} = \frac{1}{2M}\,\bar{u}(p')\left[F_{1,1} + \frac{i\sigma^{i+}k_T^i}{P^+}\,F_{1,2} + \frac{i\sigma^{i+}\Delta_T^i}{P^+}\,F_{1,3} + \frac{i\sigma^{ij}k_T^i\Delta_T^j}{M^2}\,F_{1,4}\right]u(p)$$

$\longrightarrow$ GTMDs are complex functions: $F_{1,n} = F_{1,n}^e + iF_{1,n}^o$

  - Parameterization for all twists
  - By-product: first full classification of GPDs beyond leading twist
  - Relations between GPDs/TMDs and GTMDs worked out
  - Leading twist GTMDs computed in scalar diquark model

Wigner distributions

- Phase-space distribution in classical mechanics $\rho(\vec{k}, \vec{r})$
- Phase-space distribution in quantum mechanics (Wigner distribution) $W(\vec{k}, \vec{r})$
  - Relation to probability density in position and momentum space

$$|\psi(\vec{r})|^2 = \int d^3\vec{k}\, W(\vec{k}, \vec{r}) \qquad |\psi(\vec{k})|^2 = \int d^3\vec{r}\, W(\vec{k}, \vec{r})$$

- Fourier transform of GTMDs $(\xi = 0)$ (Ji, 2003 / Belitsky, Ji, Yuan, 2003)

$$\mathrm{WD}(x, \vec{k}_T, \vec{b}_T) \simeq \int d^2\vec{\Delta}_T\, e^{-i\vec{\Delta}_T \cdot \vec{b}_T}\, \mathrm{GTMD}(x, \vec{k}_T, \vec{\Delta}_T)$$

  - Relation with GPDs and TMDs

$$\mathrm{GPD}(x, \vec{b}_T) \simeq \int d^2\vec{k}_T\, \mathrm{WD}(x, \vec{k}_T, \vec{b}_T) \qquad \mathrm{TMD}(x, \vec{k}_T) \simeq \int d^2\vec{b}_T\, \mathrm{WD}(x, \vec{k}_T, \vec{b}_T)$$

  - No handle on longitudinal position of parton
  - $\vec{b}_T$ and $\vec{k}_T$ are not Fourier conjugate variables

Impact parameter representation of GPDs

- Fourier transform of GPD-correlator $(\xi = 0)$ (Burkardt, 2000)

$$F^{[\gamma^+]}(x, \vec{\Delta}_T) = \frac{1}{2} \int \frac{dz^-}{2\pi} \, e^{ik\cdot z} \, \langle p' \,|\, \bar{\psi}\Big(-\frac{z}{2}\Big) \, \gamma^+ \, \mathcal{W}_{GPD} \, \psi\Big(\frac{z}{2}\Big) \,|\, p \rangle \Big|_{z^+ = z_T = 0}$$

$$\begin{aligned} \mathcal{F}^{[\gamma^+]}(x, \vec{b}_T) &= \int \frac{d^2\vec{\Delta}_T}{(2\pi)^2} \, e^{-i\vec{\Delta}_T\cdot\vec{b}_T} \, F^{[\gamma^+]}(x, \vec{\Delta}_T) \\ &= \mathcal{H}^q(x, \vec{b}_T^{\,2}) + \frac{\epsilon_T^{ij} b_T^i S_T^j}{M} \Big(\mathcal{E}^q(x, \vec{b}_T^{\,2})\Big)' \end{aligned}$$

$$\text{with} \quad \mathcal{H}^q(x, \vec{b}_T^{\,2}) = \int \frac{d^2\vec{\Delta}_T}{(2\pi)^2} \, e^{-i\vec{\Delta}_T\cdot\vec{b}_T} \, H^q(x, 0, -\vec{\Delta}_T^{\,2})$$

- Distortion of GPD-correlator in impact parameter space

$$d^{q,i} = \int dx \int d^2\vec{b}_T \, b_T^i \, \mathcal{F}^q(x, \vec{b}_T; S) = -\frac{\epsilon_T^{ij} S_T^j}{2M} \int dx \, E^q(x, 0, 0) = -\frac{\epsilon_T^{ij} S_T^j}{2M} \kappa^q$$

$\longrightarrow$ Flavor dipole moment of about $0.2\,\mathrm{fm}$!

- Relation between distortion and Sivers effect (Burkardt, 2002)

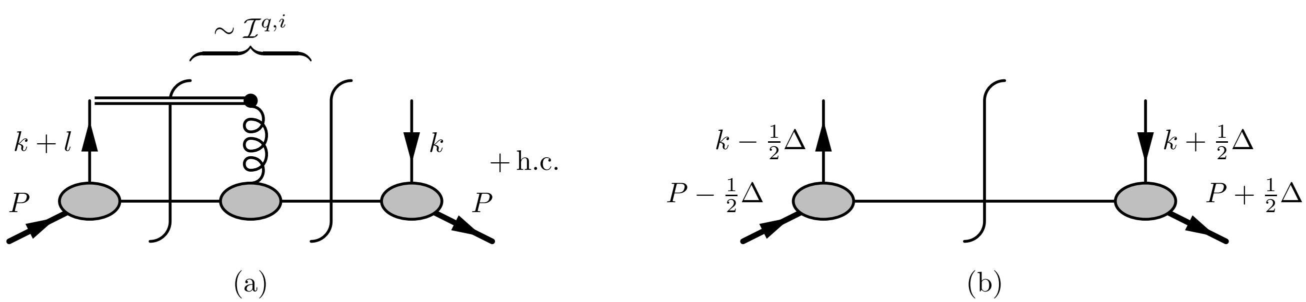
  (Not obvious because, *a priori*, QCD-description of, e.g., SIDIS not related to GPD-correlator in b_T-space)

  – Quantitative nontrivial relation in spectator model (Burkardt, Hwang, 2003)

$$\begin{aligned}
\langle k_T^{q,i}(x)\rangle_{UT} &= -\int d^2\vec{k}_T\, k_T^i\, \frac{\epsilon_T^{jk} k_T^j S_T^k}{M}\, f_{1T}^{\perp q}(x,\vec{k}_T^{\,2}) \\
&= \int d^2\vec{b}_T\, \mathcal{I}^{q,i}(x,\vec{b}_T)\, \frac{\epsilon_T^{jk} b_T^j S_T^k}{M} \Big(\mathcal{E}^q(x,\vec{b}_T^{\,2})\Big)'
\end{aligned}$$

  – Interpretation

$$\boxed{\text{Sivers effect} = \text{Distortion} \otimes \text{FSI}}$$

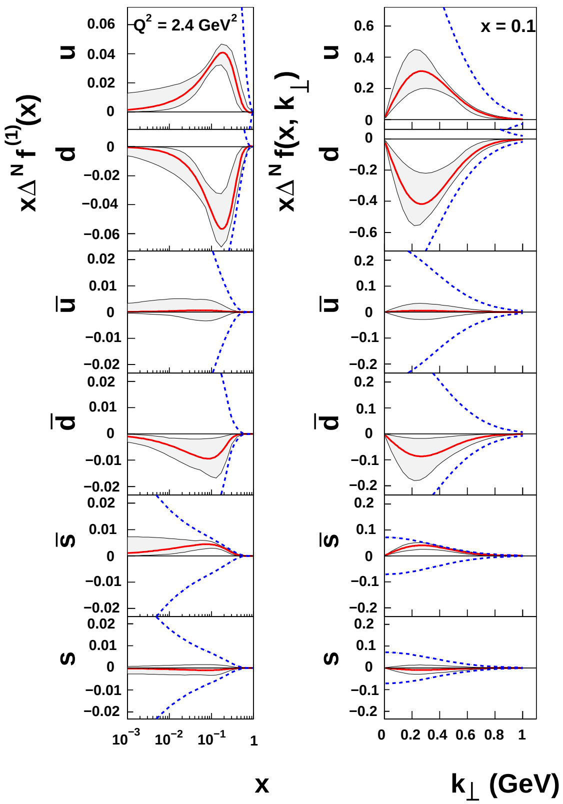


– Prediction

$$f_{1T}^{\perp u/p} \sim -\,0.8\, f_{1T}^{\perp d/p} < 0$$

$\longrightarrow$ relative (but not absolute) sign also from large N_c-analysis (Pobylitsa, 2003)

$\longrightarrow$ extraction of $\Delta^N f = -\frac{2k_T}{M} f_{1T}^{\perp}$ by Anselmino et al., 2008



$\longrightarrow$ nice agreement between qualitative picture and extraction

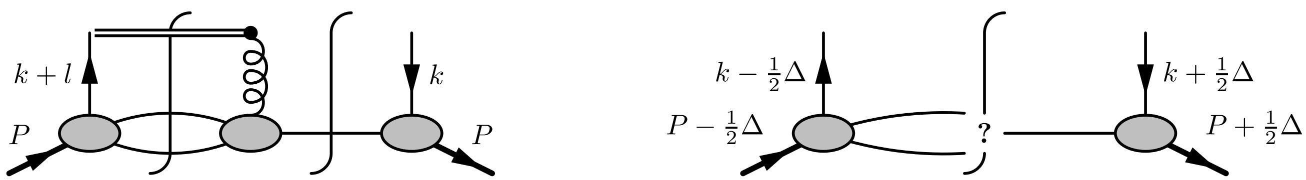
$\longrightarrow$ agreement with large N_c-analysis and qualitative picture supports interpretation of nonzero signal for $A_{UT}^{\sin(\Phi-\Phi_S)}$ as Sivers effect

– Sign reversal of the Sivers function (Collins, 2002)

$$f_{1T}^{\perp}\Big|_{\mathrm{DY}} = -f_{1T}^{\perp}\Big|_{\mathrm{SIDIS}}$$

$\longrightarrow$ sign reversal provided in qualitative picture:
lensing function $\mathcal{I}^q$ changes sign (attractive vs repulsive interaction)

– Higher order contributions spoil relation (Meißner, Metz, Goeke, 2007)



(a) (b)

– Spectator model: neglecting relation-breaking higher order graphs, making certain kinematical approximation, and introducing IR-regulator, provides to all orders

$$\boxed{\text{Sivers effect} = \text{Distortion} \otimes \text{FSI}}$$

(Gamberg, Schlegel, 2009)

Comparing GPD- and TMD-correlator

- Additional relations by comparing GPD-correlator with TMD-correlator (Diehl, Hägler, 2005)

$$
\begin{aligned}
\Phi^{[\gamma^+]}(x, \vec{k}_T) &= f_1^q(x, \vec{k}_T^{\,2}) - \frac{\epsilon_T^{ij} k_T^i S_T^j}{M} f_{1T}^{\perp q}(x, \vec{k}_T^{\,2}) \\
\mathcal{F}^{[\gamma^+]}(x, \vec{b}_T) &= \mathcal{H}^q(x, \vec{b}_T^{\,2}) + \frac{\epsilon_T^{ij} b_T^i S_T^j}{M} \Big(\mathcal{E}^q(x, \vec{b}_T^{\,2}) \Big)'
\end{aligned}
$$

- Comparison allows one to find analogy:

$$
f_{1T}^{\perp q} \leftrightarrow -\Big(\mathcal{E}^q \Big)'
$$

- Comparison can be extended to other quark and gluon distributions

- No relation for GPDs $\tilde{E}, \ \tilde{E}_T$ (drop out for $\xi = 0$) and TMDs $g_{1T}, \ h_{1L}^{\perp}$

- Relations of first type

$$f_1^{q/g} \leftrightarrow \mathcal{H}^{q/g} \qquad g_{1L}^{q/g} \leftrightarrow \tilde{\mathcal{H}}^{q/g}$$

$$\left(h_{1T}^q + \frac{\vec{k}_T^2}{2M^2}\,h_{1T}^{\perp q}\right) \leftrightarrow \left(\mathcal{H}_T^q - \frac{\vec{b}_T^2}{M^2}\,\Delta\tilde{\mathcal{H}}_T^q\right)$$

- Relations of second type

$$f_{1T}^{\perp q/g} \leftrightarrow -\left(\mathcal{E}^{q/g}\right)' \qquad h_1^{\perp q} \leftrightarrow -\left(\mathcal{E}_T^q + 2\tilde{\mathcal{H}}_T^q\right)'$$

$$\left(h_{1T}^g + \frac{\vec{k}_T^2}{2M^2}\,h_{1T}^{\perp g}\right) \leftrightarrow -2\left(\mathcal{H}_T^g - \frac{\vec{b}_T^2}{M^2}\,\Delta\tilde{\mathcal{H}}_T^g\right)'$$

- Relations of third type

$$h_{1T}^{\perp q} \leftrightarrow 2\left(\tilde{\mathcal{H}}_T^q\right)'' \qquad h_1^{\perp g} \leftrightarrow 2\left(\mathcal{E}_T^g + 2\tilde{\mathcal{H}}_T^g\right)''$$

- Relation of fourth type

$$h_{1T}^{\perp g} \leftrightarrow -4\left(\tilde{\mathcal{H}}_T^g\right)'''$$

- Some consequences
  - Relation for Boer-Mulders function $h_1^{\perp q}$ expected to match with the one for $f_{1T}^{\perp q}$

$$
\begin{aligned}
\langle k_T^{q,i}(x) \rangle_{UT} &= -\int d^2\vec{k}_T \, k_T^i \, \frac{\epsilon_T^{jk} k_T^j S_T^k}{M} \, f_{1T}^{\perp q}(x, \vec{k}_T^{\,2}) \\
&= \int d^2\vec{b}_T \, \mathcal{I}^{q,i}(x, \vec{b}_T) \, \frac{\epsilon_T^{jk} b_T^j S_T^k}{M} \left(\mathcal{E}^q(x, \vec{b}_T^{\,2}) \right)'
\end{aligned}
$$

$$
\begin{aligned}
\langle k_T^{q,i}(x) \rangle_{TU}^j &= -\int d^2\vec{k}_T \, k_T^i \, \frac{\epsilon_T^{kj} k_T^k}{M} \, h_1^{\perp q}(x, \vec{k}_T^{\,2}) \\
&= \int d^2\vec{b}_T \, \mathcal{I}^{q,i}(x, \vec{b}_T) \, \frac{\epsilon_T^{kj} b_T^k}{M} \left(\mathcal{E}_T^q(x, \vec{b}_T^{\,2}) + 2\tilde{\mathcal{H}}_T^q(x, \vec{b}_T^{\,2}) \right)'
\end{aligned}
$$

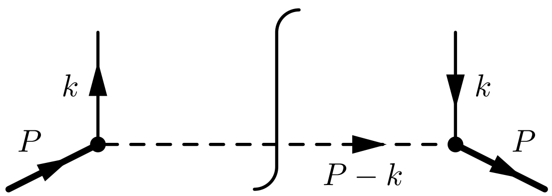
(Burkardt, 2005 / Meißner, Metz, Goeke, 2007)

$\longrightarrow$ information on chiral odd GPDs (Pasquini, Pincetti, Boffi, 2005 / QCDSF, 2007)
$\longrightarrow$ implies: $h_1^{\perp u/p} < 0 \qquad h_1^{\perp d/p} < 0$
$\longrightarrow$ agrees, e.g., with spectator model calculations
(Gamberg, Goldstein, Schlegel, 2007 / Bacchetta, Conti, Radici, 2008 / etc.)

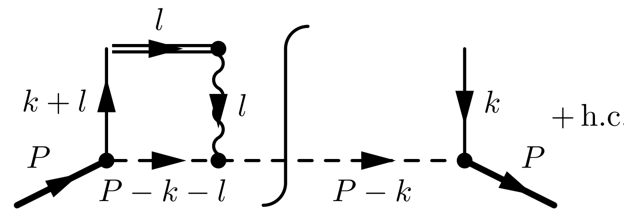
  - Relation for $h_{1T}^{\perp q}$ expected to be different

Model results, continued

- Scalar diquark spectator model of the nucleon

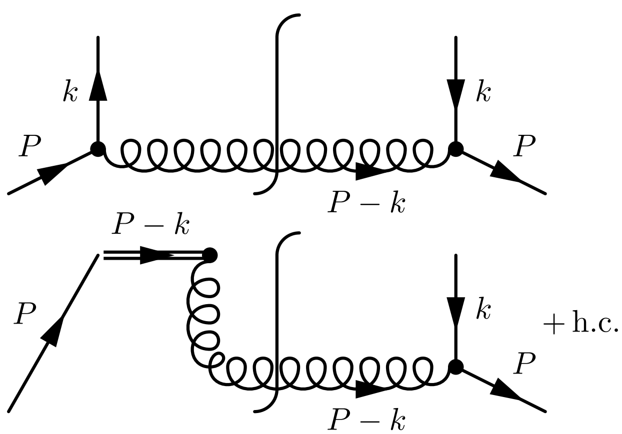


(a)

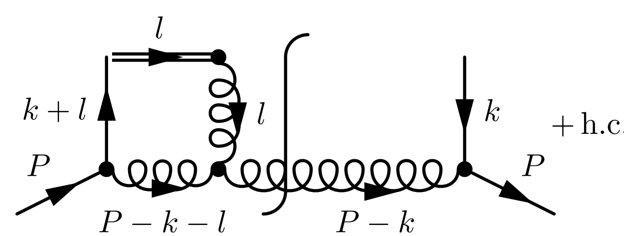


(b)

- Quark target model in QCD



(a)



(b)

- Moments of GPDs and TMDs

$$X^{(n)}(x) = \frac{1}{2M^2}\int d^2\vec{\Delta}_T \left(\frac{\vec{\Delta}_T^2}{2M^2}\right)^{n-1} X\left(x,0,-\frac{\vec{\Delta}_T^2}{(1-x)^2}\right)$$

$$Y^{(n)}(x) = \int d^2\vec{k}_T \left(\frac{\vec{k}_T^2}{2M^2}\right)^{n} Y(x,\vec{k}_T^2)$$

- Relations of second type

$$f_{1T}^{\perp q\,(n)}(x) = H_2(n)\,\frac{1}{1-x}\,E^{q\,(n)}(x) \qquad (0 \le n \le 1)$$

  - $H_2(n)$ depends on model
  - formula holds for all the relations of second type
  - particular cases

$$f_{1T}^{\perp q\,(0)}(x) = \frac{\pi e_q e_s}{48(1-x)}\,E^q(x,0,0) \qquad \text{(Lu, Schmidt, 2006)}$$

$$f_{1T}^{\perp q\,(1)}(x) = \frac{e_q e_s}{4(2\pi)^2\,(1-x)}\,E^{q\,(1)}(x) \qquad \text{(Burkardt, Hwang, 2003)}$$

- Relations of third type

$$h_{1T}^{\perp q\,(n)}(x) = H_3(n)\,\frac{1}{(1-x)^2}\,\tilde{H}_T^{q\,(n)}(x) \qquad\qquad (0 \le n \le 1)$$

  - $H_3(n)$ is the same in both models
  - Formula holds for all the relations of third type
  - Particular cases

$$h_{1T}^{\perp q\,(0)}(x) \;=\; \int d^2\vec{k}_T\, h_{1T}^{\perp q}(x, \vec{k}_T^{\,2}) = \frac{3}{(1-x)^2}\,\tilde{H}_T^q(x,0,0)$$

$$h_{1T}^{\perp q\,(1)}(x) \;=\; \int d^2\vec{k}_T\,\frac{\vec{k}_T^{\,2}}{2M^2}\, h_{1T}^{\perp q}(x, \vec{k}_T^{\,2}) = \int d^2\vec{b}_T\,\frac{\vec{b}_T^{\,2}}{2M^2}\,2\Big(\tilde{\mathcal{H}}_T^q(x, \vec{b}_T^{\,2})\Big)''$$

  - No immediate evidence for breakdown of relations of third type

- Relation of fourth type
  - Trivially satisfied because

$$h_{1T}^{\perp g} = \tilde{\mathcal{H}}_T^g = 0$$

GTMDs and nontrivial GPD-TMD relations

- Relate GPDs/TMDs to mother distributions (GTMDs)
- Which GPDs and TMDs have the same mother distributions ?
- Implications for potential nontrivial relations
  - Relations of second type

$$
\begin{aligned}
E(x,0,\vec{\Delta}_T^2) &= \int d^2\vec{k}_T \left[-F^e_{1,1} + 2\left(\frac{\vec{k}_T \cdot \vec{\Delta}_T}{\vec{\Delta}_T^2} F^e_{1,2} + F^e_{1,3} \right) \right] \\
f_{1T}^{\perp}(x,\vec{k}_T^2) &= -F^o_{1,2}(x,0,\vec{k}_T^2,0,0)
\end{aligned}
$$

$\longrightarrow$ no model-independent nontrivial relation between E and $f_{1T}^{\perp}$ possible

$\longrightarrow$ relation in spectator model due to simplicity of the model

$\longrightarrow$ no information on numerical violation of relation

$\longrightarrow$ likewise for nontrivial relation involving $h_1^{\perp}$

– Relation of third type

$$\tilde{H}_T(x,0,\vec{\Delta}_T^2) = \int d^2\vec{k}_T \Bigg[\Bigg(\frac{\vec{k}_T\cdot\vec{\Delta}_T}{\vec{\Delta}_T^2}\,H_{1,1}^e + H_{1,2}^e\Bigg) - 2\Bigg(\frac{2(\vec{k}_T\cdot\vec{\Delta}_T)^2 - \vec{k}_T^2\vec{\Delta}_T^2}{(\vec{\Delta}_T^2)^2}\,H_{1,4}^e + \frac{\vec{k}_T\cdot\vec{\Delta}_T}{\vec{\Delta}_T^2}\,H_{1,5}^e + H_{1,6}^e\Bigg)\Bigg]$$

$$h_{1T}^{\perp}(x,\vec{k}_T^2) = H_{1,4}^e(x,0,\vec{k}_T^2,0,0)$$

$\longrightarrow$ no model-independent nontrivial relation between $\tilde{H}_T$ and $h_{1T}^{\perp}$ possible

$\longrightarrow$ relation in spectator model

$$H_{1,1}^e(\xi=0) = H_{1,5}^e(\xi=0) = 0$$

$$2\tilde{H}_{1,2}^e(x) - 4\tilde{H}_{1,6}^e(x) = (1-x)^2\,\tilde{H}_{1,4}^e(x)$$

$\longrightarrow$ relation in light-front constituent quark model
(Pasquini, Cazzaniga, Boffi, 2008, 2009)

$$\int d^2\vec{k}_T\,h_{1T}^{\perp}(x,\vec{k}_T^2) = \frac{2}{(1-x)^2}\,\tilde{H}_T(x,0,0) \qquad \text{instead of}$$

$$\int d^2\vec{k}_T\,h_{1T}^{\perp}(x,\vec{k}_T^2) = \frac{3}{(1-x)^2}\,\tilde{H}_T(x,0,0) \qquad \text{in spectator model}$$

Further developments/applications of GTMDs

- Overlap representation of GTMDs in terms of light-front wave functions
(Lorcè, Pasquini, Vanderhaeghen, 2010)

- GTMDs (and Wigner distributions) have been computed in models
  - Quark models
  (Lorcè, Pasquini, Vanderhaeghen, 2010, 2011)
  - Spectator-type model
  (Müller, DIS2011)

- GTMDs may be used to estimate certain higher twist contributions
to DVCS and HEMP

- GTMDs for gluons have already been used to describe exclusive diffractive
processes in the high energy limit
(e.g., Martin, Ryskin, Teubner, 1999 / Khoze, Martin, Ryskin, 2000)

Summary

- Classification of Generalized TMDs (and Wigner distributions) for nucleon exists

- GTMD analysis can be applied to study potential nontrivial GPD-TMD relations

- Various quantitative nontrivial GPD-TMD relations in simple spectator models

- GTMD analysis: none of those relations can have model-independent status
  (analysis also for subleading twist)

- Additional developments and further applications