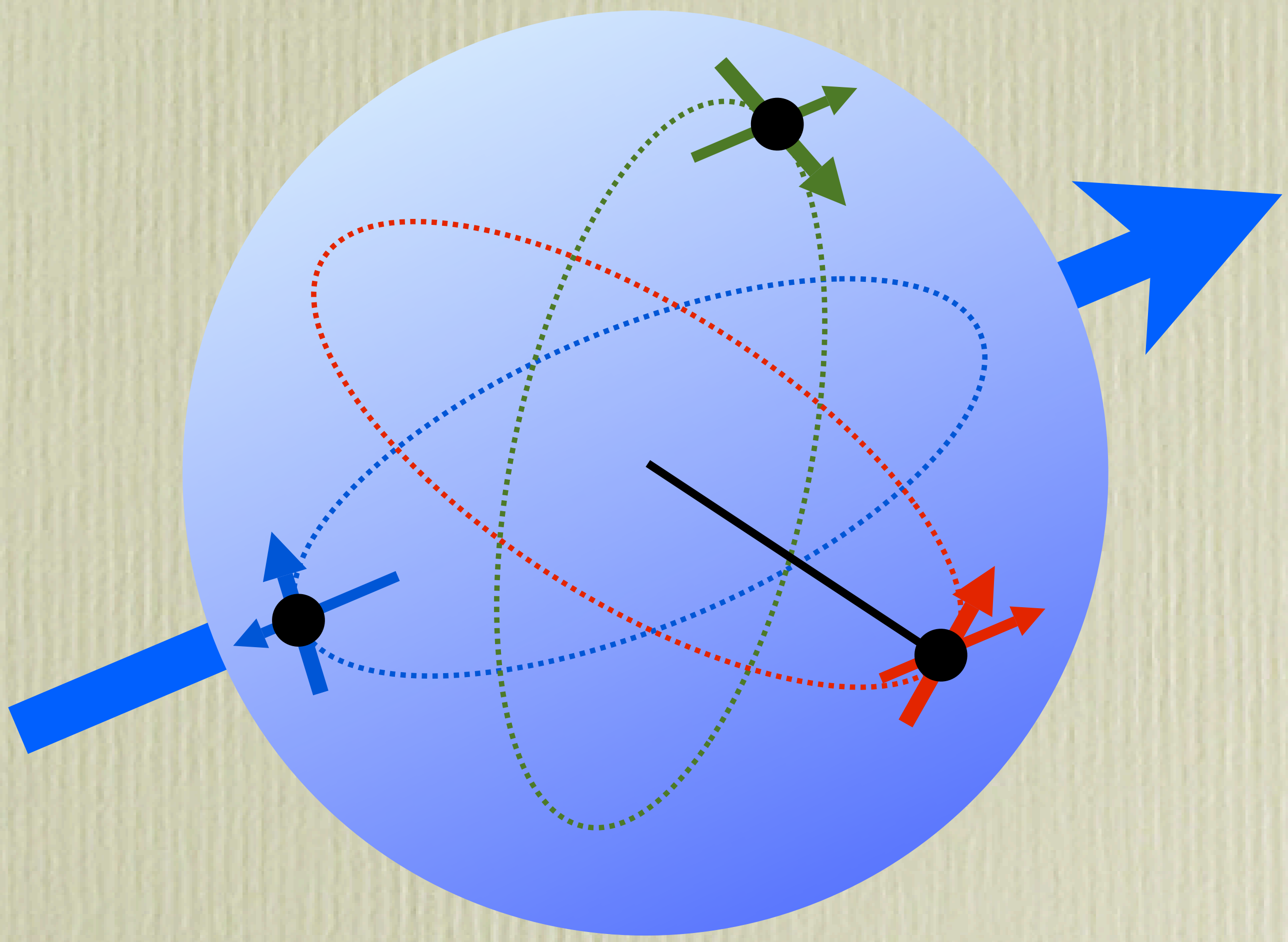
# Status (critique) of TMD phenomenology

## 3-D partonic momentum distributions

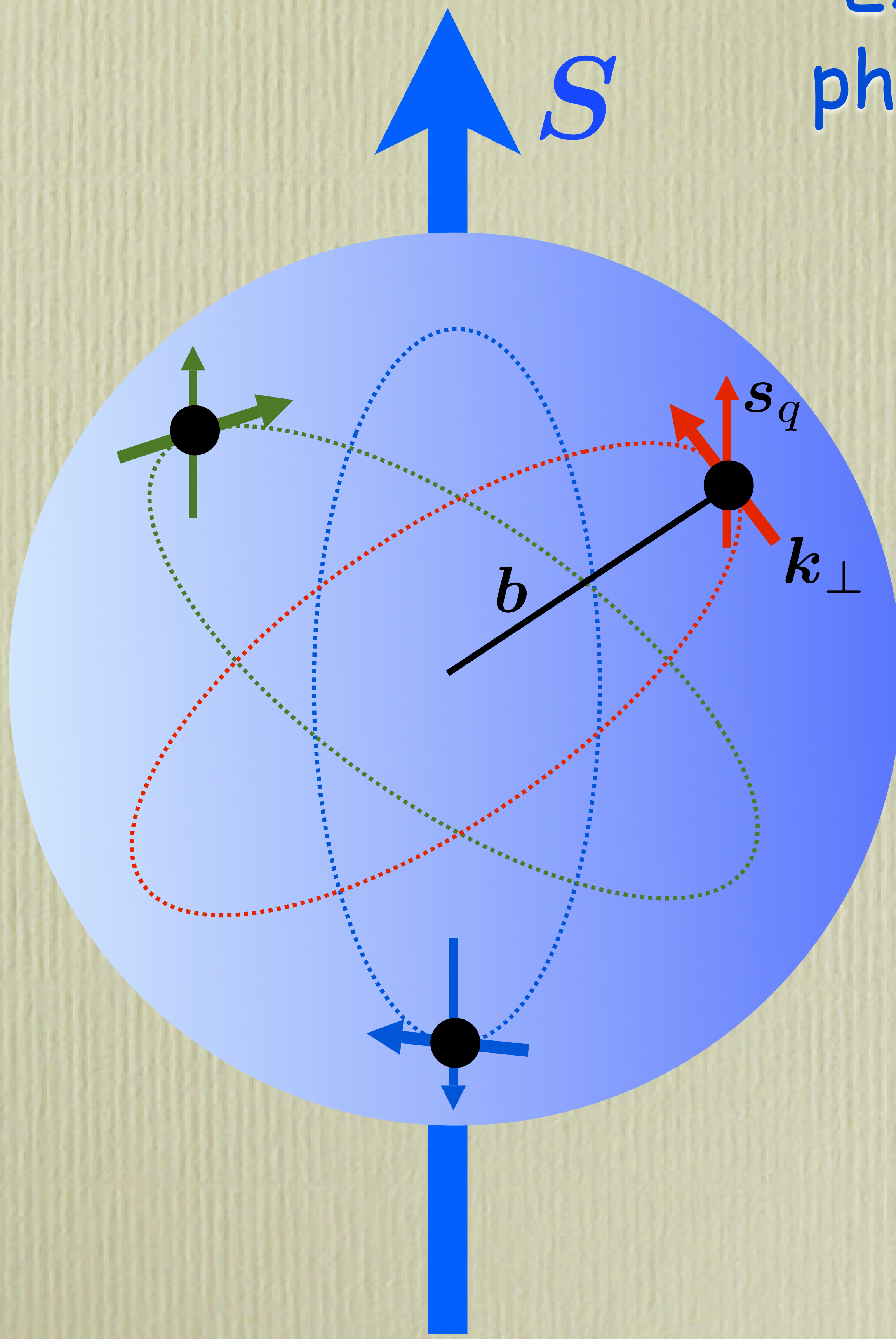
what would we like to know?
what do we know and how?



Opportunities for Drell-Yan Physics at RHIC
May 11-13, 2011, RIKEN BNL

Mauro Anselmino, Torino University & INFN

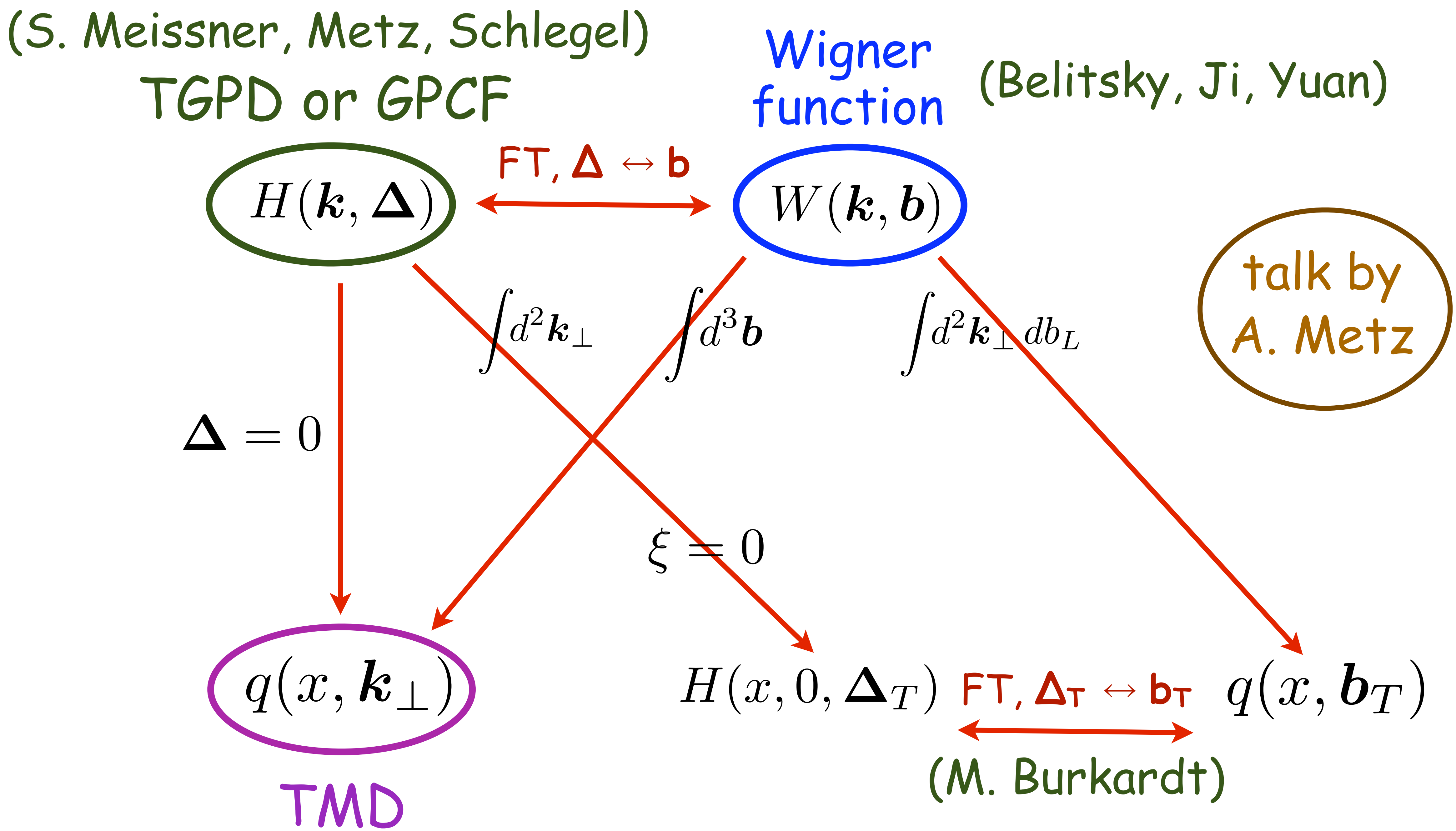
# Exploring the 3-dimensional phase-space structure of the nucleon




phase-space (**k**-**b**) distribution of partons in nucleons; parton intrinsic motion; spin-$k_\perp$ correlations? orbiting quarks?

information encoded in GPDs and TMDs (exclusive and inclusive processes)

# phase-space parton distribution, $W(\boldsymbol{k}, \boldsymbol{b})$

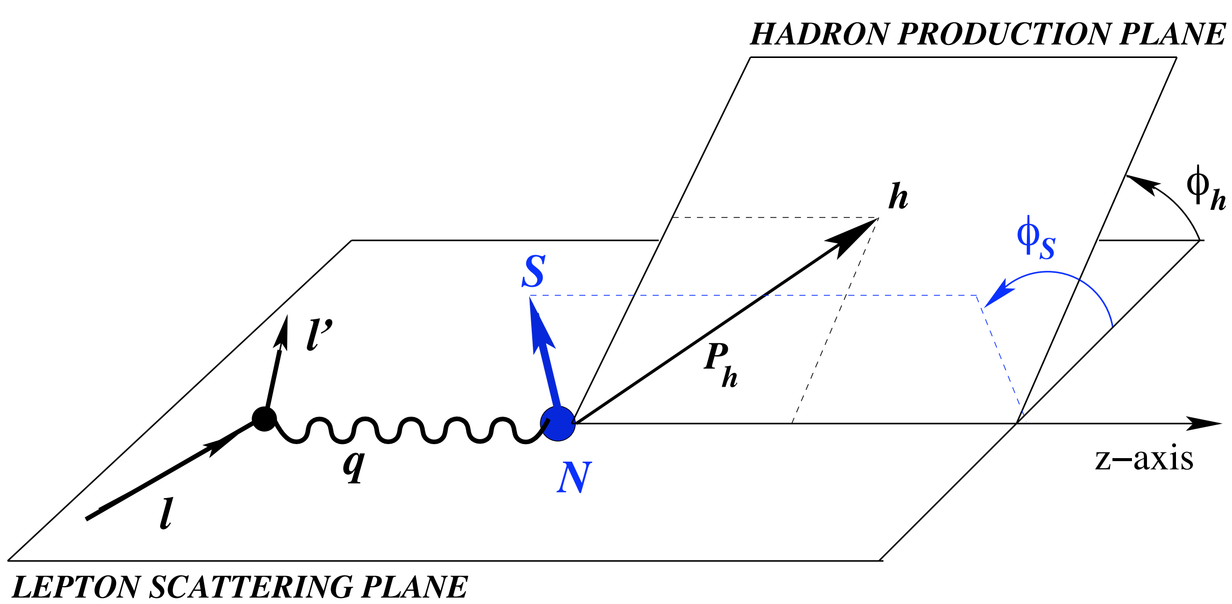



$$\int d^2\boldsymbol{k}_\perp \, H(\boldsymbol{k}, \boldsymbol{\Delta}) = H(x, \xi, \boldsymbol{\Delta}_T)$$

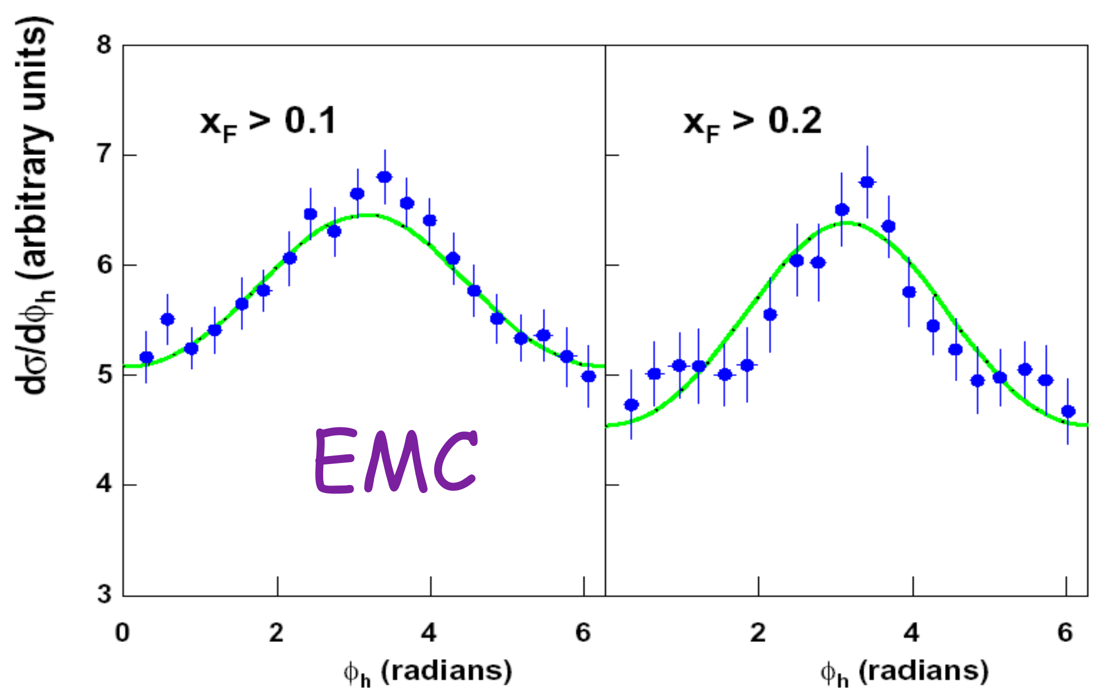
# first hints at quark transverse motion from data

Feynman, Field and Fox, 1978

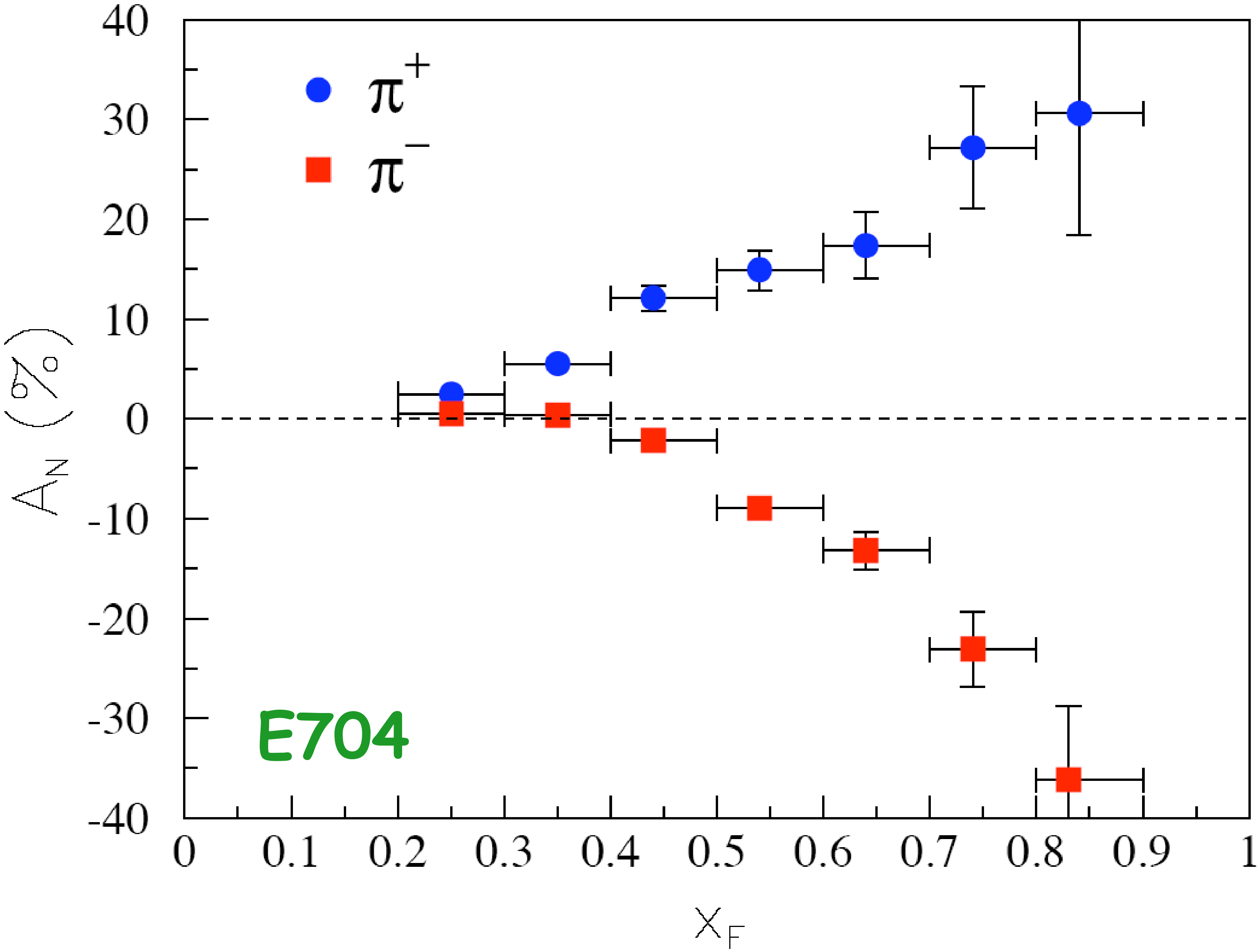
intrinsic motion increases cross section in p p $\rightarrow \pi$ X large $p_T$ processes (easier to scatter a quark at large angles if it has already some transverse momentum)




Cahn effect, 1978, azimuthal dependence due to quark intrinsic motion



# large SSAs in p p → π X, ~ 1990 and before




Sivers effect, 1990

Collins effect, 1993

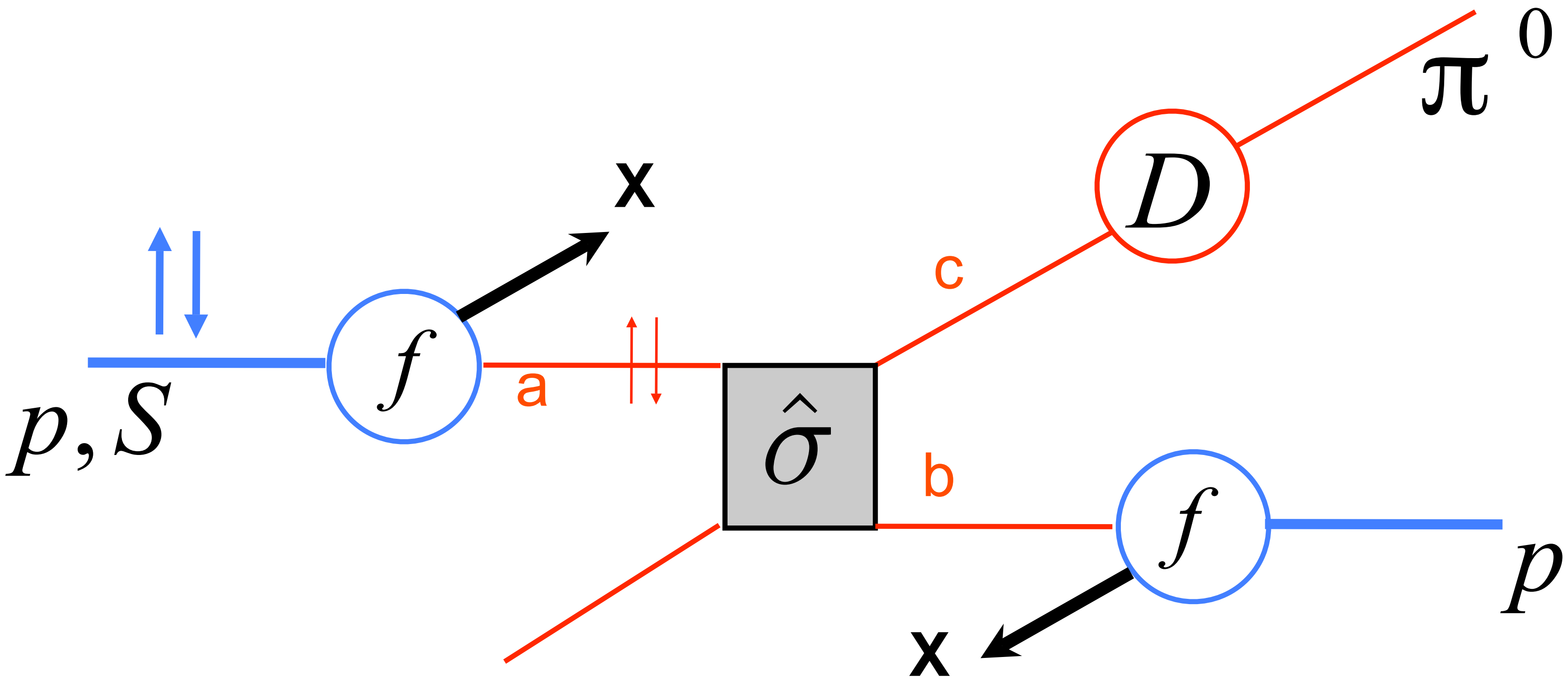
Boer-Mulders effect, 1998

$$A_N \equiv \frac{d\sigma^{\uparrow} - d\sigma^{\uparrow}}{d\sigma^{\uparrow} + d\sigma^{\uparrow}}$$

E704 √s = 20 GeV

0.7 < $p_T$ < 2.0

# no SSA in leading-twist collinear factorization

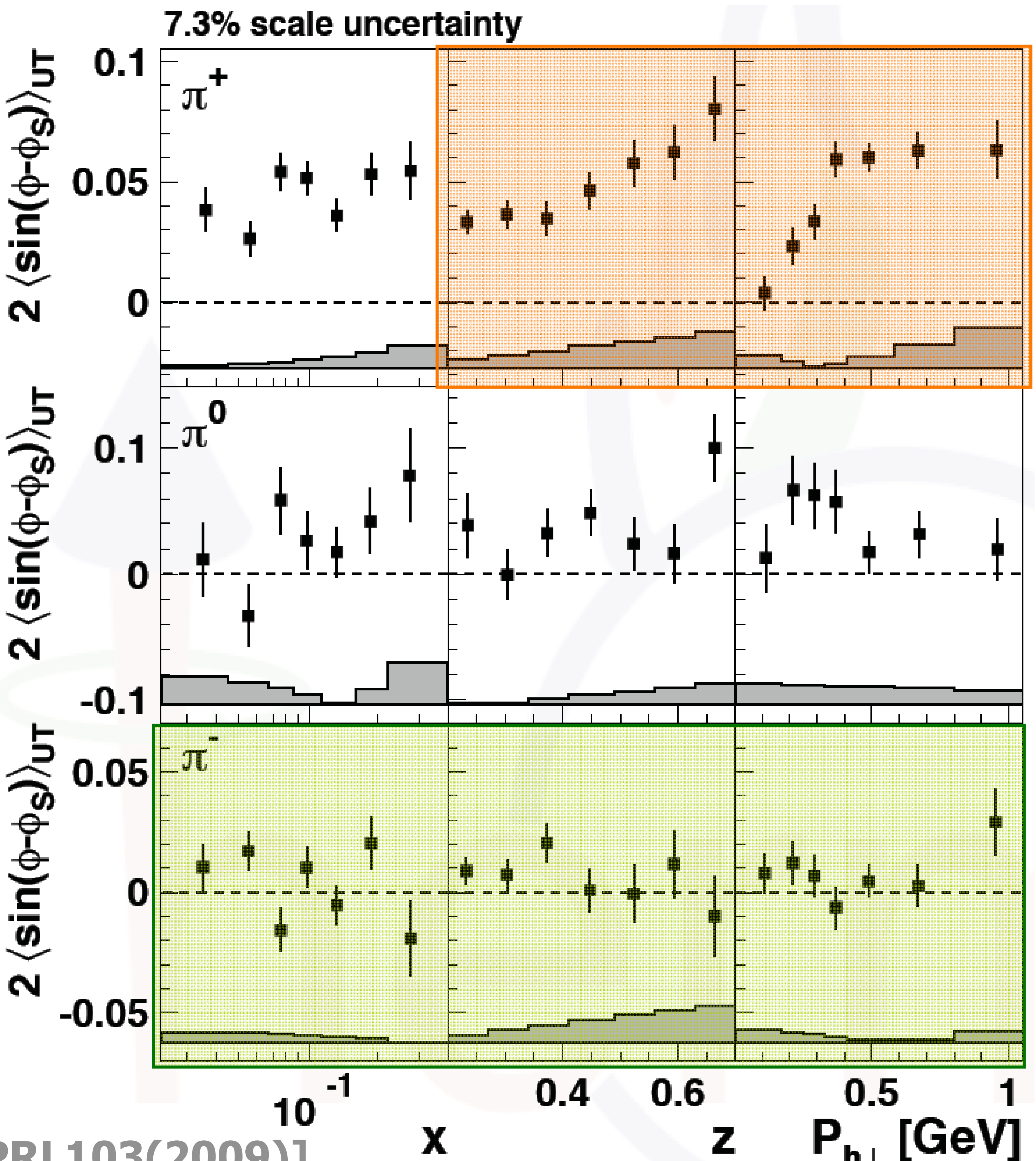


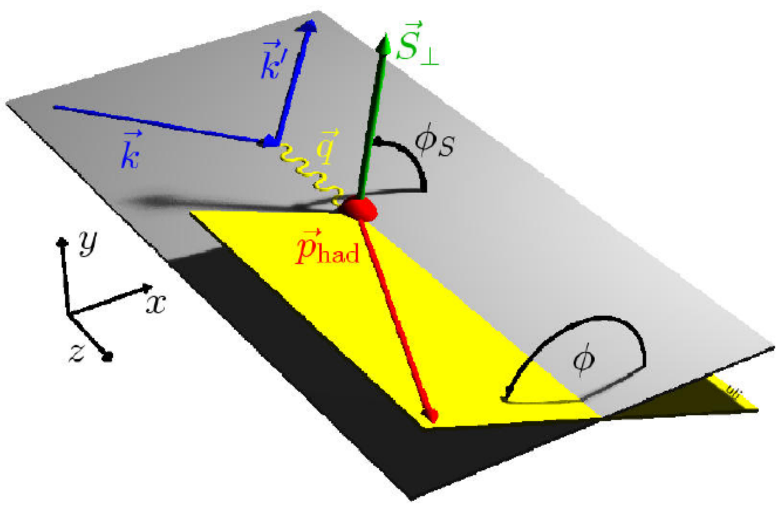

$$d\sigma^{\uparrow} - d\sigma^{\downarrow} = \sum_{a,b,c,d=q,\bar{q},g} \underbrace{\Delta_T f_a}_{\text{transversity}} \otimes f_b \otimes \underbrace{[\hat{\sigma}^{\uparrow} - \hat{\sigma}^{\downarrow}]}_{\text{pQCD elementary SSA}} \otimes \underbrace{D_{\pi/c}}_{\text{FF}}$$

$$A_N = \frac{\mathrm{d}\sigma^{\uparrow} - \mathrm{d}\sigma^{\downarrow}}{\mathrm{d}\sigma^{\uparrow} + \mathrm{d}\sigma^{\downarrow}} \propto \hat{a}_N \propto \frac{m_q}{E_q}\,\alpha_s$$

was considered almost a theorem

# polarized SIDIS azimuthal asymmetries, from 2004




$$2\,\langle\sin(\phi-\phi_S)\rangle = A_{UT}^{\sin(\phi-\phi_S)}$$

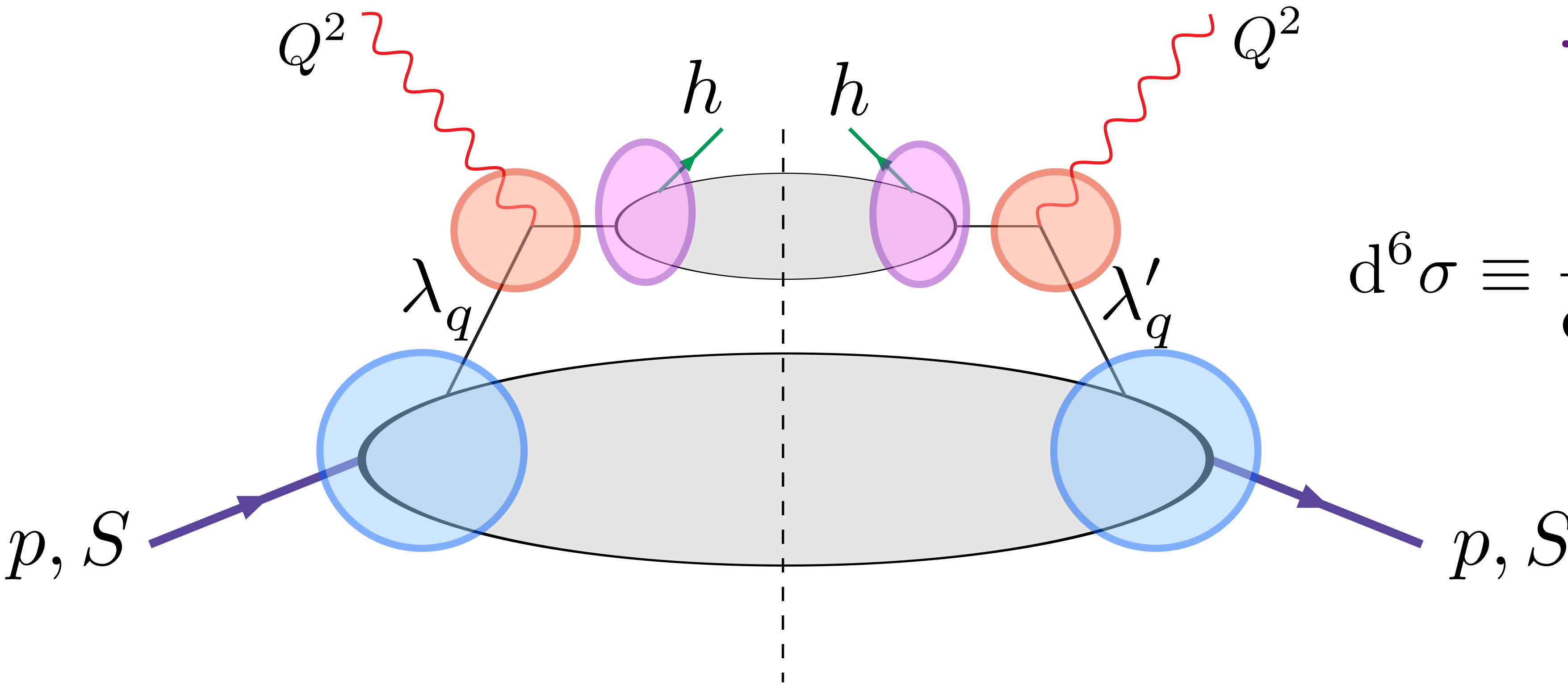
$$\equiv 2\,\frac{\int \mathrm{d}\phi\,\mathrm{d}\phi_S\,(\mathrm{d}\sigma^\uparrow - \mathrm{d}\sigma^\downarrow)\,\sin(\phi-\phi_S)}{\int \mathrm{d}\phi\,\mathrm{d}\phi_S\,(\mathrm{d}\sigma^\uparrow + \mathrm{d}\sigma^\downarrow)}$$

[PRL103(2009)]

plenty of more data accumulated recently in SIDIS, e+e- and NN inclusive processes, more expected....

all these effects can be (are) somewhat related to parton intrinsic motion;
how to interpret data within a QCD - parton model framework and extract unambiguous information?
Great progress in the last years

# TMDs in SIDIS




talk by G. Schnell

$$d^6\sigma \equiv \frac{d^6\sigma^{\ell p^\uparrow \to \ell h X}}{dx_{_B}\, dQ^2\, dz_h\, d^2\boldsymbol{P}_T\, d\phi_S}$$

TMD factorization holds at large $Q^2$, and $P_T \approx k_\perp \approx \Lambda_{\rm QCD}$

Two scales: $P_T \ll Q^2$

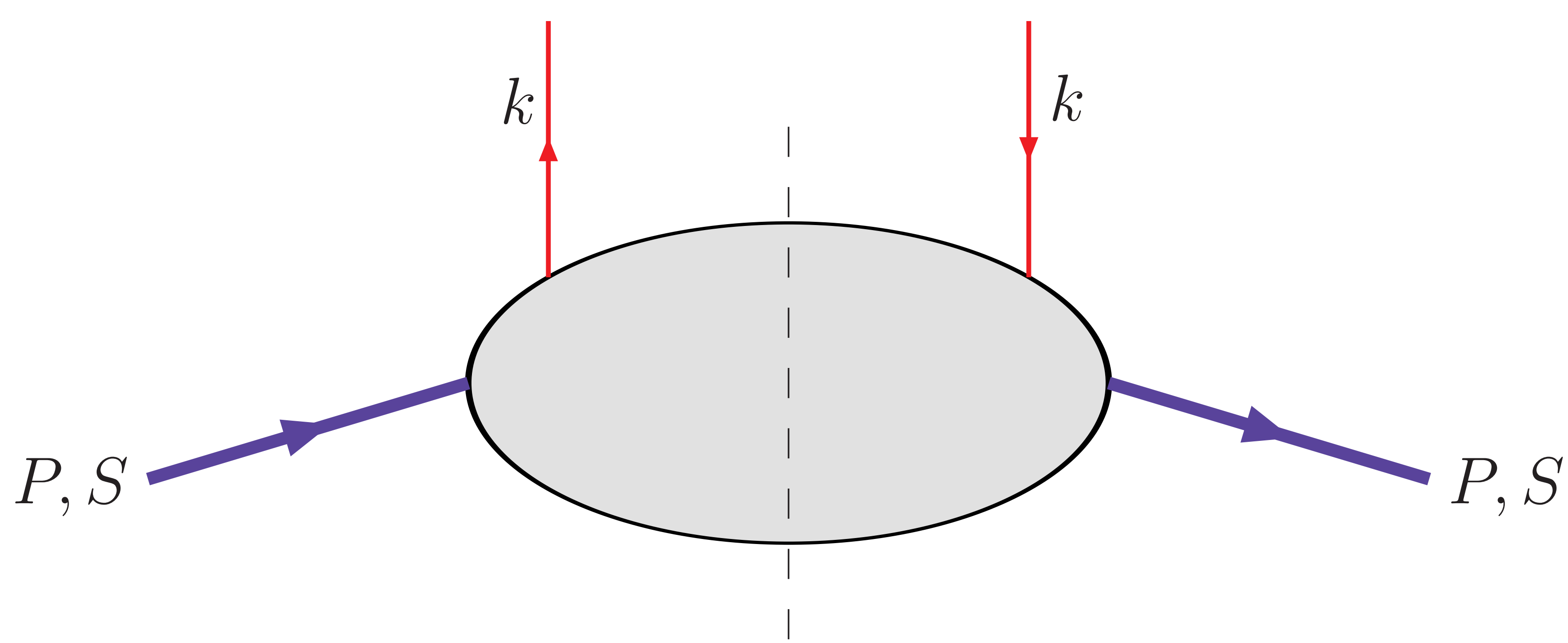
$$\boldsymbol{p}_\perp \simeq \boldsymbol{P}_T - z_h\,\boldsymbol{k}_\perp$$

$$d\sigma^{\ell p \to \ell h X} = \sum_q f_q(x, \boldsymbol{k}_\perp; Q^2) \otimes d\hat\sigma^{\ell q \to \ell q}(y, \boldsymbol{k}_\perp; Q^2) \otimes D_q^h(z, \boldsymbol{p}_\perp; Q^2)$$

(Collins, Soper, Ji, J.P. Ma, Yuan, Qiu, Vogelsang, Collins, Metz)

# TMDs: the leading-twist correlator, with intrinsic $k_\perp$, contains 8 independent functions

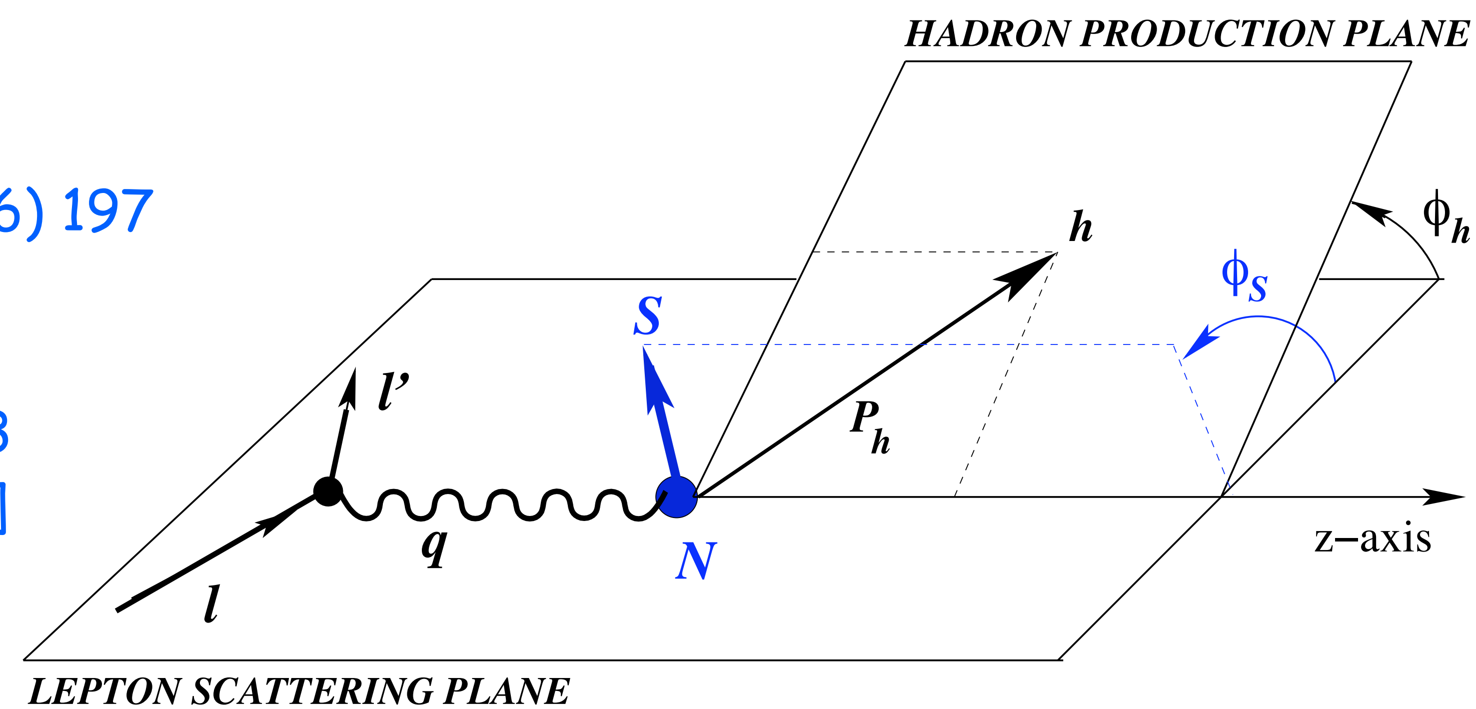
$$
\begin{aligned}
\Phi(x, \boldsymbol{k}_\perp) \quad = \quad & \frac{1}{2}\Bigg[f_1 \not{n}_+ + f_{1T}^\perp \frac{\epsilon_{\mu\nu\rho\sigma}\gamma^\mu n_+^\nu k_\perp^\rho S_T^\sigma}{M} + \left(S_L g_{1L} + \frac{\boldsymbol{k}_\perp \cdot \boldsymbol{S}_T}{M} g_{1T}^\perp\right)\gamma^5 \not{n}_+ \\
+ \quad & h_{1T} i\sigma_{\mu\nu}\gamma^5 n_+^\mu S_T^\nu + \left(S_L h_{1L}^\perp + \frac{\boldsymbol{k}_\perp \cdot \boldsymbol{S}_T}{M} h_{1T}^\perp\right)\frac{i\sigma_{\mu\nu}\gamma^5 n_+^\mu k_\perp^\nu}{M} \\
+ \quad & h_1^\perp \frac{\sigma_{\mu\nu} k_\perp^\mu n_+^\nu}{M}\Bigg]
\end{aligned}
$$



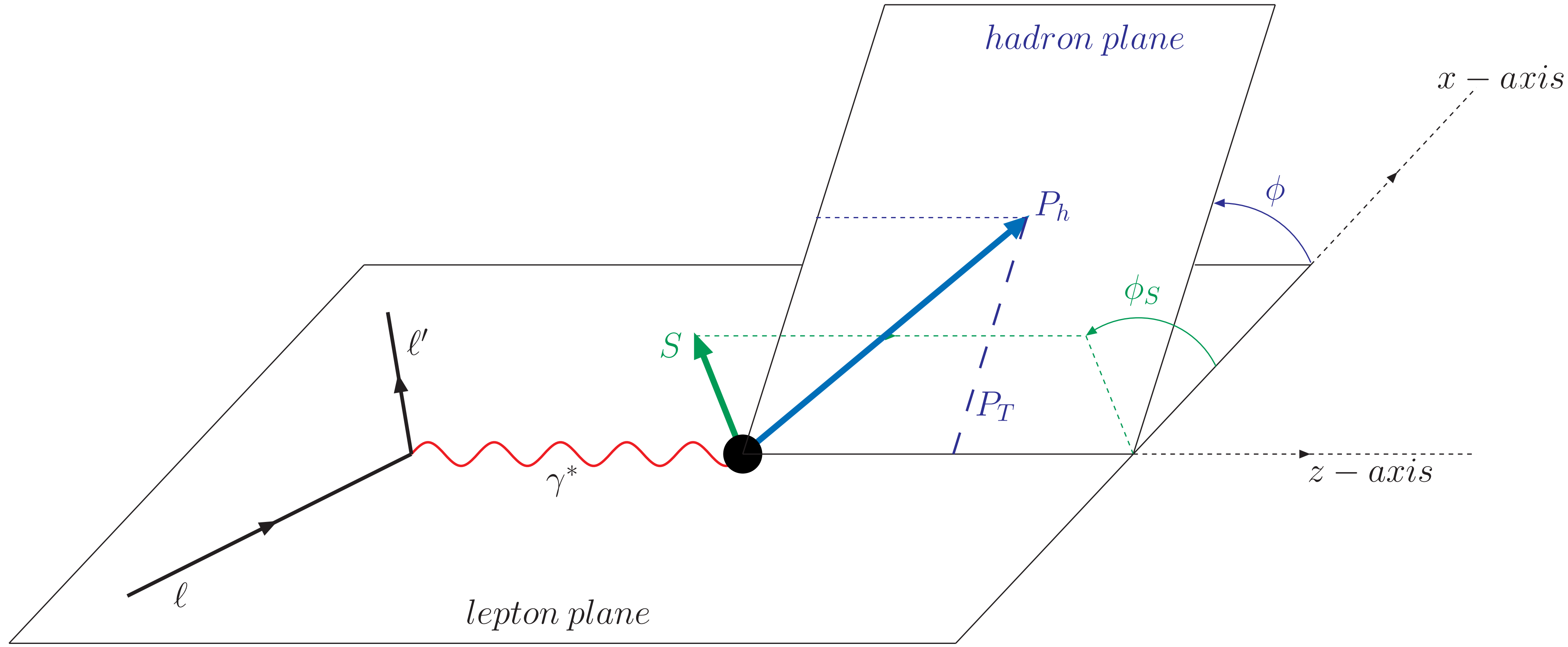

with partonic interpretation

$$
\begin{aligned}
\frac{\mathrm{d}\sigma}{\mathrm{d}\phi} = \; & F_{UU} + \cos(2\phi)\, F_{UU}^{\cos(2\phi)} + \frac{1}{Q}\, \cos\phi\, F_{UU}^{\cos\phi} + \lambda\, \frac{1}{Q}\, \sin\phi\, F_{LU}^{\sin\phi} \\
+ \; & S_L \left\{ \sin(2\phi)\, F_{UL}^{\sin(2\phi)} + \frac{1}{Q}\, \sin\phi\, F_{UL}^{\sin\phi} + \lambda \left[ F_{LL} + \frac{1}{Q}\, \cos\phi\, F_{LL}^{\cos\phi} \right] \right\} \\
+ \; & S_T \Bigg\{ \sin(\phi - \phi_S)\, F_{UT}^{\sin(\phi-\phi_S)} + \sin(\phi + \phi_S)\, F_{UT}^{\sin(\phi+\phi_S)} + \sin(3\phi - \phi_S)\, F_{UT}^{\sin(3\phi-\phi_S)} \\
+ \; & \frac{1}{Q} \left[ \sin(2\phi - \phi_S)\, F_{UT}^{\sin(2\phi-\phi_S)} + \sin\phi_S\, F_{UT}^{\sin\phi_S} \right] \\
+ \; & \lambda \left[ \cos(\phi - \phi_S)\, F_{LT}^{\cos(\phi-\phi_S)} + \frac{1}{Q} \left( \cos\phi_S\, F_{LT}^{\cos\phi_S} + \cos(2\phi - \phi_S)\, F_{LT}^{\cos(2\phi-\phi_S)} \right) \right] \Bigg\}
\end{aligned}
$$


Kotzinian, **NP B441** (1995) 234
Mulders and Tangermann, **NP B461** (1996) 197
Boer and Mulders, **PR D57** (1998) 5780
Bacchetta et al., **PL B595** (2004) 309
Bacchetta et al., **JHEP 0702** (2007) 093
Anselmino et al., arXiv:1101.1011 [hep-ph]


the $F^{(\dots)}_{S_B S_T}$ contain the TMDs




$$\left.\begin{array}{ll} F_{UU} \sim \sum_a e_a^2\, f_1^a \otimes D_1^a & F_{LT}^{\cos(\phi-\phi_S)} \sim \sum_a e_a^2\, g_{1T}^{\perp a} \otimes D_1^a \\ F_{LL} \sim \sum_a e_a^2\, g_{1L}^a \otimes D_1^a & F_{UT}^{\sin(\phi-\phi_S)} \sim \sum_a e_a^2\, f_{1T}^{\perp a} \otimes D_1^a \end{array}\right\} \text{chiral-even TMDs}$$

$$\left.\begin{array}{ll} F_{UU}^{\cos(2\phi)} \sim \sum_a e_a^2\, h_1^{\perp a} \otimes H_1^{\perp a} & F_{UT}^{\sin(\phi+\phi_S)} \sim \sum_a e_a^2\, h_{1T}^a \otimes H_1^{\perp a} \\ F_{UL}^{\sin(2\phi)} \sim \sum_a e_a^2\, h_{1L}^{\perp a} \otimes H_1^{\perp a} & F_{UT}^{\sin(3\phi-\phi_S)} \sim \sum_a e_a^2\, h_{1T}^{\perp a} \otimes H_1^{\perp a} \end{array}\right\} \text{chiral-odd TMDs}$$

$$f \otimes D \sim \int d^2\boldsymbol{k}_\perp\, d^2\boldsymbol{p}_\perp\, \delta^{(2)}(\boldsymbol{P}_T - z_h \boldsymbol{k}_\perp - \boldsymbol{p}_\perp)\, w(\boldsymbol{k}_\perp, \boldsymbol{P}_T)\, f(x_B, k_\perp)\, D(z_h, p_\perp)$$

# Siver function phenomenology in SIDIS

M.Anselmino, M.Boglione, J.C.Collins, U.D'Alesio, A.V.Efremov, K.Goeke, A.Kotzinian, S.Menzel, A.Metz, F.Murgia, A.Prokudin, P.Schweitzer, W.Vogelsang, F.Yuan

$$2\,\langle\sin(\phi-\phi_S)\rangle = A_{UT}^{\sin(\phi-\phi_S)} \equiv 2\,\frac{\overbrace{\int \mathrm{d}\phi\,\mathrm{d}\phi_S\,(\mathrm{d}\sigma^{\uparrow}-\mathrm{d}\sigma^{\downarrow})\,\sin(\phi-\phi_S)}^{F_{UT}^{\sin(\phi-\phi_S)}}}{\int \mathrm{d}\phi\,\mathrm{d}\phi_S\,(\mathrm{d}\sigma^{\uparrow}+\mathrm{d}\sigma^{\downarrow})}$$

extraction of Sivers function based on very simple parameterization, with x and $\mathbf{k}_\perp$ factorization. Typically:

$$\Delta^N f_{q/p^\uparrow}(x,k_\perp) = -\frac{2k_\perp}{M} f_{1T}^{\perp q}(x,k_\perp) = N x^\alpha (1-x)^\beta\, h(k_\perp) f_{q/p}(x,k_\perp)$$

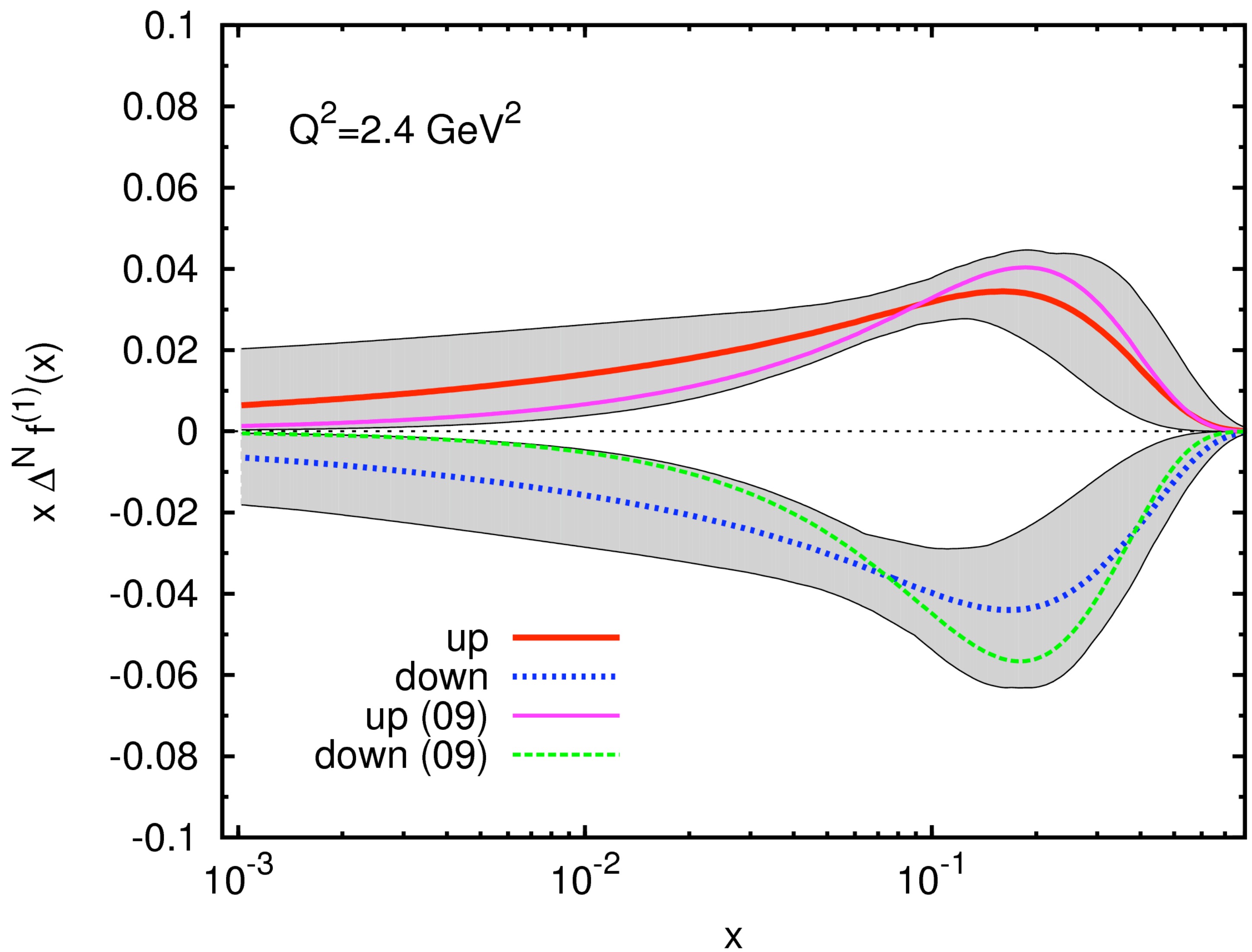
with

$$f_{q/p}(x,k_\perp) = f_q(x)\,\frac{1}{\pi\langle k_\perp^2\rangle}\,e^{-k_\perp^2/\langle k_\perp^2\rangle}$$

$\langle k_\perp^2\rangle$ constant and flavour independent simple Sivers functions for u and d quarks are sufficient to fit the available SIDIS data

large and very small x dependence not constrained by data

talk by A. Prokudin



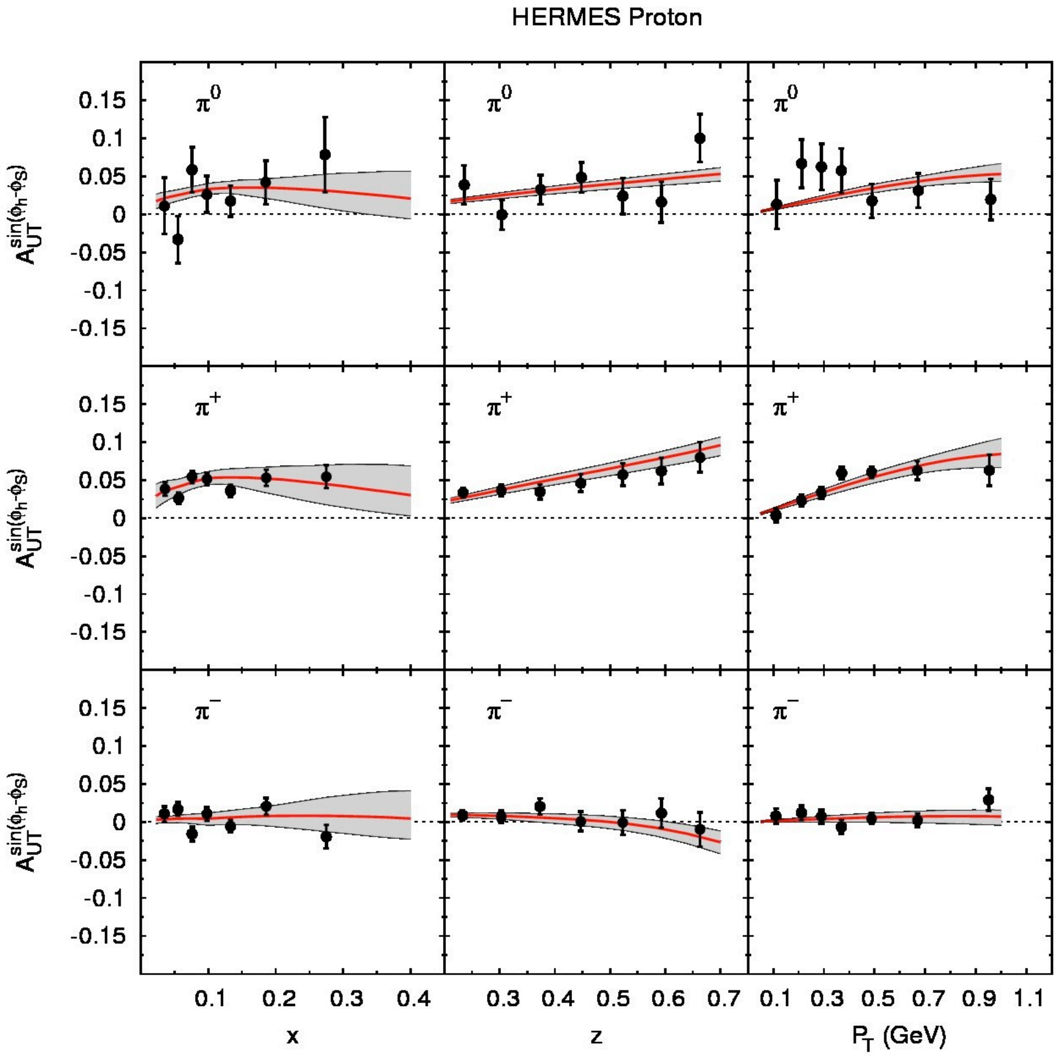

new and previous extraction of u and d Sivers functions

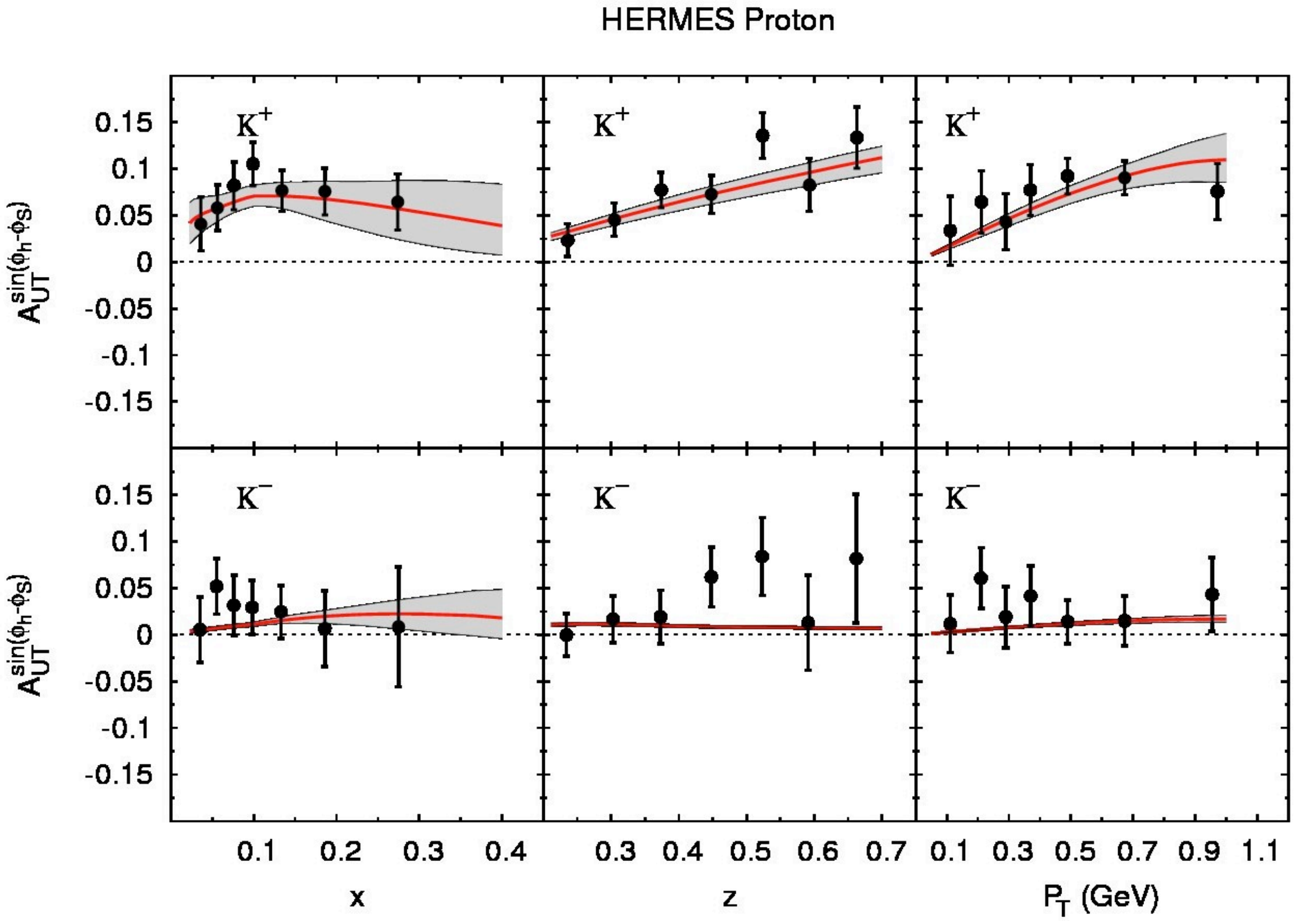
S. Melis and A. Prokudin, preliminary results

Anselmino et al. Eur. Phys. J. A39,89 (2009)

# S. Melis, talk at DIS 2011

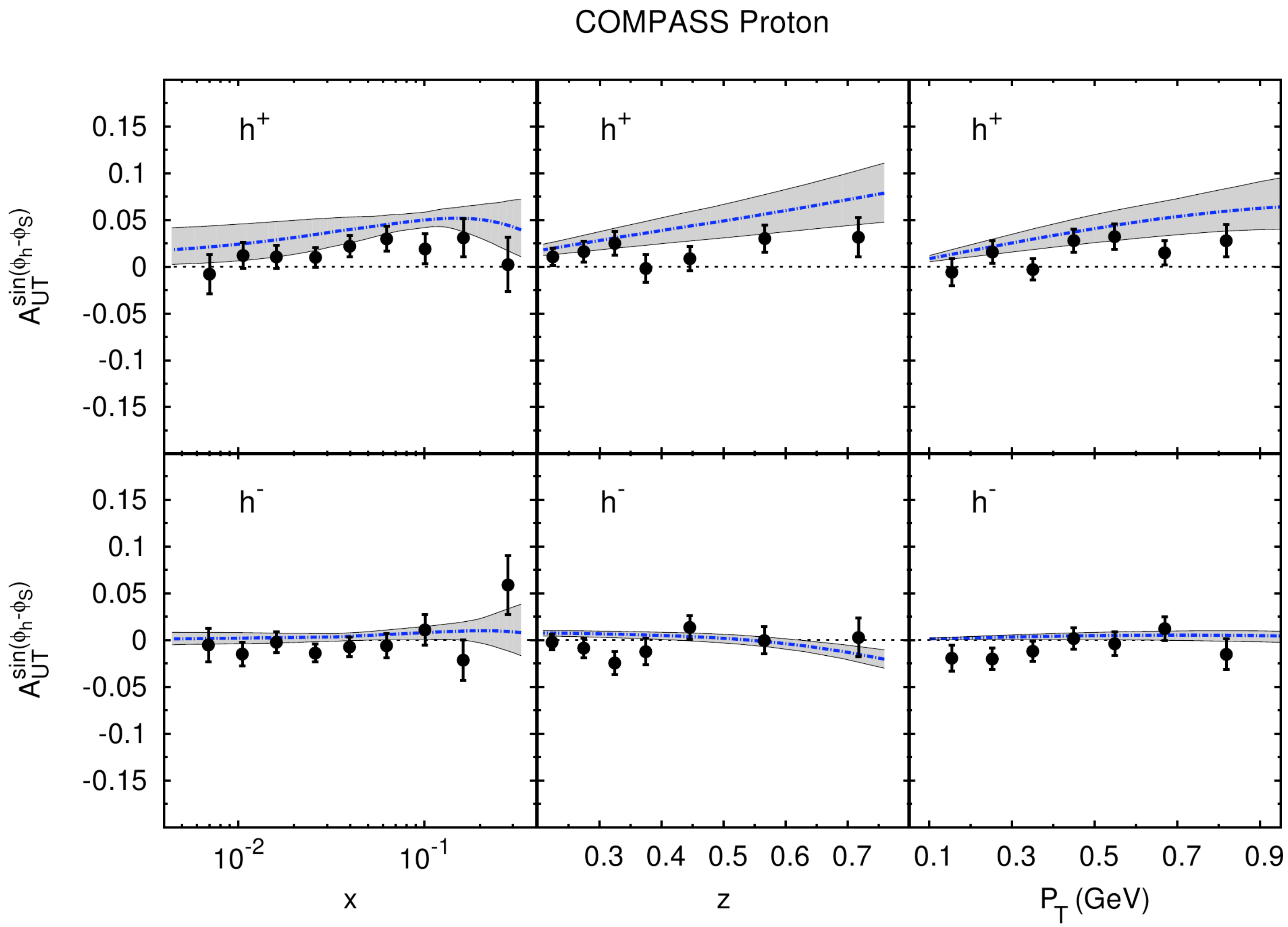
FIT u & d only



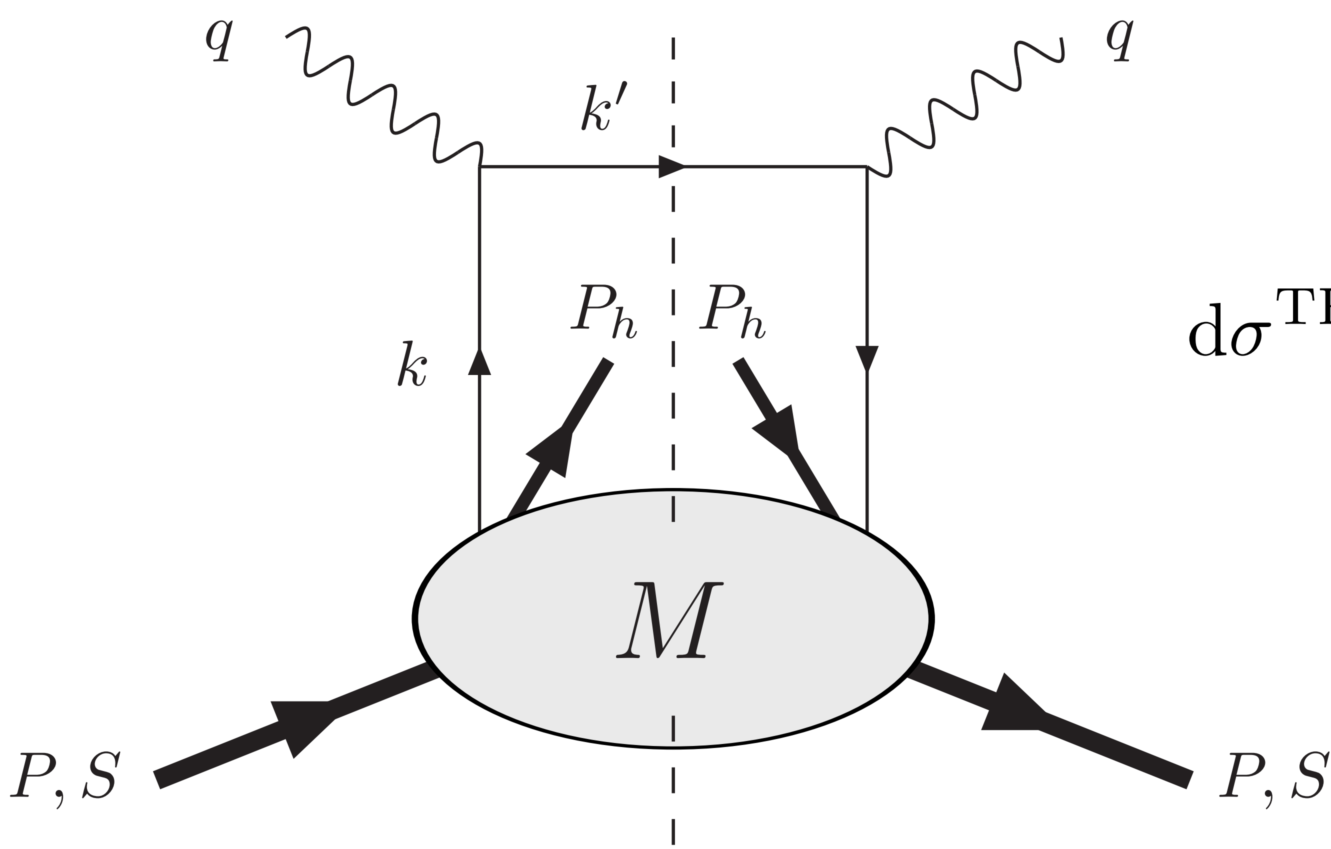
# S. Melis, talk at DIS 2011

FIT u & d only



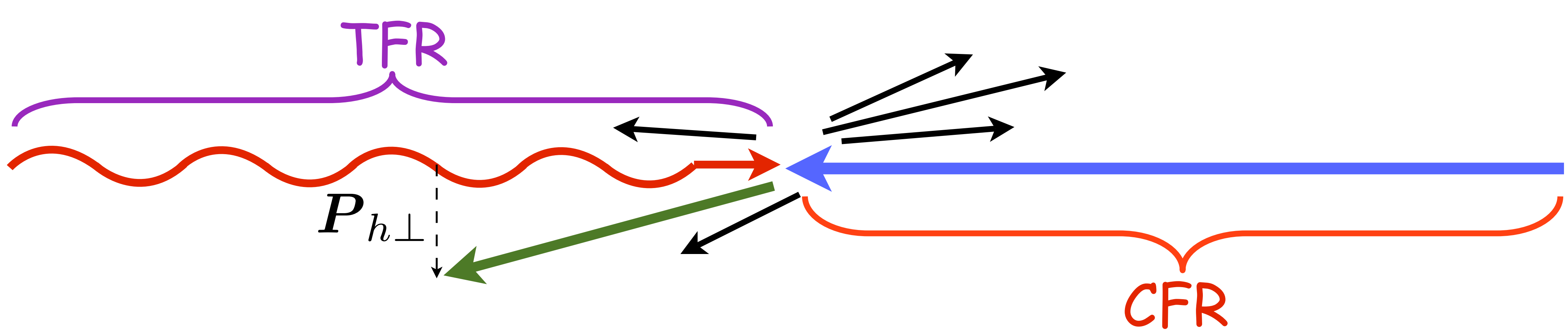
# azimuthal dependences from target fragmentation region

(fracture functions, talk by A. Kotzinian)




$$\mathrm{d}\sigma^{\mathrm{TFR}} = \sum_a \underbrace{M_a(x_B, \zeta, \boldsymbol{P}_{h\perp}^2)}_{\text{fracture functions}} \otimes \mathrm{d}\hat{\sigma}(y)$$

$$\zeta \simeq \frac{E_h}{E} \simeq (1 - x_B)|x_F|$$



# azimuthal modulations in TFR

(M.A, V. Barone, A. Kotzinian, PL B699 (2011) 108 )

cross section for lepto-production of an unpolarized or spinless hadron in the TFR

$$
\begin{aligned}
\frac{\mathrm{d}\sigma^{\mathrm{TFR}}}{\mathrm{d}x_B\,\mathrm{d}y\,\mathrm{d}\zeta\,\mathrm{d}^2\boldsymbol{P}_{h\perp}\,\mathrm{d}\phi_S} = \frac{2\alpha_{\mathrm{em}}^2}{Q^2 y}\,\Bigg\{\left(1-y+\frac{y^2}{2}\right) \\
\times \sum_a e_a^2 \left[M(x_B,\zeta,\boldsymbol{P}_{h\perp}^2) - |\boldsymbol{S}_\perp|\,\frac{|\boldsymbol{P}_{h\perp}|}{m_h}\,M_T^h(x_B,\zeta,\boldsymbol{P}_{h\perp}^2)\,\sin(\phi_h-\phi_S)\right] \\
+\lambda_l\,y\left(1-\frac{y}{2}\right)\sum_a e_a^2\left[S_\parallel\,\Delta M_L(x_B,\zeta,\boldsymbol{P}_{h\perp}^2)\right. \\
\left. +\ |\boldsymbol{S}_\perp|\,\frac{|\boldsymbol{P}_{h\perp}|}{m_h}\,\Delta M_T^h(x_B,\zeta,\boldsymbol{P}_{h\perp}^2)\,\cos(\phi_h-\phi_S)\right]\Bigg\}\ .
\end{aligned}
$$

possible Sivers-like azimuthal dependence from target fragmentation region the azimuthal dependence induced by intrinsic motion in unpolarized SIDIS (Cahn effect) has been confirmed (EMC, HERMES, COMPASS, CLAS)

phenomenolgical analysis and data needs improvement (Schweitzer, Teckentrup, Metz; Boglione, Melis, Prokudin, ....)

## Gaussian $k_\perp$ distribution of TMDs?

$$\langle k_\perp^2 \rangle(x, Q^2) \quad \langle p_\perp^2 \rangle(z, Q^2)$$

x, z dependence?

flavour dependence?

energy dependence?

$k_\perp$ dependence of $\Delta q$ vs. q?

......................

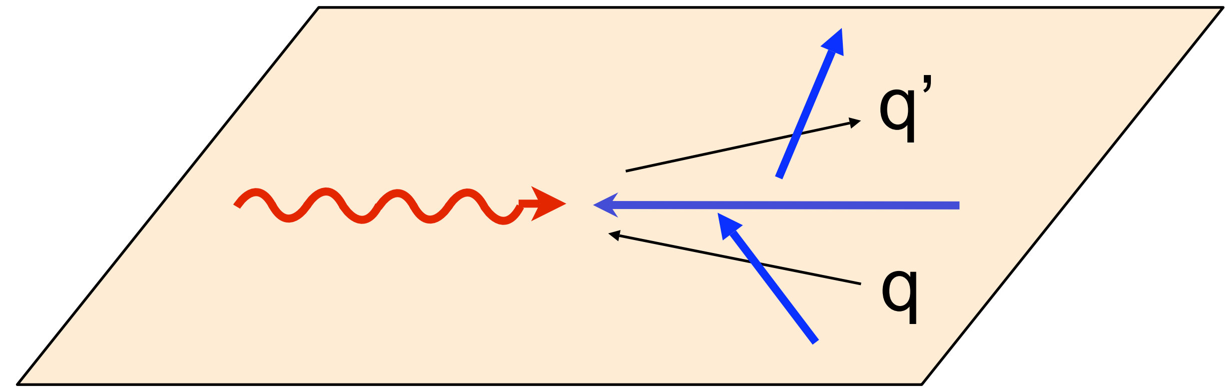
Sivers effect now observed by two experiments (+ HALL-A $A_{UT}$ on neutrons), but needs further measurements

great improvement in study of QCD evolution
(Aybat, Rogers, arXiv:1101.5057)

$Q^2$ of data not so high, role of higher twists?
clear separation of TFR and CFR needed...
more sophisticated parameterization...
universality of Sivers function?...
(talk by P. Mulders)

# Collins effect in SIDIS - $F_{UT}^{\sin(\phi+\phi_S)}$

$$D_{h/q,\boldsymbol{s}_q}(z,\boldsymbol{p}_\perp) = D_{h/p}(z,p_\perp) + \frac{1}{2}\Delta^N D_{h/q^\uparrow}(z,p_\perp)\,\boldsymbol{s}_q \cdot (\hat{\boldsymbol{p}}_q \times \hat{\boldsymbol{p}}_\perp)$$

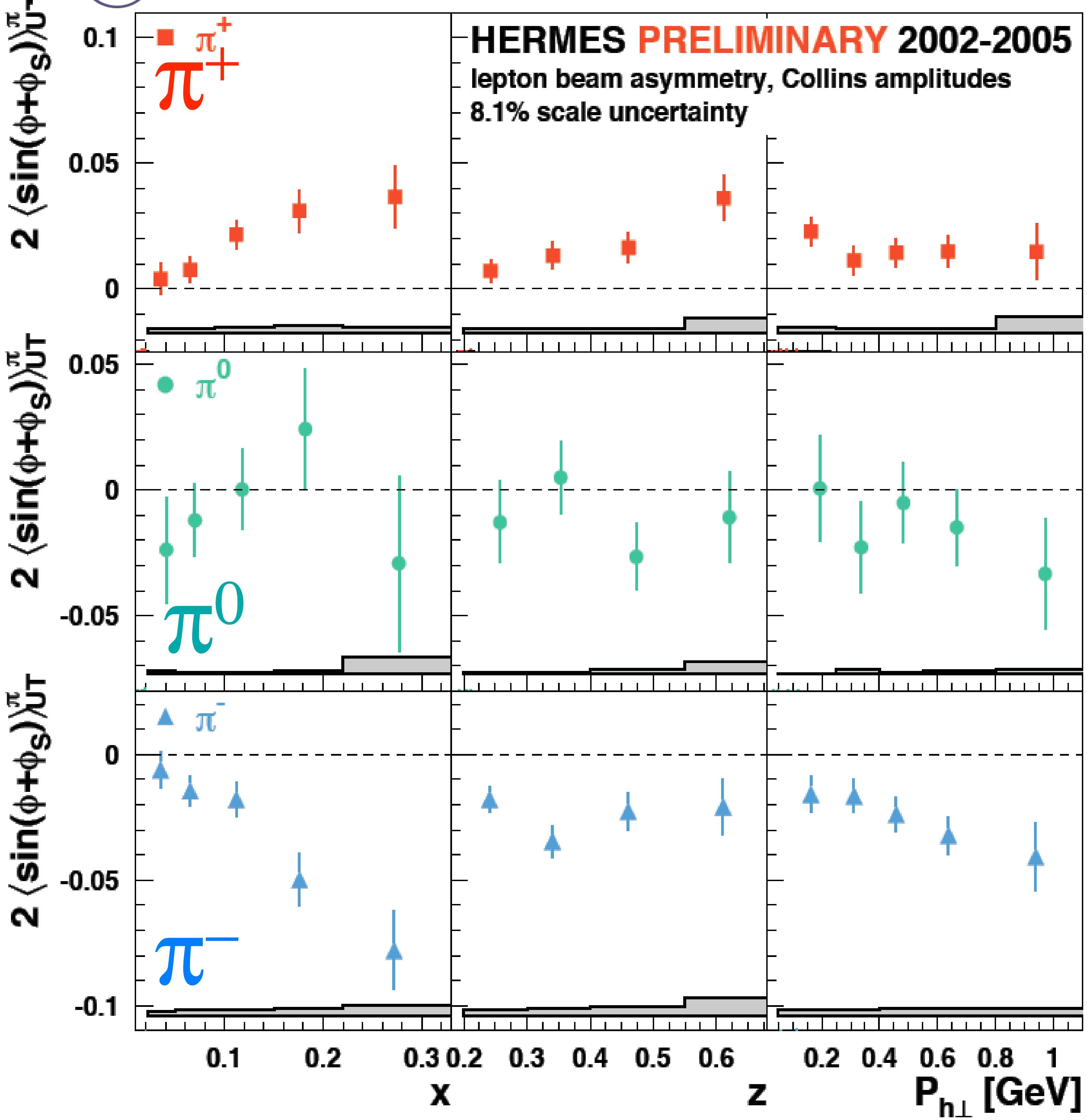


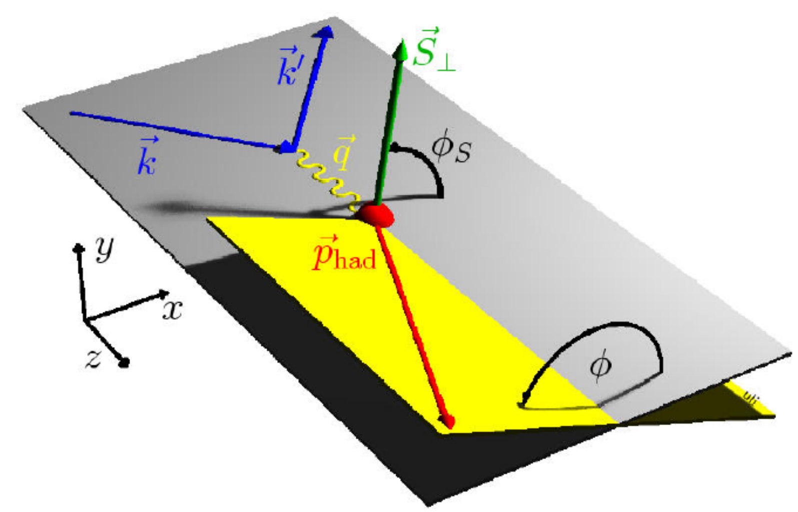

$$\mathrm{d}\sigma^\uparrow - \mathrm{d}\sigma^\downarrow = \sum_q h_{1q}(x,k_\perp) \otimes \mathrm{d}\Delta\hat{\sigma}(y,\boldsymbol{k}_\perp) \otimes \Delta^N D_{h/q^\uparrow}(z,\boldsymbol{p}_\perp)$$

$$A_{UT}^{\sin(\phi+\phi_S)} \equiv 2\,\frac{\int \mathrm{d}\phi\,\mathrm{d}\phi_S\,[\mathrm{d}\sigma^\uparrow - \mathrm{d}\sigma^\downarrow]\,\sin(\phi+\phi_S)}{\int \mathrm{d}\phi\,\mathrm{d}\phi_S\,[\mathrm{d}\sigma^\uparrow + \mathrm{d}\sigma^\downarrow]}$$

$$\mathrm{d}\Delta\hat{\sigma} = \mathrm{d}\hat{\sigma}^{\ell q^\uparrow \to \ell q^\uparrow} - \mathrm{d}\hat{\sigma}^{\ell q^\uparrow \to \ell q^\downarrow}$$

Collins effect in SIDIS couples to transversity ep↑→ π X  $E_b$=27GeV,  √s~7 GeV

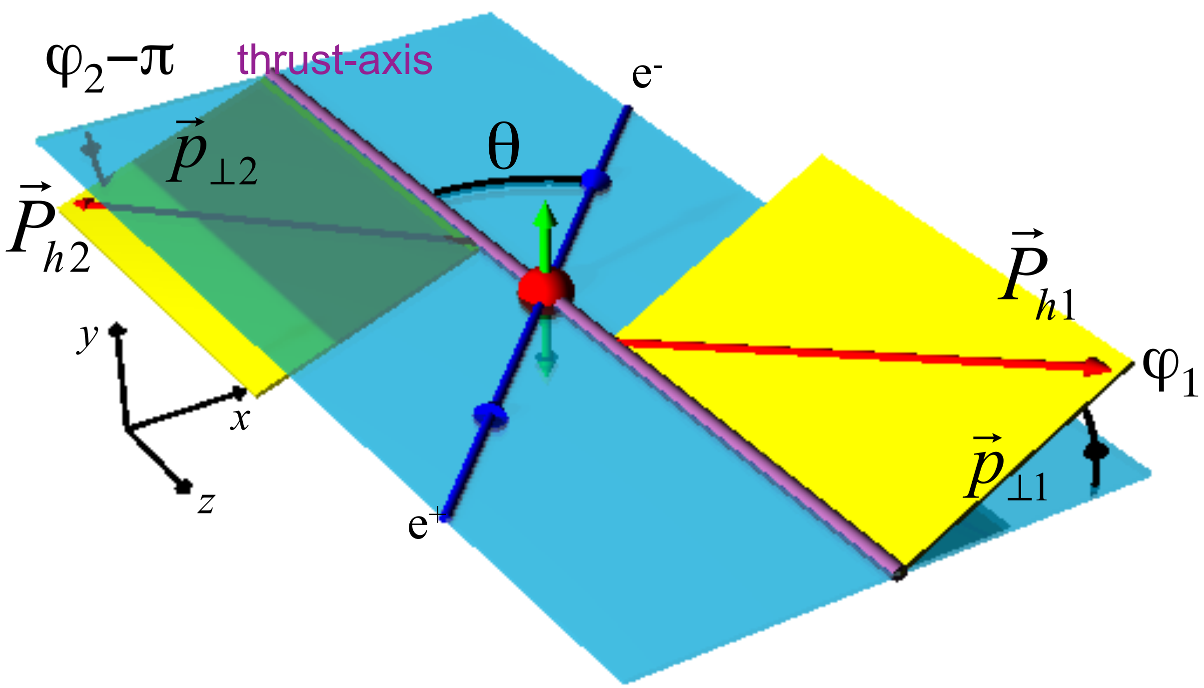


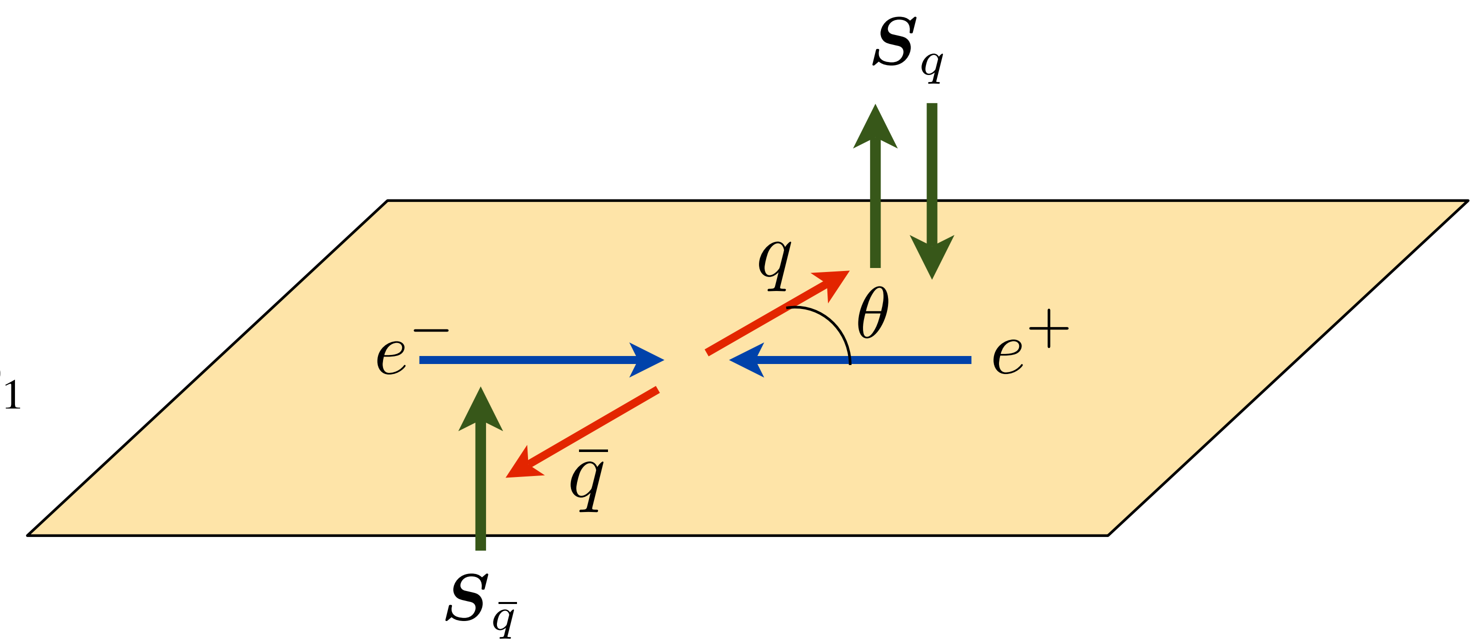




HERMES
Collins
asymmetry

# independent information on Collins function from $e^+e^-$ processes

## BELLE @ KEK




$$A_{12}(z_1, z_2, \theta, \varphi_1+\varphi_2) \equiv \frac{1}{\langle d\sigma \rangle} \frac{d\sigma^{e^+e^- \to h_1 h_2 X}}{dz_1\, dz_2\, d\cos\theta\, d(\varphi_1+\varphi_2)}$$

$$= 1 + \frac{1}{4} \frac{\sin^2\theta}{1+\cos^2\theta} \cos(\varphi_1+\varphi_2) \times \frac{\sum_q e_q^2 \Delta^N D_{h_1/q^\uparrow}(z_1) \Delta^N D_{h_2/\bar{q}^\uparrow}(z_2)}{\sum_q e_q^2 D_{h_1/q}(z_1)\, D_{h_2/\bar{q}}(z_2)}$$

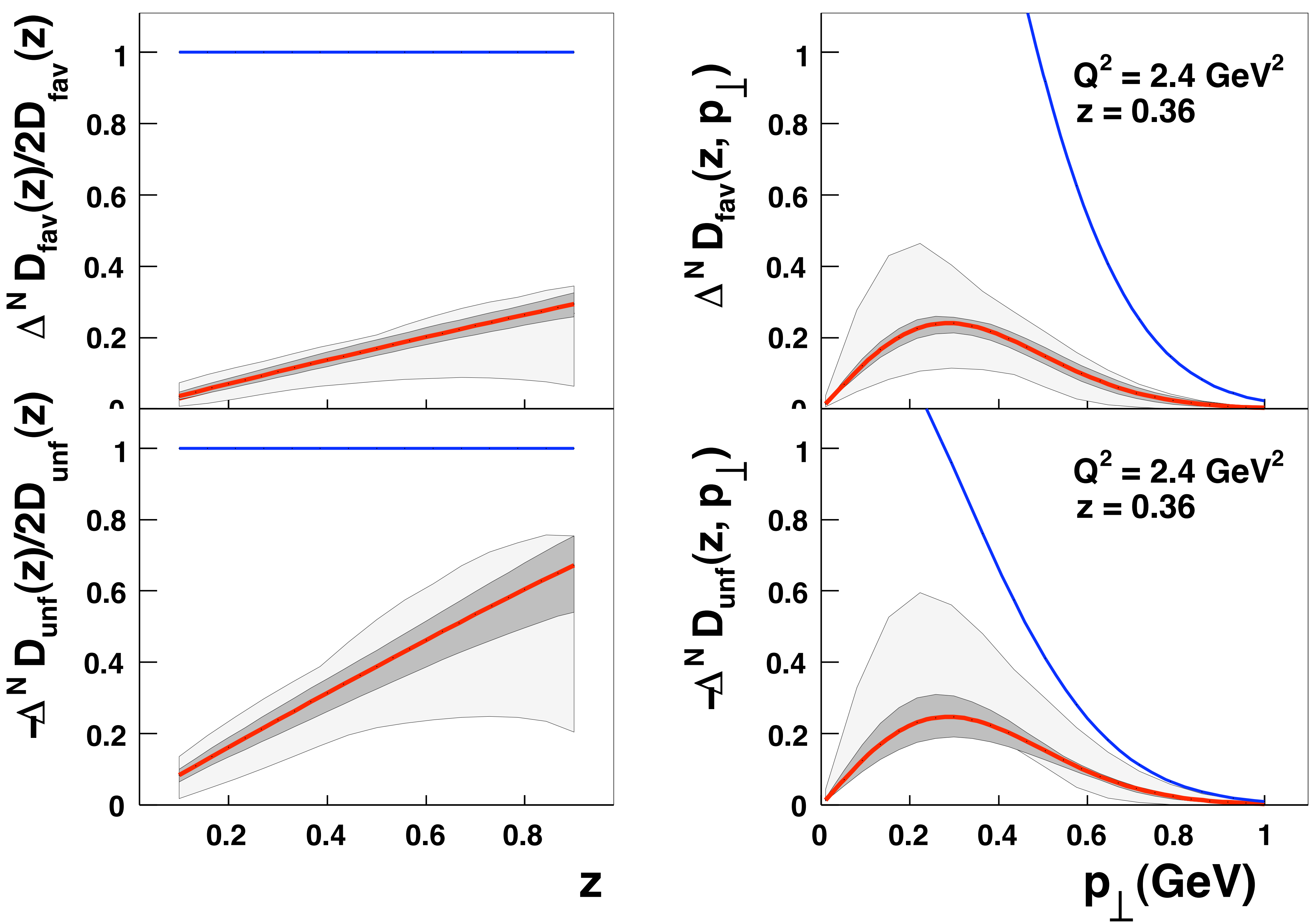
# Transversity & Collins function phenomenology in SIDIS and e+e-

Same simple parametrization as for Sivers, but Collins effect has been clearly observed by three independent experiments: HERMES, COMPASS and BELLE

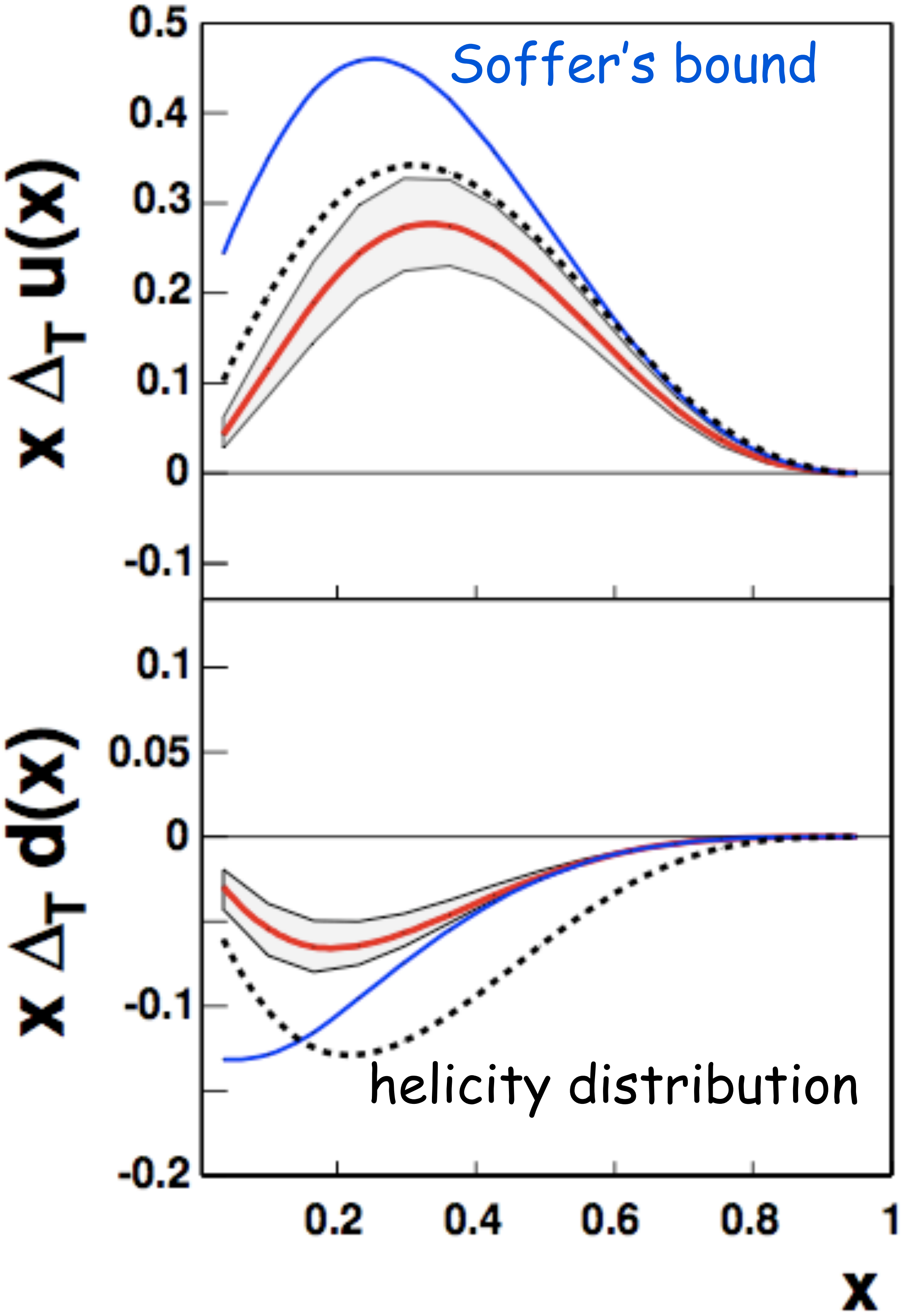
Collins function expected to be universal

QCD evolution important, as BELLE data are at a much higher energy than SIDIS data

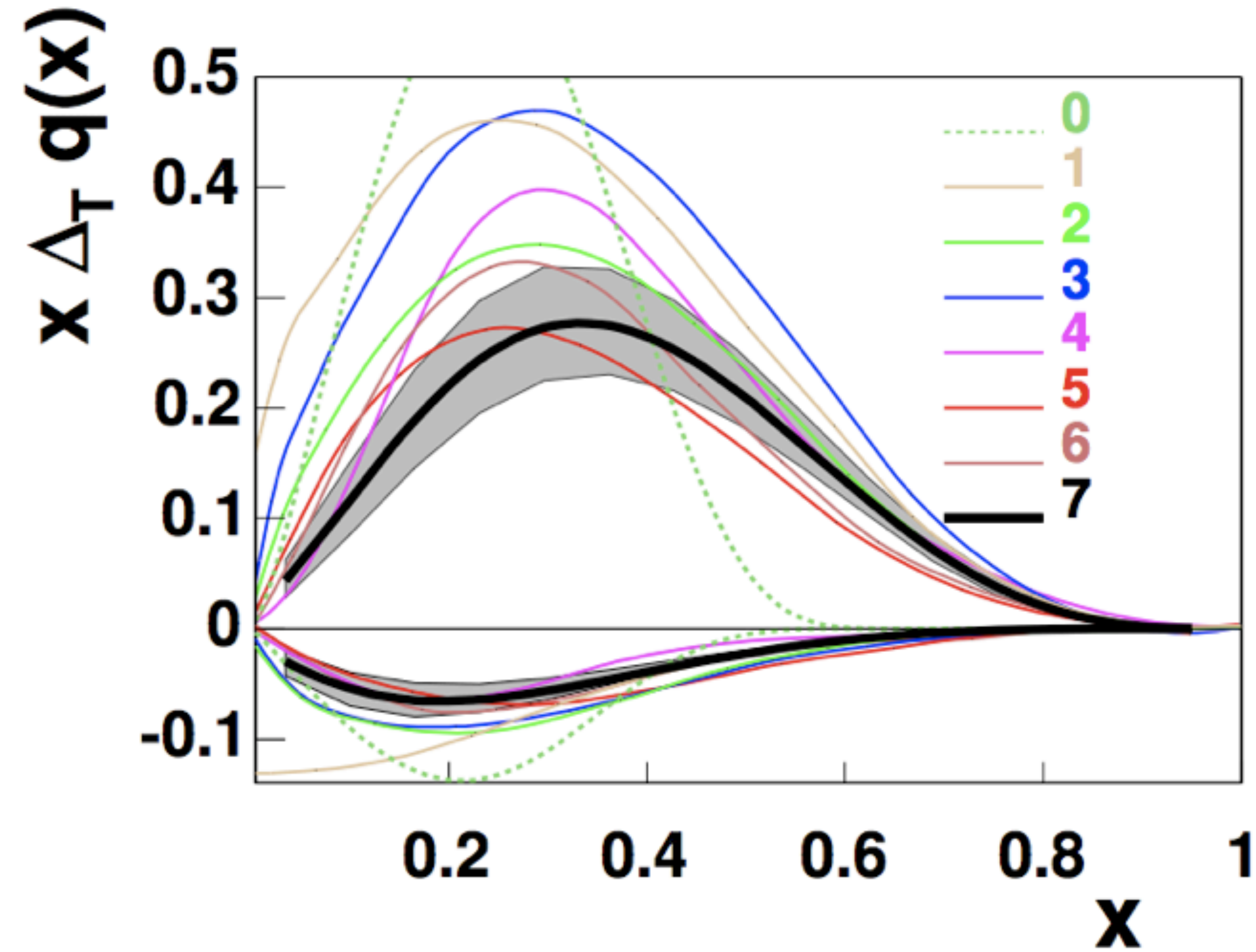
# extracted Collins functions



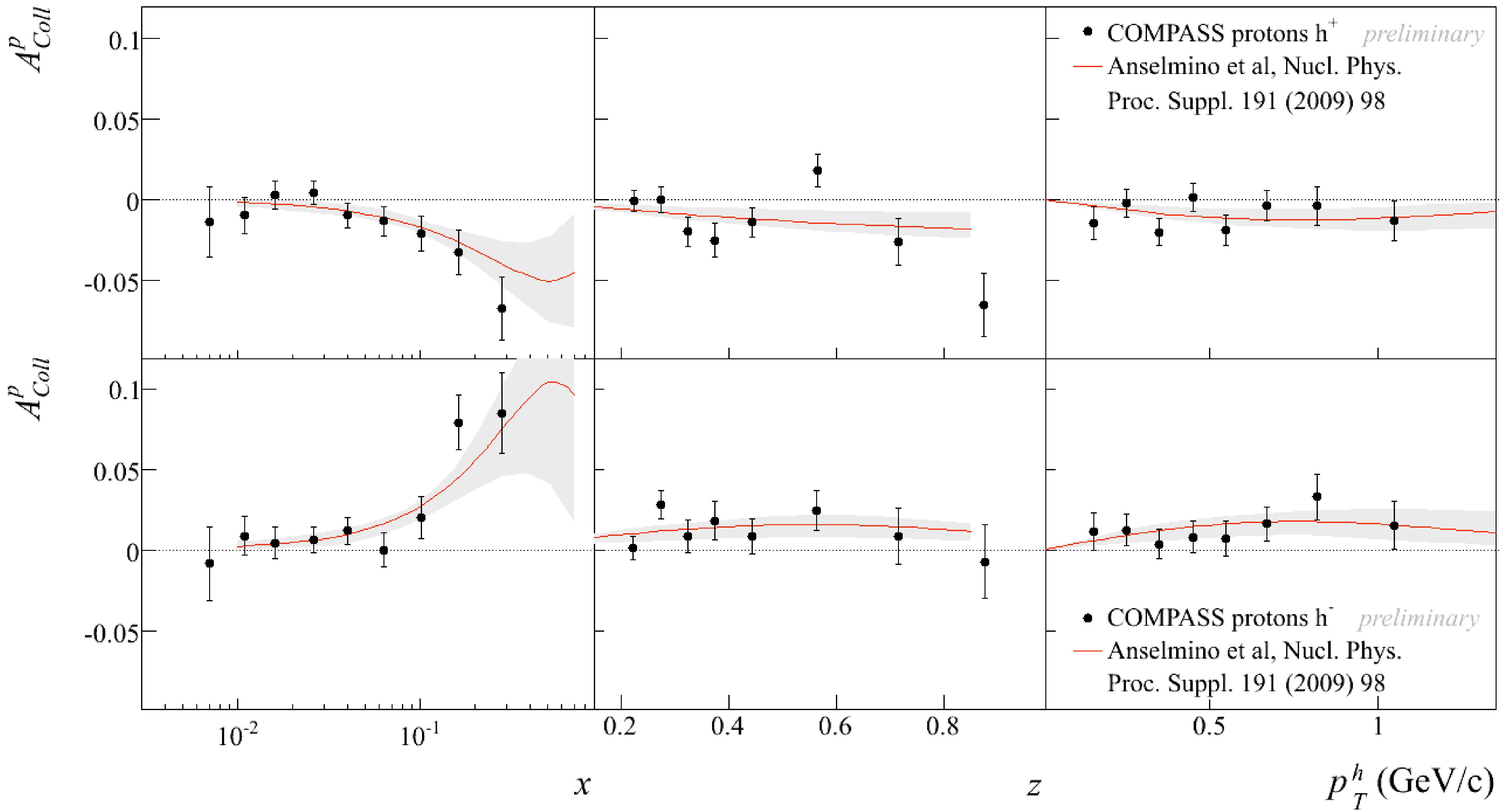

M.A., M. Boglione, U. D'Alesio, A. Kotzinian, S. Melis, F. Murgia, A. Prokudin, C. Türk




extracted transversity and comparison with models

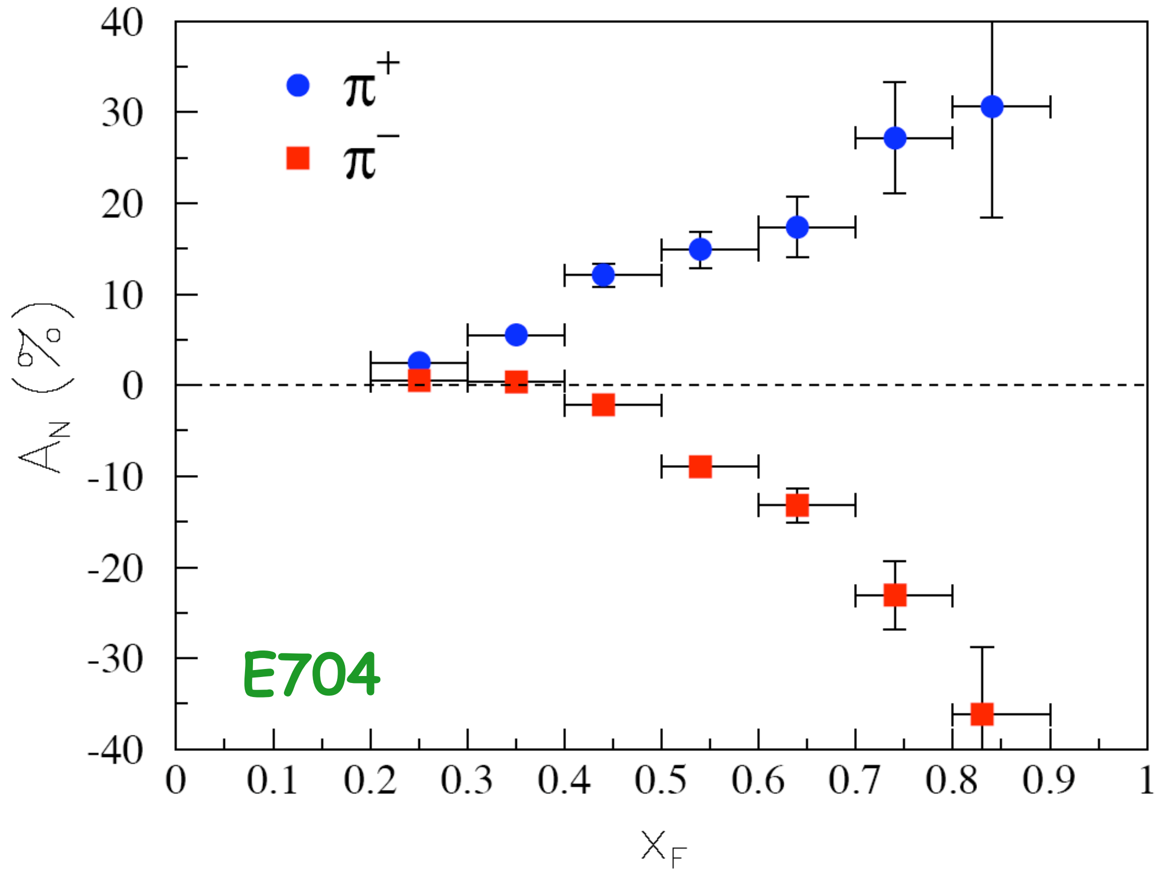


# Predictions for COMPASS, with a proton target, and comparison with data




A. Martin, DIS2010

# $A_N$ in p p → $\pi$ X, the big challenge

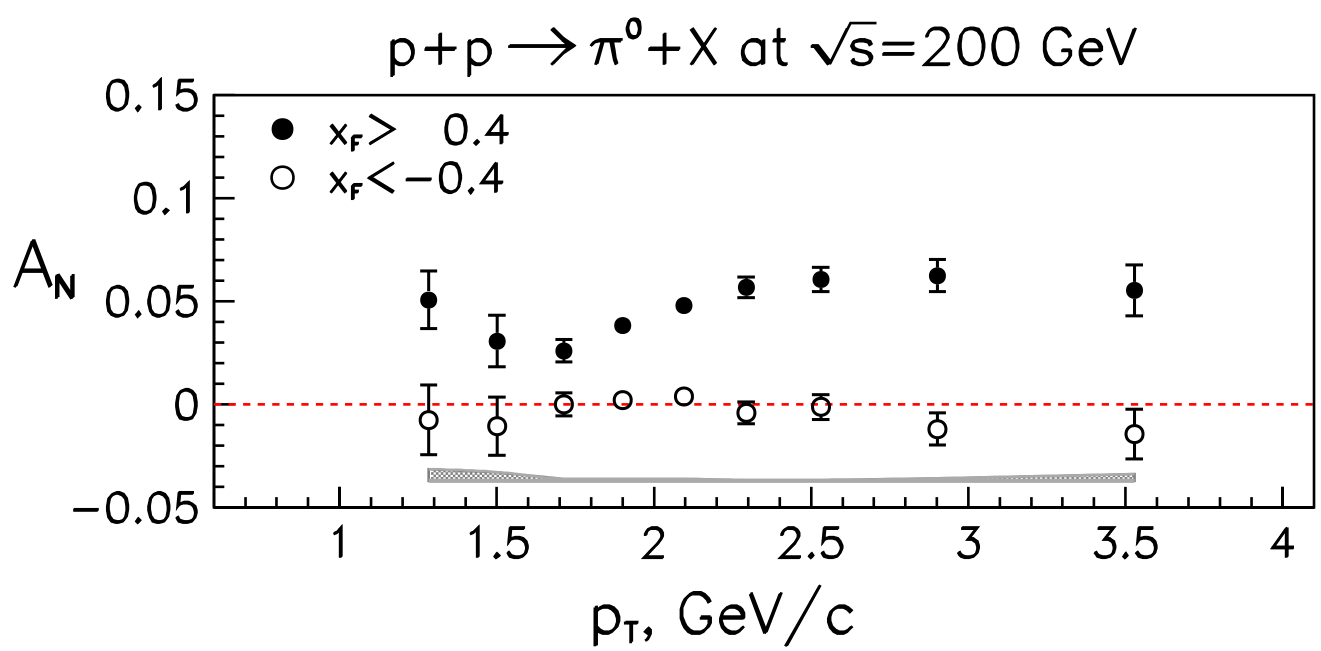



$$A_N \equiv \frac{d\sigma^{\uparrow} - d\sigma^{\uparrow}}{d\sigma^{\uparrow} + d\sigma^{\uparrow}}$$

E704 $\sqrt{s}$ = 20 GeV

0.7 < $p_T$ < 2.0

and all beautiful RHIC data, persisting at high energy...

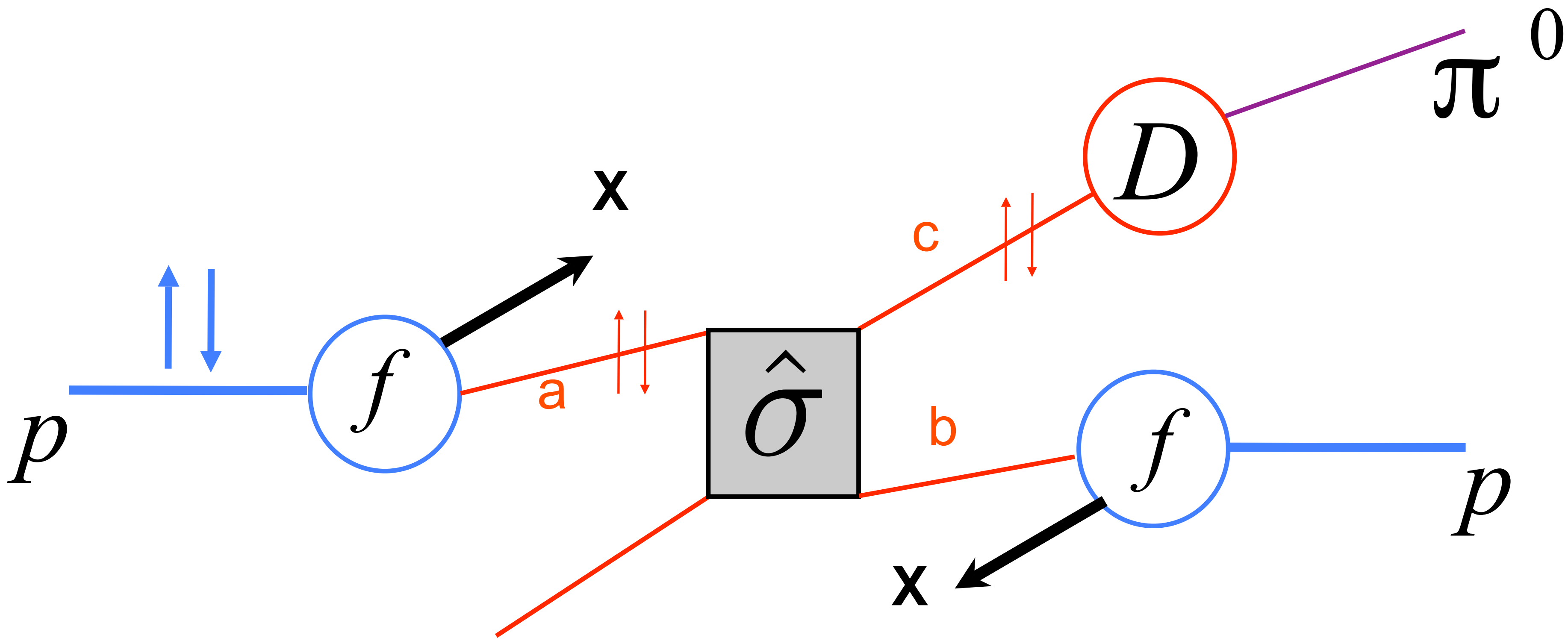


# Only one large scale, $P_T$. Any role for TMDs?

## TMD factorization not proven

talk by Rogers

### 1. Generalization of collinear scheme (assuming factorization)




$$\mathrm{d}\sigma^{\uparrow} = \sum_{a,b,c=q,\bar{q},g} \underbrace{f_{a/p^{\uparrow}}(x_a, \boldsymbol{k}_{\perp a})} \otimes \underbrace{f_{b/p}(x_b, \boldsymbol{k}_{\perp b})} \otimes \mathrm{d}\hat{\sigma}^{ab\to cd}(\boldsymbol{k}_{\perp a}, \boldsymbol{k}_{\perp b}) \otimes \underbrace{D_{\pi/c}(z, \boldsymbol{p}_{\perp \pi})}$$

single spin effects in TMDs

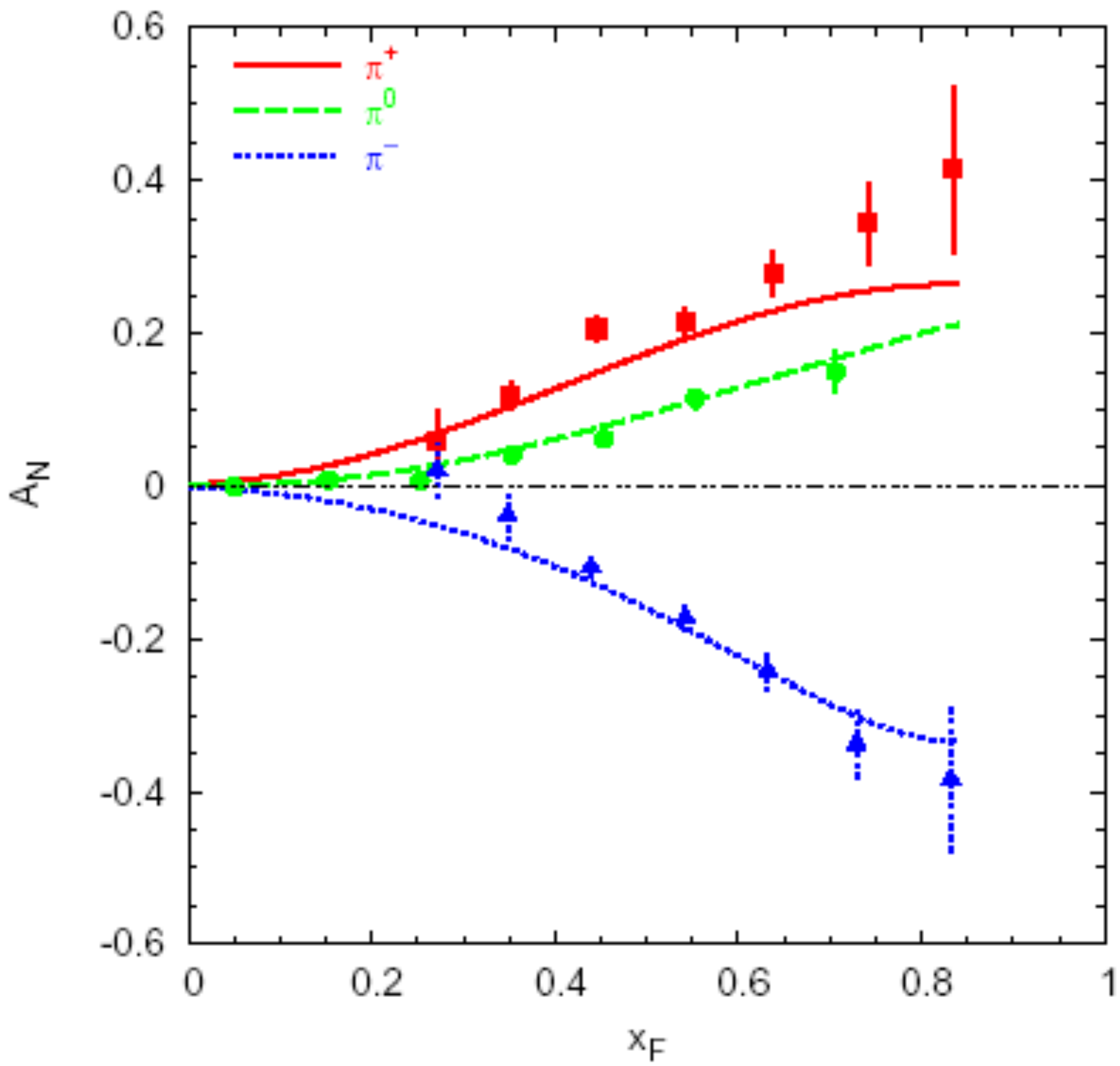
M.A., M. Boglione, U. D'Alesio, E. Leader, S. Melis, F. Murgia, A. Prokudin, ...

(Field-Feynman in unpolarized case)

# TMD factorization at work ....

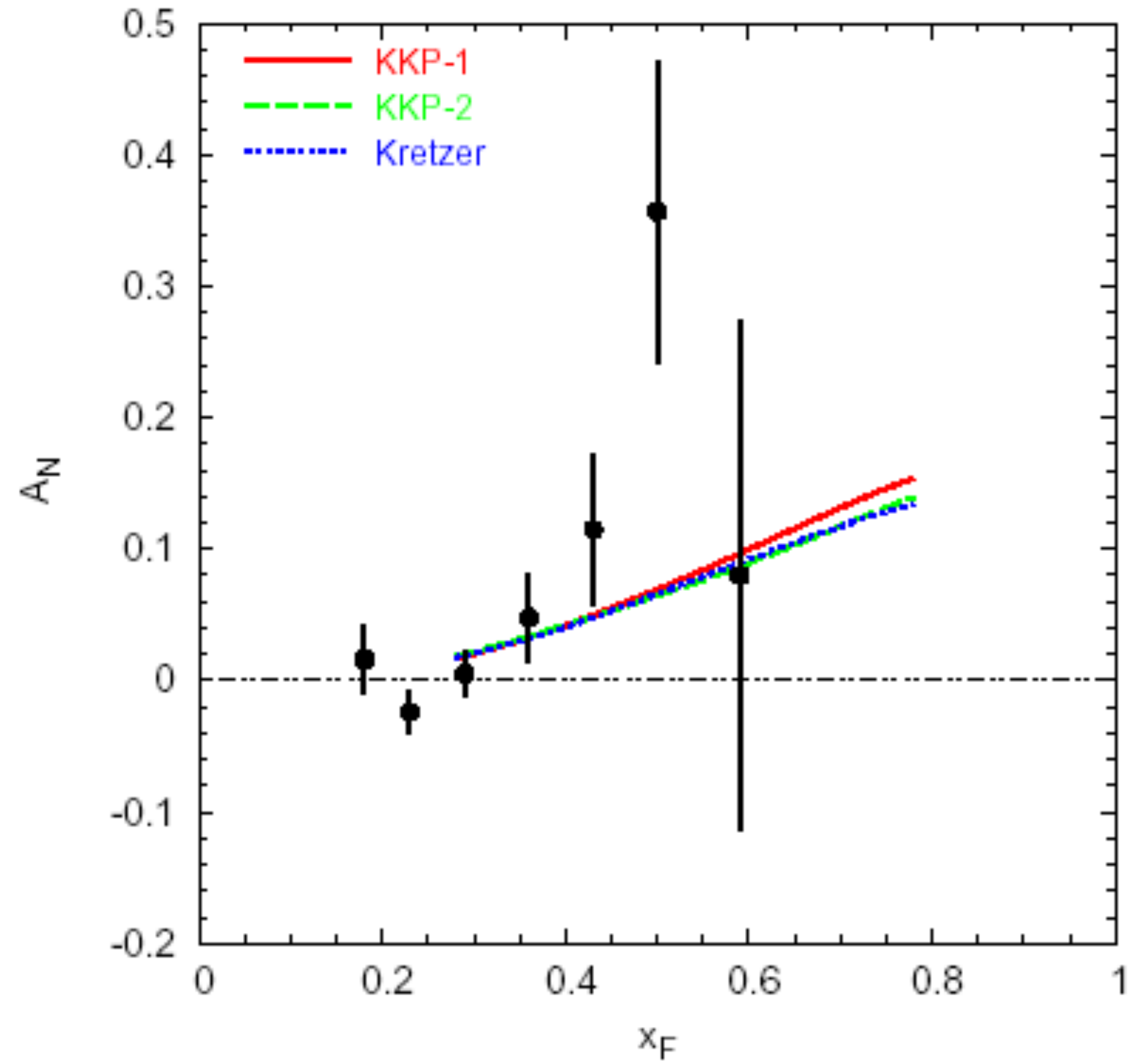
U. D'Alesio, F. Murgia

E704 data




fit

STAR data




prediction

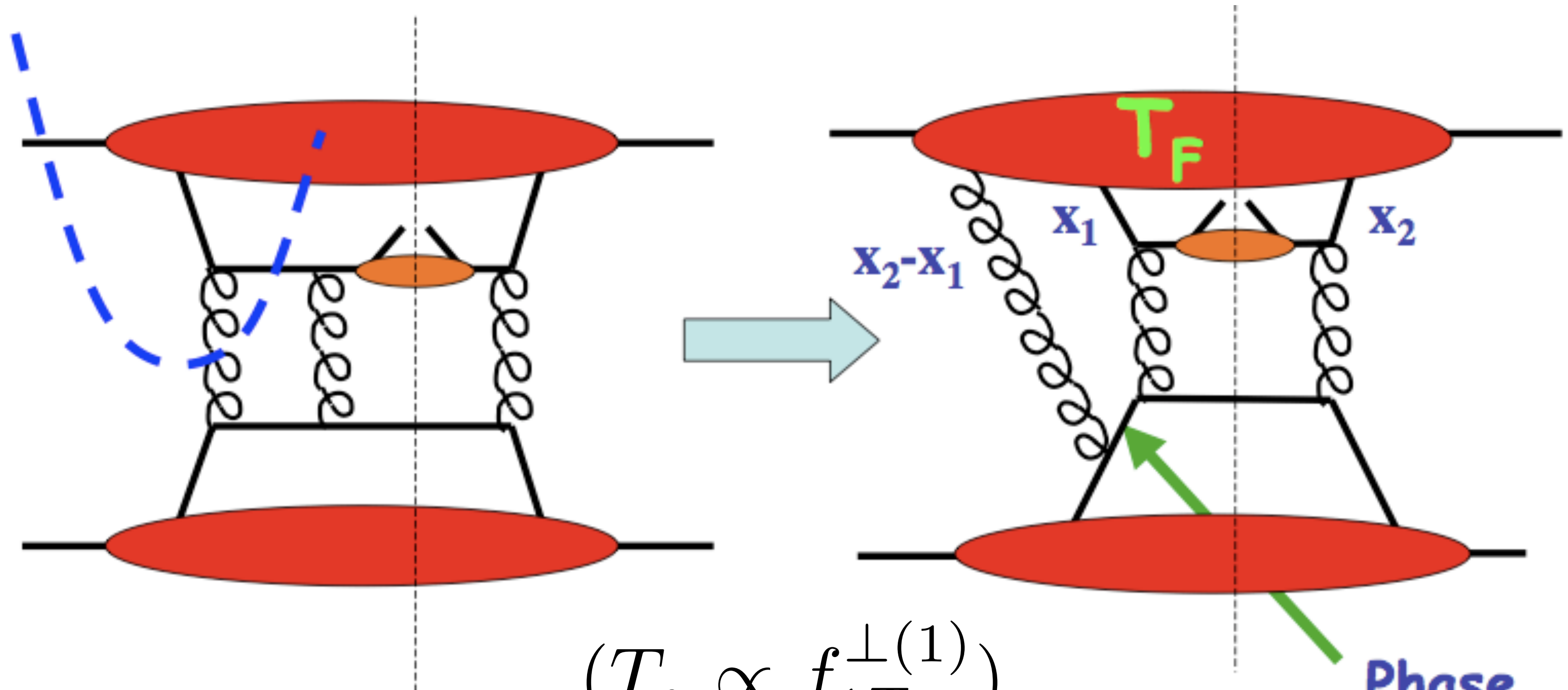
Sivers effect $p\,p \to \pi\,X$

# 2. Higher-twist partonic correlations

(Efremov, Teryaev; Qiu, Sterman; Kouvaris, Vogelsang, Yuan; Bacchetta, Bomhof, Mulders, Pijlman; Koike ... )

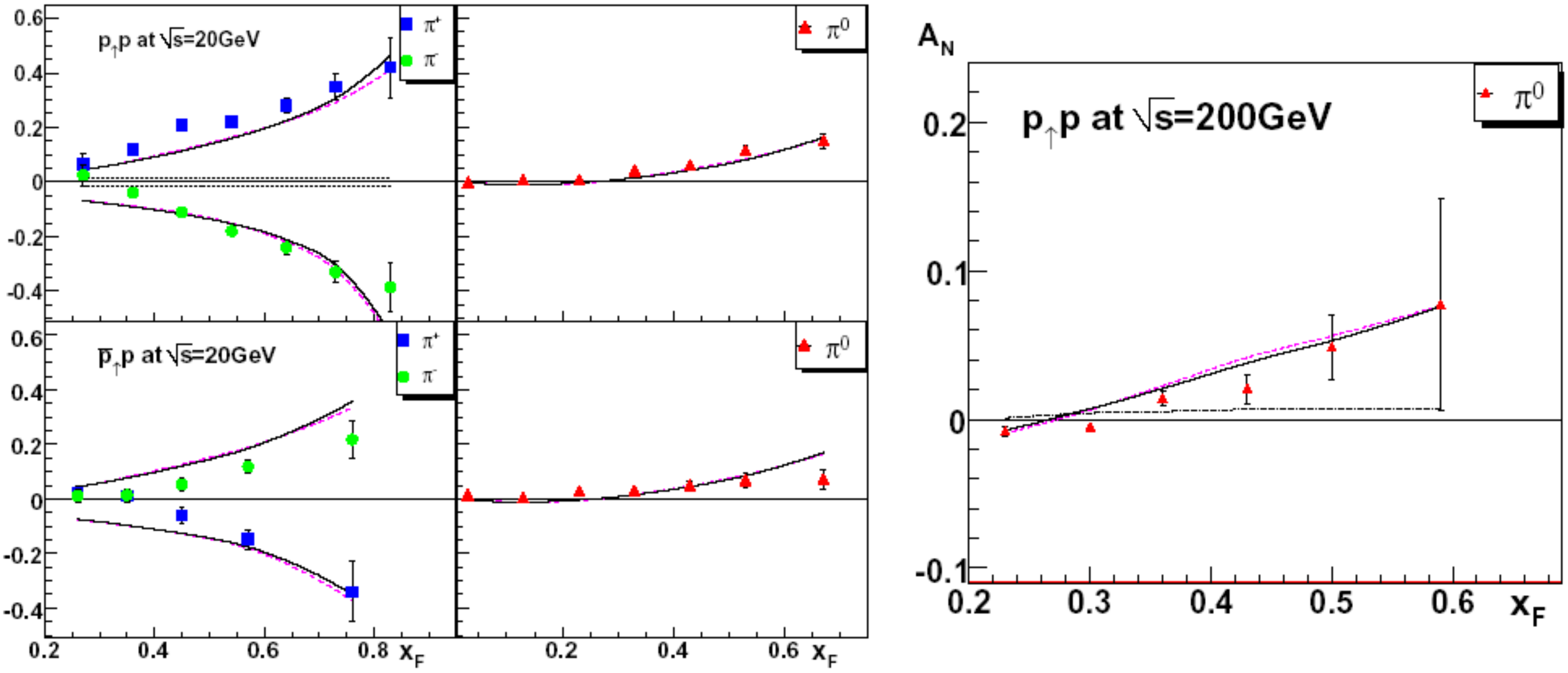
higher-twist partonic correlations - factorization OK

$$\mathrm{d}\Delta\sigma \propto \sum_{a,b,c} \underbrace{T_a(k_1, k_2, \boldsymbol{S}_\perp)}_{\text{twist-3 functions}} \otimes f_{b/B}(x_b) \otimes \underbrace{H^{ab\to c}(k_1, k_2)}_{\text{hard interaction, not a cross section}} \otimes D_{h/c}(z)$$




$(T_a \propto f_{1T}^{\perp(1)})$

possible project: compute $T_a$ using SIDIS extracted Sivers functions (talk by Z. Kang)




fits of E704 and STAR data

Kouvaris, Qiu, Vogelsang, Yuan

# sign mismatch

(Kang, Qiu, Vogelsang, Yuan)

compare

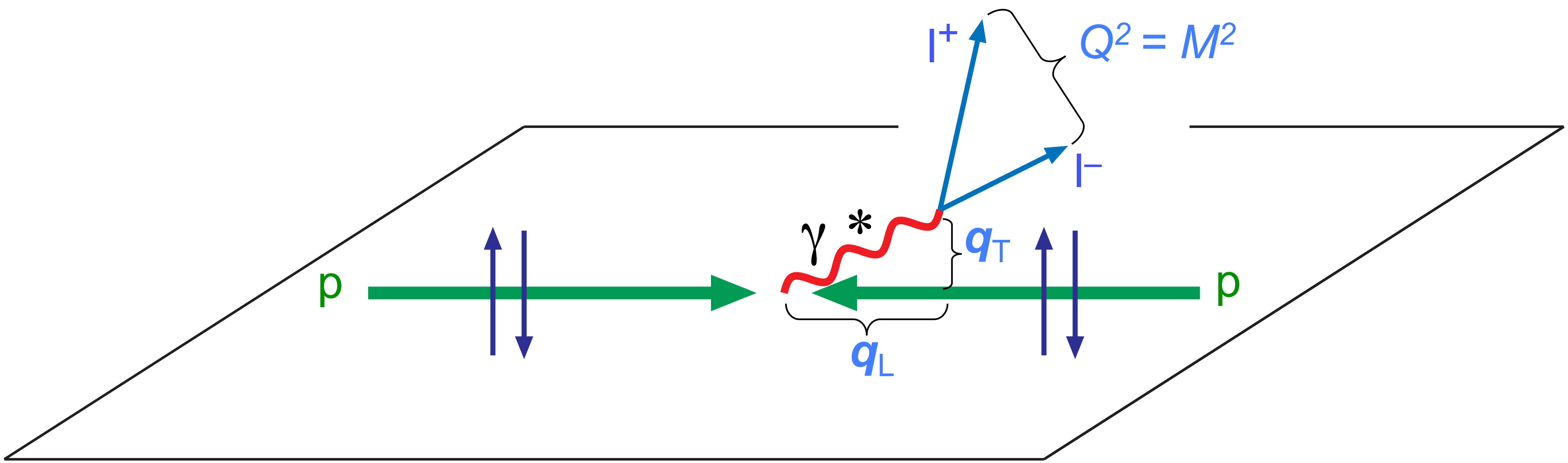
$$gT_{q,F}(x,x) = -\int d^2k_\perp \frac{|k_\perp|^2}{M} f_{1T}^{\perp q}(x,k_\perp^2)|_{\mathrm{SIDIS}}$$

as extracted from fitting $A_N$ data, with that obtained by inserting in the the above relation the SIDIS extracted Sivers functions

**similar magnitude, but opposite sign!**

the same mismatch does not occurr adopting TMD factorization; the reason is that the hard scattering part in higher-twist factorization is negative

# TMDs in Drell-Yan processes




factorization holds, two scales, $M^2$, and $q_T \ll M$

$$d\sigma^{D-Y} = \sum_a f_q(x_1, \boldsymbol{k}_{\perp 1}; Q^2) \otimes f_{\bar{q}}(x_2, \boldsymbol{k}_{\perp 2}; Q^2)\, d\hat{\sigma}^{q\bar{q}\to\ell^+\ell^-}$$

direct product of TMDs, no fragmentation process

(many talks)

# cross-section: most general pp leading-twist expression

$$\frac{d\sigma}{d^4q\,d\Omega} = \frac{\alpha_{em}^2}{F\,q^2}\times$$

S. Arnold, A. Metz and M. Schlegel, arXiv:0809.2262 [hep-ph]

$$\begin{aligned}
&\Big\{\Big((1+\cos^2\theta)\,F_{UU}^1 + (1-\cos^2\theta)\,F_{UU}^2 + \sin 2\theta\cos\phi\,F_{UU}^{\cos\phi} + \sin^2\theta\cos 2\phi\,F_{UU}^{\cos 2\phi}\Big)\\
&+ S_{aL}\Big(\sin 2\theta\sin\phi\,F_{LU}^{\sin\phi} + \sin^2\theta\sin 2\phi\,F_{LU}^{\sin 2\phi}\Big)\\
&+ S_{bL}\Big(\sin 2\theta\sin\phi\,F_{UL}^{\sin\phi} + \sin^2\theta\sin 2\phi\,F_{UL}^{\sin 2\phi}\Big)\\
&+ |\vec{S}_{aT}|\Big[\sin\phi_a\Big((1+\cos^2\theta)\,F_{TU}^1 + (1-\cos^2\theta)\,F_{TU}^2 + \sin 2\theta\cos\phi\,F_{TU}^{\cos\phi} + \sin^2\theta\cos 2\phi\,F_{TU}^{\cos 2\phi}\Big)\\
&\qquad + \cos\phi_a\Big(\sin 2\theta\sin\phi\,F_{TU}^{\sin\phi} + \sin^2\theta\sin 2\phi\,F_{TU}^{\sin 2\phi}\Big)\Big]\\
&+ |\vec{S}_{bT}|\Big[\sin\phi_b\Big((1+\cos^2\theta)\,F_{UT}^1 + (1-\cos^2\theta)\,F_{UT}^2 + \sin 2\theta\cos\phi\,F_{UT}^{\cos\phi} + \sin^2\theta\cos 2\phi\,F_{UT}^{\cos 2\phi}\Big)\\
&\qquad + \cos\phi_b\Big(\sin 2\theta\sin\phi\,F_{UT}^{\sin\phi} + \sin^2\theta\sin 2\phi\,F_{UT}^{\sin 2\phi}\Big)\Big]\\
&+ S_{aL}\,S_{bL}\Big((1+\cos^2\theta)\,F_{LL}^1 + (1-\cos^2\theta)\,F_{LL}^2 + \sin 2\theta\cos\phi\,F_{LL}^{\cos\phi} + \sin^2\theta\cos 2\phi\,F_{LL}^{\cos 2\phi}\Big)\\
&+ S_{aL}\,|\vec{S}_{bT}|\Big[\cos\phi_b\Big((1+\cos^2\theta)\,F_{LT}^1 + (1-\cos^2\theta)\,F_{LT}^2 + \sin 2\theta\cos\phi\,F_{LT}^{\cos\phi} + \sin^2\theta\cos 2\phi\,F_{LT}^{\cos 2\phi}\Big)\\
&\qquad + \sin\phi_b\Big(\sin 2\theta\sin\phi\,F_{LT}^{\sin\phi} + \sin^2\theta\sin 2\phi\,F_{LT}^{\sin 2\phi}\Big)\Big]\\
&+ |\vec{S}_{aT}|\,S_{bL}\Big[\cos\phi_a\Big((1+\cos^2\theta)\,F_{TL}^1 + (1-\cos^2\theta)\,F_{TL}^2 + \sin 2\theta\cos\phi\,F_{TL}^{\cos\phi} + \sin^2\theta\cos 2\phi\,F_{TL}^{\cos 2\phi}\Big)\\
&\qquad + \sin\phi_a\Big(\sin 2\theta\sin\phi\,F_{TL}^{\sin\phi} + \sin^2\theta\sin 2\phi\,F_{TL}^{\sin 2\phi}\Big)\Big]\\
&+ |\vec{S}_{aT}|\,|\vec{S}_{bT}|\Big[\cos(\phi_a+\phi_b)\Big((1+\cos^2\theta)\,F_{TT}^1 + (1-\cos^2\theta)\,F_{TT}^2 + \sin 2\theta\cos\phi\,F_{TT}^{\cos\phi} + \sin^2\theta\cos 2\phi\,F_{TT}^{\cos 2\phi}\Big)\\
&\qquad + \cos(\phi_a-\phi_b)\Big((1+\cos^2\theta)\,\bar{F}_{TT}^1 + (1-\cos^2\theta)\,\bar{F}_{TT}^2 + \sin 2\theta\cos\phi\,\bar{F}_{TT}^{\cos\phi} + \sin^2\theta\cos 2\phi\,\bar{F}_{TT}^{\cos 2\phi}\Big)\\
&\qquad + \sin(\phi_a+\phi_b)\Big(\sin 2\theta\sin\phi\,F_{TT}^{\sin\phi} + \sin^2\theta\sin 2\phi\,F_{TT}^{\sin 2\phi}\Big)\\
&\qquad + \sin(\phi_a-\phi_b)\Big(\sin 2\theta\sin\phi\,\bar{F}_{TT}^{\sin\phi} + \sin^2\theta\sin 2\phi\,\bar{F}_{TT}^{\sin 2\phi}\Big)\Big]\Big\}
\end{aligned}$$

Drell-Yan

48 structure functions ...

# Cahn effect in unpolarized D-Y

M. Boglione, S. Melis, arXiv:1103.2084

access to $\langle k_\perp^2 \rangle$

$$\frac{d\sigma^{unp}}{d^4q\,d\Omega'} = \frac{\alpha^2}{6M^2 s}\sum_q e_q^2\; f^q_{a/A}(x_a)\;\bar{f}^q_{b/B}(x_b)\,\frac{e^{-q_T^2/\langle q_T^2\rangle}}{\pi\langle q_T^2\rangle}\left\{(1+\cos^2\theta') + \underbrace{\frac{q_T}{M}\,\frac{\langle k^2_{\perp a}\rangle - \langle k^2_{\perp b}\rangle}{\langle q_T^2\rangle}\sin 2\theta'\cos\phi'}_{\text{Cahn effect}}\right\}$$

$$\langle k^2_{\perp a}\rangle + \langle k^2_{\perp b}\rangle \equiv \langle q_T^2\rangle \qquad\qquad \boldsymbol{q}_T = \boldsymbol{k}_{\perp a} + \boldsymbol{k}_{\perp b}$$

$$f_{a/A}(x_a, k_{\perp a}) = f_{a/A}(x_a)\,\frac{e^{-k^2_{\perp a}/\langle k^2_{\perp a}\rangle}}{\pi\langle k^2_{\perp a}\rangle}$$

gaussian $k_\perp$ dependence

no effect if $\langle k^2_{\perp a}\rangle = \langle k^2_{\perp b}\rangle$

same conclusion holds for non gaussian distributions

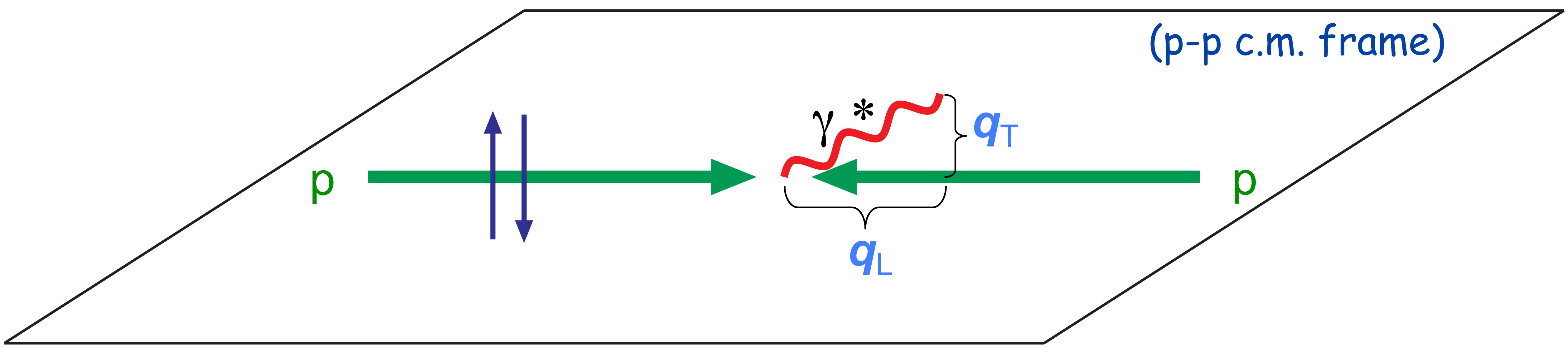
# Sivers effect in D-Y processes

By looking at the $d^4\sigma/d^4q$ cross section one can single out the Sivers effect in D-Y processes

$$\mathrm{d}\sigma^{\uparrow} - \mathrm{d}\sigma^{\downarrow} \propto \sum_q \Delta^N f_{q/p^\uparrow}(x_1, \boldsymbol{k}_\perp) \otimes f_{\bar{q}/p}(x_2) \otimes \mathrm{d}\hat{\sigma}$$

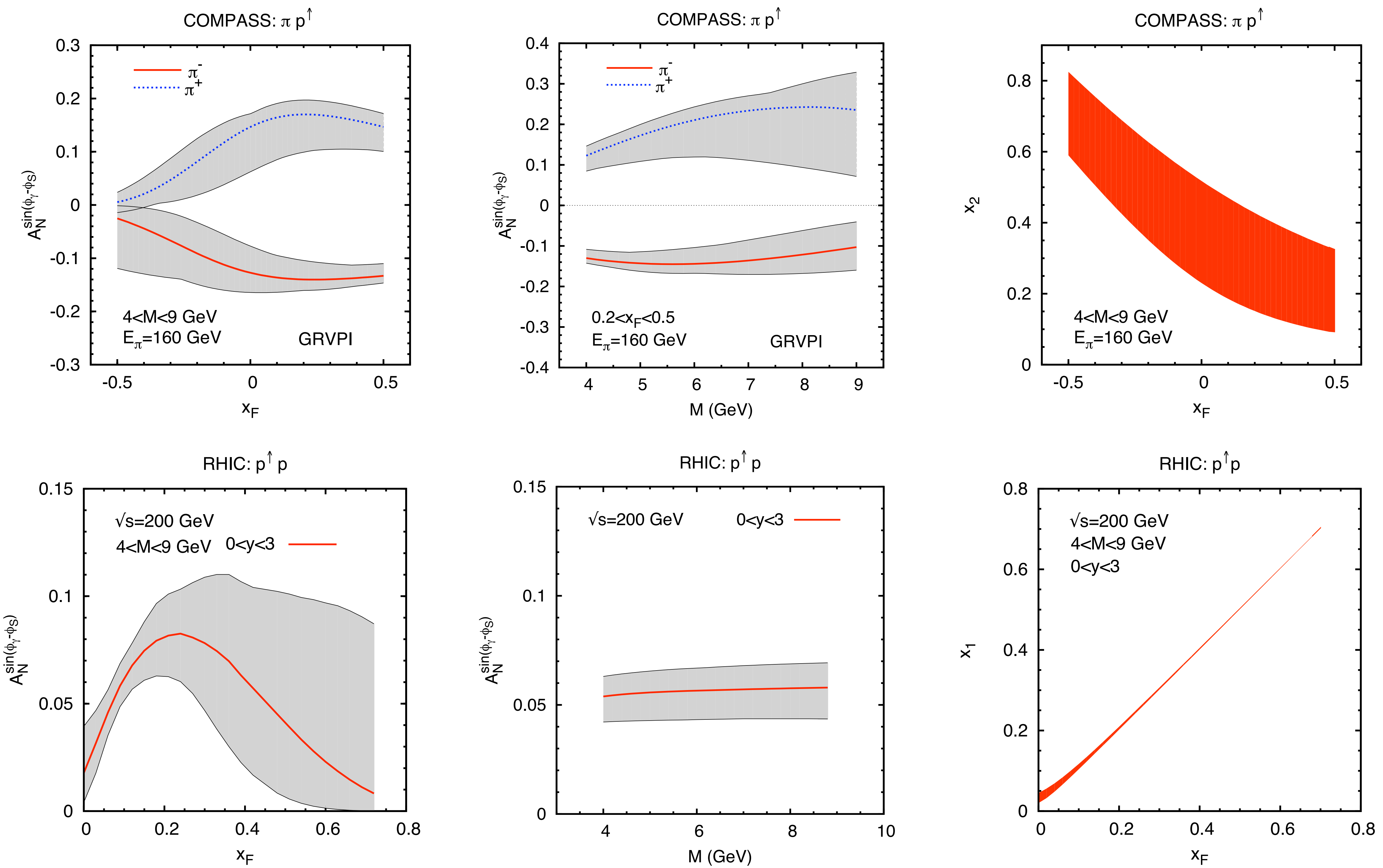
$$q = u, \bar{u}, d, \bar{d}, s, \bar{s}$$

$$A_N^{\sin(\phi_S-\phi_\gamma)} \equiv \frac{2\int_0^{2\pi}\mathrm{d}\phi_\gamma\,[\mathrm{d}\sigma^\uparrow - \mathrm{d}\sigma^\downarrow]\,\sin(\phi_S-\phi_\gamma)}{\int_0^{2\pi}\mathrm{d}\phi_\gamma\,[\mathrm{d}\sigma^\uparrow + \mathrm{d}\sigma^\downarrow]}$$



# Predictions for $A_N$

## Sivers functions as extracted from SIDIS data, **with opposite sign**




M.A., M. Boglione, U. D'Alesio, S. Melis, F. Murgia, A. Prokudin, e-Print: arXiv:0901.3078

# Conclusions

The3-dimensional exploration of the nucleon has just started: collect as much data as possible and try to reconstruct the nucleon phase-space structure

TMDs describe the momentum distribution; the actual knowledge covers limited kinematical regions, and assumes (too) simple functional forms

The properties of the Sivers function and its different role in different processes, have to be investigated

and much more to do .....

Varenna School on the 3-dimensional partonic structure of the nucleon, June 28 - July 8, 2011