Workshop on Opportunities for Drell Yan at RHIC
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The basic idea of PDFs is achieving a factorized description with soft and hard parts, soft parts being portable and hard parts being calculable. In the leading contributions at high energies, the PDFs can be interpreted as probabilities. Beyond the collinear treatment one considers not only the dependence on partonic momentum fractions x, but also the dependence on the transverse momentum p_T of the partons. Experimentally, transverse momentum dependent functions (TMDs) provide a rich phenomenology of azimuthal asymmetries for produced hadrons or jet-jet asymmetries. Furthermore inclusion of transverse momentum dependence provides an explanation for single spin asymmetries.

An important issue is the universality of TMDs, which we study for some characteristic hard processes, where we focus on the pecularities coming from the color flow in the hard part. This color flow in the hard process gives rises to a variety of Wilson lines in the description of the cross section. These give rise to color entanglement, in particular in situations that the color flow is not just a simple transfer of color from initial or final state.

We argue that these Wilson lines can be combined into the appropriate gauge links for TMD correlators in cases where only the transverse momentum of partons in a single (incoming) hadron is relevant (1-parton un-integrated or 1PU processes). Such a situation occurs in single weighted cross sections, which consists of a sum of 1PU processes or if absence of any polarization makes all explicit transverse momentum effects vanish. For 1PU processes one finds TMDs with a complex gauge link structure depending on the color flow of the hard process. In the case of single weighted cross sections the results are the gluonic pole or Qiu-Sterman matrix elements appearing with calculable color factors.

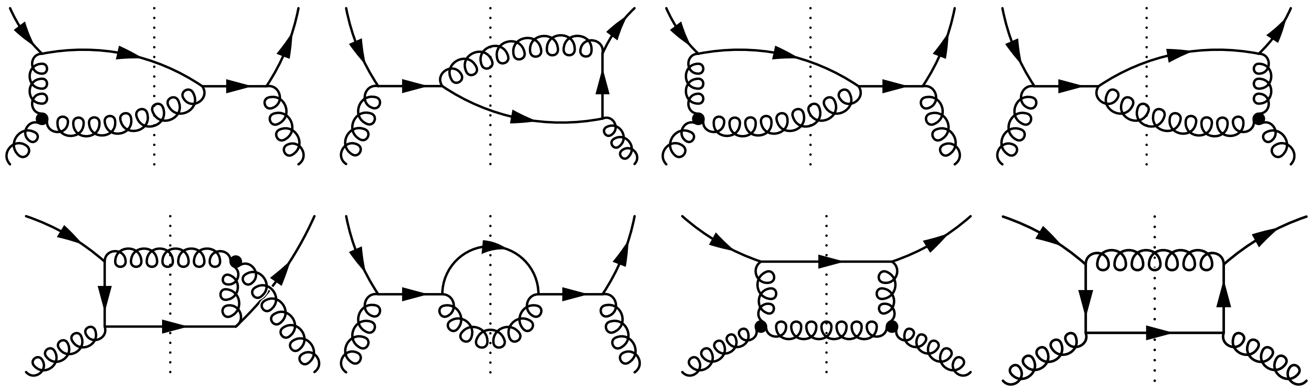
Introduction

- Isolating hard process (factorization)
  - Study of quark and gluon structure of hadrons
  - Account for hadronic physics to study hard process
- Beyond collinear approach
  - Include mismatch of parton momentum p and xP (fraction of hadron momentum)
  - TMDs with novel features
- Operator structure of TMDs
  - Color gauge invariance as guiding principle
  - Appearance of TMDs in hard processes
  - Gauge links in 1-particle un-integrated (1PU) processes

Hard part: QCD & Standard Model

- QCD framework (including electroweak theory) provides the machinery to calculate transition amplitudes, e.g. $g^*q \to q$, $q\bar{q} \to g^*$, $g^* \to q\bar{q}$, $qq \to qq$, $qg \to qg$, etc.
- Example:

  $qg \to qg$

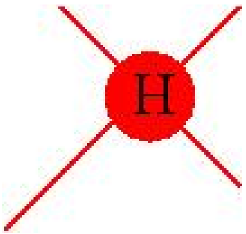


- Calculations work for plane waves

$$\left\langle 0 \middle| \psi_i^{(s)}(\xi) \middle| p, s \right\rangle = u_i(p,s)\, e^{-ip.\xi}$$

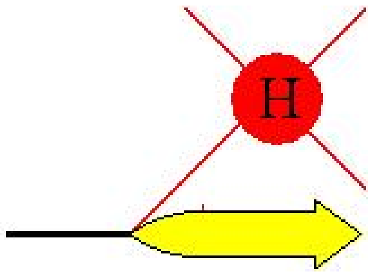
- External particles: $u_i(p,s)\,\bar{u}_j(p,s) = (\not{p} + m)_{ij}$

Soft part: hadronic matrix elements



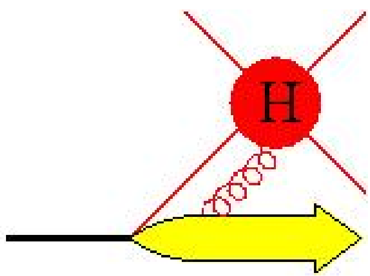
- For hard scattering process involving electrons and photons the link to external particles is, indeed, the ‘one-particle wave function’

$$\left\langle 0 \left| \psi_i\,(\xi) \right| p,s \right\rangle = u_i(p,s)\, e^{-ip.\xi}$$



- Confinement, however, leads to hadrons as ‘sources’ for quarks

$$\left\langle X \left| \psi_i\,(\xi) \right| P \right\rangle e^{+ip.\xi}$$



- ... and ‘source’ for quarks + gluons

$$\left\langle X \left| \psi_i\,(\xi) A^\mu(\eta) \right| P \right\rangle e^{+i(p-p_1).\xi+ip_1.\eta}$$

- ... and ....

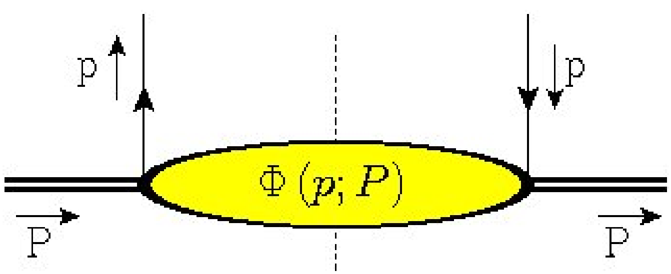
Soft part: hadronic matrix elements

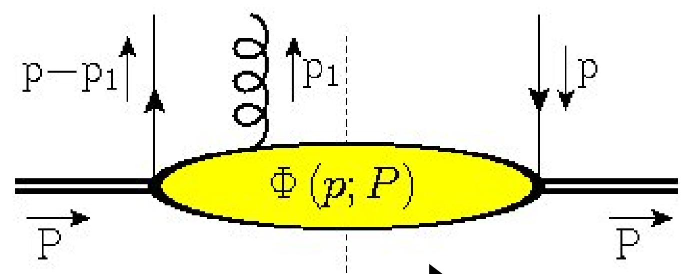
Thus, the nonperturbative input for calculating hard processes involves [instead of $u_i(p)\bar{u}_j(p)$] forward matrix elements of the form

$$\Phi_{ij}(p,P) = \int \frac{d^3P_X}{(2\pi)^3 2E_X} < P\,|\,\overline{\psi}_j(0)\,|\,X >< X\,|\,\psi_i(0)\,|\,P > \delta(P - P_X - p)$$

quark momentum

$$= \frac{1}{(2\pi)^4}\int d^4\xi\; e^{i\,p.\xi} < P\,|\,\overline{\psi}_j(0)\,\psi_i(\xi)\,|\,P >$$





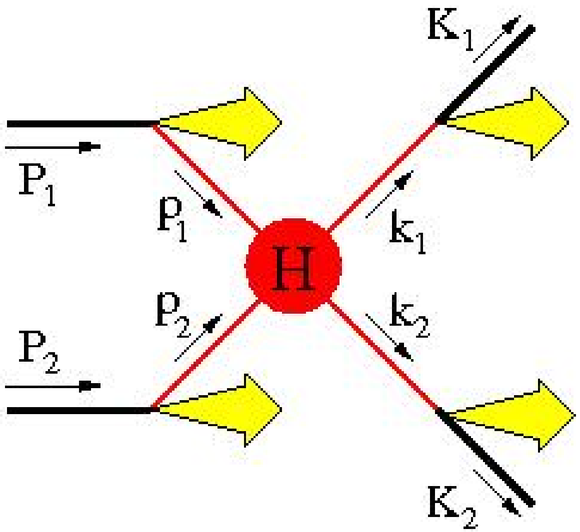
$$< P\,|\,\overline{\psi}_j(0) A^\alpha(\eta)\,\psi_i(\xi)\,|\,P >$$

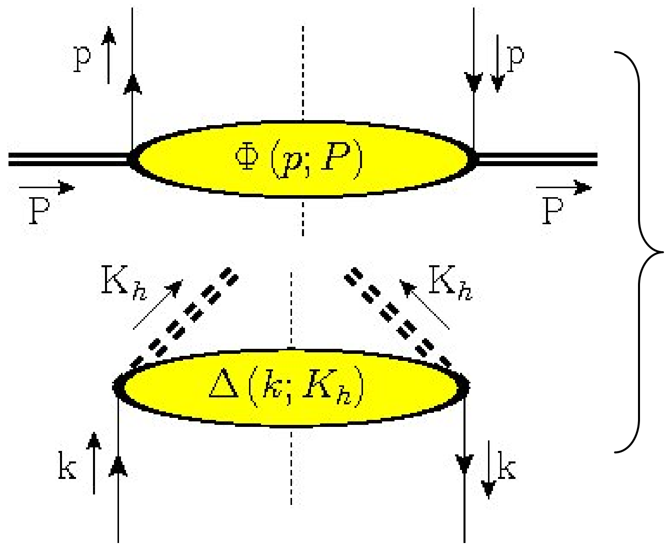
PDFs and PFFs

Basic idea of PDFs is to get a full factorized description of high energy scattering processes

$$\hat{\sigma} = |\, H(p_1, p_2, \ldots)\, |^2$$

calculable





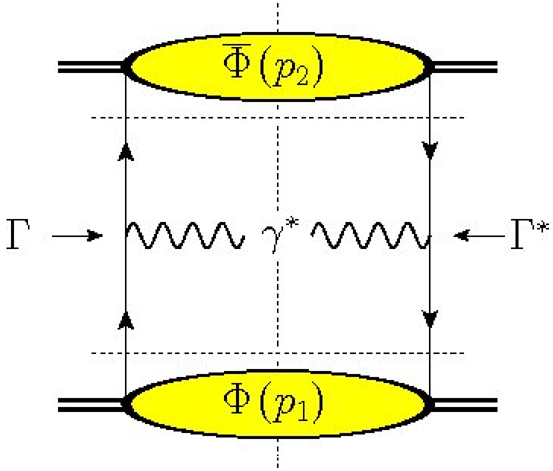
defined (!)
&
portable

$$\sigma(P_1, P_2, \ldots) = \iiint \ldots dp_1 \ldots \Phi_a(p_1, P_1; \mu) \otimes \Phi_b(p_2, P_2; \mu)$$

$$\otimes\, \hat{\sigma}_{ab,c\ldots}(p_1, p_2, \ldots; \mu) \otimes \Delta_c(k_1, K_1; \mu) \ldots.$$

Give a meaning to integration variables!

Example: Drell-Yan process



$$\sum_s u(p_1,s)\,\overline{u}(p_1,s)$$

$$\Rightarrow\ \Phi(p_1,P_1) \sim (\not{p}_1 + m)\,f(p_1)$$

- High energy limits number of soft matrix elements that contribute (twist expansion).
- Expand parton momenta (for DY take e.g. $n = P_2/P_1.P_2$)

$$p = xP^\mu + p_T^\mu + \sigma n^\mu$$

~ Q ~ M ~ M²/Q

$$x = p^+ = p.n \sim 1$$

$$\sigma = p.P - xM^2 \sim M^2$$

- For meaningful separation of hard and soft, integrate over p.P and look at $\Phi(x,p_T)$. This shows that separation fails beyond 'twist 3'.

Jaffe (1984), Diehl & Gousset (1998), ...

Integrated quark correlators: collinear and TMD

- Rather than considering general correlator Φ(p,P,...), one integrates over p.P = p^- ($\sim M_R^2$, which is of order M^2)

$$\Phi_{ij}^{q}(x, p_T; n) = \int \frac{d(\xi.P)\, d^2\xi_T}{(2\pi)^3}\, e^{i\,p.\xi}\, \langle P | \bar{\psi}_j(0)\, \psi_i(\xi) | P \rangle_{\xi.n=0}$$

TMD

lightfront

- and/or p_T (which is of order 1)

$$\Phi_{ij}^{q}(x; n) = \int \frac{d(\xi.P)}{(2\pi)}\, e^{i\,p.\xi}\, \langle P | \bar{\psi}_j(0)\, \psi_i(\xi) | P \rangle_{\xi.n=\xi_T=0}$$

collinear

lightcone

- The integration over p^- = p.P makes time-ordering automatic. This works for Φ(x) and $\Phi(x,p_T)$
- This allows the interpretation of soft (squared) matrix elements as forward antiquark-target amplitudes (untruncated!), which satisfy particular analyticity and support properties, etc.

Relevance of transverse momenta?

$$p_1 \approx x_1 P_1 + p_{1T}$$
$$p_2 \approx x_2 P_2 + p_{2T}$$

- At high energies fractional parton momenta fixed by kinematics (external momenta)

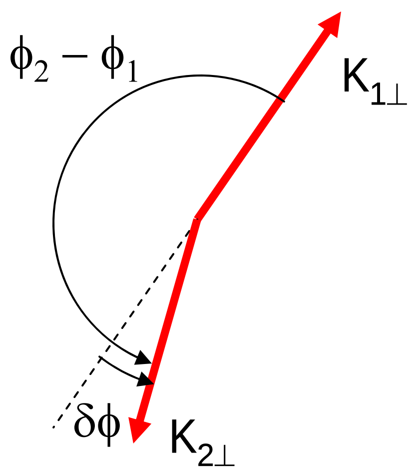
DY $$x_1 = p_1.n = \frac{p_1.P_2}{P_1.P_2} = \frac{q.P_2}{P_1.P_2}$$

- Also possible for transverse momenta of partons

DY $$q_T = q - x_1 P_1 - x_2 P_2 = p_{1T} + p_{2T}$$

2-particle inclusive hadron-hadron scattering

$$q_T = z_1^{-1} K_1 + z_2^{-1} K_2 - x_1 P_1 - x_2 P_2$$
$$= p_{1T} + p_{2T} - k_{1T} - k_{2T}$$



pp-scattering

Care is needed: we need more than one hadron and knowledge of hard process(es)!

Second scale!

Oppertunities of TMDs

- TMD quark correlators (leading part, unpolarized) including T-odd part

$$\boxed{\Phi^{[\pm]q}(x,p_T) = \left(f_1^q(x,p_T^2) \pm i\,h_1^{\perp q}(x,p_T^2)\,\frac{\not{p}_T}{M}\right)\frac{\not{P}}{2}}$$

- Interpretation: quark momentum distribution $f_1^q(x,p_T)$ and its transverse spin polarization $h_1^{\perp q}(x,p_T)$ both in an unpolarized hadron
- The function $h_1^{\perp q}(x,p_T)$ is T-odd (momentum-spin correlations!)
- TMD gluon correlators (leading part, unpolarized)

$$\boxed{\Phi_g^{\mu\nu}(x,p_T) = \frac{1}{2x}\left(-g_T^{\mu\nu}\,f_1^g(x,p_T^2) + \left(\frac{p_T^\mu p_T^\nu + \frac{1}{2}g_T^{\mu\nu}}{M^2}\right) h_1^{\perp g}(x,p_T^2)\right)}$$

- Interpretation: gluon momentum distribution $f_1^g(x,p_T)$ and its linear polarization $h_1^{\perp g}(x,p_T)$ in an unpolarized hadron (both are T-even)

Twist expansion of (non-local) correlators

- Dimensional analysis to determine importance of matrix elements (just as for local operators)
- maximize contractions with n to get leading contributions

$$\dim[\overline{\psi}(0)\not{n}\,\psi(\xi)] = 2$$

$$\dim[F^{n\alpha}(0)F^{n\beta}(\xi)] = 2$$

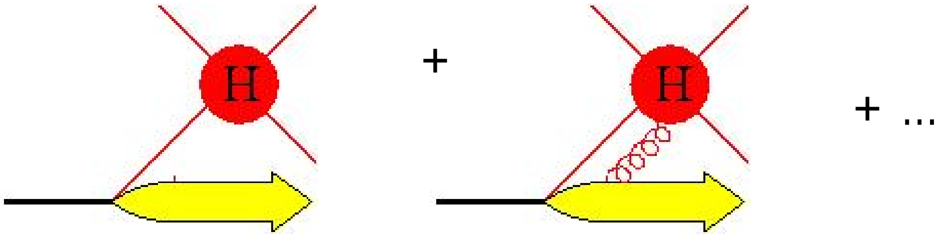
- 'Good' fermion fields and 'transverse' gauge fields
- and in addition any number of n.A(ξ) = $A^n(x)$ fields (dimension zero!) but in color gauge invariant combinations

dim 0: $i\partial^n \rightarrow iD^n = i\partial^n + gA^n$

dim 1: $i\partial_T^\alpha \rightarrow iD_T^\alpha = i\partial_T^\alpha + gA_T^\alpha$

- Transverse momentum involves 'twist 3'.

Soft parts: gauge invariant definitions



- Matrix elements containing A_μ (gluon) fields produce gauge link

$$U^{[C]}_{[0,\xi]} = \mathscr{P}\exp\left(-ig\int_0^\xi ds^\mu A_\mu\right)$$

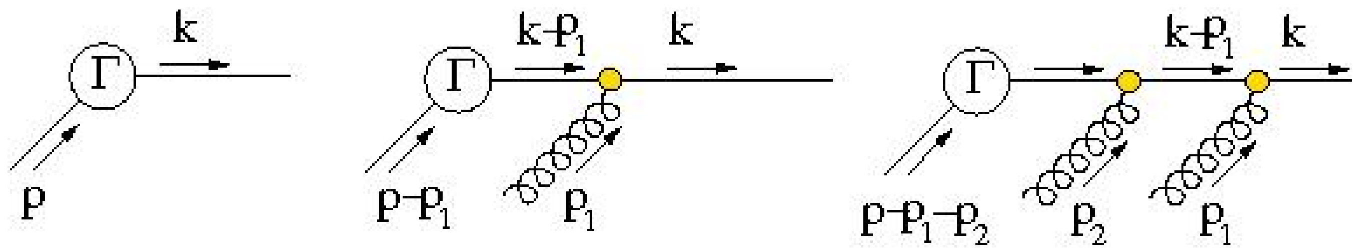
Any path yields a (different) definition

- ... essential for color gauge invariant definition

$$\boxed{\Phi^{[C]}_{ij}(p;P) = \int \frac{d^4\xi}{(2\pi)^4} e^{i\,p.\xi}\left\langle P\right|\bar{\psi}_j(0)\,U^{[C]}_{[0,\xi]}\,\psi_i(\xi)\left|P\right\rangle}$$

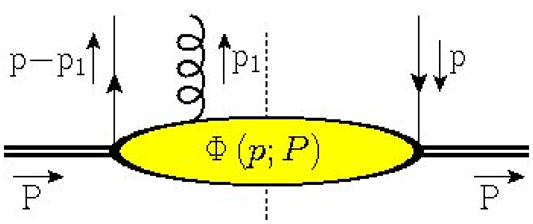
A.V. Belitsky, X.Ji, F. Yuan, NPB 656 (2003) 165
D. Boer, PJM, F. Pijlman, NPB 667 (2003) 201

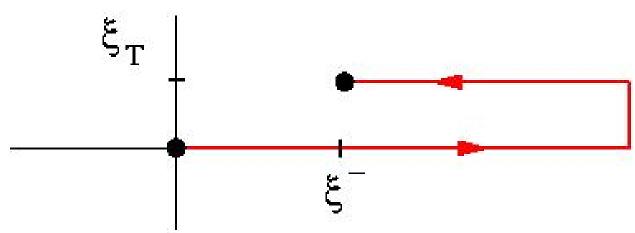
Gauge link results from leading gluons



Expand gluon fields and reshuffle a bit:

$$A^{\mu}(p_1) = n.A(p_1)\,\frac{P^{\mu}}{n.P} + i\,A_T^{\mu}(p_1) + \ldots \;=\; \frac{1}{p_1.n}\Big[A^{n}(p_1)\;p_1^{\mu} + iG_T^{n\mu}(p_1) + \ldots\Big]$$





Coupling only to final state partons, the collinear gluons add up to a U_+ gauge link, (with transverse connection from $A_T^{\alpha} \rightarrow G^{n\alpha}$ reshuffling)

Gauge-invariant definition of TMDs: which gauge links?

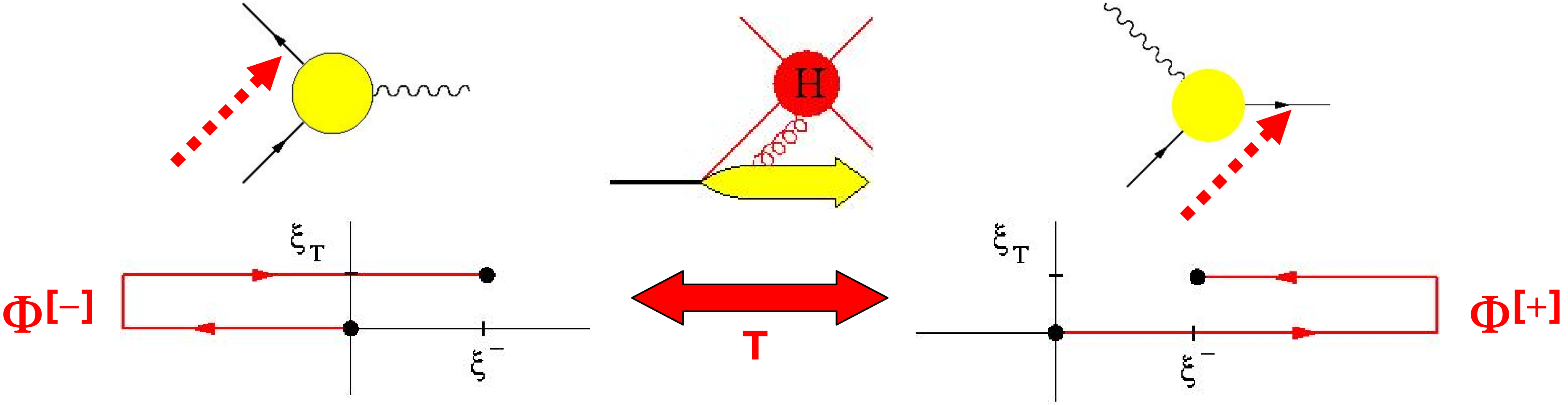
$$\Phi_{ij}^{q[C]}(x, p_T; n) = \int \frac{d(\xi.P)\,d^2\xi_T}{(2\pi)^3}\, e^{i\,p.\xi} \left\langle P \left| \overline{\psi}_j(0)\, U_{[0,\xi]}^{[C]}\, \psi_i(\xi) \right| P \right\rangle_{\xi.n=0}$$

TMD

$$\Phi_{ij}^{q}(x; n) = \int \frac{d(\xi.P)}{(2\pi)}\, e^{i\,p.\xi} \left\langle P \left| \overline{\psi}_j(0)\, U_{[0,\xi]}^{[n]}\, \psi_i(\xi) \right| P \right\rangle_{\xi.n=\xi_T=0}$$

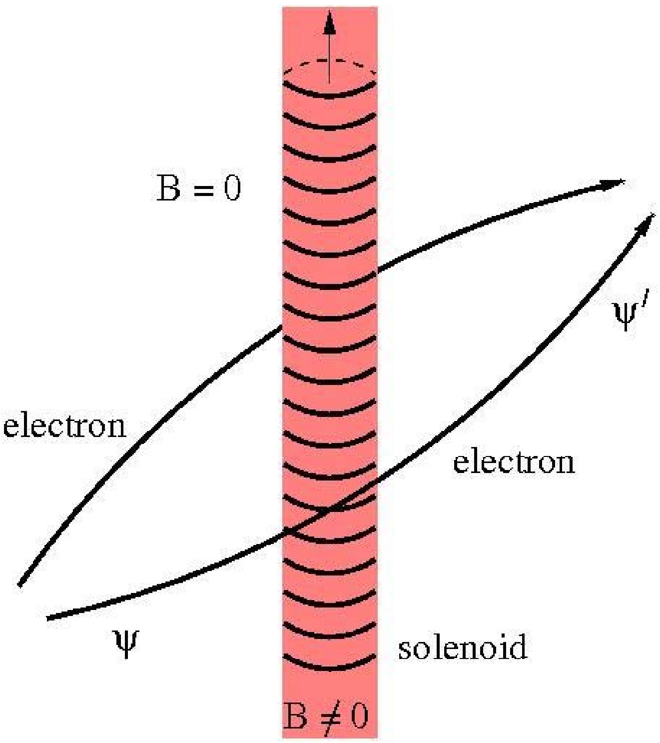
collinear

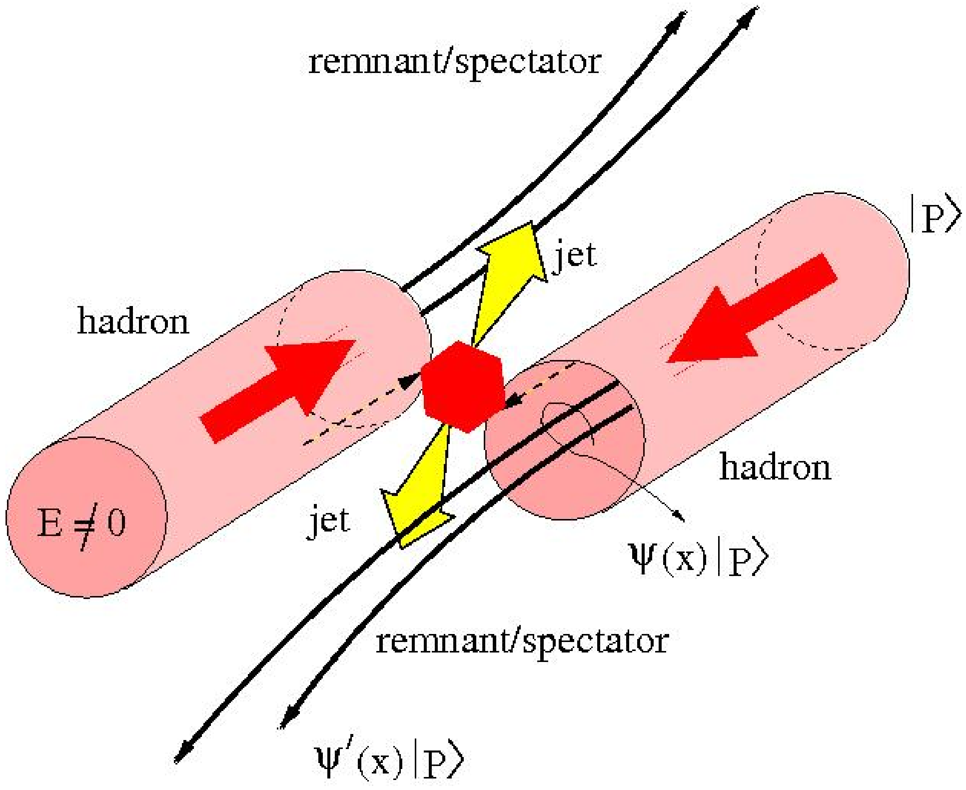
- Even simplest links for TMD correlators non-trivial:



These merge into a ‘simple’ Wilson line in collinear (p_T-integrated) case

Featuring: phases in gauge theories





$$\psi' = P e^{ie\int ds.A} \psi$$

$$\psi_i(x)\left|P\right\rangle = P e^{-ig\int_x^{x'} ds_\mu A^\mu} \psi_i(x')\left|P\right\rangle$$

C Bomhof, PJM, F Pijlman; EPJ C 47 (2006) 147
F Dominguez, B-W Xiao, F Yuan, PRL 106 (2011) 022301

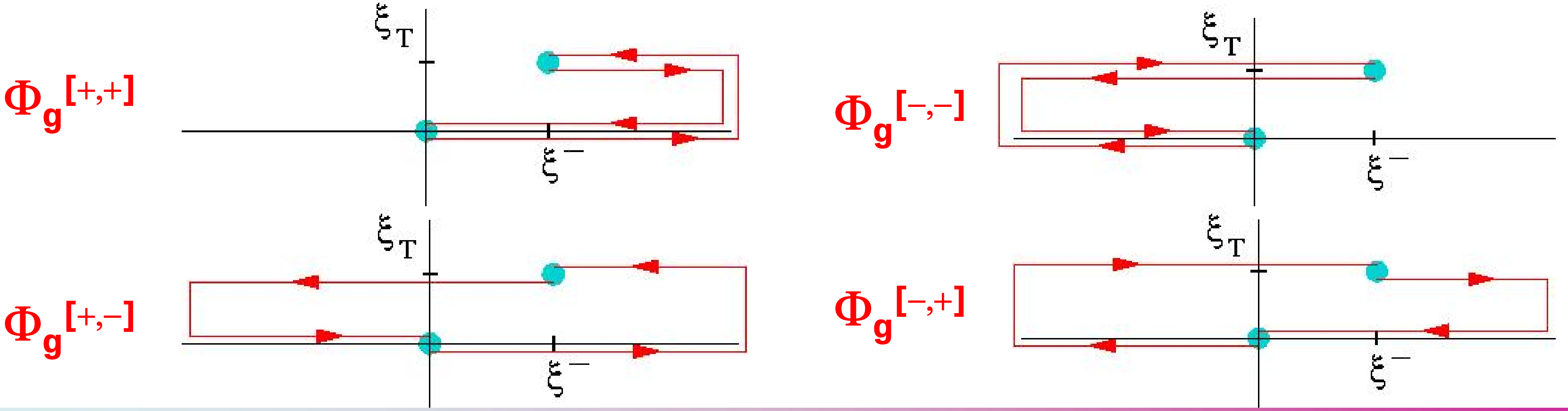
TMD correlators: gluons

$$\Phi_g^{\alpha\beta[C,C']}(x, p_T; n) = \int \frac{d(\xi.P)d^2\xi_T}{(2\pi)^3} e^{i\,p.\xi} \left\langle P \middle| U_{[\xi,0]}^{[C]} F^{n\alpha}(0) U_{[0,\xi]}^{[C']} F^{n\beta}(\xi) \middle| P \right\rangle_{\xi.n=0}$$

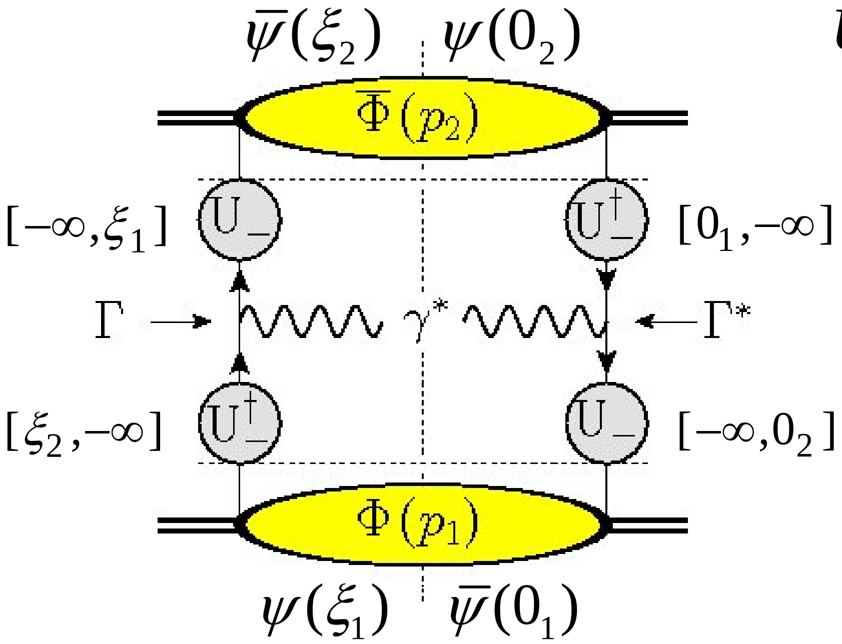
- The most general TMD gluon correlator contains two links, which in general can have different paths.
- Note that standard field displacement involves C = C'

$$F^{\alpha\beta}(\xi) \to U_{[\eta,\xi]}^{[C]} F^{\alpha\beta}(\xi) U_{[\xi,\eta]}^{[C]}$$

- Basic (simplest) gauge links for gluon TMD correlators:



Gauge invariance for DY



$$U_{[0_1,-\infty]}U_{[-\infty,\xi_1]}U_{[\xi_2,-\infty]}U_{[-\infty,0_2]}$$

$$= W^{[n]}_{-[0_1,\xi_1]} W^{[n]\dagger}_{-[0_2,\xi_2]} = W^{[n]}_{-}[p_1] W^{[n]\dagger}_{-}[p_2]$$

Strategy:
transverse moments

$$d\sigma_{DY} = Tr_c[W_-^{[p_2]}[p_1]\Phi_q(x_1,p_{1T})]\,Tr_c[\Phi_{\bar{q}}(x_2,p_{2T})W_-^{[p_1]\dagger}[p_2]]\,\tfrac{1}{N_c}\Gamma\Gamma^*$$

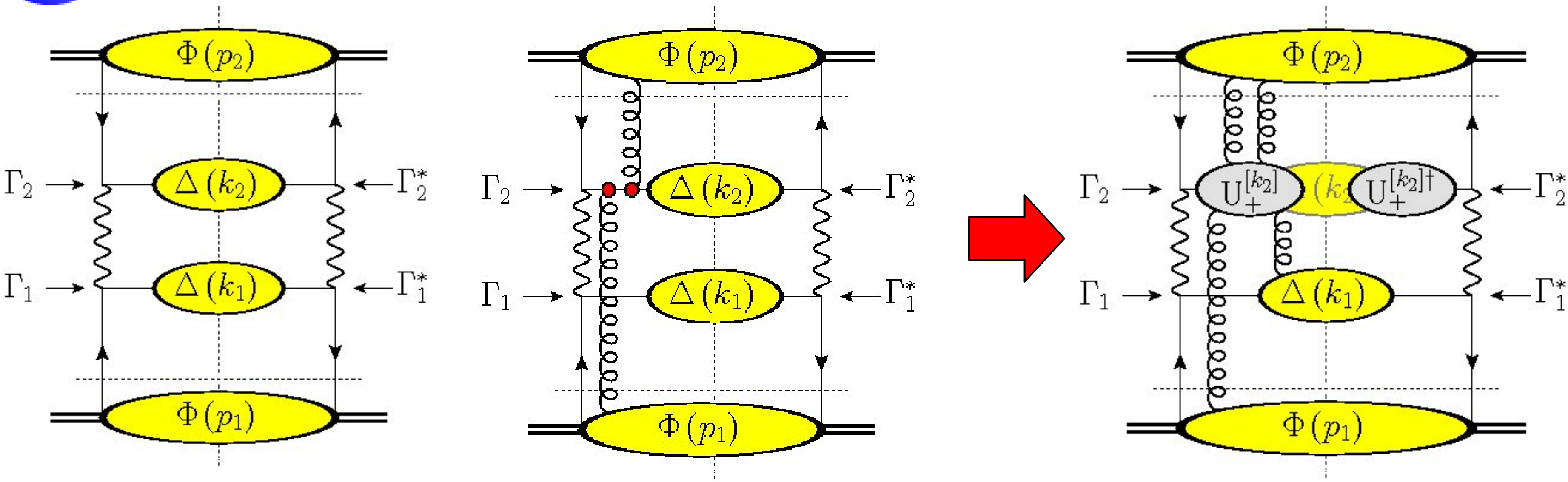
$$= \Phi_q^{[-]}(x_1,p_{1T})\,\Phi_{\bar{q}}^{[-\dagger]}(x_2,p_{2T})\,\hat{\sigma}_{q\bar{q}\to\gamma}$$

Employing simple color flow possibilities, e.g. in gg → $\gamma\gamma$

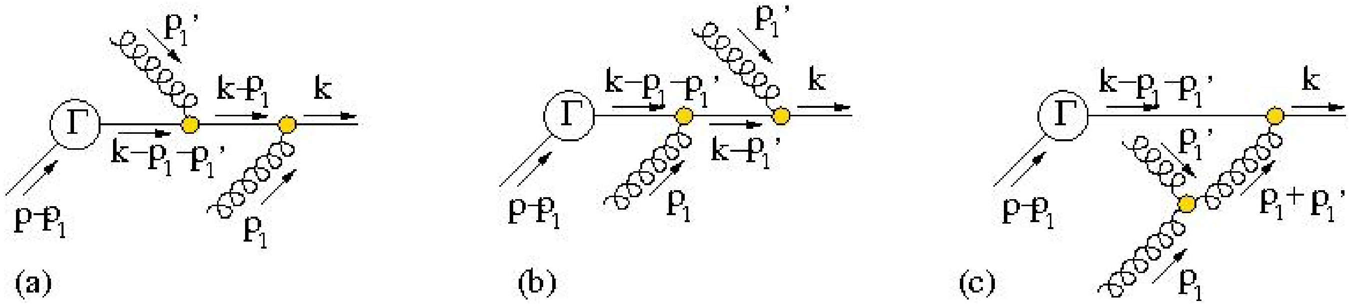
J. Qiu, M. Schlegel, W. Vogelsang, ArXiv 1103.3861 (hep-ph)

T.C. Rogers, PJM, PR D81 (2010) 094006

Complications (example: qq → qq)

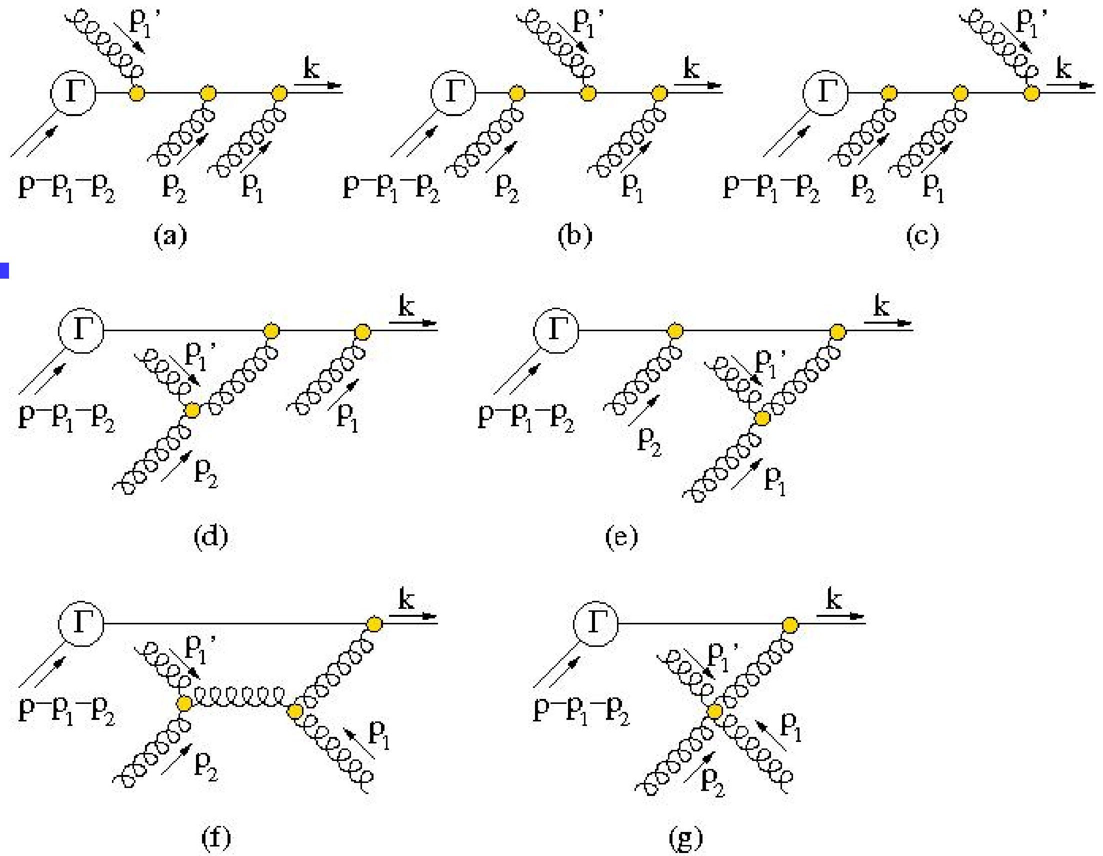


$U_+^{[n]}[p_1,p_2,k_1]$ modifies color flow, spoiling universality (and factorization)



$$U_{+\infty}^{[k](11)}(p,p')\ldots\Gamma\ldots\psi(p)\ldots\psi(p') = \frac{1}{2}\left\{U_{+\infty}^{[k](1)}(p), U_{+\infty}^{[k](1)}(p')\right\}\ldots\Gamma\ldots\psi(p)\ldots\psi(p')$$

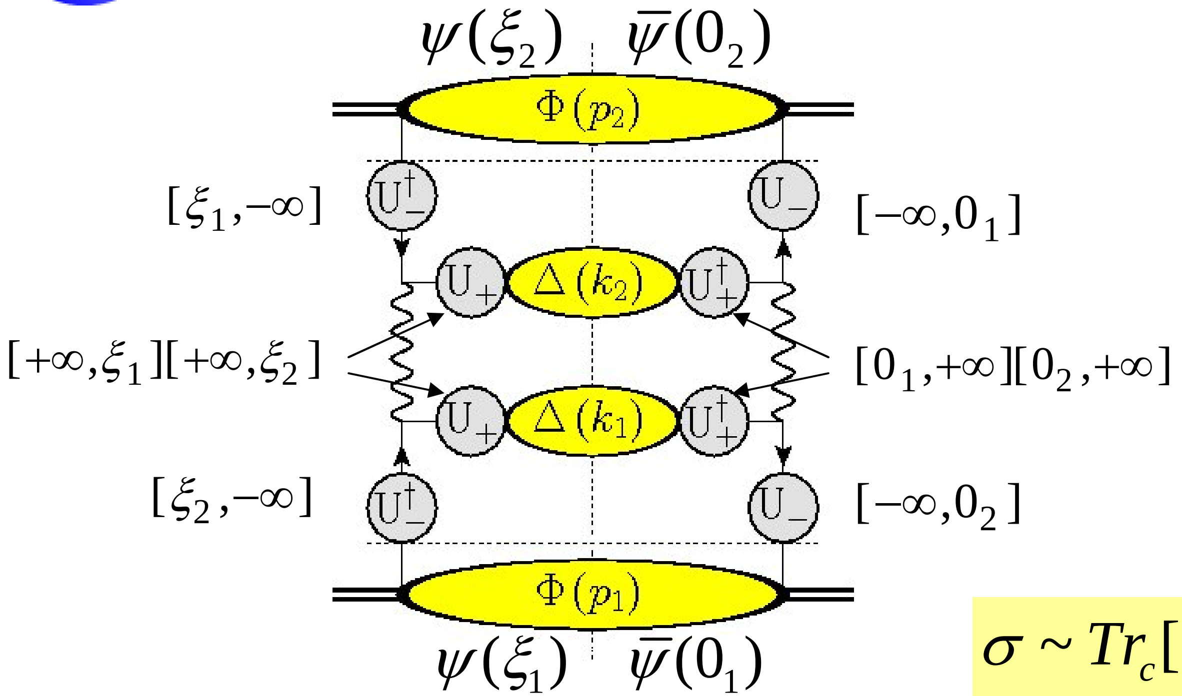
Color entanglement



$$U_-^{[k](21)}(p,p') = \frac{1}{4}U_-^{[k](2)}(p)U_-^{[k](1)}(p') + \frac{1}{4}U_-^{[k](1)}(p)U_-^{[k](1)}(p')U_-^{[k](1)}(p) + \frac{1}{4}U_-^{[k](1)}(p')U_-^{[k](2)}(p)$$

M.G.A. Buffing, PJM; in preparation

Color disentanglement for 1PU



Collinear treatment for all-but-one parton (p_1):

$$\xi_{2t} \rightarrow 0_{2T}$$

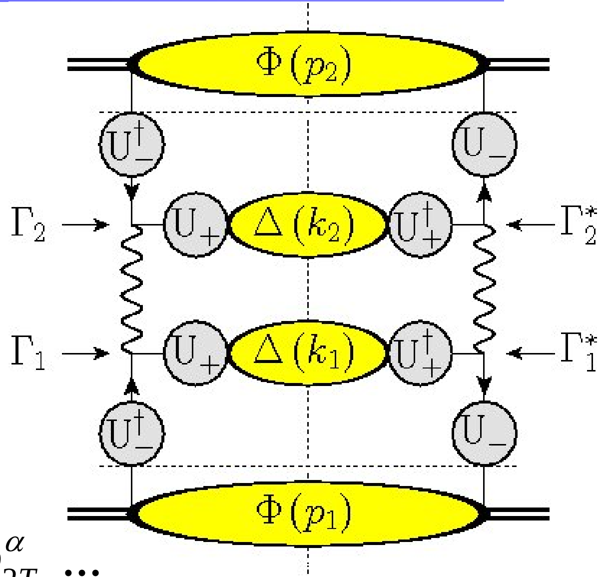
$$\sigma \sim Tr_c[\Phi^{[(\Box)+]}(x_1, p_{1T})\Gamma^*_b \Delta(z_1)\Gamma_a] \times Tr_c[\Phi(x_2)\Gamma^{b*}\Delta(z_2)\Gamma^a]$$

$$U_{[0_2,+\infty][0_1,+\infty]}U_{[+\infty,\xi_2][+\infty,\xi_1]}U_{[\xi_1,-\infty]}U_{[-\infty,0_1]} = W^{[n]}_{[0_2,\xi_2]}W^{[n]}_{+[0_1,\xi_1]}W^{[n]\dagger}_{-[0_1,\xi_1]} = W^{[n]}[p_2]W^{[n]}_{\Box}[p_1]$$

$$U_{[-\infty,0_2]}U_{[0_1,+\infty][0_2,+\infty]}U_{[+\infty,\xi_1][+\infty,\xi_2]}U_{[\xi_2,-\infty]} = U_{[0_1,+\infty]}U_{[+\infty,\xi_1]} = W^{[n]}_{+[0_1,\xi_1]}$$

1-parton unintegrated

- Resummation of all phases spoils universality
- Transverse moments (p_T-weighting) feels entanglement
- Special situations for only one transverse momentum, as in single weighted asymmetries

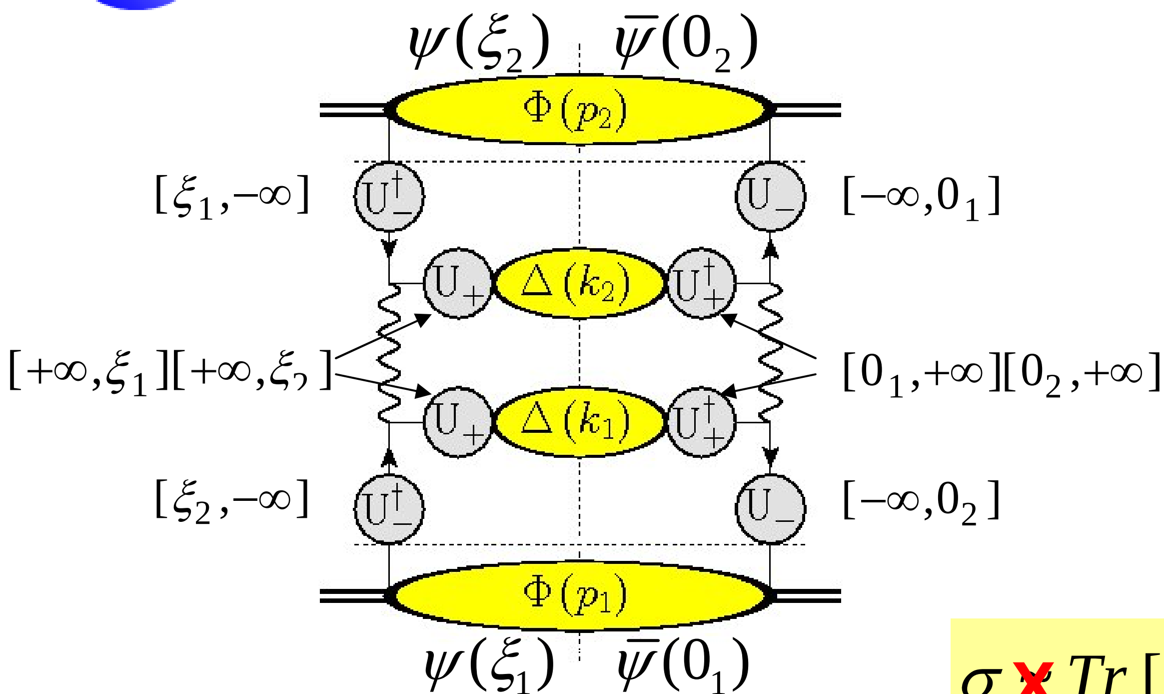


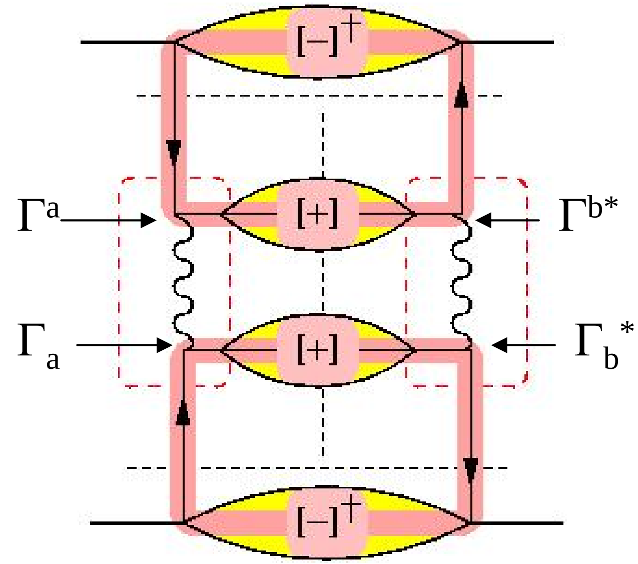
$$\int d^2q_T\ q_T^\alpha \ldots \int d^2p_{1T} \int d^2p_{2T} \ldots \delta^2(q_T - p_{1T} - p_{2T})$$
$$= \int d^2p_{1T}\ \ p_{1T}^\alpha \int d^2p_{2T} \ldots\ + \int d^2p_{1T} \int d^2p_{2T}\ \ p_{2T}^\alpha\ \ldots$$

- But: it does produces 'complex' gauge links
- Applications of 1PU is looking for gluon $h_1^{\perp g}$ (linear gluon polarization) using jet or heavy quark production in ep scattering (e.g. EIC), D. Boer, S.J. Brodsky, PJM, C. Pisano, PRL 106 (2011) 132001

M.G.A. Buffing, PJM; in preparation

Full color disentanglement? NO!





$$\sigma \not\propto Tr_c[\Phi^{[(\square)+]}(p_1)\Gamma^*_b\Delta^{[(\square)-^\dagger]}(k_1)\Gamma_a]$$
$$\times Tr_c[\Phi^{[(\square)+]}(p_2)\Gamma^{b*}\Delta^{[(\square)-^\dagger]}(k_2)\Gamma^a]$$

Loop 1: $U_{[0_2,+\infty][0_1,+\infty]}U_{[+\infty,\xi_2][+\infty,\xi_1]}U_{[\xi_1,-\infty]}U_{[-\infty,0_1]} = W^{[n]}_{+[0_2,\xi_2]}W^{[n]}_{+[0_1,\xi_1]}W^{[n]\dagger}_{-[0_1,\xi_1]}$

$$= W^{[n]}_+[p_2]W^{[n]}_\square[p_1]$$

Result for integrated cross section

$$\frac{d\sigma}{d^2 p_{1T}} \sim \sum_{D,abc} \Phi_a^{[C_1(D)]}(x_1, p_{1T})\,\Phi_b(x_2)\,\hat{\sigma}^{[D]}_{ab\to c\ldots}\,\Delta_c(z_1)\ldots \qquad \text{(1PU)}$$

Collinear cross section

$$\Phi^{\cancel{[C]}}(x) = \int d^2 p_T\ \ \Phi^{[C]}(x, p_T)$$

Gauge link structure becomes irrelevant!

$$\sigma \sim \sum_{abc} \Phi_a(x_1)\Phi_b(x_2)\,\hat{\sigma}_{ab\to\, c\ldots}\,\Delta_c(z_1)\ldots$$

$$\hat{\sigma}_{ab\to c\ldots} = \sum_D \hat{\sigma}^{[D]}_{ab\to c\ldots}$$

(partonic cross section)

Result for single weighted cross section

$$\frac{d\sigma}{d^2 p_{1T}} \sim \sum_{D,abc} \Phi_a^{[C_1(D)]}(x_1, p_{1T})\,\Phi_b(x_2)\,\hat{\sigma}_{ab\to c\ldots}^{[D]}\,\Delta_c(z_1)\ldots \qquad \text{(1PU)}$$

Single weighted cross section (azimuthal asymmetry)

$$\Phi_\partial^{\alpha[C]}(x) = \int d^2 p_T\;\; p_T^\alpha\,\Phi^{[C]}(x, p_T)$$

$$\left\langle p_{1T}^\alpha \sigma \right\rangle \sim \sum_{D,abc} \Phi_{\partial\, a}^{\alpha[C(D)]}(x_1)\Phi_b(x_2)\,\hat{\sigma}_{ab\to c\ldots}^{[D]}\,\Delta_c(z_1)\ldots$$

Result for single weighted cross section

$$\frac{d\sigma}{d^2 p_{1T}} \sim \sum_{D,abc} \Phi_a^{[C_1(D)]}(x_1, p_{1T})\,\Phi_b(x_2)\,\hat{\sigma}_{ab\to c\ldots}^{[D]}\,\Delta_c(z_1)\ldots \qquad \text{(1PU)}$$

$$\left\langle p_{1T}^{\alpha}\sigma\right\rangle \sim \sum_{D,abc} \Phi_{\partial\, a}^{\alpha[C(D)]}(x_1)\Phi_b(x_2)\,\hat{\sigma}_{ab\to c\ldots}^{[D]}\,\Delta_c(z_1)\ldots$$

$$\left\langle p_{1T}^{\alpha}\sigma\right\rangle \sim \sum_{D,abc} \Phi_{\partial\, a}^{\alpha[C(D)]}(x_1)\Phi_b(x_2)\,\hat{\sigma}_{ab\to c\ldots}^{[D]}\,\Delta_c(z_1)\ldots$$

$$\left\langle p_{1T}^{\alpha}\sigma\right\rangle \sim \sum_{D,abc} \Phi_{\partial\, a}^{\alpha[C(D)]}(x_1)\Phi_b(x_2)\,\hat{\sigma}_{ab\to c\ldots}^{[D]}\,\Delta_c(z_1)\ldots$$

$$\left\langle p_{1T}^{\alpha}\sigma\right\rangle \sim \sum_{D,abc} \Phi_{\partial\, a}^{\alpha[C(D)]}(x_1)\Phi_b(x_2)\,\hat{\sigma}_{ab\to c\ldots}^{[D]}\,\Delta_c(z_1)\ldots$$

Qiu, Sterman; Koike; Brodsky, Hwang, Schmidt, ...

Result for single weighted cross section

$$\frac{d\sigma}{d^2 p_{1T}} \sim \sum_{D,abc} \Phi_a^{[C_1(D)]}(x_1, p_{1T})\,\Phi_b(x_2)\,\hat{\sigma}_{ab\to c\ldots}^{[D]}\,\Delta_c(z_1)\ldots \qquad \text{(1PU)}$$

$$\left\langle p_{1T}^{\alpha}\,\sigma \right\rangle \sim \sum_{D,abc} \Phi_{\partial\, a}^{\alpha[C(D)]}(x_1)\Phi_b(x_2)\,\hat{\sigma}_{ab\to c\ldots}^{[D]}\,\Delta_c(z_1)\ldots$$

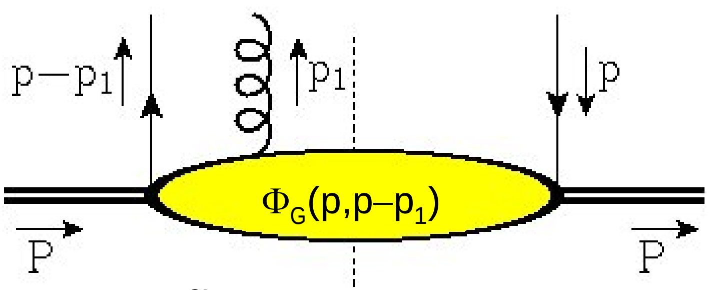
$$\Phi_{\partial}^{\alpha[C]}(x) = \tilde{\Phi}_{\partial}^{\alpha[\cancel{C}]}(x) + C_G^{[U(C)]}\pi\,\Phi_G^{\alpha[\cancel{C}]}(x,x)$$

T-even

universal matrix elements

T-odd

(operator structure)



$$\Phi_G^{\alpha}(x, x - x_1)$$

$\Phi_G(x,x)$ is gluonic pole ($x_1 = 0$) matrix element (color entangled!)

Result for single weighted cross section

$$\boxed{\frac{d\sigma}{d^2 p_{1T}} \sim \sum_{D,abc} \Phi_a^{[C_1(D)]}(x_1, p_{1T})\,\Phi_b(x_2)\,\hat{\sigma}^{[D]}_{ab\to c\ldots}\,\Delta_c(z_1)\ldots \qquad \text{(1PU)}}$$

$$\left\langle p_{1T}^{\alpha}\,\sigma \right\rangle \sim \sum_{D,abc} \Phi_{\partial\, a}^{\alpha[C(D)]}(x_1)\,\Phi_b(x_2)\,\hat{\sigma}^{[D]}_{ab\to c\ldots}\,\Delta_c(z_1)\ldots$$

$$\Phi_{\partial}^{\alpha[C]}(x) = \tilde{\Phi}_{\partial}^{\alpha[\cancel{C}]}(x) + C_G^{[U(C)]}\,\pi\,\Phi_G^{\alpha[\cancel{C}]}(x,x)$$

universal matrix elements

Examples are:

$C_G^{[U^+]} = 1$, $C_G^{[U^-]} = -1$, $C_G^{[W\,U^+]} = 3$, $C_G^{[\mathrm{Tr}(W)U^+]} = N_c$

Result for single weighted cross section

$$\frac{d\sigma}{d^2 p_{1T}} \sim \sum_{D,abc} \Phi_a^{[C_1(D)]}(x_1, p_{1T})\,\Phi_b(x_2)\,\hat{\sigma}_{ab\to c\ldots}^{[D]}\,\Delta_c(z_1)\ldots$$ (1PU)

$$\left\langle p_{1T}^{\alpha}\,\sigma\right\rangle \sim \sum_{D,abc} \Phi_{\partial\,a}^{\alpha[C(D)]}(x_1)\Phi_b(x_2)\,\hat{\sigma}_{ab\to c\ldots}^{[D]}\,\Delta_c(z_1)\ldots$$

$$\Phi_{\partial}^{\alpha[C]}(x) = \tilde{\Phi}_{\partial}^{\alpha[\cancel{C}]}(x) + C_G^{[U(C)]}\pi\,\Phi_G^{\alpha[\cancel{C}]}(x,x)$$

$$\left\langle p_{1T}^{\alpha}\,\sigma\right\rangle \sim \sum_{abc} \tilde{\Phi}_{\partial}^{\alpha}(x_1)\Phi_b(x_2)\,\hat{\sigma}_{ab\to c\ldots}\,\Delta_c(z_1)\ldots$$

$$+\sum_{abc} \pi\Phi_{G\,a}^{\alpha}(x_1,x_1)\Phi_b(x_2)\,\hat{\sigma}_{[a]b\to c\ldots}\,\Delta_c(z_1)\ldots$$ T-odd part

$$\hat{\sigma}_{[a]b\to c\ldots} = \sum_D C_G^{[U(C(D))]}\hat{\sigma}_{ab\to c\ldots}^{[D]}$$ (gluonic pole cross section)

Higher p_T moments

- Higher transverse moments

$$\Phi^{[N]\alpha_1\ldots\alpha_N}(x) = \int d^2 p_T \,(p_T^{\alpha_1}\ldots p_T^{\alpha_1} - traces)\,\Phi(x, p_T)$$

- involve yet more functions

$$\tilde{\Phi}_{\partial\partial}^{\alpha\beta}(x),\ \tilde{\Phi}_{\partial G}^{\alpha\beta}(x,x),\ \Phi_{GG}^{\alpha\beta}(x,x,x)$$

- Important application: there are no complications for fragmentation, since the 'extra' functions Δ_G, Δ_{GG}, ... vanish. using the link to 'amplitudes';

  L. Gamberg, A. Mukherjee, PJM, PRD 83 (2011) 071503 (R)

- In general, by looking at higher transverse moments at tree-level, one concludes that transverse momentum effects from different initial state hadrons cannot simply factorize.

Conclusions

- Color gauge invariance produces a jungle of Wilson lines attached to all parton legs, although the gauge connections themselves have a nicely symmetrized form
- Easy cases are collinear and 1-parton un-integrated (1PU) processes, with in the latter case for the TMD a complex gauge link, depending on the color flow in the tree-level hard process
- Example of 1PU processes are the terms in the sum of contributions to single weighted cross sections
- Single weighted cross sections involve T-even ‘normal weighting’ and T-odd gluonic pole matrix elements (SSA’s)
- Gluonic pole matrix elements in fragmentation correlators vanish, thus treatment of fragmentation TMDs is universal (physical picture: observation of jet direction)
- Furthermore, there is the issue of factorization! (next talk)

Thank you