Introduction: Collinear and TMD Factorization for Drell-Yan Production

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This talk describes some general considerations to help set the stage for the workshop. Most of what is included applies to both spin averaged and spin-dependent cross sections. In summary: Factorization in quantum field theory is closely related to classical considerations. Differences between initial- and final-state gauge links are consistent with this factorization. There is a well-developed theory of factorization for Drell-Yan, including transverse momentum (Q_T) dependence. The 'QCD-inclusive' nature of Drell-Yan production maintains the underlying factorization. Nonperturbative effects play an essential role at low Q_T and should be thought of as an integral part of the formalism. The stage is set for a new phenomenology to explore the transverse-momentum dependent and spin-sensitive parton distributions.

I. Drell-Yan Production in the Parton Model

- **The original 'collinear factorization'**
- **In the parton model (1970).**
  Drell and Yan: look for the annihilation of quark pairs into virtual photons of mass Q . . . any electroweak boson in NN scattering.

$$\frac{d\sigma_{NN\to\mu\bar{\mu}+X}(Q,p_1,p_2)}{dQ^2 d\ldots} \sim$$

$$\int d\xi_1 d\xi_2 \sum_{a=\mathrm{q}\bar{\mathrm{q}}} \frac{d\sigma^{\mathrm{EW,\,Born}}_{\mathrm{a}\bar{\mathrm{a}}\to\mu\bar{\mu}}(Q,\xi_1 p_1,\xi_2 p_2)}{dQ^2 d\ldots}$$
$$\times(\text{probability to find parton a}(\xi_1)\text{ in }N)$$
$$\times(\text{probability to find parton }\bar{\mathrm{a}}(\xi_2)\text{ in }N)$$

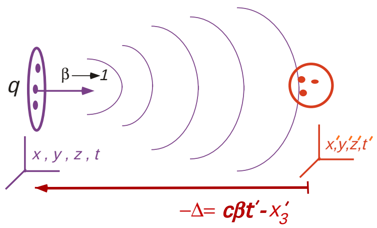
The probabilities are $\phi_{q/N}(\xi_i)$'s from DIS

2. The Physical Basis of Factorization

- 'Collinear factorization' for hadron-hadron scattering for a hard, inclusive process with momentum transfer M to produce final state $F+X$:

$$d\sigma_{\mathrm{H_1H_2}}(p_1,p_2,M) = \sum_{a,b}\int_0^1 d\xi_a\, d\xi_b d\hat{\sigma}_{ab\to F+X}\,(\xi_a p_1, \xi_b p_2, M, \mu) \times \phi_{a/H_1}(\xi_a,\mu)\,\phi_{b/H_2}(\xi_b,\mu)$$

- Factorization proofs: justifying the "universality" of the parton distributions.



<u>field</u>	<u>x frame</u>	<u>x' frame</u>
scalar	$\frac{q}{\lvert\vec{x}\rvert}$	$\frac{q}{(x_T^2+\gamma^2\Delta^2)^{1/2}} \sim \frac{1}{\gamma}$
gauge (0)	$A^0(x) = \frac{q}{\lvert\vec{x}\rvert}$	$A'^0(x') = \frac{-q\gamma}{(x_T^2+\gamma^2\Delta^2)^{1/2}} \sim \gamma^0$
field strength	$E_3(x) = \frac{q}{\lvert\vec{x}\rvert^2}$	$E_3'(x') = \frac{-q\gamma\Delta}{(x_T^2+\gamma^2\Delta^2)^{3/2}} \sim \frac{1}{\gamma^2}$

- **The “gluon field” A'^μ is enhanced, yet is a total derivative:**

$$A'^\mu = q\frac{\partial}{\partial x'_\mu}\ \ln\left(\Delta(t', x_3')\right) + \mathcal{O}(1-\beta) \sim A'^-$$

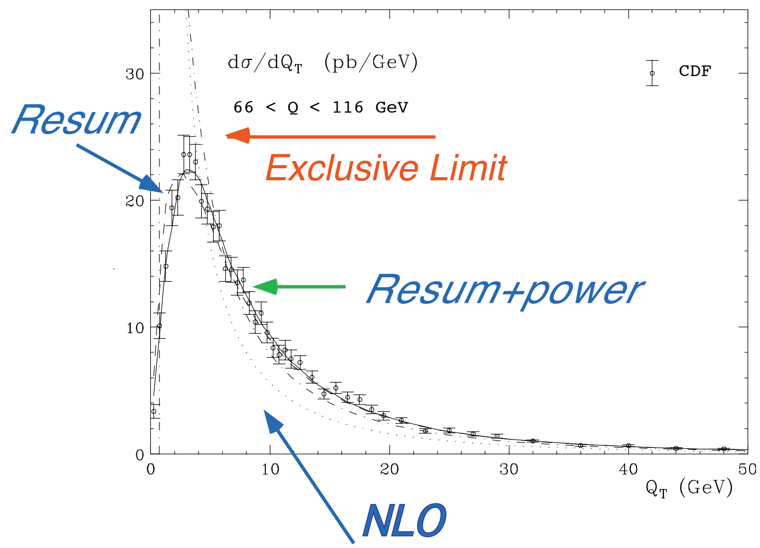
- **The “large” part of A'^μ can be removed by a gauge transformation!**

4. TMD Factorization for Drell-Yan Production

- Q_T factorized cross sections: the motivation
- Low Q_T Drell-Yan & Higgs at leading log (LL) $(\alpha_s{}^n \ln^{2n-1} Q_T)$

$$\frac{d\sigma(Q)}{dQ_T} \sim \frac{d}{dQ_T} \exp\left[-\frac{\alpha_s}{\pi} C_F \ln^2\left(\frac{Q}{Q_T}\right)\right]$$

$(C_F = 4/3)$



• **Window to nonperturbative distributions:**

$$
\begin{aligned}
E^{\rm soft} &= \frac{1}{2\pi}\int_0^{\mu_I^2}\frac{d^2k_T}{k_T^2}\,A_q(\alpha_s(k_T))\,\ln\left(\frac{Q^2}{k_T^2}\right)\left(e^{i\mathrm{b}\cdot\mathrm{k}_T}-1\right)\\
&\sim -\int_0^{\mu_I^2}\frac{dk_T^2}{k_T^2}\,(\mathrm{b}\cdot\mathrm{k}_T)^2A_q(\alpha_s(k_T))\,\ln\left(\frac{Q^2}{k_T^2}\right)+\cdots\\
&\sim -\,b^2\int dk_T^2\,A_q(\alpha_s(k_T))\,\ln\left(\frac{Q^2}{k_T^2}\right)
\end{aligned}
$$

$\theta(k_T-1/b)\Rightarrow(e^{i\mathrm{b}\cdot\mathrm{k}_T}-1)$; **in fact, correct to all orders,**

Note the expansion is for b " small enough" only.