Generalized TMDs and Wigner Distributions

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ABSTRACT: The first complete parameterization of Generalized TMDs (GTMDs) for a spin-$\frac{1}{2}$ target is presented. The Fourier transform of GTMDs has a strong similarity to Wigner distributions, which are the quantum mechanical analogues of classical phase space distributions.

Many nontrivial relations between GPDs and TMDs have been found in simple spectator models. Since GTMDs contain the GPDs and the TMDs in certain limits, one can use them in order to study the status of the nontrivial GPD-TMD relations. Such an analysis reveals that none of those relations can be promoted to a model-independent status. The talk also briefly addresses more recent developments on the GTMD field as well as some potential further applications of these objects.

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Parameterization of GTMDs

- GTMD-correlator

$$W^{[\Gamma]} = \frac{1}{2}\int \frac{dz^-}{2\pi}\,\frac{d^2\vec{z}_T}{(2\pi)^2}\,e^{ik\cdot z}\,\langle p'\,|\,\bar{\psi}\Big(-\frac{z}{2}\Big)\,\Gamma\,\mathcal{W}_{GTMD}\,\psi\Big(\frac{z}{2}\Big)\,|\,p\rangle\,\Big|_{z^+=0}$$

$\longrightarrow$ $W^{[\Gamma]}$ appears, e.g., in handbag diagram of DVCS

- Projection onto GPDs and TMDs

$$\begin{aligned} F^{[\Gamma]} &= \frac{1}{2}\int \frac{dz^-}{2\pi}\,e^{ik\cdot z}\,\langle p'\,|\,\bar{\psi}\Big(-\frac{z}{2}\Big)\,\Gamma\,\mathcal{W}_{GPD}\,\psi\Big(\frac{z}{2}\Big)\,|\,p\rangle\,\Big|_{z^+=z_T=0} \\ &= \int d^2\vec{k}_T\,W^{[\Gamma]} \end{aligned}$$

$$\begin{aligned} \Phi^{[\Gamma]} &= \frac{1}{2}\int \frac{dz^-}{2\pi}\,\frac{d^2\vec{z}_T}{(2\pi)^2}\,e^{ik\cdot z}\,\langle p\,|\,\bar{\psi}\Big(-\frac{z}{2}\Big)\,\Gamma\,\mathcal{W}_{TMD}\,\psi\Big(\frac{z}{2}\Big)\,|\,p\rangle\,\Big|_{z^+=0} \\ &= W^{[\Gamma]}\Big|_{\Delta=0} \end{aligned}$$

$\longrightarrow$ GPDs and TMDs appear as certain limits of GTMDs (mother distributions)

- Parameterization of GTMD-correlator (Meißner, Metz, Schlegel, 2009)
  - Use constraints from hermiticity and parity
  - Eliminate redundant terms by means of Gordon identities, etc.

$$\det \begin{pmatrix} g^{\alpha\mu} & g^{\beta\mu} & g^{\gamma\mu} & g^{\delta\mu} & g^{\varepsilon\mu} \\ g^{\alpha\nu} & g^{\beta\nu} & g^{\gamma\nu} & g^{\delta\nu} & g^{\varepsilon\nu} \\ g^{\alpha\rho} & g^{\beta\rho} & g^{\gamma\rho} & g^{\delta\rho} & g^{\varepsilon\rho} \\ g^{\alpha\sigma} & g^{\beta\sigma} & g^{\gamma\sigma} & g^{\delta\sigma} & g^{\varepsilon\sigma} \\ g^{\alpha\tau} & g^{\beta\tau} & g^{\gamma\tau} & g^{\delta\tau} & g^{\varepsilon\tau} \end{pmatrix} = 0$$

  - Example

$$W^{[\gamma^+]} = \frac{1}{2M}\,\bar{u}(p')\left[F_{1,1} + \frac{i\sigma^{i+}k_T^i}{P^+}\,F_{1,2} + \frac{i\sigma^{i+}\Delta_T^i}{P^+}\,F_{1,3} + \frac{i\sigma^{ij}k_T^i\Delta_T^j}{M^2}\,F_{1,4}\right]u(p)$$

  $\longrightarrow$ GTMDs are complex functions: $F_{1,n} = F_{1,n}^e + iF_{1,n}^o$

  - Parameterization for all twists
  - By-product: first full classification of GPDs beyond leading twist
  - Relations between GPDs/TMDs and GTMDs worked out
  - Leading twist GTMDs computed in scalar diquark model

Wigner distributions

- Phase-space distribution in classical mechanics $\rho(\vec{k}, \vec{r})$
- Phase-space distribution in quantum mechanics (Wigner distribution) $W(\vec{k}, \vec{r})$
  - Relation to probability density in position and momentum space

$$|\psi(\vec{r})|^2 = \int d^3\vec{k}\, W(\vec{k}, \vec{r}) \qquad |\psi(\vec{k})|^2 = \int d^3\vec{r}\, W(\vec{k}, \vec{r})$$

- Fourier transform of GTMDs $(\xi = 0)$ (Ji, 2003 / Belitsky, Ji, Yuan, 2003)

$$\mathrm{WD}(x, \vec{k}_T, \vec{b}_T) \simeq \int d^2\vec{\Delta}_T\, e^{-i\vec{\Delta}_T \cdot \vec{b}_T}\, \mathrm{GTMD}(x, \vec{k}_T, \vec{\Delta}_T)$$

  - Relation with GPDs and TMDs

$$\mathrm{GPD}(x, \vec{b}_T) \simeq \int d^2\vec{k}_T\, \mathrm{WD}(x, \vec{k}_T, \vec{b}_T) \qquad \mathrm{TMD}(x, \vec{k}_T) \simeq \int d^2\vec{b}_T\, \mathrm{WD}(x, \vec{k}_T, \vec{b}_T)$$

  - No handle on longitudinal position of parton
  - $\vec{b}_T$ and $\vec{k}_T$ are not Fourier conjugate variables

Relations/Analogies between GPDs and TMDs

- Relations of first type

$$f_1^{q/g} \leftrightarrow \mathcal{H}^{q/g} \qquad g_{1L}^{q/g} \leftrightarrow \tilde{\mathcal{H}}^{q/g}$$

$$\left(h_{1T}^q + \frac{\vec{k}_T^{\,2}}{2M^2}\, h_{1T}^{\perp q} \right) \leftrightarrow \left(\mathcal{H}_T^q - \frac{\vec{b}_T^{\,2}}{M^2}\, \Delta \tilde{\mathcal{H}}_T^q \right)$$

- Relations of second type

$$f_{1T}^{\perp q/g} \leftrightarrow -\left(\mathcal{E}^{q/g} \right)' \qquad h_1^{\perp q} \leftrightarrow -\left(\mathcal{E}_T^q + 2\tilde{\mathcal{H}}_T^q \right)'$$

$$\left(h_{1T}^g + \frac{\vec{k}_T^{\,2}}{2M^2}\, h_{1T}^{\perp g} \right) \leftrightarrow -2\left(\mathcal{H}_T^g - \frac{\vec{b}_T^{\,2}}{M^2}\, \Delta \tilde{\mathcal{H}}_T^g \right)'$$

- Relations of third type

$$h_{1T}^{\perp q} \leftrightarrow 2\left(\tilde{\mathcal{H}}_T^q \right)'' \qquad h_1^{\perp g} \leftrightarrow 2\left(\mathcal{E}_T^g + 2\tilde{\mathcal{H}}_T^g \right)''$$

- Relation of fourth type

$$h_{1T}^{\perp g} \leftrightarrow -4\left(\tilde{\mathcal{H}}_T^g \right)'''$$

Summary

- Classification of Generalized TMDs (and Wigner distributions) for nucleon exists

- GTMD analysis can be applied to study potential nontrivial GPD-TMD relations

- Various quantitative nontrivial GPD-TMD relations in simple spectator models

- GTMD analysis: none of those relations can have model-independent status
  (analysis also for subleading twist)

- Additional developments and further applications