Factorized Drell-Yan cross section

❑ **TMD factorization ($q_\perp \ll Q$):**

$$\frac{d\sigma_{AB}}{d^4q} = \sigma_0 \int d^2k_{a\perp} d^2k_{b\perp} d^2k_{s\perp} \delta^2(q_\perp - k_{a\perp} - k_{b\perp} - k_{s\perp}) \mathcal{F}_{a/A}(x_A, k_{a\perp}) \mathcal{F}_{b/B}(x_B, k_{b\perp}) \mathcal{S}(k_{s\perp})$$

$$+\mathcal{O}(q_\perp/Q) \qquad x_A = \frac{Q}{\sqrt{s}} e^{y} \qquad x_B = \frac{Q}{\sqrt{s}} e^{-y}$$

The soft factor, $\mathcal{S}$, is universal, could be absorbed into the definition of TMD parton distribution

❑ **Collinear factorization ($q_\perp \sim Q\,, \quad q_\perp \gg Q$):**

$$\frac{d\sigma_{AB}}{d^4q} = \int dx_a\, f_{a/A}(x_a, \mu) \int dx_b\, f_{b/B}(x_b, \mu)\, \frac{d\hat{\sigma}_{ab}}{d^4q}(x_a, x_b, \alpha_s(\mu), \mu)$$

❑ **Spin dependence:**

The factorization arguments are independent of the spin states of the colliding hadrons

⟶ same formula with different distributions for γ^*,W/Z, H^0…

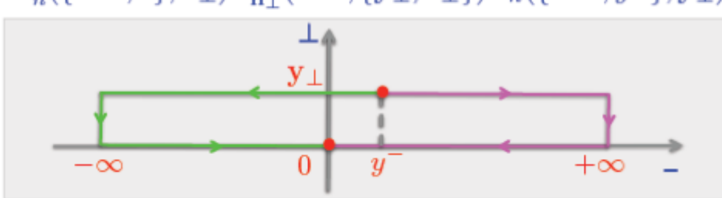
The sign change of Sivers function

❑ **TMD quark distribution:**

See talks by Collins, Kang, …

$$f_{q/h^\uparrow}(x, \mathbf{k}_\perp, \vec{S}) = \int \frac{dy^- d^2y_\perp}{(2\pi)^3} e^{ixp^+y^- - i\mathbf{k}_\perp\cdot\mathbf{y}_\perp} \langle p, \vec{S}|\overline{\psi}(0^-, \mathbf{0}_\perp) \boxed{\text{Gauge link}} \frac{\gamma^+}{2} \psi(y^-, \mathbf{y}_\perp)|p, \vec{S}\rangle$$

- SIDIS: $\Phi_n^\dagger(\{+\infty, 0\}, \mathbf{0}_\perp)\Phi_{\mathbf{n}_\perp}^\dagger(+\infty, \{\mathbf{y}_\perp, \mathbf{0}_\perp\})\Phi_n(\{+\infty, y^-\}, \mathbf{y}_\perp)$
- DY: $\Phi_n^\dagger(\{-\infty, 0\}, \mathbf{0}_\perp)\Phi_{\mathbf{n}_\perp}^\dagger(-\infty, \{\mathbf{y}_\perp, \mathbf{0}_\perp\})\Phi_n(\{-\infty, y^-\}, \mathbf{y}_\perp)$



- For a fixed spin state:

$$f^{\rm SIDIS}_{q/h^\uparrow}(x, \mathbf{k}_\perp, \vec{S}) \neq f^{\rm DY}_{q/h^\uparrow}(x, \mathbf{k}_\perp, \vec{S})$$

❑ **Parity + Time-reversal invariance:**

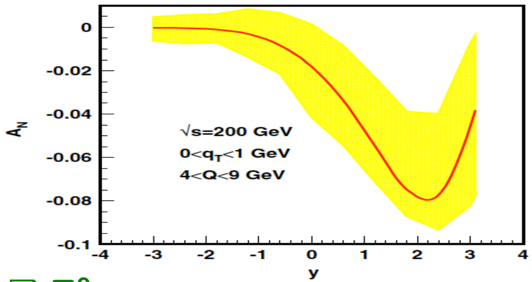
$$\Longrightarrow \quad f^{\rm Sivers}_{q/h^\uparrow}(x, k_\perp)^{\rm SIDIS} = -f^{\rm Sivers}_{q/h^\uparrow}(x, k_\perp)^{\rm DY}$$

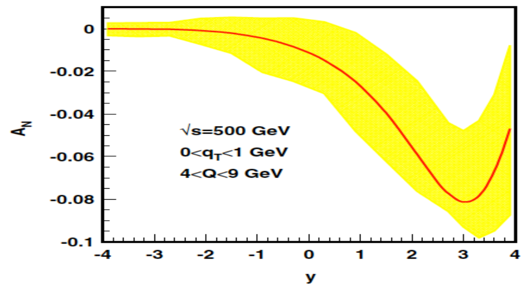
It is a critical test of TMD factorization approach

Test of the modified universality

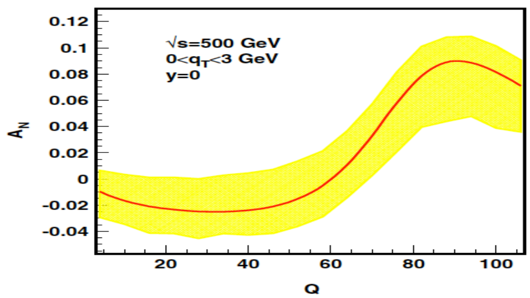
- **Drell-Yan:** $A_N^{\sin(\phi-\phi_s)} = -A_N$

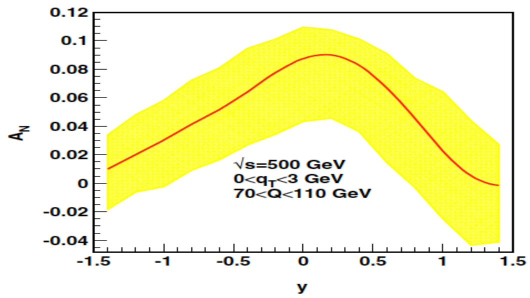
See talks by Anselmino, Rogers, …





- **Z^0:**





Theoretical challenge: Q-dependence of Sivers function?

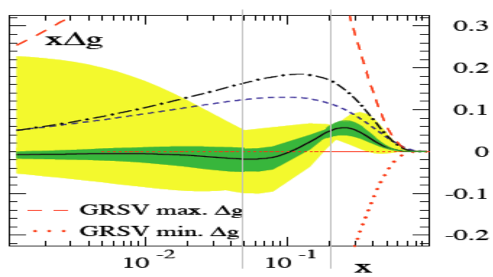
The sign "mismatch"

- **Asymmetry could have a node:**

See talks by Kang

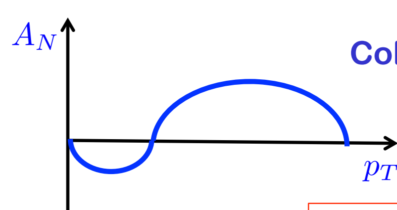
Sign change of $\Delta g(x)$:

$$A(s) \;\propto\; \sigma(s) - \sigma(-s)$$



- **A_N of Drell-Yan p_T distribution:**

We could have:



Collinear region ($p_T \sim Q$)

TMD region ($p_T \ll Q$)

Difference of two Gaussians

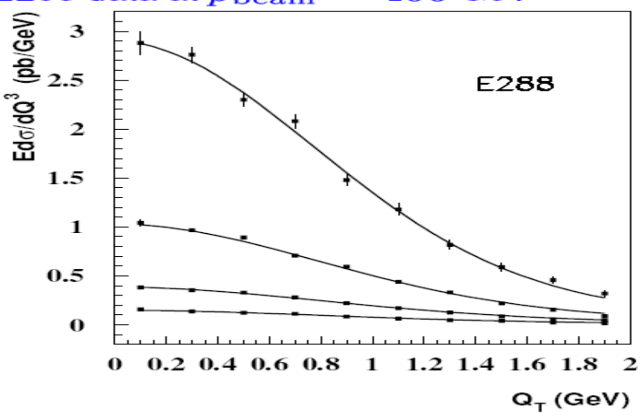
Important measurement for understanding A_N of hadronic pion

Unpolarized Drell-Yan cross section

- The denominator of the Asymmetry:

$$\frac{d\sigma}{d^4q} \qquad\qquad \frac{d\sigma}{d^4q\,d\Omega}$$

- Angular integrated Drell-Yan is under control:
  - Fermilab E288 data at $p_{\rm beam} = 400$ GeV



But, Drell-Yan lepton angular distribution needs work!

Lam-Tung relation

- Normalized Drell-Yan lepton angular distribution:

$$\frac{dN}{d\Omega} \equiv \left(\frac{d\sigma}{d^4q}\right)^{-1} \frac{d\sigma}{d^4q\,d\Omega} = \frac{3}{4\pi}\left(\frac{1}{\lambda+3}\right)\left[1 + \lambda\cos^2\theta + \mu\sin(2\theta)\cos\phi + \frac{\nu}{2}\sin^2\theta\cos(2\phi)\right]$$

- Lam-Tung relation:

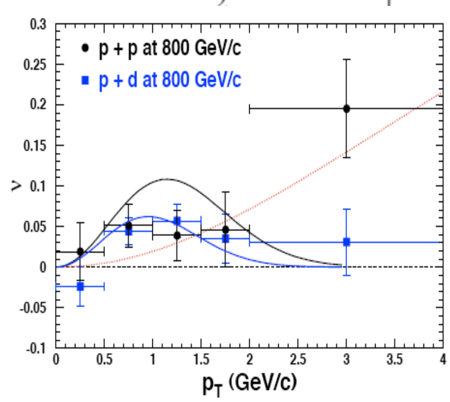
$$1 - \lambda - 2\nu = 0$$

- Collinear factorization:

$$\lambda = \frac{W_T - W_L}{W_T + W_L} \approx \frac{W_T^{\rm Resum} - W_L^{\rm Resum}}{W_T^{\rm Resum} + W_L^{\rm Resum}} = \frac{1 - \frac{1}{2}Q_\perp^2/Q^2}{1 + \frac{3}{2}Q_\perp^2/Q^2}$$

$$\nu = \frac{2W_{\Delta\Delta}}{W_T + W_L} \approx \frac{2W_{\Delta\Delta}^{\rm Resum}}{W_T^{\rm Resum} + W_L^{\rm Resum}} = \frac{Q_\perp^2/Q^2}{1 + \frac{3}{2}Q_\perp^2/Q^2}$$

Still valid after resummation of large logs



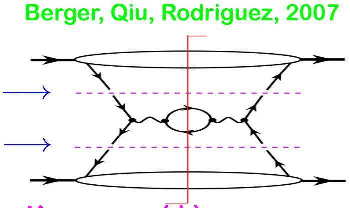
Berger, Qiu, Rodriguez, 2007

- TMD factorization:
  - Boer – Mulder function – cos(2Φ):

$$h_1^{\perp{\rm DY}}(x) = -h_1^{\perp{\rm SIDIS}}(x)$$

$\sigma^{+\alpha}k_{\perp\alpha} \longrightarrow$

$\sigma^{+\beta}k'_{\perp\beta} \longrightarrow$



Theory challenge: Q-dependence of BM function, cos(Φ), …

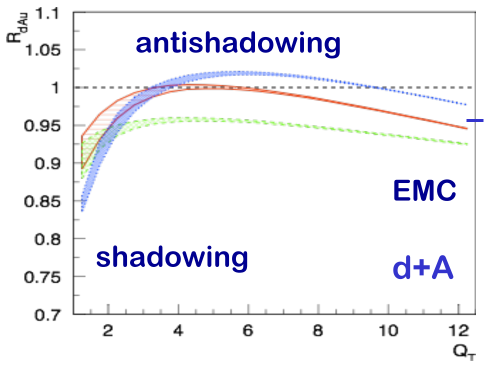
Excellent probe of gluon distribution

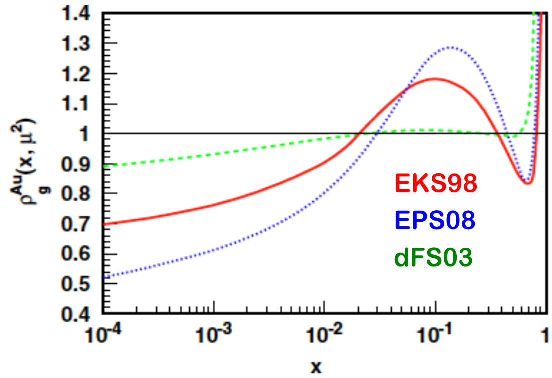
Kang, Qiu, Vogelsang, PRD 2009

- **Nuclear modification factor:**

$$R_{\rm dAu} \equiv \frac{1}{\langle N_{coll}\rangle}\frac{d^2N^{\rm dAu}/dQ_Tdy}{d^2N^{pp}/dQ_Tdy} \overset{\rm min.bias}{=} \frac{\frac{1}{2A}d^2\sigma^{\rm dAu}/dQ_Tdy}{d^2\sigma^{pp}/dQ_Tdy}$$

- **RHIC kinematics – if dominated by single scattering:**



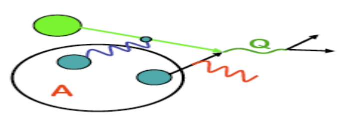


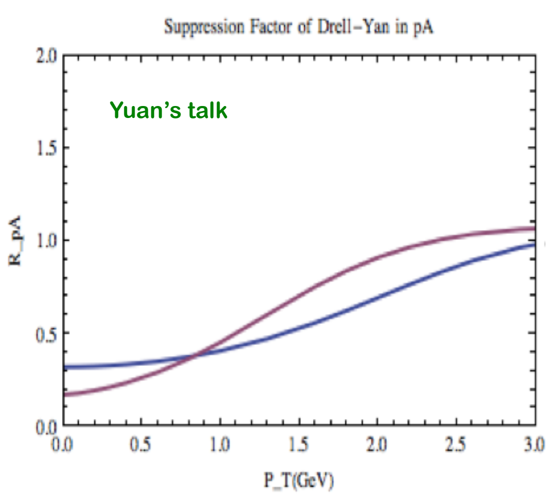
- The band is given by κ=1 (top lines) and κ=0 (bottom lines)
- Ratio follows the feature of gluon distribution if turns off isospin
- No suppression if removing isospin effect

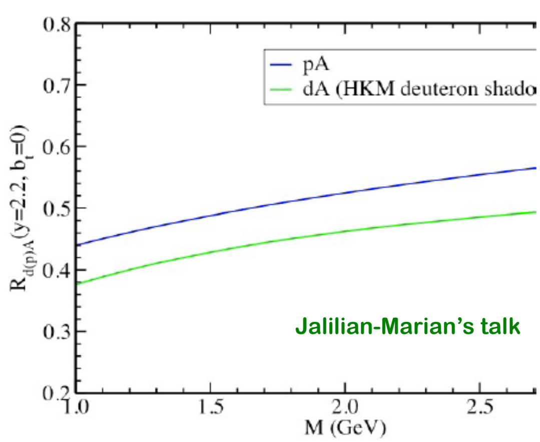
Saturation and CGC physics

- **Forward rapidity (y >> 0):**

If $Q_T \sim Q_s$, collinear factorization fails





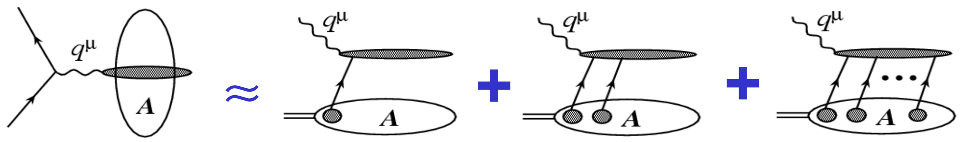


Nuclear shadowing cannot produce such suppression!

Theory challenge: Role of p_T?

Another sign change

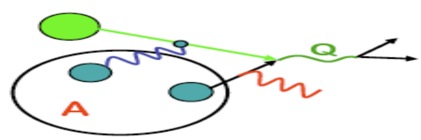
- Power correction to DIS – single scale:



$$F_{eA}(x,Q^2) = F_{eA}^{\rm LP}(x,Q^2) + \frac{1}{Q^2} F_{eA}^{\rm NLP}(x,Q^2) + \ldots$$

Negative - suppression

- Power correction to inclusive DY – single scale:



$$\frac{d\sigma_{pA}}{dQ^2} = \frac{d\sigma_{pA}^{\rm LP}}{dQ^2} + \frac{1}{Q^2}\frac{d\sigma_{pA}^{\rm NLP}}{dQ^2} + \ldots$$

Positive - enhancement

Compton gives negative contribution in CO factorization

Summary and outlook

- Drell-Yan process is one of the oldest hard process proposed to test QCD – it still a very good one!

- The proof of QCD factorization for Drell-Yan is solid (LP + NLP for collinear, LP for TMD)

- The test of the sign change of the Sivers function is a critical test of TMD factorization!

- Drell-Yan could provide much more than the sign change of Sivers function

Thank you!