From transverse momentum to b space: some thoughts

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1. Effect of limited acceptance in t
2. There is more than just cross sections
3. What if the proton breaks up

Principle

- measure cross section of exclusive process as function of t take is square root
- Fourier transform $\to$ distribution in impact parameter space
- applicable both in the GPD and the dipole formulation

more precisely:

- transverse momentum $\boldsymbol{p}$ of scattered proton in $\gamma^* p$ c.m.

$$t = t_0 - \frac{\boldsymbol{p}^2}{1-x} = -\frac{x^2\, m_p^2 + \boldsymbol{p}^2}{1-x} \qquad\qquad x = \frac{Q^2 + M_V^2}{W^2 + Q^2}$$

- impact parameter profile:

$$F(b) = \frac{1}{(2\pi)^2} \int d^2\boldsymbol{p}\, e^{-i\boldsymbol{p}\boldsymbol{b}} \sqrt{\frac{d\sigma}{dt}} = \frac{1}{2\pi} \int_0^\infty dp\, p\, J_0(pb) \sqrt{\frac{d\sigma}{dt}}$$

with $p = |\boldsymbol{p}|$ and $b = |\boldsymbol{b}|$

In real life

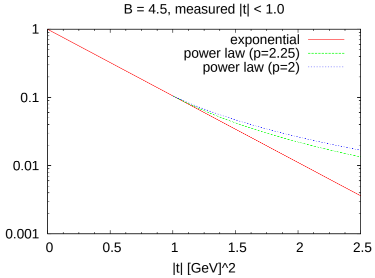
$$F(b) = \frac{1}{2\pi} \int_0^\infty dp\, p\, J_0(pb) \sqrt{\frac{d\sigma}{dt}}$$

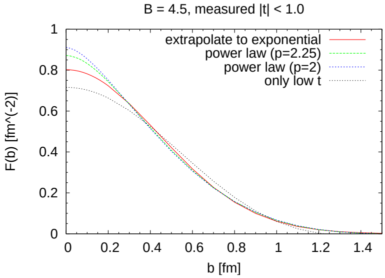
- cannot measure for arbitrary large p, or for p very small
- practical procedure: see e.g. Mueller, Munier, Stasto hep-ph/0102291
  - fit data in measured p region to given function
    try several functions if want estimate of uncertainty
  - extrapolate to larger p (and to small p if needed)
    using again different functions
  - Fourier transform
    spread in obtained $F(b)$ gives estimate of uncertainty

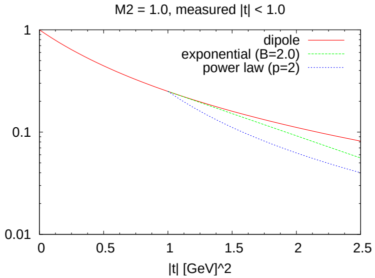
Estimate uncertainty from large p

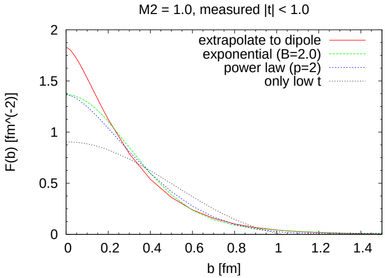
Fourier transform curves in two scenarios:

- exponential e^{Bt} for $|t| < |t|_{\text{max}}$, extrapolate with
  1. exponential (same slope)
  2. power law $\propto |t|^{-q}$ with q such that 1st derivative smooth
  3. power law $\propto |t|^{-2}$
- dipole form $\left(1 + \frac{|t|}{M^2}\right)^{-2}$ for $|t| < |t|_{\text{max}}$, extrapolate with
  1. dipole (same dipole mass)
  2. exponential with B such that 1st derivative smooth
  3. power law $\propto |t|^{-2}$
- comment on extrap. with $|t|^{-2}$:
  in practice certainly want to have smooth 1st derivative,
  but slope of curve around $|t|_{\text{max}}$ may not be known accurately
- also evaluate Fourier transf. of measured $|t|$ region only
- take $x = 0.1 \;\rightarrow\;$ effect of t_0 very small, $p \approx \sqrt{|t|}$

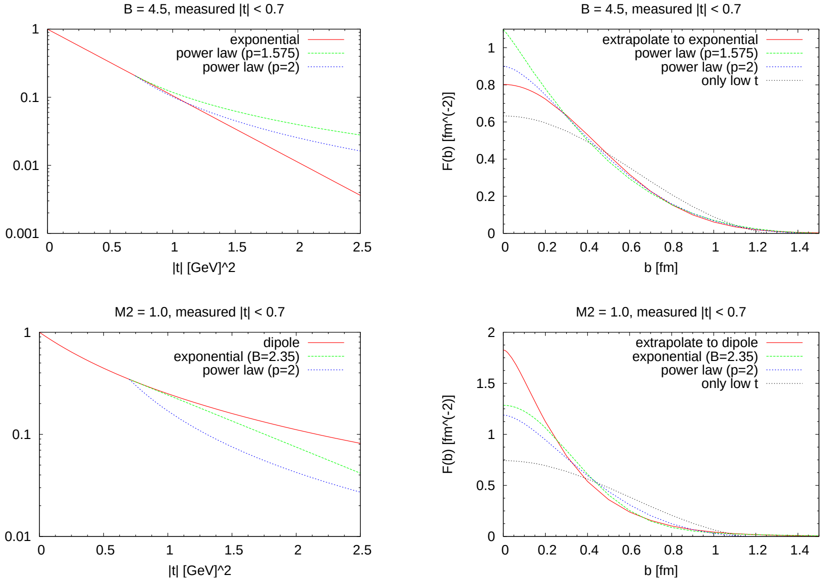




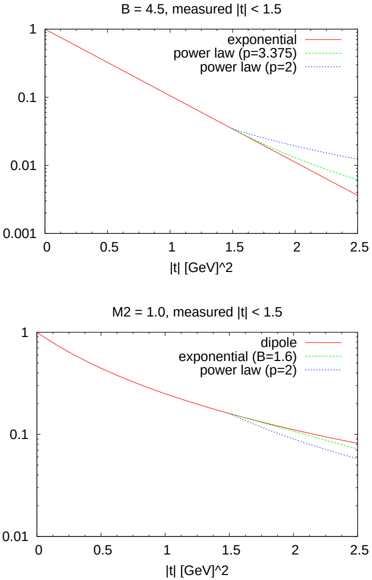




$$1/\sqrt{1\,\mathrm{GeV}^2} = 0.2\ \mathrm{fm}$$



$$1/\sqrt{0.7\,\mathrm{GeV}^2} = 1/(0.84\,\mathrm{GeV}) = 0.24\ \text{fm}$$



$$1/\sqrt{1.5\,\mathrm{GeV}^2} = 1/(1.2\,\mathrm{GeV}) = 0.16 \text{ fm}$$

Findings

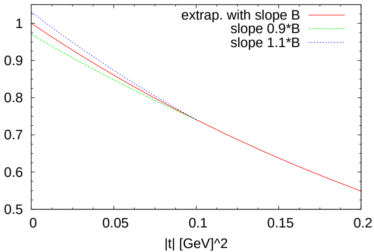
- spread of curves in b space $\to$ uncertainty of large-t extrapolation
- typically curves close together for large region of b
  below $b \sim 1/\sqrt{|t|_{\text{max}}}$ spread quickly becomes substantial
- curves from $|t| < |t|_{\text{max}}$ only: somewhat larger deviations
  in my opinion not a good error estimate
- (not shown): uncertainties slightly smaller for $B = 6\,\mathrm{GeV}^{-2}$ or $M^2 = 0.5\,\mathrm{GeV}^2$

Estimate uncertainty from low p

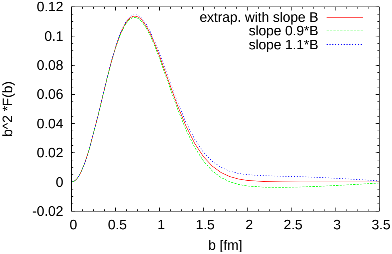
- take exponential or dipole for $|t| > |t|_{\text{min}}$
- extrapolate to $t = 0$ using same function, with 1st derivative at $|t| > |t|_{\text{min}}$
    1. remaining the same
    2. smaller by factor 0.9
    3. larger by factor 1.1

  to simulate accuracy of 10% in measuring slope at $|t| = |t|_{\text{min}}$

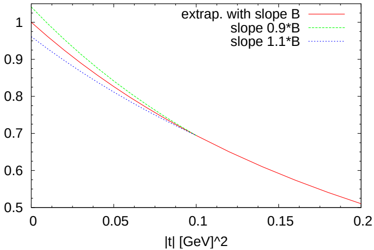
B = 6, measured |t| > 0.1



B = 6, measured |t| > 0.1



M2 = 0.5, measured |t| > 0.1



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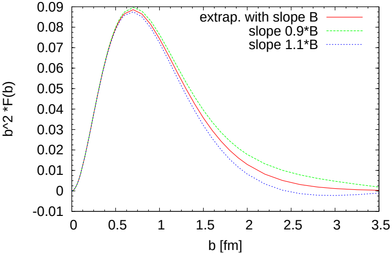
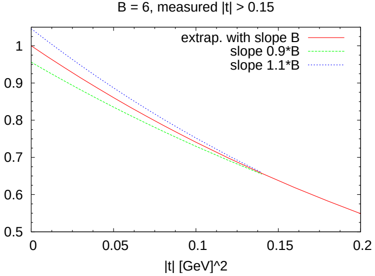
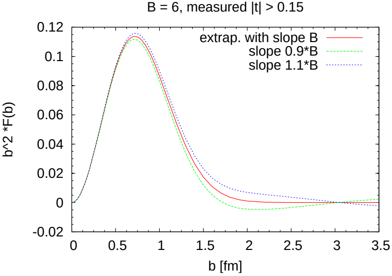
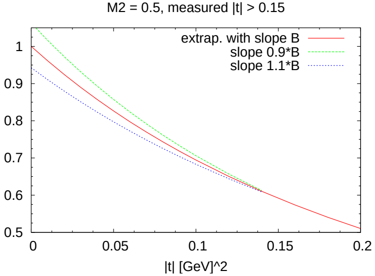
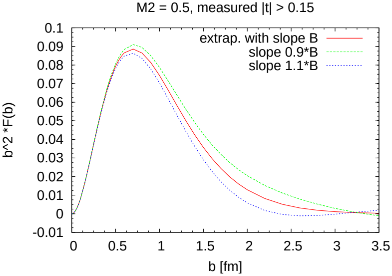


$$1/\sqrt{0.1\,\mathrm{GeV}^2} = 1/(316\,\mathrm{MeV}) = 0.63 \text{ fm}$$

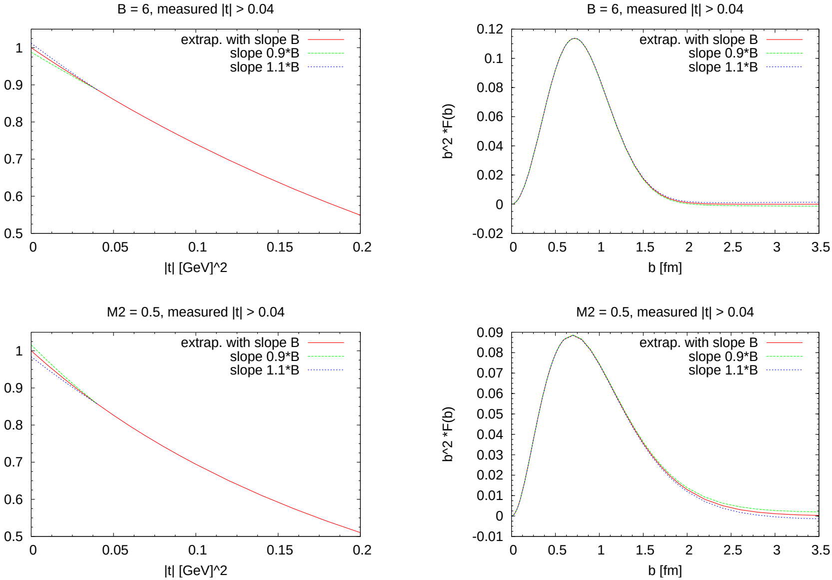








$$1/\sqrt{0.15\,\mathrm{GeV}^2} = 1/(388\,\mathrm{MeV}) = 0.52 \text{ fm}$$



$$1/\sqrt{0.04\,\mathrm{GeV}^2} = 1/(200\,\mathrm{MeV}) = 1 \text{ fm}$$

Findings

- curves start to spread at some b (typically larger than $1/\sqrt{|t|}$)
- at $b > 1$ fm can have large relative uncertainty,
  because $F(b)$ is already small
  effects barely visible if plot $F(b)$ instead of $b^2 F(b)$
- large differences between different choices of $|t|_{\text{min}}$

DVCS

$$d\sigma(ep \to ep\gamma) = d\sigma_{\mathsf{VCS}} + d\sigma_{\mathsf{BH}} + d\sigma_{\mathsf{INT}}$$

- so far considered unpolarized cross section
  valid for meson production and for $d\sigma_{\mathsf{VCS}}$

$$d\sigma/dt \propto |\mathcal{H}(t)|^2$$
$$+ \text{ kinematically suppressed contrib's from other GPDs}$$

$$\mathcal{H}(t,\xi) = \sum_{i=q,\bar{q},g} H_i(x,\xi,t) \otimes_x \text{hard scattering}(x,\xi)$$

with $\xi = \frac{\zeta}{2-\zeta}$ and $\zeta = \frac{Q^2+M_V^2}{W^2+Q^2}$

- Fourier transform of $|\mathcal{H}(t)|$ not directly a density
  but gives direct information on b distribution of partons
- alternative strategy: fit data to parameterization of $H_i(x,\xi,t)$
  (and other GPDs as needed), then Fourier transf. H_i

More information from DVCS

$$d\sigma(ep \to ep\gamma) = d\sigma_{\mathsf{VCS}} + d\sigma_{\mathsf{BH}} + d\sigma_{\mathsf{INT}}$$

- angular distribution and spin dependence: examples
- cross section difference for opposite lepton beam polarization:

$$\frac{d\sigma^{\uparrow}_{\mathsf{INT}}}{dt\, d\ldots} - \frac{d\sigma^{\downarrow}_{\mathsf{INT}}}{dt\, d\ldots} \propto \frac{\sqrt{t_0 - t}}{t}\, F_1(t)\, \mathrm{Im}\, \mathcal{H}(t) + \text{other terms}$$

kinematic prefactor includes corrections of order $\sqrt{t}/Q$

- extract $\mathrm{Im}\, \mathcal{H}(t, \xi)$ from cross section difference and then Fourier transform as before
- taking asymmetry instead of X sect. difference only useful if unpolarized X sect. strongly dominated by $d\sigma_{\mathsf{BH}}$

More information (cont'd)

- $ep \to ep\gamma$ cross section with transverse target polarization

$$\frac{d\sigma_{\mathsf{INT}}}{dt\,d\ldots} \propto \ldots \sin(\phi - \phi_S)\Big[F_2(t)\ \mathrm{Im}\,\mathcal{H}(t) - F_1(t)\ \mathrm{Im}\,\mathcal{E}(t) + \text{other terms}\Big]$$
$$+ \ldots \cos(\phi - \phi_S)\big[\ldots\big] + \text{unpolarized terms}$$

$\phi - \phi_S =$ azimuth between target spin and hadronic plane

- σ_{VCS} or meson production with transverse target polarization

$$\frac{d\sigma(ep)}{dt\,d\ldots} \propto \ldots \Big(\big|\mathcal{H}(t)\big|^2 + \text{corrections}\Big)$$
$$+ \ldots \sin(\phi - \phi_S)\frac{\sqrt{t_0 - t}}{m_p}\ \mathrm{Im}\big[\mathcal{H}^*(t)\mathcal{E}(t)\big]$$

- extract $\mathrm{Im}\,\mathcal{E}$ or $\mathcal{E} \times \cos < (\mathcal{E}, \mathcal{H})$ from data
  then Fourier transform

Proton excitation: some kinematics

- so far assumed that have elastic proton in final state
  i.e. GPDs for $p \to p$ transition
- instead of scattered p also can have baryon resonance or low-mass continuum state
- lowest-mass states:
  - Δ (requires isospin transfer, not by gluon exchange)
  - Λ, Σ (require strangeness transfer)
  - $N(1440)$

  Δ, Λ, Σ^+ decay into $N\pi$ to almost 100%
  $N(1440) \to N\pi$ with branching fraction $55 - 75\%$
- decay momenta in πN rest frame from PDF tables

$$\Lambda(1116): 101 \text{ or } 104\,\mathrm{MeV} \qquad \Sigma^+(1189): 185 \text{ or } 189\,\mathrm{MeV}$$
$$\Delta(1232): 229\,\mathrm{MeV} \qquad N(1440): 398\,\mathrm{MeV}$$

Proton excitation: some kinematics

boost to lab system

- if decaying hadron has transv. mom. $p_T = 0$

$$E_\pi = E\,\frac{\sqrt{p^2 + m_\pi^2} + p^* \cos\theta^*}{M} \qquad\qquad p_{T\pi} = p^* \sin\theta^*$$

M and E: mass and energy of decaying hadron in lab
p^* and θ^*: momentum and angle of π in πN rest frame
formula for E_π valid in relativistic kinematics: $E \gg M$, $E_\pi \gg m_\pi$

- minimum and maximum values of E_π/E:

$$\Lambda : 0.06 \ldots 0.25 \qquad\qquad \Sigma^+ : 0.04 \ldots 0.35$$

$$\Delta : 0.03 \ldots 0.40 \qquad\qquad N(1440) : 0.016 \ldots 0.56$$

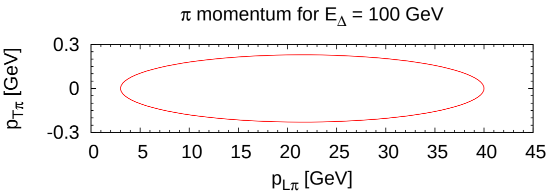
Proton excitation: some kinematics

boost to lab system

- if decaying hadron has transv. mom. $p_T = 0$

$$E_\pi = E\,\frac{\sqrt{p^2 + m_\pi^2} + p^* \cos\theta^*}{M} \qquad\qquad p_{T\pi} = p^* \sin\theta^*$$

graphically: very elongated ellipsoid



- if $p_T \neq 0$ then rotate this ellipsoid by α with $\sin\alpha = p_T/E$