From Dipoles to Quadrupoles

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- F.Dominguez, C.Marquet, BX and F. Yuan, Phys.Rev.D83:105005,2011.
- F. Dominguez, A. Mueller, S. Munier and BX, Phys.Lett. B705 (2011) 106-111.
- A. Stasto, BX and F. Yuan, arXiv:1109.1817 [hep-ph].
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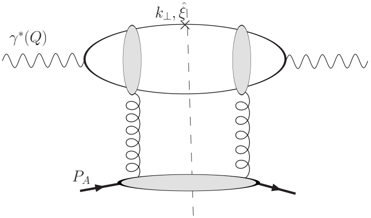
Outline

Inclusive and Semi-inclusive DIS at small-x

The Dipole Model has become the common practice of QCD calculation at small-x. Using QCD dipole model, one can write the cross section of SIDIS as

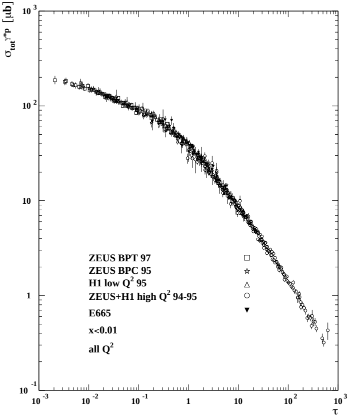


$$\frac{d\sigma_{T,L}^{\gamma^* p \to qX}}{dz dk_\perp^2} = \frac{1}{4\pi}\int d^2\mathbf{x} d^2\mathbf{y}\ \Phi_{T,L}(z,\mathbf{x},\mathbf{y},Q^2)\ e^{-ik_\perp\cdot(\mathbf{x}-\mathbf{y})}$$
$$\times \int d^2\mathbf{b}\,[1 - S_{q\bar{q}}(\mathbf{x},x) + S_{q\bar{q}}(\mathbf{y},x) + S_{q\bar{q}}(\mathbf{x}-\mathbf{y},x)]$$

- Integrating over z and $k_\perp$ gives inclusive cross section.
- Convoluting with the fragmentation $D_{h/q}(\frac{\hat{\xi}}{z})$ yields the single inclusive hadron spectrum.

Golec-Biernat Wusthoff model and Geometrical Scaling

[Golec-Biernat, Wusthoff,; 98], [Golec-Biernat, Stasto, Kwiecinski; 01]



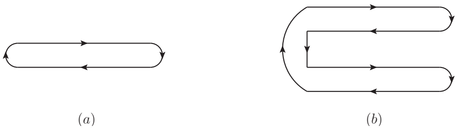
- The dipole amplitude in the GBW model

$$S_{q\bar{q}}(r_\perp) = \exp[-\frac{Q_s^2 r_\perp^2}{4}]$$

with $Q_s^2(x) = Q_{s0}^2(x_0/x)^\lambda$ where $Q_{s0} = 1\text{GeV}$, $x = 3.04 \times 10^{-3}$ and $\lambda = 0.288$.

Dipoles, Quadrupoles and higher point functions

Dipole and Quadrupole

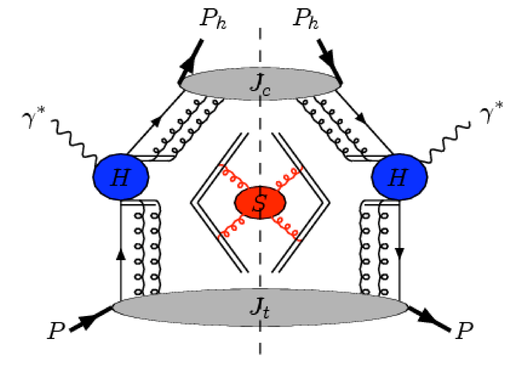


$$S^{(2)} = \frac{1}{N_c}\mathrm{Tr}\left[U(x_1)\,U^\dagger(x_2)\right]$$
$$S^{(4)} = \frac{1}{N_c}\mathrm{Tr}\left[U(x_1)\,U^\dagger(x_2)\,U(x_3)\,U^\dagger(x_4)\right]$$

- Scattering amplitudes between different projectiles and the target.
- Semi-inclusive processes only involve dipoles.
- Dijet and more exclusive processes involve dipoles, quadrupoles and higher point functions.
- For instance, the sextupole $S^{(6)} = \frac{1}{N_c}\mathrm{Tr}\left[U(x_1)\,U^\dagger(x_2)\,U(x_3)\,U^\dagger(x_4)\,U(x_5)\,U^\dagger(x_6)\right]$ appears in the $gg \to gg$ channel of dijet processes. Similar results can be obtained in *TMD* approach as well. [Bomhof, Mulders and Pijlman; 06]

Transverse Momentum Dependent (TMD) factorization

[Collins-Soper-Sterman,85], [Ji-Ma-Yuan, 04]



- (Mueller's dipole model) k_t factorization is widely used in small-x physics. TMD factorization is widely used in spin physics (large x).
- k_t factorization $=$ *TMD* factorization ? Yes!
- Universality of the TMD parton distributions? Yes and No!

The effective k_t factorization

For pA (dilute-dense system) collisions, there is an effective k_t factorization.

$$\boxed{\frac{d\sigma^{pA\to qfX}}{d^2P_\perp d^2q_\perp dy_1 dy_2} = x_p q(x_p, \mu^2) xf(x, q_\perp^2) \frac{1}{\pi}\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\hat{t}}.}$$

Remarks:

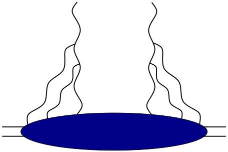
- For pp, AA collisions, there is no k_t factorization[Rogers, Mulders; 10].
- Penalty: K_t dependent Parton distributions $xf(x, q_\perp^2)$ are not universal. $xf(x, q_\perp^2)$ here can be the quark or gluon distributions of the dense target. It contains the anomalous terms after resummation.
- $x_p q(x_p, \mu^2)$ is the Feyman parton distribution of the dilute projectile.
- Thanks to the nuclear enhancement, soft gluon exchange from the dilute proton can be neglected.
- Assuming small-x limit, namely, $s \to \infty$, fixed Q^2, $x \to 0$.
- For DIS, this k_t factorization works as well. The question is which gluon distributions one should use.

A Tale of Two Gluon Distributions

In small-x physics, two gluon distributions are widely used:

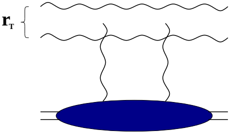
I. Weizsäcker Williams gluon distribution (MV model):[Mueller, Kovchegov, 98], [McLerran, Venugopalan, 99]

$$xG^{(1)} = \frac{S_\perp}{\pi^2 \alpha_s} \frac{N_c^2 - 1}{N_c} \times \int \frac{d^2 r_\perp}{(2\pi)^2} \frac{e^{-ik_\perp \cdot r_\perp}}{r_\perp^2} \left(1 - e^{-\frac{r_\perp^2 Q_{sg}^2}{4}}\right) \Leftarrow$$



II. Color Dipole gluon distributions:

$$xG^{(2)} = \frac{S_\perp N_c}{2\pi^2 \alpha_s} \times \int \frac{d^2 r_\perp}{(2\pi)^2} e^{-ik_\perp \cdot r_\perp} \nabla_{r_\perp}^2 e^{-\frac{r_\perp^2 Q_{sq(g)}^2}{4}} \Leftarrow$$



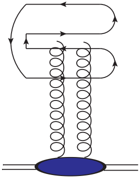
Remarks:

- The WW gluon distribution simply counts the number of gluons.
- The Color Dipole gluon distribution often appears in calculations. $N(r_\perp)$ is the color dipole amplitude. It is now in fundamental representation.
- Does this mean that gluon distributions are non-universal? Yes and No!
- These two distributions are used in R_{pA} calculation. [Kharzeev, Kovchegov, Tuchin; 03].

A Tale of Two Gluon Distributions

I. Weizsäcker Williams gluon distribution (MV model):

$$xG^{(1)} = \frac{S_\perp}{\pi^2\alpha_s}\frac{N_c^2-1}{N_c} \qquad \Leftarrow$$
$$\times \int \frac{d^2 r_\perp}{(2\pi)^2}\frac{e^{-ik_\perp\cdot r_\perp}}{r_\perp^2}\left(1-e^{-\frac{r_\perp^2 Q_{sg}^2}{4}}\right)$$



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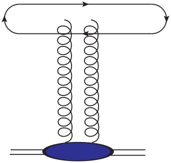
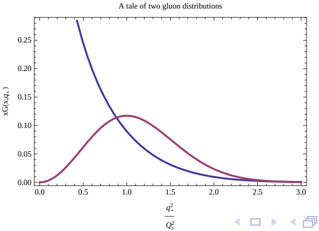




A Tale of Two Gluon Distributions

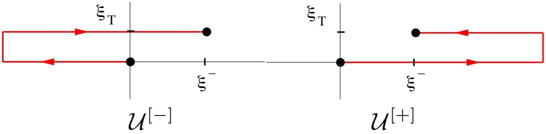
The operator definitions of these two gluon distributions: [Bomhof, Mulders and Pijlman; 06][F. Dominguez, BX and F. Yuan, 11]

I. Weizsäcker Williams gluon distribution:

$$xG^{(1)} = 2\int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+\xi^- - ik_\perp\cdot\xi_\perp} \mathrm{Tr}\langle P|F^{+i}(\xi^-,\xi_\perp)\mathcal{U}^{[+]\dagger}F^{+i}(0)\mathcal{U}^{[+]}|P\rangle.$$

II. Color Dipole gluon distributions:

$$xG^{(2)} = 2\int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+\xi^- - ik_\perp\cdot\xi_\perp} \mathrm{Tr}\langle P|F^{+i}(\xi^-,\xi_\perp)\mathcal{U}^{[-]\dagger}F^{+i}(0)\mathcal{U}^{[+]}|P\rangle.$$



Remarks:

- The WW gluon distribution is the conventional gluon distributions. In light-cone gauge, it is the gluon density. (Only final state interactions.)
- The dipole gluon distribution has no such interpretation. (Initial and final state interactions.)
- Both definitions are gauge invariant.
- Same after integrating over $q_\perp$.
- Same perturbative tail.

A Tale of Two Gluon Distributions

The operator definitions of these two gluon distributions: [Bomhof, Mulders and Pijlman; 06][F. Dominguez, BX and F. Yuan, 11]

I. Weizsäcker Williams gluon distribution (Easy to evaluate in the MV model):

$$xG^{(1)} = 2\int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+\xi^- - ik_\perp\cdot\xi_\perp} \mathrm{Tr}\langle P|F^{+i}(\xi^-,\xi_\perp)\mathcal{U}^{[+]\dagger}F^{+i}(0)\mathcal{U}^{[+]}|P\rangle.$$

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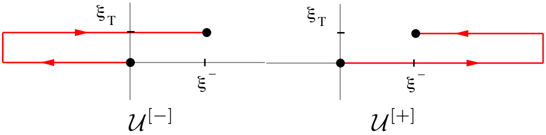
$$xG^{(2)} = 2\int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+\xi^- - ik_\perp\cdot\xi_\perp} \mathrm{Tr}\langle P|F^{+i}(\xi^-,\xi_\perp)\mathcal{U}^{[-]\dagger}F^{+i}(0)\mathcal{U}^{[+]}|P\rangle.$$

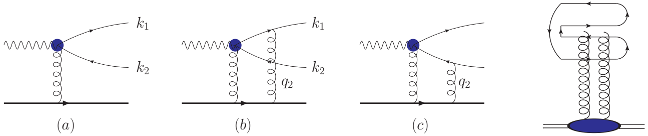


Questions:

- Can we distinguish these two gluon distributions in physical processes?
- How to measure $xG^{(1)}$ directly? DIS dijet.
- How to measure $xG^{(2)}$ directly? Direct γ+Jet in pA collisions. Maybe single-inclusive particle production in pA (Subtle).
- What happens in gluon+jet production in pA collisions? It's complicated!

DIS dijet in the TMD and CGC approaches

Resummation of the gauge links in TMD approach.



TMD factorization approach:

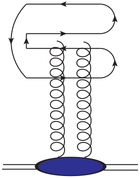
$$\frac{d\sigma^{\gamma_T^* A\to q\bar{q}+X}}{dy_1 dy_2 d^2P_\perp d^2q_\perp} = \delta(x_{\gamma^*}-1)x_g G^{(1)}(x_g, q_\perp)H_{\gamma_T^* g\to q\bar{q}}.$$

CGC approach:[Jalilian-Marian, Gelis, 02]

$$\frac{d\sigma^{\gamma_T^* A\to q\bar{q}+X}}{d\mathcal{P}.\mathcal{S}.} \propto N_c\alpha_{em}e_q^2\int\frac{\mathrm{d}^2x}{(2\pi)^2}\frac{\mathrm{d}^2x'}{(2\pi)^2}\frac{\mathrm{d}^2b}{(2\pi)^2}\frac{\mathrm{d}^2b'}{(2\pi)^2}e^{-ik_{1\perp}\cdot(x-x')}$$
$$\times e^{-ik_{2\perp}\cdot(b-b')}\sum\psi_T^*(x-b)\psi_T(x'-b')$$
$$\left[1+S_{x_g}^{(4)}(x,b;b',x')-S_{x_g}^{(2)}(x,b)-S_{x_g}^{(2)}(b',x')\right],$$

Two independent calculations agree perfectly in the correlation limit (Large $P_\perp$ and Small $q_\perp$).[F. Dominguez, BX and F. Yuan, 11]

DIS dijet



In the dijet correlation limit, where $u = x - b$ and $v = zx + (1 - z)b$

$$\left[1 + S^{(4)}_{x_g}(x, b; b', x') - S^{(2)}_{x_g}(x, b) - S^{(2)}_{x_g}(b', x')\right]$$

$$\simeq \quad -u_i u'_j \frac{1}{N_c} \langle \mathrm{Tr}\left[\partial^i U(v)\right] U^\dagger(v') \left[\partial^j U(v')\right] U^\dagger(v)\rangle_{x_g}$$

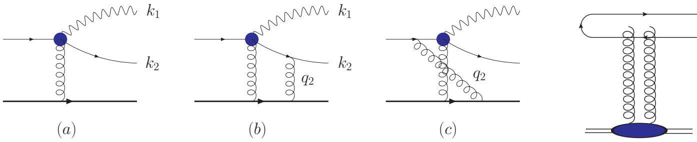
$$= \quad u_i u'_j \frac{g^2}{N_c} \int_{-\infty}^{\infty} \mathrm{d}v^+ \mathrm{d}v'^+ \left\langle \mathrm{Tr}\left[F^{i-}(v)\mathcal{U}^{[+]\dagger} F^{j-}(v')\mathcal{U}^{[+]}\right]\right\rangle_{x_g}$$

Remarks:

- Dijet in DIS is the only physical process which can directly measure Weizsäcker Williams gluon distributions.
- Golden measurement for the Weizsäcker Williams gluon distributions of nuclei at small-x.
- EIC will provide us a perfect machine to study the strong gluon fields in nuclei.

γ+Jet in pA collisions

The direct photon + jet production in pA collisions. (Drell-Yan Process follows the same factorization.)



TMD factorization approach:

$$\frac{d\sigma^{(pA\to\gamma q+X)}}{d\mathcal{P}.\mathcal{S}.} = \sum_f x_1 q(x_1, \mu^2) x_g G^{(2)}(x_g, q_\perp) H_{qg\to\gamma q}.$$

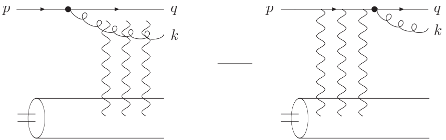
Remarks:

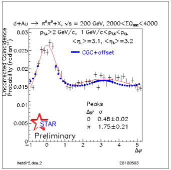
- Independent CGC calculation gives the identical result in the correlation limit. [Jalilian-Marian, Gelis, 02],[Dominguez, Marquet, BX, Yuan, 11]
- This process can be calculated exactly for all range of azimuthal angles.
- Direct measurement of the Color Dipole gluon distribution.
- The RHIC and future LHC experiments shall provide us some information on this.

Existing calculations on dijet production

Let us first look back, and re-examine the existing calculations on dijet productions.

Quark+Gluon channel [Marquet, 07] and [Albacete, Marquet, 10]



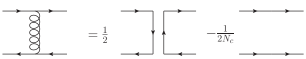


- Prediction of saturation physics.
- All the framework is correct, but over-simplified 4-point function.
- Improvement [F. Dominguez, C. Marquet, BX and F. Yuan, 11.]

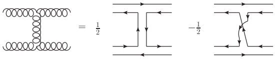
$$S^{(4)}_{x_g}(x_1,x_2;x_2',x_1') \simeq e^{-\frac{C_F}{2}[\Gamma(x_1-x_2)+\Gamma(x_2'-x_1')]} - \frac{F(x_1,x_2;x_2',x_1')}{F(x_1,x_2';x_2,x_1')}\left(e^{-\frac{C_F}{2}[\Gamma(x_1-x_2)+\Gamma(x_2'-x_1')]} - e^{-\frac{C_F}{2}[\Gamma(x_1-x_1')+\Gamma(x_2'-x_2)]}\right)$$

Dijet processes in the large N_c limit

The Fierz identity:

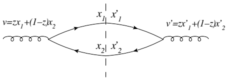


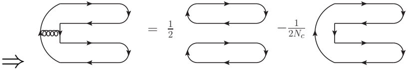
and



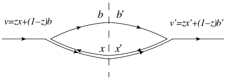
Graphical representation of dijet processes

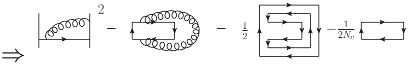
$g \to q\bar{q}$:



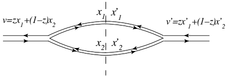


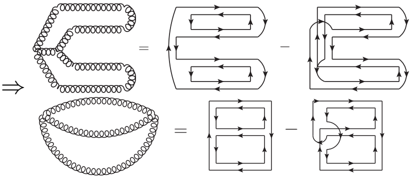
$q \to qg$





$g \to gg$





The Octupole and the Sextupole are suppressed.

Gluon+quark jets correlation

Including all the $qg \to qg$, $gg \to gg$ and $gg \to q\bar{q}$ channels, a lengthy calculation gives

$$\frac{d\sigma^{(pA\to \text{Dijet}+X)}}{d\mathcal{P}.\mathcal{S}.} = \sum_q x_1 q(x_1,\mu^2)\frac{\alpha_s^2}{\hat{s}^2}\left[\mathcal{F}_{qg}^{(1)}H_{qg}^{(1)}+\mathcal{F}_{qg}^{(2)}H_{qg}^{(2)}\right]$$
$$+x_1 g(x_1,\mu^2)\frac{\alpha_s^2}{\hat{s}^2}\left[\mathcal{F}_{gg}^{(1)}\left(H_{gg\to q\bar{q}}^{(1)}+\frac{1}{2}H_{gg\to gg}^{(1)}\right)\right.$$
$$\left.+\mathcal{F}_{gg}^{(2)}\left(H_{gg\to q\bar{q}}^{(2)}+\frac{1}{2}H_{gg\to gg}^{(2)}\right)+\mathcal{F}_{gg}^{(3)}\frac{1}{2}H_{gg\to gg}^{(3)}\right],$$

with the various gluon distributions defined as

$$\mathcal{F}_{qg}^{(1)} = xG^{(2)}(x,q_\perp),\quad \mathcal{F}_{qg}^{(2)} = \int xG^{(1)}\otimes F\,,$$
$$\mathcal{F}_{gg}^{(1)} = \int xG^{(2)}\otimes F,\quad \mathcal{F}_{gg}^{(2)} = -\int \frac{q_{1\perp}\cdot q_{2\perp}}{q_{1\perp}^2}xG^{(2)}\otimes F\,,$$
$$\mathcal{F}_{gg}^{(3)} = \int xG^{(1)}(q_1)\otimes F\otimes F\,,$$

where $F=\int\frac{d^2r_\perp}{(2\pi)^2}e^{-iq_\perp\cdot r_\perp}\frac{1}{N_c}\left\langle \text{Tr}U(r_\perp)U^\dagger(0)\right\rangle_{x_g}$.

Remarks:

- Only the term in NavyBlue color was known before.
- This can help us understand the dihadron correlation data.

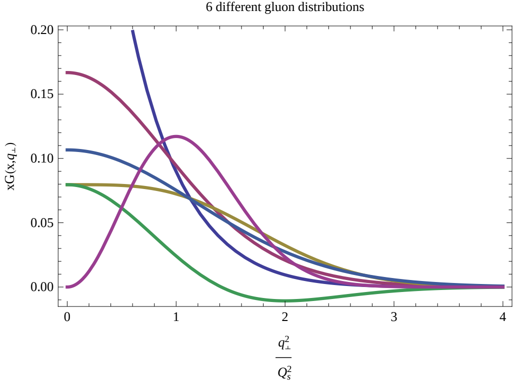
Illustration of gluon distributions

The various gluon distributions:

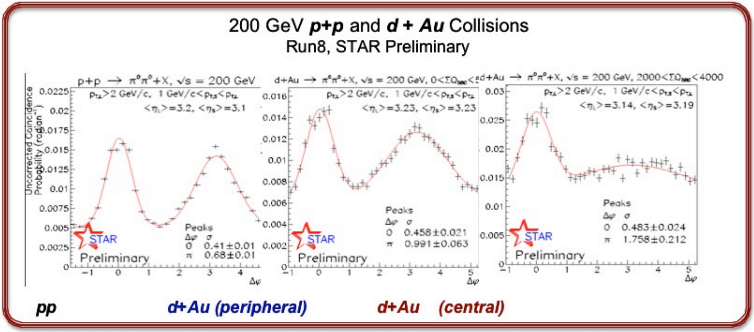
$$xG^{(1)}_{\mathrm{WW}}(x, q_\perp), \quad \mathcal{F}^{(1)}_{qg} = xG^{(2)}(x, q_\perp),$$

$$\mathcal{F}^{(1)}_{gg} = \int xG^{(2)} \otimes F, \quad \mathcal{F}^{(2)}_{gg} = -\int \frac{q_{1\perp} \cdot q_{2\perp}}{q_{1\perp}^2} xG^{(2)} \otimes F,$$

$$\mathcal{F}^{(3)}_{gg} = \int xG^{(1)}(q_1) \otimes F \otimes F, \quad \mathcal{F}^{(2)}_{qg} = \int xG^{(1)} \otimes F$$



STAR measurement on di-hadron correlation in *dA* collisions

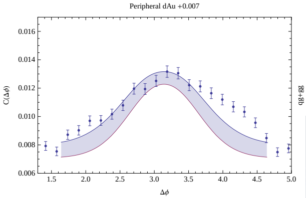


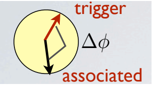
- There is no sign of suppression in the $p + p$ and $d + Au$ peripheral data.
- The suppression and broadening of the away side jet in $d + Au$ central collisions is due to the multiple interactions between partons and dense nuclear matter (CGC).
- Probably the best evidence for saturation.
- Dissect the data into three features:
  Width σ of peaks, Pedestal P and Peak suppression.

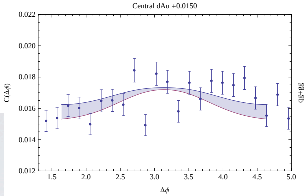
Comparing to STAR data

[A. Stasto, BX, F. Yuan, 11]

For away side peak in both peripheral and central dAu collisions







- The framework does not work for pp since the saturation scale Q_s is too low at $\sqrt{s} = 200$GeV.
- Use Golec-Biernat Wusthoff model for the saturation momentum, $Q_s^2(x) = Q_{s0}^2(x_0/x)^\lambda$.
- Adding the nuclear and impact factor dependence: $Q_{sA}^2 = c(b)A^{1/3}Q_s^2(x)$.
- Peripheral $b = 6.8 \pm 1.7$fm with $c(b) = 0.45$ and width $\sigma \simeq 0.99$;
- Central $b = 2.7 \pm 1.3$fm with $c(b) = 0.85$ and width $\sigma \simeq 1.6$.

The small-x evolution equation of quadrupoles

- The Balitsky-Kovchegov equation for dipoles

$$\frac{\partial}{\partial Y} S = \int P_{d\to dd}\left(SS - S\right),$$

with $P_{d\to dd} = \frac{\alpha_s N_c}{2\pi^2} \frac{d^2 z_\perp (x_\perp - y_\perp)^2}{(x_\perp - z_\perp)^2 (z_\perp - y_\perp)^2}$.

- The quadrupole evolution equation: Large N_c [Jalilian-Marian and Kovchegov, 04], Finite N_c [Dominguez, Mueller, Munier, BX, 11], [Iancu, Triantafyllopoulos, 11]

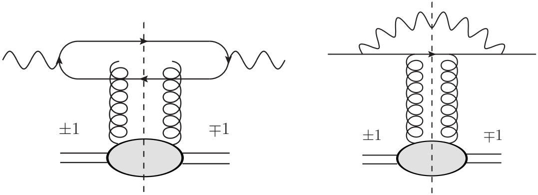
$$\frac{\partial}{\partial Y} Q = \int P_{q\to qd}\left(QS - Q\right) + \int P_{q\to dd}\left(SS - Q\right).$$

with $P_{q\to qd} + P_{q\to dd} > 0$.

- Dipoles and quadupoles are very alike in terms of evolution.
- Follow BFKL in the dilute limit. Both have geometrical scaling. Confirmed by the numerical study[Dumitru, Jalilian-Marian, Lappi, Schenke and Venugopalan,11]
- Saturate to a stable fixed point in the dense limit.

The Linearly polarized gluon distribution

The linearly polarized gluon distribution effectively measures an averaged quantum interference between a scattering amplitude with an active gluon polarized along the x(or y)-axis and a complex conjugate amplitude with an active gluon polarized along the y(or x)-axis inside an unpolarized hadron. [Mulders and Rodrigues, 01], [Boer, Brodsky, Mulders, Pisano, 10], [Metz and Zhou,10], [Dominguez, Qiu, BX and Yuan, 10]

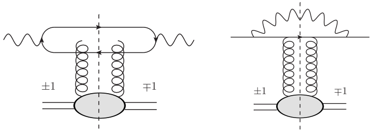


$$\frac{d\sigma^{\gamma_T^* A\to q\bar{q}+X}}{dy_1 dy_2 d^2\tilde{P}_\perp d^2 q_\perp} = \delta(x_{\gamma^*}-1) H_{\gamma_T^* g\to q\bar{q}} \times \left[x_g G^{(1)}(x_g, q_\perp) - \frac{2\epsilon_f^2 \tilde{P}_\perp^2}{\epsilon_f^4 + \tilde{P}_\perp^4} \cos(2\Delta\phi)\, x h_\perp^{(1)}(x, q_\perp) \right],$$

where $\Delta\phi = \phi_{\tilde{P}_\perp} - \phi_{q_\perp}$ and $\epsilon_f^2 = z(1-z)Q^2 \neq 0$. There is similar cross section for DY-type dijet.

The Linearly polarized gluon distribution

[Metz and Zhou,10], [Dominguez, Qiu, BX and Yuan, 10]



For DIS dijet: W.W. gluon distribution

$$2\int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+\xi^- - ik_\perp\cdot\xi_\perp} \mathrm{Tr}\langle P|F^{+i}(\xi^-,\xi_\perp)\mathcal{U}^{[+]\dagger}F^{+j}(0)\mathcal{U}^{[+]}|P\rangle_{x_g}$$

$$= \frac{1}{2}\delta^{ij}xG^{(1)}(x,q_\perp) + \frac{1}{2}\left(\frac{2q_\perp^i q_\perp^j}{q_\perp^2} - \delta^{ij}\right) xh_\perp^{(1)}(x,q_\perp).$$

For DY-type dijet processes: Dipole gluon distribution

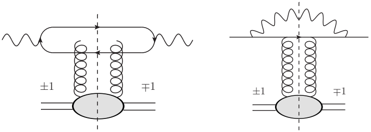
$$2\int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+\xi^- - iq_\perp\cdot\xi_\perp} \langle P|\mathrm{Tr}\left[F^{+i}(\xi^-,\xi_\perp)\mathcal{U}^{[-]\dagger}F^{+j}(0)\mathcal{U}^{[+]}\right]|P\rangle\,,$$

$$= \frac{1}{2}\delta^{ij}xG^{(2)}(x,q_\perp) + \frac{1}{2}\left(\frac{2q_\perp^i q_\perp^j}{q_\perp^2} - \delta^{ij}\right) xh_\perp^{(2)}(x,q_\perp).$$

- Diagonal and off-diagonal part of the dipole and quadrupole amplitudes.
- $xG^{(1)}(x,q_\perp) \geq xh_\perp^{(1)}(x,q_\perp)$ and $xG^{(2)}(x,q_\perp) = xh_\perp^{(2)}(x,q_\perp)$

The Linearly polarized gluon distribution

[Dominguez, Qiu, BX and Yuan, 10]



A few more comments

- The linearly polarized gluon distributions come from quantum interference and have no probability interpretation. They are different from the usual polarized gluon distribution$\left(\frac{1}{2}\left[\epsilon_{+}^{*i}\epsilon_{+}^{j}-\epsilon_{-}^{*i}\epsilon_{-}^{j}\right]\right)$.
- Need non-vanishing virtuality to have non-vanishing contribution in the dilute-dense factorization.
- The small-x evolution of the W.W. type linearly polarized gluon distribution ($\left\langle \mathrm{Tr}\left[\partial^i U(v)\right] U^\dagger(v')\left[\partial^j U(v')\right] U^\dagger(v)\right\rangle_Y$) is related to the quadrupole evolution equation, while the dipole type linearly polarized gluon distribution follows BK equation.
- They follow BFKL evolution in the dilute regime, and receive exponential growth in terms of rapidity. They also have geometrical scaling.
- They also saturate in the dense regime.

Conclusion

- The effective factorization for collisions between a dilute projectile and a dense target in the large N_c limit.
- DIS dijet provides direct information of the WW gluon distributions.
- Modified Universality for Gluon Distributions:

	Inclusive	Single Inc	DIS dijet	γ +jet	g+jet
$xG^{(1)}$	$\times$	$\times$	$\surd$	$\times$	$\surd$
$xG^{(2)}$, F	$\surd$	$\surd$	$\times$	$\surd$	$\surd$

$\times \Rightarrow$ Do Not Appear. $\surd \Rightarrow$ Apppear.

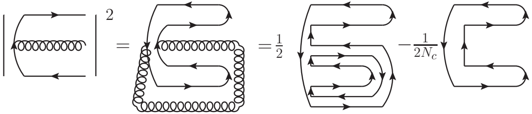
- Two fundamental gluon distributions which are related to the quadrupole and dipole amplitudes, respectively. Other gluon distributions are just different combinations and convolutions of these two.
- Dihadron correlation calculation and comparison with the STAR data.
- The small-x evolution of the quadrupole and the WW gluon distribution, a different equation from Balitsky-Kovchegov equation.
- Linearly polarized gluon distributions provide us the off-diagonal information of scattering amplitudes.

Outlook

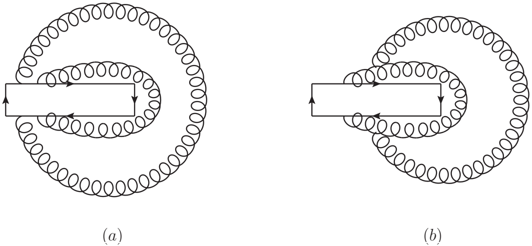
[Dominguez, Marquet, Stasto, BX and Yuan, in preparation]

- The three-jet production processes in the large N_c limit:

$q\bar{q}g$-jet



qgg-jet



- Conjecture: In the large N_c limit at small-x, the dipole and quadrupole amplitudes are the only two fundamental objects in the cross section of multiple-jet production processes up to all order.
- Other higher point functions, such as sextupoles, octupoles, decapoles and duodecapoles, etc. are suppressed by factors of $\frac{1}{N_c^2}$.