HIGH-PRECISION ℓN COLLISIONS: BASICS OF RADIATIVE CORRECTIONS

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EIC Workshop

with updates for BNL, March 2011

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MOTIVATION

The goal:

High-precision measurements of the 'nucleon structure'

➜ measure form factors, structure functions, (generalized) parton distribution functions, ...

- at low Q^2 elastic and quasi-elastic scattering
  ➜ form factors, polarizabilities, ...
- at high Q^2 deep inelastic scattering
  ➜ parton distribution functions, GPDs, GDAs, ...

The interesting physics is encoded in FFs, PDFs, ...
test the dynamics of the strong interaction
QCD precision physics — the main topic of this workshop

Lepton scattering: only via electromagnetic and weak interaction

➜ well-controlled and separable perturbative treatment

Measure FFs, PDFs, etc by comparing data with theoretical predictions:

$$\sigma_{\text{exp}} = \sigma_{\text{theory}}[F_n(x, Q^2, \ldots)]$$

High precision requires knowledge of higher-order corrections

$$\sigma_{\text{theory}} = \sigma^{(0)} + \alpha_{\text{em}}\sigma^{(1)} + \ldots$$

- Emission of real photons
  experimentally often not distinguished from non-radiative processes:
  soft photons, collinear photons
  ➜ ”radiative corrections”
- Virtual corrections: loop diagrams
  needed to cancel infrared divergences (Bloch-Nordsieck)
- Electroweak effects
  Z-, W-boson exchange ($O(G_F)$)
  and higher-order electroweak corrections ($O(\alpha G_F)$)

Radiative corrections have to be 'removed' to uncover the interesting physics and radiative corrections often considered the uninteresting part

but:

- radiation from the nucleon: DVCS deeply virtual Compton scattering, γ-PDF, is part of the 1-photon radiative corrections
- 2γ exchange (and γZ exchange) is part of the 1-loop photonic corrections: box diagrams
  corresponding infrared divergences cancel with the interference between photon radiation from the lepton and from the nucleon
- Z-exchange gives rise to P- and C-violating interactions, charge and polarization asymmetries

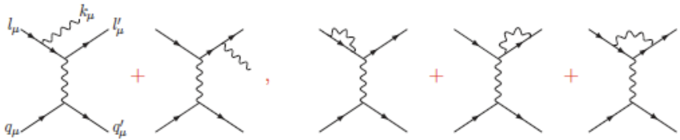
CLASSIFICATION OF $O(\alpha)$ CORRECTIONS

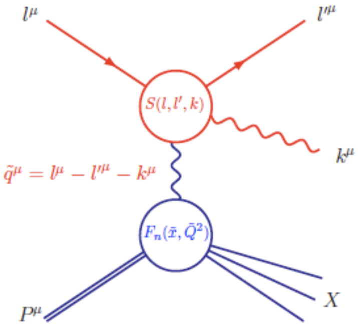
- Radiation from the lepton
  model independent (universal)
- vacuum polarization (boson self energy)
  universal
- Radiation from the hadronic initial/final state
  parton model: radiation from quarks
  to be considered as a part of the nucleon structure
- Interference of leptonic and hadronic radiation
  2γ exchange
  new structure
- purely weak corrections

Note: for NC-scattering straightforward separation
Rule: respect gauge invariance
IR divergences: need to combine real and virtual radiation

Feynman diagrams for leptonic radiation at $O(\alpha)$ (NC)

for *eq* scattering:





radiative leptonic tensor $S_{\mu\nu}(l, l', k)$ is

- gauge invariant
- infrared finite
- universal

(includes Born + loops: $\delta^{(4)}(k^\mu)$)

Observed cross section: convolution of true cross section $\otimes$ radiator function

$$\mathrm{d}\sigma^{\mathrm{obs}}(p,q) = \int \frac{d^3k}{2k^0} R(l,l',k)\mathrm{d}\sigma^{\mathrm{true}}(p,q-k)$$

or, for the structure functions:

$$F_n^{\mathrm{obs}}(x,Q^2) = \int \mathrm{d}\tilde{x}\mathrm{d}\tilde{Q}^2 R_n(x,Q^2;\tilde{x},\tilde{Q}^2) F_n^{\mathrm{true}}(\tilde{x},\tilde{Q}^2)$$

Can be extended to include higher-order effects: multi-photon emission, soft-photon exponentiation, e^+e^--pair creation,
R_n known analytically to second order, $O(\alpha^2)$

In turn: determination of the true F_n = unfolding, may be ill-defined
Difficult to treat radiative and detector effects separately (acceptance cuts, efficiencies, ...)

$$F_n^{\mathrm{obs}}(x, Q^2) = \int \mathrm{d}\tilde{x}\mathrm{d}\tilde{Q}^2 R_n(x, Q^2; \tilde{x}, \tilde{Q}^2) F_n^{\mathrm{true}}(\tilde{x}, \tilde{Q}^2)$$

Note: shifted kinematics, e.g.,

$$Q^2 = -(l - l')^2 \to \tilde{Q}^2 = -(l - l' - k)^2$$

➜ expect strong dependence on experimental prescriptions for measuring kinematic variables

- leptonic variables: measure E and θ of scattered lepton ➜ x and Q^2
- hadronic variables: measure E, θ from hadronic final state ➜ $\tilde{x}$ and $\tilde{Q}^2$
- mixed variables: combine information from leptonic and hadronic final state

➜ need full Monte-Carlo modelling

with partial fractioning, write: $R(l, l', k) = \frac{I}{k \cdot l} + \frac{F}{k \cdot l'} + \ldots$

- initial state radiation, $k \cdot l$ small for $\sphericalangle(e_{\text{in}}, \gamma) \to 0$
- final state radiation, $k \cdot l'$ small for $\sphericalangle(e_{\text{out}}, \gamma) \to 0$

narrow peaks, width $\simeq \sqrt{m_e/E_e}$: collinear or mass singularities

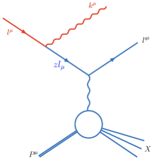
upon angular integration: large logarithm $\propto \frac{\alpha}{\pi} \log \frac{Q^2}{m_e^2} \simeq 10\,\%$

Note: $E^2_{\gamma,\max} \propto Q^2 \frac{1-x}{x}$

➜ large corrections at large Q^2 and at small x

➜ Radiation suppressed at small Q^2 and at large x,
large negative corrections from uncancelled virtual contributions

e.g., for initial-state radiation:
assume $k^\mu = (1-z)l^\mu$
➜ Radiator function



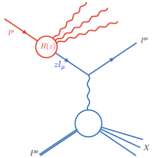
$$R_{\rm ISR} = \frac{\alpha}{2\pi}\frac{1+z^2}{1-z}\log\frac{Q^2}{m_e^2}$$

$(+\delta(1-z)$ from loops $\rightarrow$ +-distribution $1/(1-z)_+)$

$$\mathrm{d}\sigma_{\rm ISR} = \int \frac{\mathrm{d}z}{z} R_{\rm ISR}(z)\,\mathrm{d}\sigma_{\rm Born}(l^\mu \rightarrow zl^\mu)$$

(similar for final-state radiation)

Can be extended to include multi-photon emission:



$$R^{(2)}_{\rm ISR}(z) = \int_z^1 \frac{\mathrm{d}z'}{z'} R^{(1)}_{\rm ISR}(z') R^{(1)}_{\rm ISR}(z/z') + \ldots$$

Solution of evolution equations like DGLAP
Known at $O(\alpha^2)$ (complete) and partially at $O(\alpha^3)$

Corrections due to soft photons are universal

sum of real and virtual contributions: δ^{IR} (finite and gauge invariant)

$$1 + \delta^{\mathrm{tot}} = 1 + \delta^{\mathrm{IR}} + \delta^{\mathrm{fin}} \to \exp(\delta^{\mathrm{IR}})(1 + \delta^{\mathrm{fin}})$$

δ^{IR} contains $\log(E_\gamma^{\mathrm{max}})$ and $L_e = \log(m_e^2/Q^2)$:

$$1 + \frac{\alpha}{2\pi}(L_e - 1)\ln\frac{E_\gamma^{\mathrm{max}}}{E_e} + \ldots \to \left(\frac{E_\gamma^{\mathrm{max}}}{E_e}\right)^{\frac{\alpha}{2\pi}(L_e-1)} (1 + \ldots)$$

(in the $\gamma^* p$ cms: $E_\gamma^{\mathrm{max}} = \frac{1}{2}\sqrt{y(1-x)S}$, i.e. important at low y and large x)

Yennie, Frautschi, Suura, 1961

Radiation of (hard) photons ➜ shifted kinematic variables:

$$Q^2 = -(l - l')^2 \to \tilde{Q}^2 = -(l - l' - k)^2$$

and

$$x = \frac{Q^2}{2P \cdot (l - l')} \to \tilde{x} = \frac{\tilde{Q}^2}{2P \cdot (l - l' - k)}$$

Radiator function is folded with

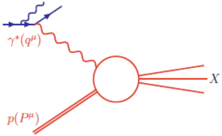
$$d\sigma(\tilde{x}, \tilde{Q}^2) \propto \frac{1}{\tilde{Q}^2}$$

➜ correction factor $d\sigma_{O(\alpha)}(x, Q^2)/d\sigma_{\mathrm{Born}}(x, Q^2)$ enhanced by $Q^2/\tilde{Q}^2$
Note: $\tilde{Q}^2 \ll Q^2$ possible: $\tilde{Q}^2_{\mathrm{min}} = \frac{x^2}{1-x} M_N^2$

➜ radiative tail, Compton peak

back to photoproduction

➜ γ-PDF



$r_c(y) = d\sigma_{O(\alpha)}(y)/d\sigma_{\rm Born}(y) - 1$, various x-ranges

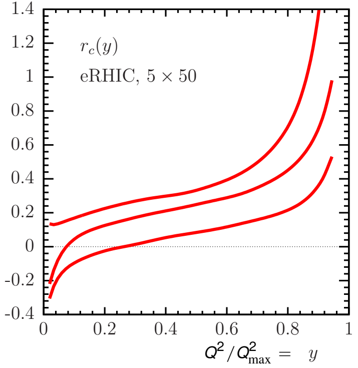
$e^- p$ at 5×50:

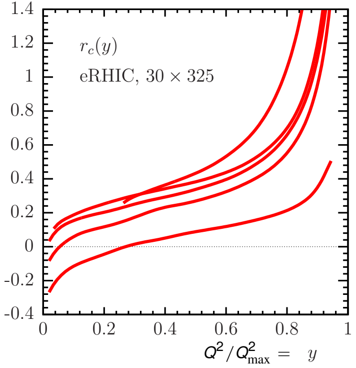
$10^{-3} \leq x_{Bj} \leq 10^{-2}$,
$10^{-2} \leq x_{Bj} \leq 10^{-1}$,
$0.1 \leq x_{Bj} \leq 0.4$,

$Q^2 > 1$ GeV^2

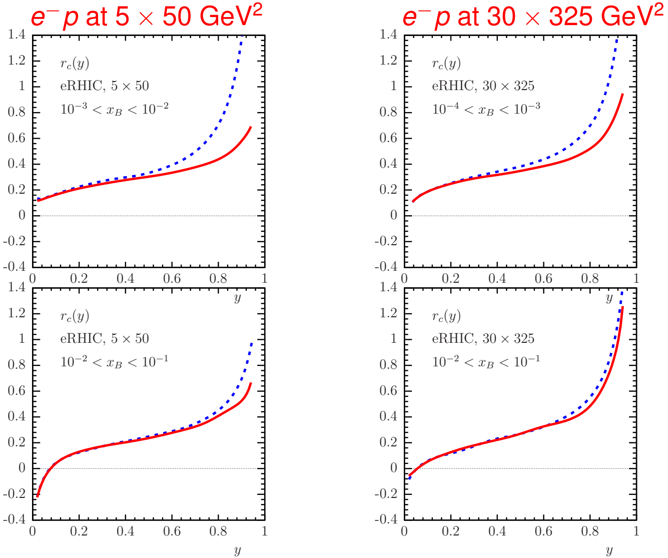
$e^- p$ at 30×325 GeV^2:

$10^{-5} \leq x_{Bj} \leq 10^{-4}$,
$10^{-4} \leq x_{Bj} \leq 10^{-3}$,
$10^{-3} \leq x_{Bj} \leq 10^{-2}$,
$10^{-2} \leq x_{Bj} \leq 10^{-1}$,
$0.1 \leq x_{Bj} \leq 0.4$,

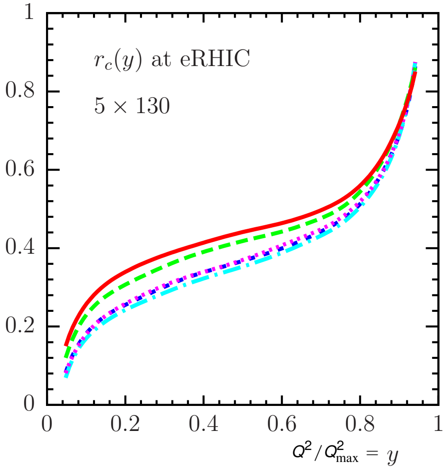




$r_c(y) = d\sigma_{O(\alpha)}(y)/d\sigma_{\mathrm{Born}}(y) - 1$, influence of a cut on $W_{had} > 1.4$ GeV



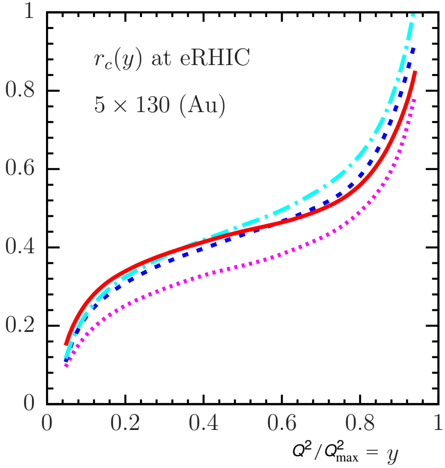
$r_c(y) = d\sigma_{O(\alpha)}(y)/d\sigma_{\mathrm{Born}}(y) - 1$
for e^-N at 5×130 GeV^2, $10^{-3} \leq x_{Bj} \leq 10^{-2}$,



different targets

- full red: Au, CTEQ61M with EPS09
- dashed green: Fe, CTEQ61M with EPS09
- dotted magenta: He, CTEQ61M with EPS09
- dash-dotted light-blue: Au, CTEQ61M, *p*PDF
- dashed blue: *p*, CTEQ61M

$r_c(y) = d\sigma_{O(\alpha)}(y)/d\sigma_{\mathrm{Born}}(y) - 1$
for e^-Au at 5×130 GeV^2, $10^{-3} \leq x_{Bj} \leq 10^{-2}$,

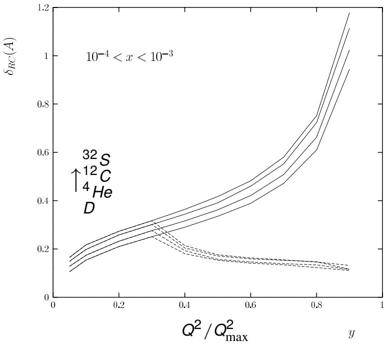


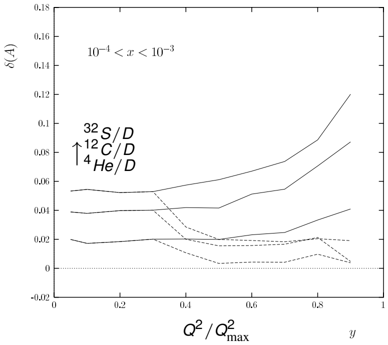
different nuclear PDFs

- full red: CTEQ61M with EPS09
- dash-dotted light-blue: CTEQ61M with EPS08
- dashed blue: CTEQ61M with EKS98
- dotted magenta: HKN

$\delta_{RC}(A) = d\sigma_{O(\alpha)}(A)/d\sigma_{\rm Born}(A) - 1$ for nuclei with $A = 2Z$ and
$\delta(A)$: corrections to the ratio $d\sigma_{O(\alpha)}(A)/d\sigma_{O(\alpha)}(D)$

Contribution from inelastic tail:

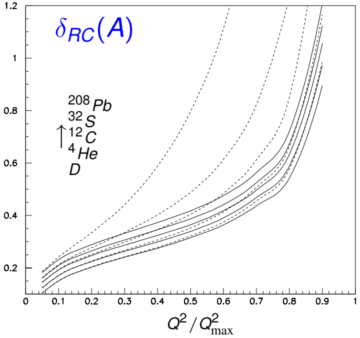




Full lines: leptonic variables without cuts (inelastic tail only)

dashed lines: with cuts on $E - p_z$ and $p_{T,\rm had}$ HERA workshop 1991

Contribution from elastic and quasi-elastic tails
(scattering off the nucleus or off individual nucleons)



Full lines: inelastic contribution
dashed lines: fully inclusive corrections, incl. elastic and quasi-elastic tails

➜ cut on mass of hadronic final state: $W_A^2 = Q^2(1-x)/x + M_A^2$

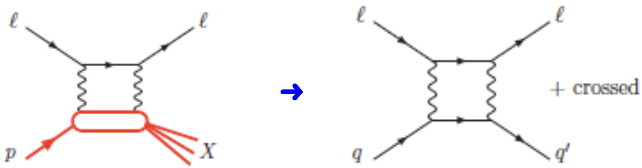
TWO-PHOTON EXCHANGE

(deep) inelastic *ep*:

factorized into

PDFs $\otimes$ 2γ-box

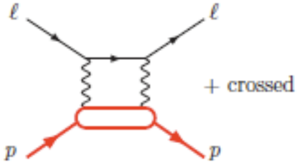
for *eq* scattering



Need interference of radiation from the lepton and the hadron to obtain IR-finite result

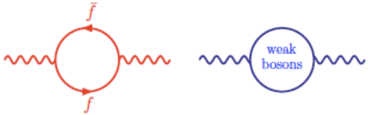
elastic *ep*:

- assume dominance of a few intermediate states: *p* + resonances
- assume factorization into GPDs $\otimes$ partonic scattering



Dedicated precision measurements to determine 2γ contributions: lepton charge asymmetry (*Re*) and lepton polarization asymmetry (*Im*)

Self energy diagrams of the exchanged boson (γ and Z)



$\propto \log \frac{Q^2}{m_f^2} \to O(10\,\%)$ small, $O(1\,\%)$

Photon self energy = vacuum polarization, absorbed in the running fine structure constant:

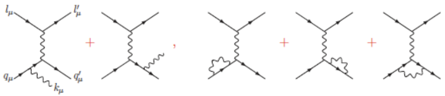
$$\alpha \to \alpha(Q^2) = \frac{\alpha}{1 - \Pi_\gamma(Q^2)}$$

Z-boson self energy: a small correction if written in terms of:

$$\frac{\alpha}{s_W^2 c_W^2} \to \frac{M_Z^2 G_\mu \sqrt{2}}{\pi} \frac{1 - \Delta r}{1 - \Pi_Z(Q^2)}$$

(with s_W^2, c_W^2: sin and cos of the weak mixing angle; G_μ the muon decay constant; Δr one-loop corrections to the muon decay: renormalization)

at large Q^2: DIS, parton model
emission of photons
like emission of gluons



infrared divergences (soft photons / gluons) cancel with loops, collinear emission gives rise to corrections $\frac{\alpha}{2\pi} \log m_q^2$, but quark masses are ill-defined
➜ factorize and absorb collinear divergences into parton distribution functions

$$d\sigma = \sum_f d\hat{\sigma}_f (1 + \delta_f(Q^2; m_q^2)) q_f(x)$$

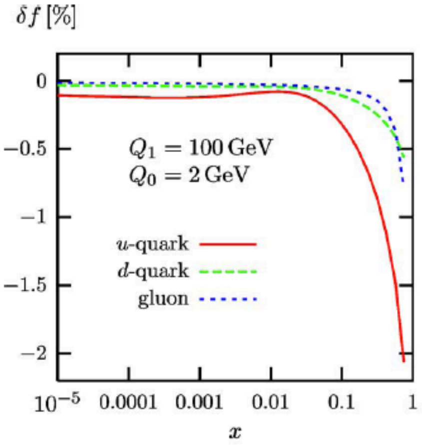
$$d\sigma = \sum_f d\hat{\sigma}_f (1 + \delta_f(Q^2; m_q^2)) q_f(x) = \sum_f d\hat{\sigma}_f \hat{q}_f(x, Q^2)$$

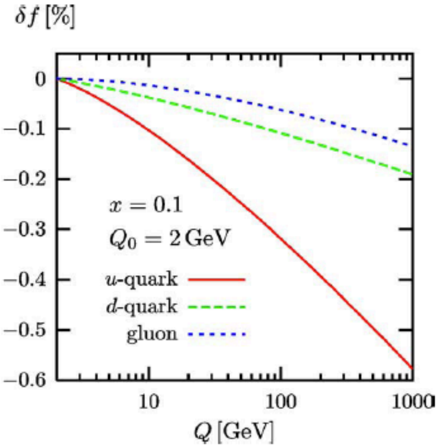
renormalized parton distribution functions

$$\hat{q}_f(x, Q^2) = (1 + \delta_f(Q^2; m_q^2)) q_f(x)$$

➜ modified scaling violations

well-known in QCD, $\overline{\text{MS}}$ factorization different charges of *u*- and *d*-quarks ➜ isospin-violating effect

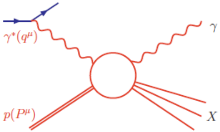




implemented in MRST2004 HS; Roth, Weinzierl, PLB590

relevant for precision predictions, e.g. W production at the LHC

Different point of view:
not as part of radiative corrections
but with observed photon



➜ DVCS: deeply virtual Compton scattering

➜ direct photons . . .

MONTE-CARLO IMPLEMENTATION

Classical analytical approach: Mo, Tsai
often used in 'private' implementations of experimental collaborations

Full Monte-Carlo approach:
HERACLES: complete electroweak corrections at $O(\alpha)$ (parton model) for NC and CC scattering at HERA, including polarization

Full event generation:
DJANGO: universal leptonic corrections at $O(\alpha)$, interface to QCD-based event generation of jets, parton showers, hadronic final state, includes models for low Q^2 behaviour: elastic tail, SOPHIA for low-mass hadronic final states

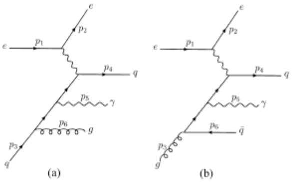
Specialized: VANDERHAEGHEN ET AL. QED $O(\alpha)$ corrections to virtual Compton scattering

... and many more tools: TERAD by Bardin et al.,
HEKTOR, KRONOS, FRANEQ, RADGEN...

MONTE-CARLO: FUTURE IMPROVEMENTS ?

More dedicated efforts needed to include:

- IR/soft photon exponentiation and radiator functions at $O(\alpha^2)$ ➜ multi-photon emission
- radiation from quarks:
  subtraction and modified parton showers
  including $q \to q + \gamma$
  (mixed QED+QCD corrections
  lepton-hadron interference and
  2-photon exchange)



- Simulation of the hadronic final state for scattering off heavy nuclei:
  nucleus break-up
  ➜ interface to dmpjet, etc.

SUMMARY

- High precision needs careful treatment of radiative corrections
- Closely related to experimental conditions

- Photon radiation provides access to interesting physics: direct photons, DVCS, γ-PDF, 2γ-box
- Impact on possible precision for electroweak effects: $\sin^2\theta_W$