Multiparticle production processes from the Information Theory point of view

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RHIC SCHOOL 04

4th BUDAPEST
WINTER SCHOOL ON
HEAVY ION PHYSICS

Dec. 1 - 3 (Wed - Fri),
Budapest, Hungary

Csontvary Kosztka, T. : Storm on the puszta

Content:

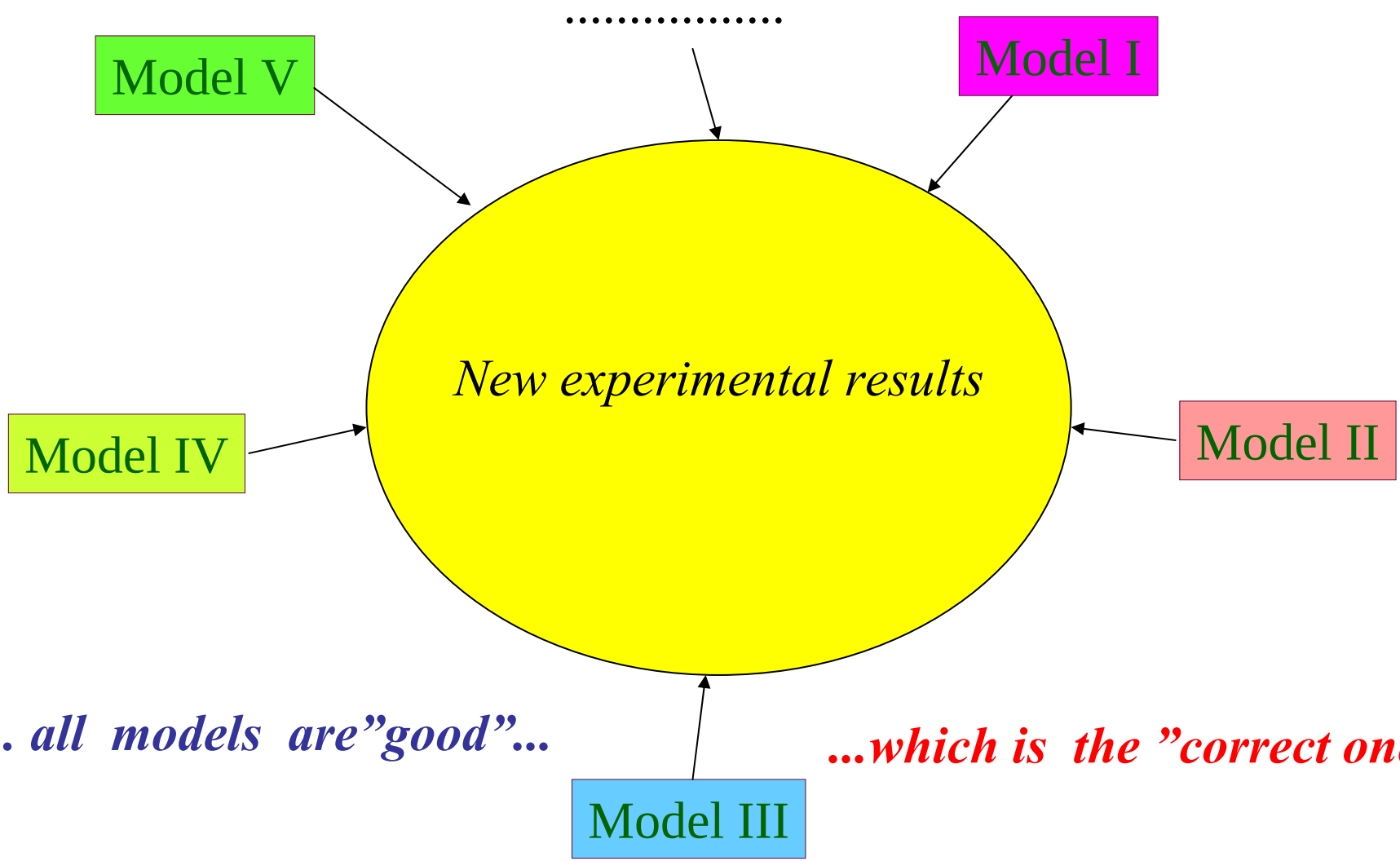
(*) **Why** ***and*** **When** ***of information theory in multiparticle production***

(*) **How** ***to use it:***

- ***"extensively"***
- ***"nonextensively"***

(*) ***Examples of applications to*** $\bar{p}p$, AA, e^+e^- ***collisions***

Why *and* When *of information theory in multiparticle production*



... all models are"good"...

...which is the "correct one"?...

- to some extend –

.....

All of them!

(*) To quantify this problem one uses notion of

information

(*) and resorts to

information theory

based on Shannon information entropy

$$S = - \Sigma p_i \ln p_i$$

where $\{p_i\}$ denotes probability distribution of quantity of interest

This probability distribution must satisfy the following:

(*) to be normalized to unity: $\Sigma p_i = 1$

(*) to reproduce results of experiment: $\Sigma p_i R_k(x_i) = \langle R_k \rangle$

(*) to maximize (under the above constraints) information entropy

$$\Rightarrow \quad p_i = \exp[-\sum \lambda_k \bullet R_k(x_i)]$$

with λ_k *uniquely given by the experimental constraint equations*

The Truth

is

here !

$$p_i = \exp[\, - \sum \lambda_k \bullet R_k(x_i) \,]$$

$$p_i = \exp[-\sum \lambda_k \bullet R_k(x_i)]$$

This is distribution which:

(*) tells us "the truth, the whole truth" about our experiment, i.e., it reproduces known information

(*) tells us "nothing but the truth" about our experiment, i.e., it conveys the least information (= only those which is given in this experiment, nothing else)

⇨ it contains maximum missing information

G.Wilk, Z.Włodarczyk, Phys. Rev. 43 (1991) 794

Notice:

If some new data occur and they turn out to disagree with

$$p_i = \exp[-\sum \lambda_k \bullet R_k(x_i)]$$

it means that there is more information which must be accounted for :

(a) either by some new $\lambda = \lambda_{k+1}$

(b) or by recognizing that system is nonextensive and needs a new form of $\exp(...) \rightarrow \exp_q(...)$

(c) or both .....

Some examples

(*) Knowledge of only $\langle n \rangle$ *and that particles are:* — *the most probable* $P(n)$ *is:*

- *distinguishable* ➡ *geometrical (Bose-Einstein)*
- *nondistinguishable* ➡ *Poissonian*
- *coming from* k *independent, equally strongly emitting sources* ➡ *Negative Binomial*

(*) Knowledge of $\langle n \rangle$ *and* $\langle n^2 \rangle$ ➡ *the most probable* $P(n)$ *is Gaussian*

➡ *Additional information converts the resultant P(n) from the most broad to the most narrow : geometrical* $(D \approx \langle n \rangle)$ ⇨ *Poissonian* $(D \approx \sqrt{\langle n \rangle})$

Example *(from Y.-A. Chao, Nucl. Phys. B40 (1972) 475)*

Question: what have in common such successful models as:

(*) multi-Regge model

(*) uncorrelated jet model

(*) thermodynamical model

(*) hydrodynamical model

(*) .....

Answer: ***they all share common (explicite or implicite) dynamical assumptions that:***

(*) only part of initial energy of reaction is used for production of particles ↔ existence of inelasticity of reaction, K~0.5

(*) transverse momenta of produced secondaries are cut-off ↔ dominance of the longitudinal phase-space

Suppose we hadronize mass M into N secondaries with mean transverese mass μ_T each in longitudinal phase space:

(*) we are looking for

$$f_N(y) = \frac{1}{N}\frac{dN}{dy} = \frac{1}{\sigma_N N}\int d^2 p_T \left[\frac{E d\sigma}{d^3 p}\right]$$

(*) by maximizing

$$S = -\int_{-Y_M}^{Y_M} dy \cdot f_N(y) \cdot \ln\left[f_N(y)\right]$$

(*) under conditions:

$$\int_{-Y_M}^{Y_M} dy \cdot f_N(y) = 1$$

$$\int_{-Y_M}^{Y_M} dy \cdot E \cdot f_N(y) = \mu_T \int_{-Y_M}^{Y_M} dy \cdot \cosh(y) \cdot f_N(y) = \frac{M}{N}$$

where

$$Y_M = \ln\left\{\frac{M'}{2\mu_T}\left[1 + \left(1 - \frac{4\mu_T^2}{M'^2}\right)^{1/2}\right]\right\}$$

As most probable distribution we get

$$f_N(y) = \frac{1}{Z} \cdot \exp[-\beta\mu_T \cdot \cosh(y)]$$

where $Z = Z(M, N, \mu_T) = \int_{-Y_M}^{Y_M} dy \cdot \exp[-\beta\mu_T \cdot \cosh(y)]$

and $\beta = \beta(M; N, \mu_T)$ *is obtained by solving equation:*

$$\int_0^{Y_M} dy \left[\cosh(y) - \frac{M}{\mu_T N} \right] \cdot \exp[-\beta\mu_T \cdot \cosh(y)] = 0$$

Notice: ***Z and β are not free parameters, therefore this is not a thermal model!***

From: G.Wilk, Z.Włodarczyk, Phys. Rev. 43 (1991) 794

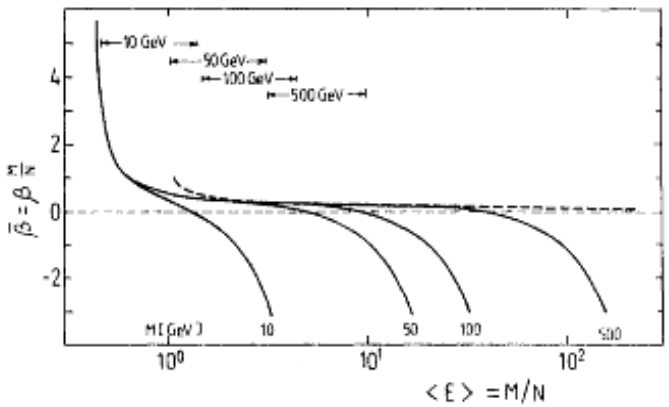


FIG. 1. The parameter $\bar{\beta}=\beta(M/N)$ in one-dimensional case [Eqs. (14), (16), and (19)] as a function of the mean available energy $\langle E \rangle = M/N$ for different energies M; $\mu_T = 0.4$ GeV. The dashed curve corresponds to $\bar{\beta}$ being a solution of the approximate Eq. (22). The regions of $\langle E \rangle$ corresponding to the $N_{\pm} = \bar{N}(M) \pm \sigma$ as given in Table I are also indicated.

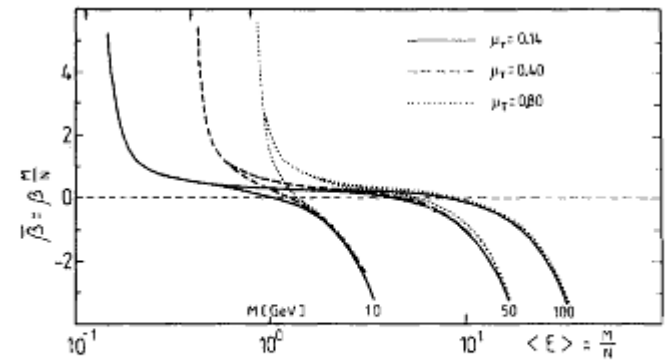


FIG. 2. Example of the sensitivity of $\bar{\beta}$ as defined in Fig. 1 on different choices of μ_T for different energies. Arrows indicate values of $\langle E \rangle$ $(=\mu_T)$ for which $\bar{\beta} \to +\infty$.

(*) for $N \to 2$ one has $\beta \to -\infty$ and $f(y)=(1/2)[\,\delta(y-Y_M)+\delta(y+Y_M)\,]$

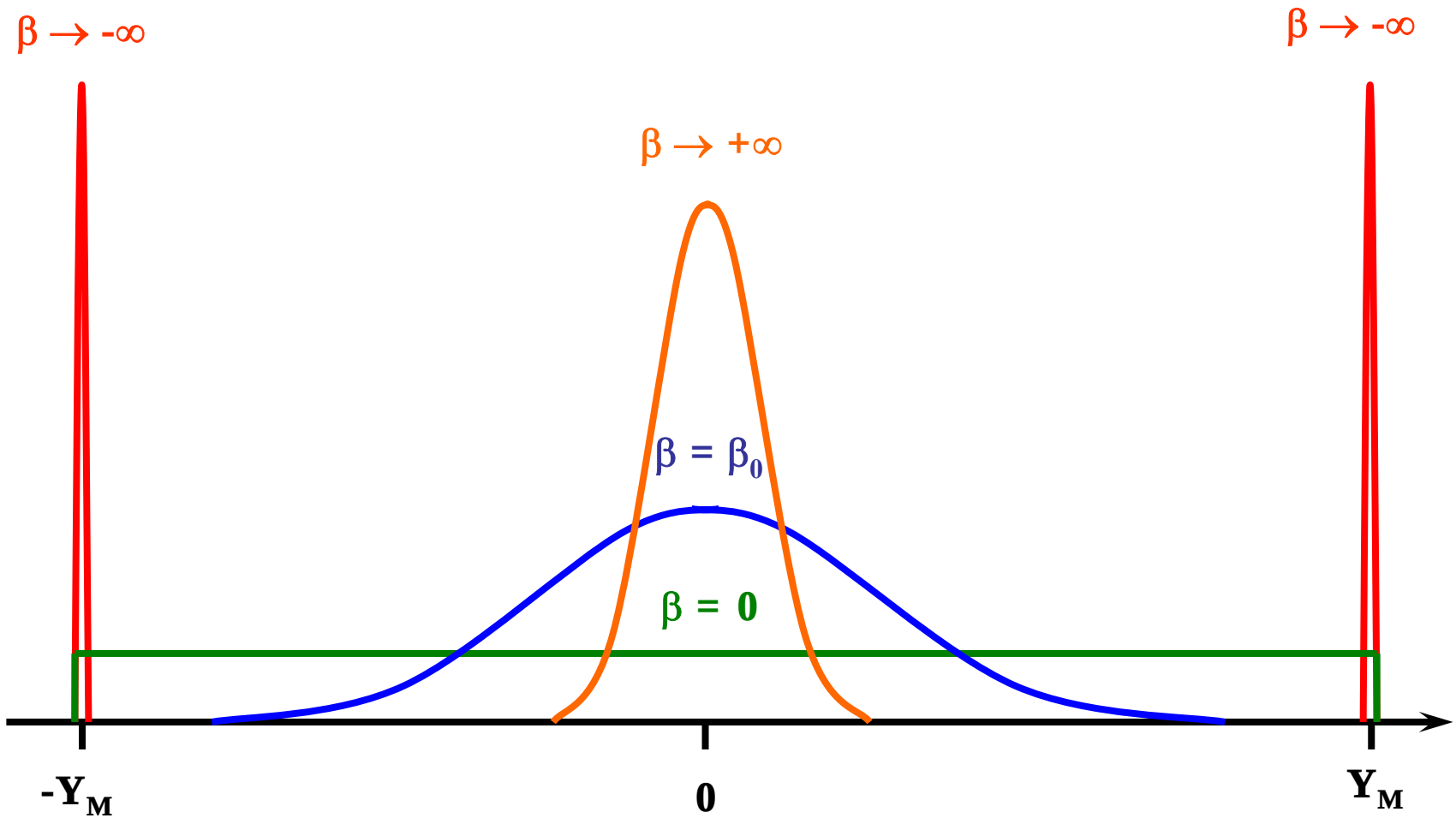
(*) for $N=N_0=N_0(M;N,\mu_T) \approx 2\ln[M/\mu_T] = 2\ln(N_{max})$ one has $\beta=0$ and $f(y) = const$

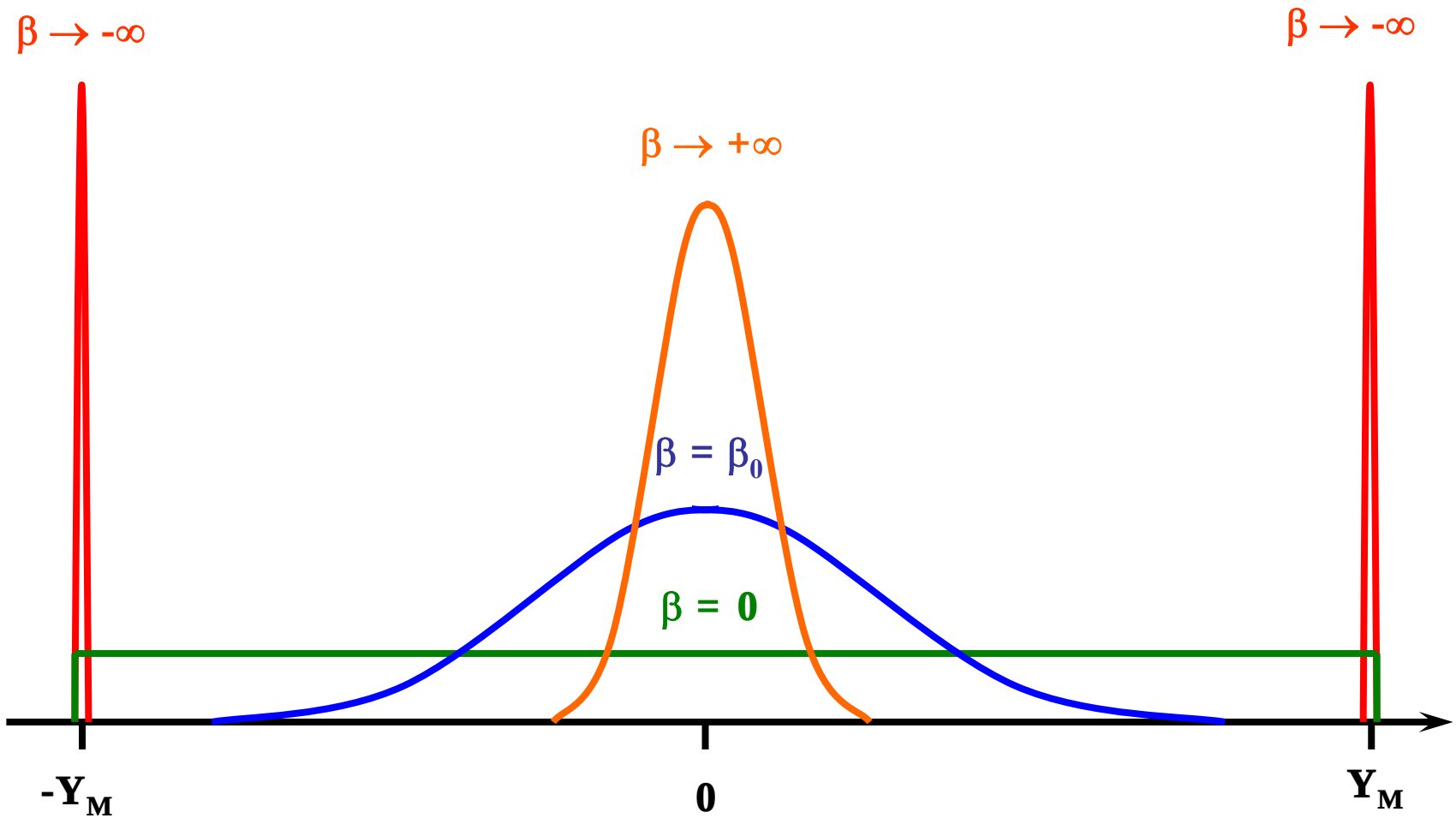
(*) for $N>N_0$ one has $\beta>0$ and

Scaling !

$$f_N(y) = \frac{1}{Z} \cdot \exp\left[-\bar{\beta}\, \frac{\mu_T \cdot \cosh(y)}{\langle E \rangle} \right]$$

(*) for $N \to N_{max}$ one has $\beta \to (3/2\mu_T)[1/(1-N/N_{max})]$ and $f(y) \to \delta(y)$


β → -∞
β → -∞
β → +∞
β = β0
β = 0
-YM
0
YM


β → -∞
β → -∞
β → +∞
β = β0
β = 0
-YM
0
YM

Other point of view .....

(*) **Fact:** *In multiparticle production processes many observables follow simple exponential form:*

$$f(X) \approx \exp[\ -X/\Lambda\ ]$$

➔ *"thermodynamics" (i.e., $\Lambda \approx T$)?*

(*) **Reason:** *because:*

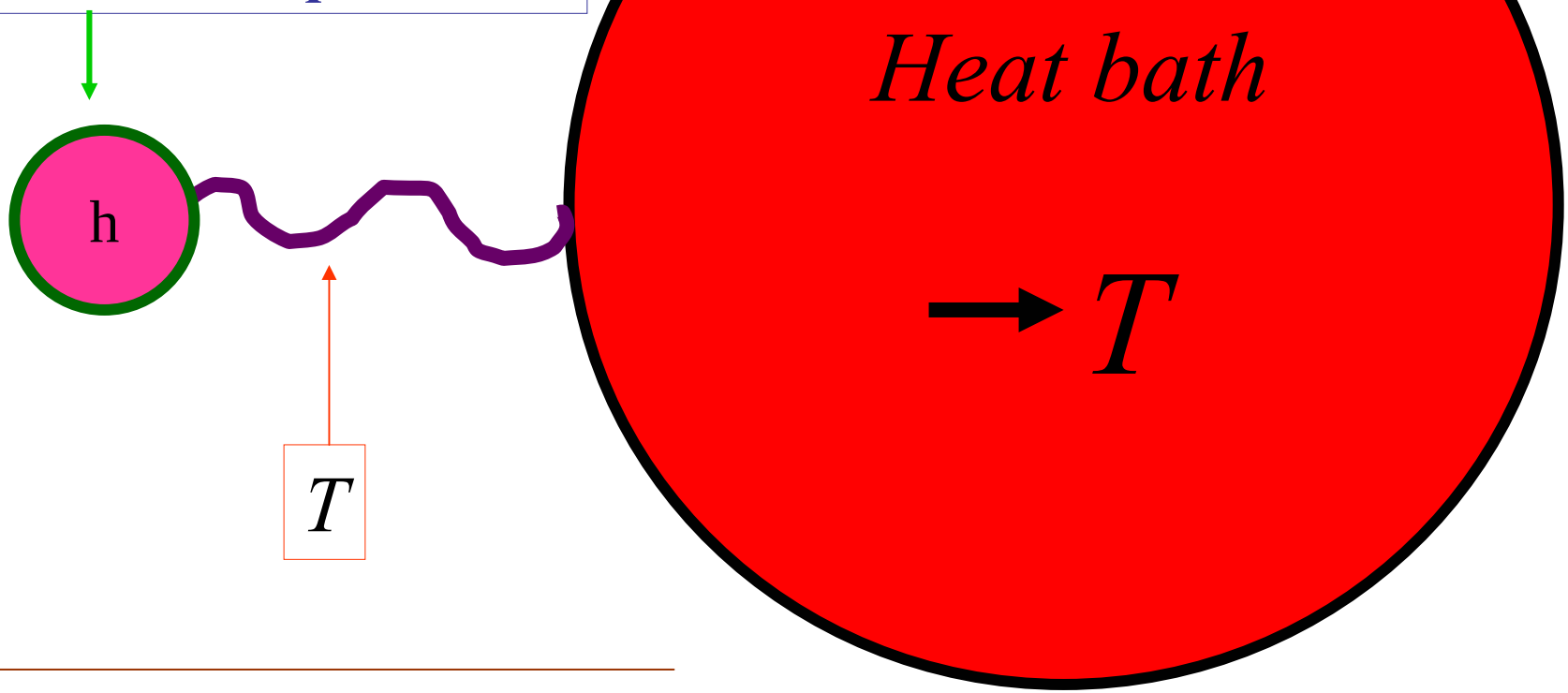
- *in an event many particles are produced*
- *but only a part of them is registered*
- *out of which usually only one is analysed (inclusive distributions of many sorts...)*

➔ *the rest acts therefore as a kind of heat bath*

N-particle system ⇒

N-1 particle "heath bath"

and 1 observed particle



L.Van Hove, Z.Phys. C21 (1985) 93,
Z.Phys. C27 (1985) 135.

(*) Nonextensivity – what it is and why to bother about it ...more...

However, in real life:

(*) in "thermodynamical" approach :

one has to remember tacit assumptions of infinity and homogenity concerning introduction of the "heath bath" concept, otherwise it will not be characterised by only one parameter - the "temperature" T

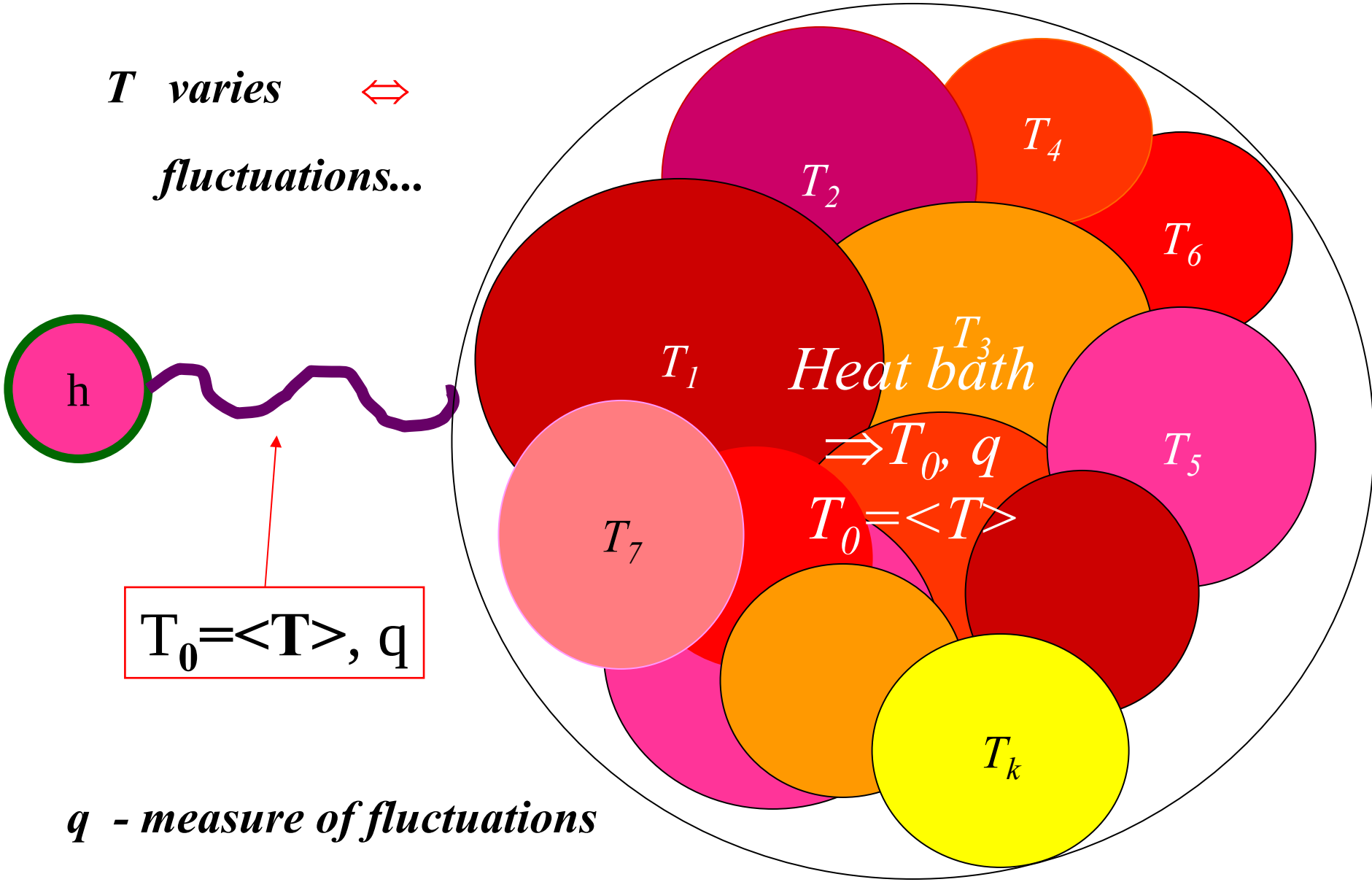
(*) in information theory approach :

one has to remember that there are other measures of information (other entropy functionals) possible

(*) in both cases it means departure from the simple exponential form as given above

(*) Nonextensivity – its possible origins .... "thermodynamics"

T* varies ⇔ *fluctuations...



***q* - measure of fluctuations**

(*) Nonextensivity – its possible origins .... information theory approach

... Other measures of information possible in addition to Shannon entropy accounting for some features of the physical systems, like:

(*) existence of long range correlations

(*) memory effect

(*) fractality of the available phase space

(*) intrinsic fluctuations existing in the system

(*)..... others ....

In particular one can use Tsallis entropy

$$S_q = - \Sigma(1-p_i^q)/(1-q)$$

$$\Rightarrow - \Sigma p_i \ln p_i \quad (\text{for } q \rightarrow 1)$$

(*) *This leads to:* **Non Extensive Statistics**

(*) *Tsallis entropy :*

$$S = \frac{1 - \sum_a p_a^q}{q - 1}$$

is nonextensive because

$$S(A + B) = S(A) + S(B) - (q - 1) S(A) S(B)$$

➔ *q-biased probabilities:*

$$\tilde{p}_a = \frac{p_a^q}{\sum_a p_a^q}$$

q-biased averages:

$$E = \sum_a \tilde{p}_a E_a$$

$$N = \sum_a \tilde{p}_a N_a$$

Historical example:

(*) observation of deviation from the expected exponential behaviour

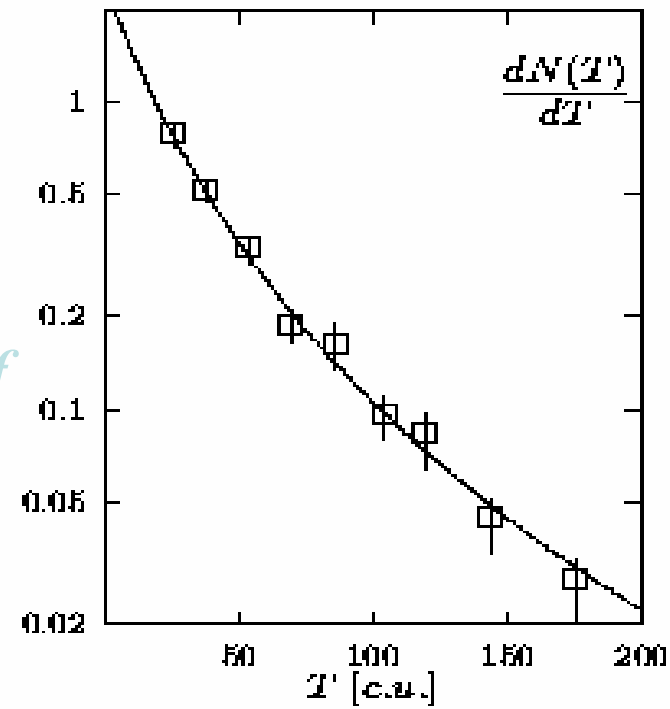
(*) successfully intrepreted [(*)] in terms of cross-section fluctuation:

$$\omega = \frac{<\sigma^2> - <\sigma>^2}{<\sigma>^2} \geq 0.2$$

(*) can be also fitted by:

$$\frac{dN}{dT} = const \cdot \exp\left(-\frac{T}{\lambda}\right) \Rightarrow$$

$$\frac{dN}{dT} = const \cdot \left[1-(1-q)\frac{T}{\lambda}\right]^{\frac{1}{1-q}}; q = 1.3$$



Depth distributions of starting points of cascades in Pamir lead chamber Cosmic ray experiment (WW, NPB (Proc.Suppl.) A75 (1999) 191

(*) WW, PRD50 (1994) 2318

(*) immediate conjecture:

q ⇦⇨ fluctuations present in the system

Summarizing: 'extensive' ⇔ 'nonextensive'

$$L = \exp\left(-\frac{x}{\lambda_0}\right) \Rightarrow L_q = \exp_q\left(-\frac{x}{\lambda_0}\right) = \left\langle \exp\left(-\frac{x}{\lambda}\right)\right\rangle$$

*where **q** measures amound of fluctuations*

and

<….> *denotes averaging over (Gamma) distribution in (1/λ)*

$$q = 1 \pm \omega = 1 \pm \frac{\left\langle \left(\frac{1}{\lambda}\right)^2 \right\rangle - \left\langle \frac{1}{\lambda} \right\rangle^2}{\left\langle \frac{1}{\lambda} \right\rangle^2}$$

(*) Possible origin of q: fluctuations present in the system...final

(*) Where gamma function comes from?

⇨ *WW, PRL 84 (2000) 2770; Chaos,Solitons and Fractals 13/3 (2001) 581*

(*) In general: Superstatistics (with other forms of function f)

⇨ *C.Beck, E.G.D. Cohen, Physica A322 (2003) 267*

(*) Fluctuations of temperature: $\lambda=T \Leftrightarrow$ ***if we allow for energy dependent***

$$T=T(E)=T_0+a(E-E_0)$$

with $a=1/C_V$***, then the equation on probability P(E) that a system A interacting with the heat bath A' with temperature T has energy E changes in the following way (q=1+a):***

$$d\ln[P(E)] \approx \frac{dE}{T} \Rightarrow P(E) \approx \exp\left(-\frac{E}{T}\right) \rightarrow$$

$$d\ln[P(E)] \approx \frac{dE}{T_0 + a(E-E_0)} \Rightarrow P(E) \approx \left[1-(1-q)\frac{E-E_0}{T_0}\right]^{\frac{1}{1-q}}$$

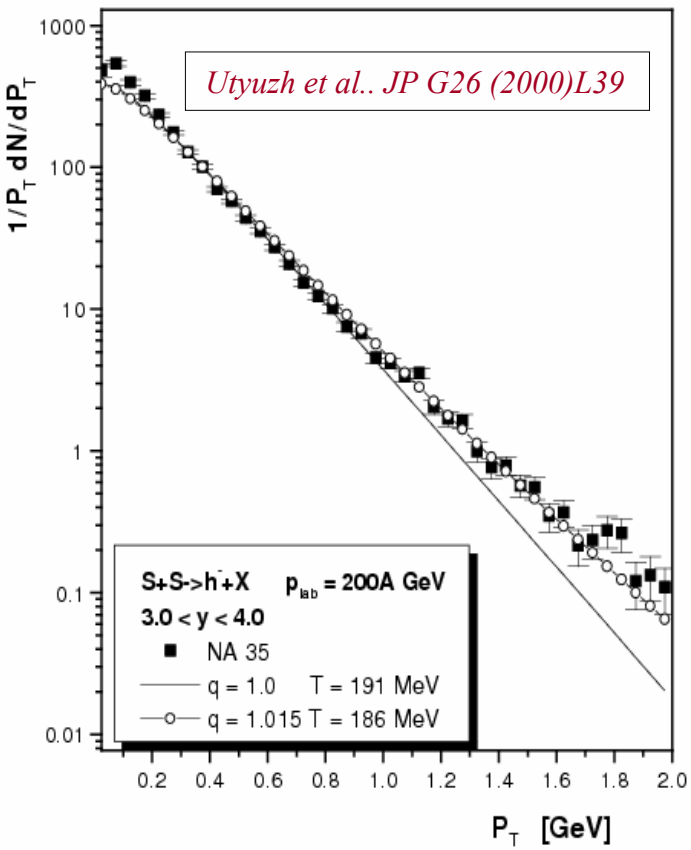


Fig.2

Some comments on T-fluctuations:

(*) Common expectation: slopes of p_T distributions $\Rightarrow$ information on T

(*) Only true for q=1 case, otherwise it is <T>, |q-1| provides us additional information

(*) Example: $|q-1|=0.015 \Rightarrow \Delta T/T \approx 0.12$

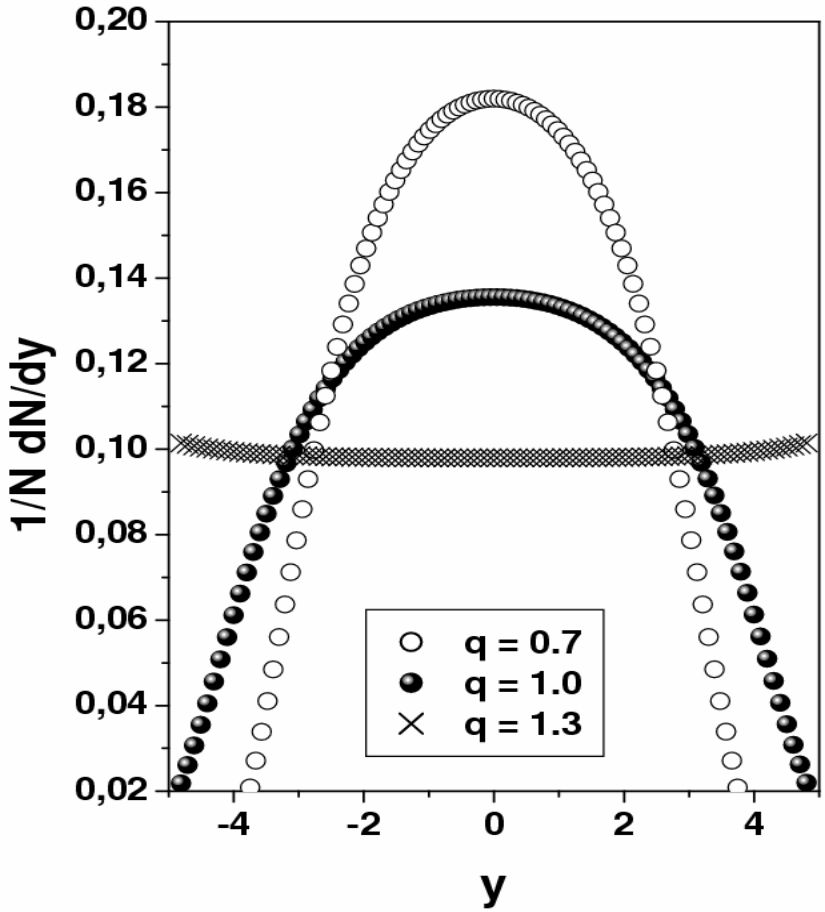
(*) Important: these are fluctuations existing in small parts of the hadronic system with respect to the whole system rather than of the event-by-event type for which $\Delta T/T = 0.06/\sqrt{N} \rightarrow 0$ for large N

Such fluctuations are potentially very interesting because they provide a direct measure of the total heat capacity of the system

$$\frac{\sigma^2(\beta)}{<\beta>^2} = \frac{1}{C} = \omega = q-1$$

Prediction: C ~ volume of reaction V, therefore q(hadronic)>>q(nuclear)

Rapidity distributions:



$$\frac{dN}{dy} = \frac{1}{Z_q} \cdot \left[1-(1-q)\cdot \beta_q \cdot \mu_T \cosh y\right]^{\frac{1}{1-q}}$$

$$Z_q = \int dy \left[1-(1-q)\cdot \beta_q \cdot \mu_T \cosh y\right]^{\frac{1}{1-q}}$$

Features:

(*) two parameters: $\beta_q = 1/T_q$ and q

$\Rightarrow$ shape and height are strongly correlated

(*) in usual application only $\beta = 1/T$

- but in reality [♦] $1/Z_{q=1}$ is always used as another independent parameter $\Rightarrow$ height and shape are fitted independently

(*) in q-approch they are correlated

[♦] T.T.Chou, C.N.Yang, PRL 54 (1985) 510; PRD32 (1985) 1692

NUWW PRD67 (2003) 114002

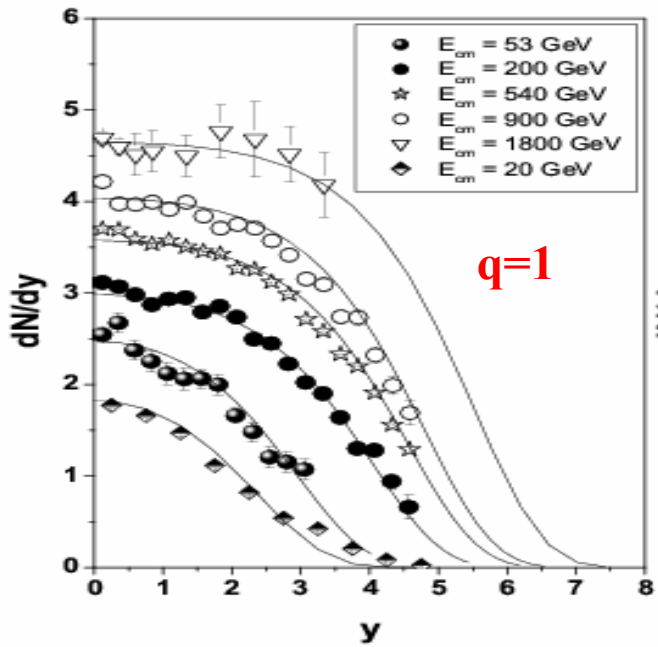


Figure 2:

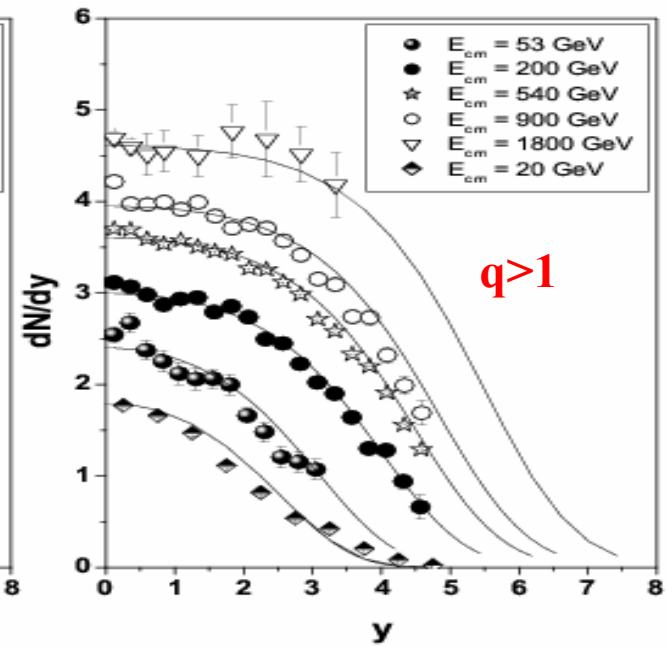


Figure 3:

Rapidity spectra for $p\bar{p}$ obtained by UA5 (53-900 GeV: R.Baltrusaitis et al., *Phys. Rev. Lett.* 52(1993) 1380) and Tevatron (1800 GeV: F.Abe et al., *Phys. Rev.* D41 (1990) 2330) fitted by formula with $q = 1$ (left panel) and with $q > 1$ (right panel). For completeness results for pp data at 20 GeV (C.De Marzo et al., *Phys. Rev.* D26 (1982) 1019 and D29 (1984) 2476) are also shown. The corresponding values of inelasticities K and κ_q and nonextensivity parameter q are listed in Table ??.

(*) Input: $\sqrt{s}$, μ_T, $\langle N_{charged} \rangle$
(*) Fitted parameter: q, q-inelasticity κ_q

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$$p_q(y) = \frac{1}{N}\frac{dN_q}{dy} = \frac{1}{Z_q}\exp_q\left(-\beta_q \mu_T \cosh y\right)$$

$$\int_{-Y_m}^{Y_m} dy \cdot \mu_T \cosh y \cdot \left[p_q(y)\right]^q = \frac{\kappa_q \sqrt{s}}{N} \Rightarrow \beta_q$$

$$K_q = \frac{N}{\sqrt{s}}\langle E \rangle_q = \frac{N}{\sqrt{s}}\int_{-Y_m}^{Y_m} dy \cdot \mu_T \cosh y \cdot \left[p_q(y)\right] \approx \frac{\kappa_q}{3-2q}$$

(*) Inelasticity K:= fraction of the total energy $\sqrt{s}$, which goes into observed secondaries produced in the central region of reaction
$\Rightarrow$very important quantity in cosmic ray research and statistical models

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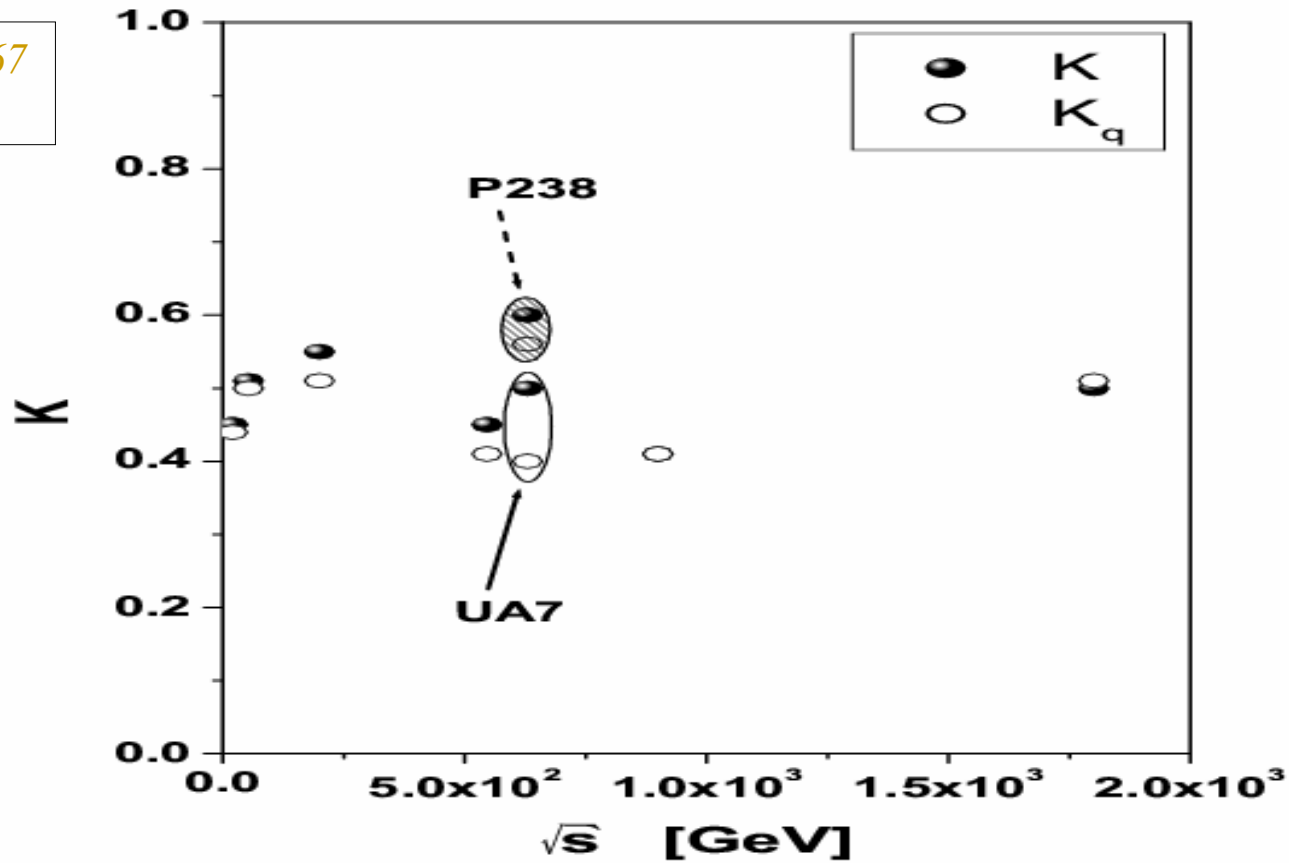

Figure 5:

Energy dependences of inelasticities obtained in extensive (K) and nonextensive (K_q). Notice that results for P238 data do not follow the overall trend.

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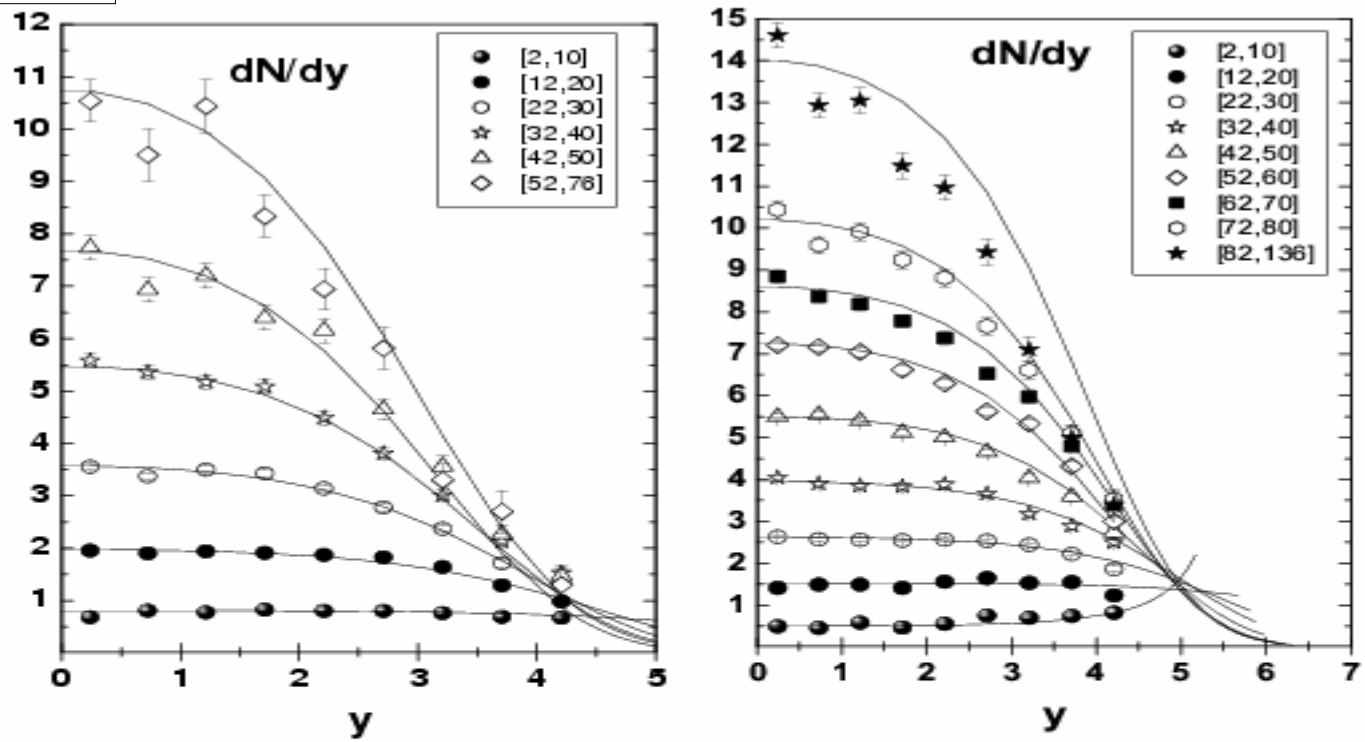

Figure 7:

Fits using nonextensive distributions $\frac{dN_s}{dy}$ for different rapidity bins for $\sqrt{s} = 200$ and 900 GeV (left and right panels, respectively, data from G.J.Alner et al. (UA5 Collab.), *Z. Phys.* C33 (1986) 1).

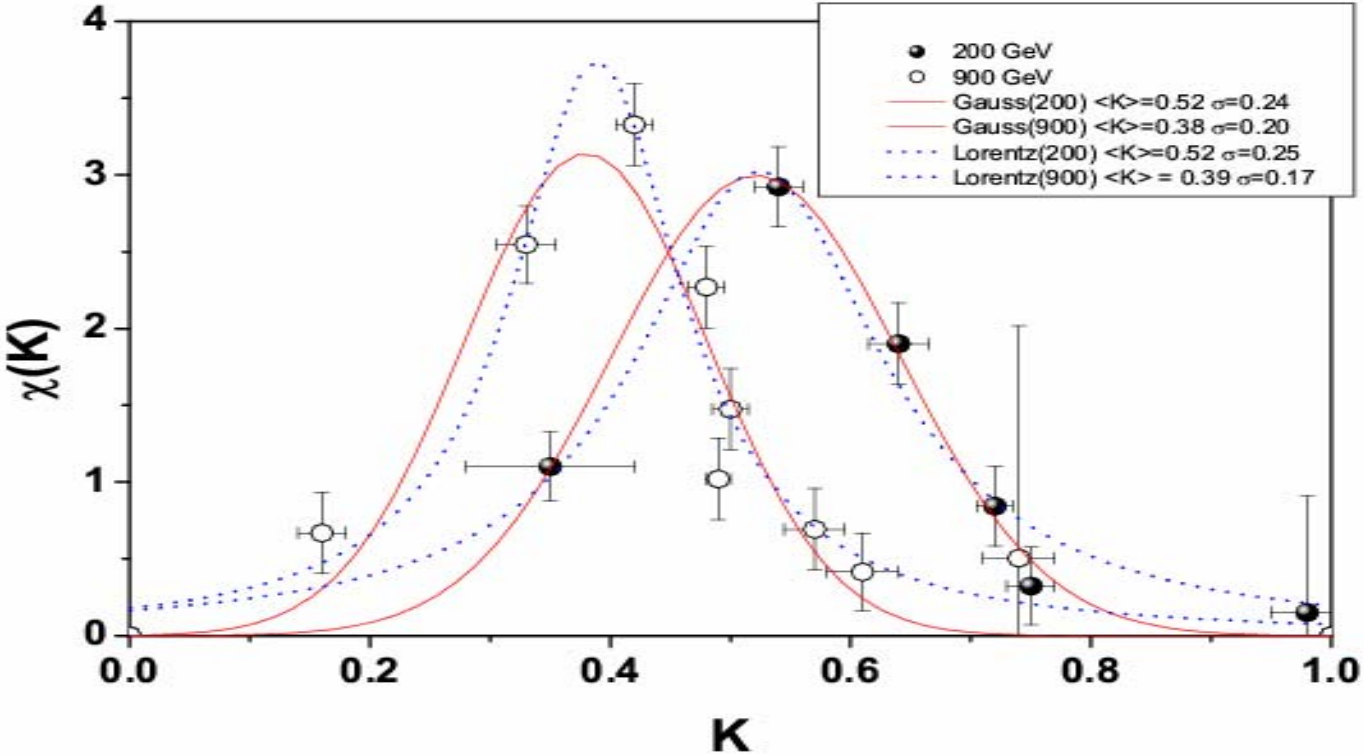

Figure 8:

Inelasticity distributions $\chi(K = K_q)$ (normalized to unity) obtained from data in rapidity bins for $\sqrt{s} = 200$ GeV and $\sqrt{s} = 900$ GeV. K_q is estimated from the q and κ_q parameters in Table ??. To guide the eyes obtained results were fitted by simple gaussian

$$\chi(K) \simeq \exp\left[-(K - \langle K \rangle)/(2a^2)\right]$$

and lorentzian

$$\chi(K) \simeq \sigma/[4(K - \langle K \rangle)^2 + \sigma^2]$$

formulas, respectively, with σ obtained for $K \in (0, 1)$).

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Navarra et al.., NC 24C (2001)

The only parameter here is q

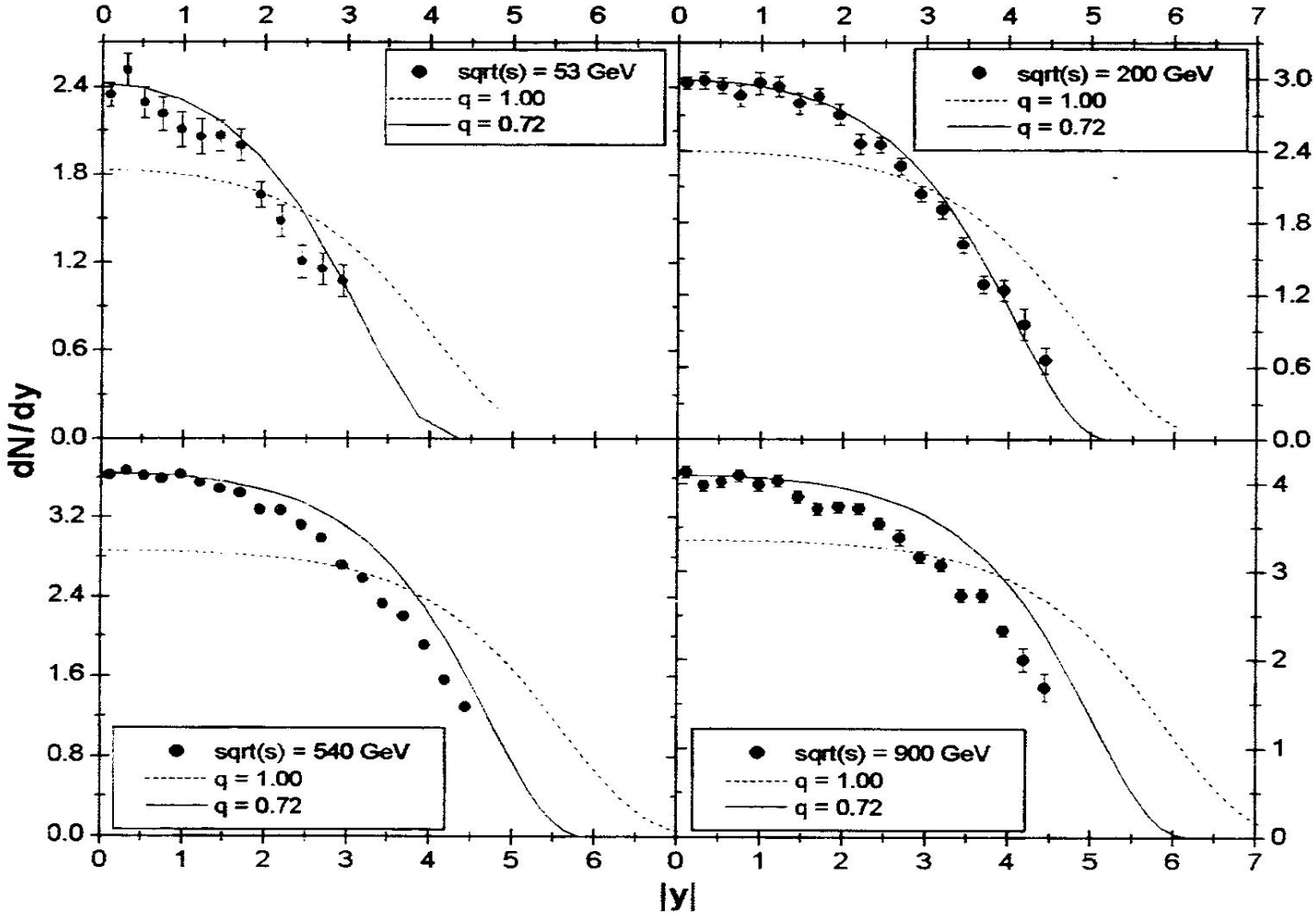

Fig. 1. – Comparison of eq. (5) with UA5 [10] data for $q = 1$ and $q = 072$.

(5) $$\frac{dN}{dy} = \frac{< N(s) >}{Z}\left[1-(1-q)\beta\mu_T \cosh y\right]^{\frac{1}{1-q}}; \beta = \beta(\sqrt{s}, \frac{3}{2} < N(s) >)$$

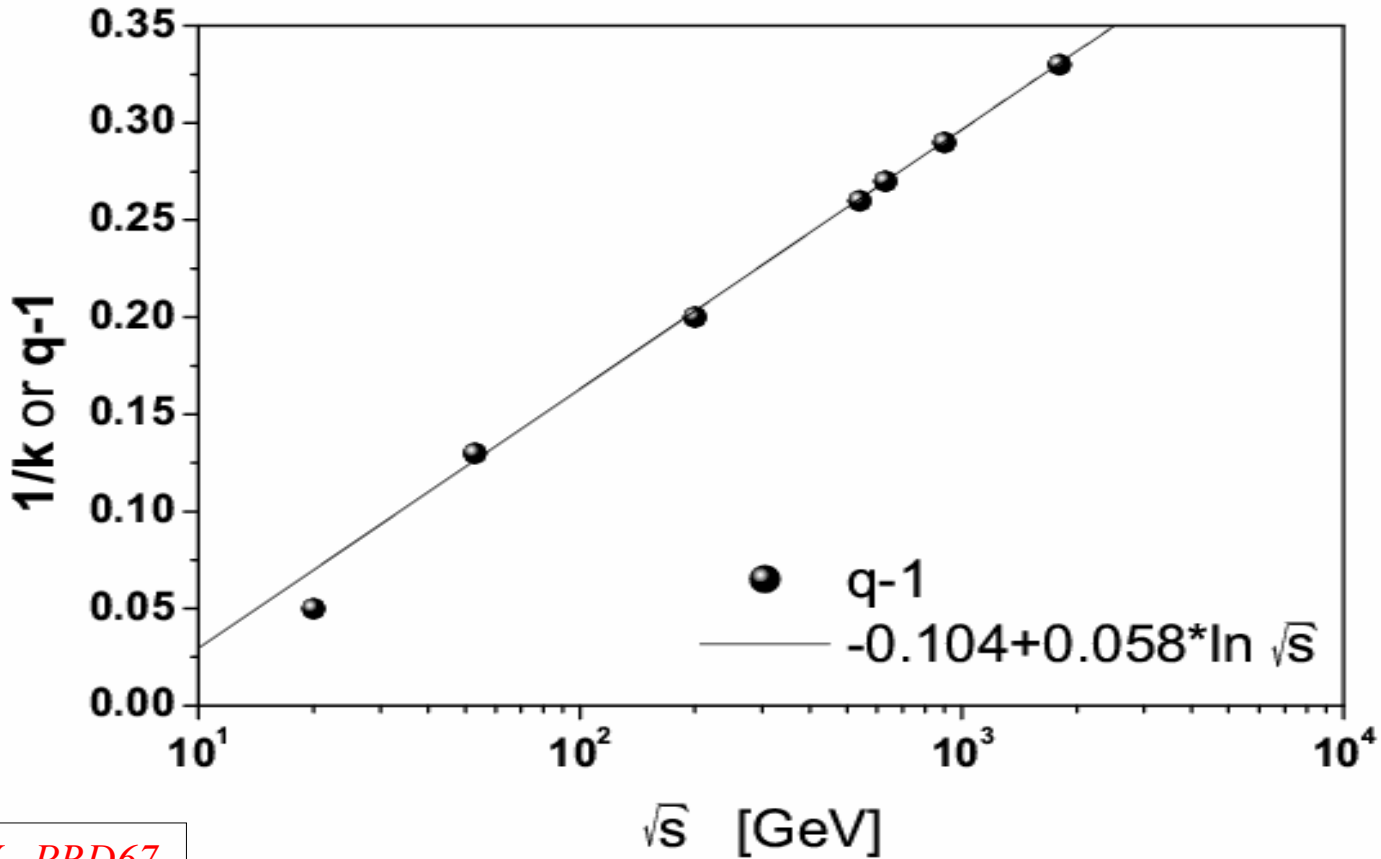

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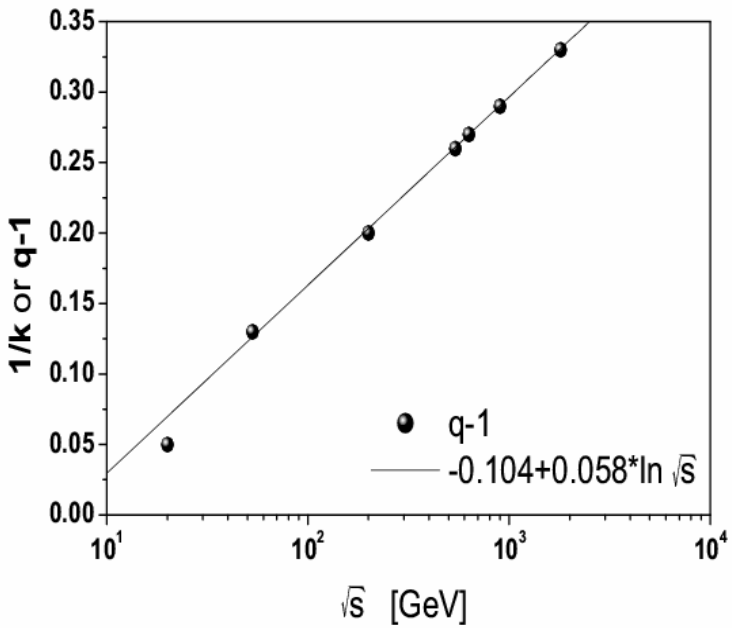
Figure 6:

The values of the nonextensivity parameter q obtained in fits shown here and listed in Table I compared with the values of the parameter k of Negative Binomial distribution fit to the corresponding multiplicity distributions (as given by C.Geich-Gimbel, *Int. J. Mod. Phys.* A4 (1989) 1527).

Possible meaning of parameter q in rapidity distributions



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(*) From fits to rapidity distribution data one gets systematically q>1 with some energy dependence

(*) What is now behind this q?

(*) y-distributions ⇔ 'partition temperature' $T \approx K \cdot \sqrt{s}/N$

(*) q ⇔ fluctuating T ⇔ fluctuating N

(*) Conjecture: q-1 should measure amount of fluctuation in P(N)

(*) It does so, indeed, see Fig. where data on q obtained from fits are superimposed with fit to data on parameter k in Negative Binomial Distribution!

Parameter q as measure of dynamical fluctuations in P(N)

(*) Experiment: P(N) is adequately described by NBD depending on <N> and k (k≥1) affecting its width:

$$\frac{1}{k} = \frac{\sigma^2(N)}{<N>^2} - \frac{1}{<N>}$$

(*) If 1/k is understood as measure of fluctuations of <N> then

$$P(N) = \int_0^\infty d\bar{n}\, \frac{\bar{n}^n \exp(-\bar{n})}{n!} \cdot \frac{\gamma^k \bar{n}^{k-1} \exp(-\gamma\bar{n})}{\Gamma(k)}$$

$$= \frac{\Gamma(k+n)}{\Gamma(1+n)\Gamma(k)} \cdot \frac{\gamma^k}{(\gamma+1)^{k+n}}$$

(P.Carruthers,C.C.Shih, Int.J.Phys. A4 (1989)5587)

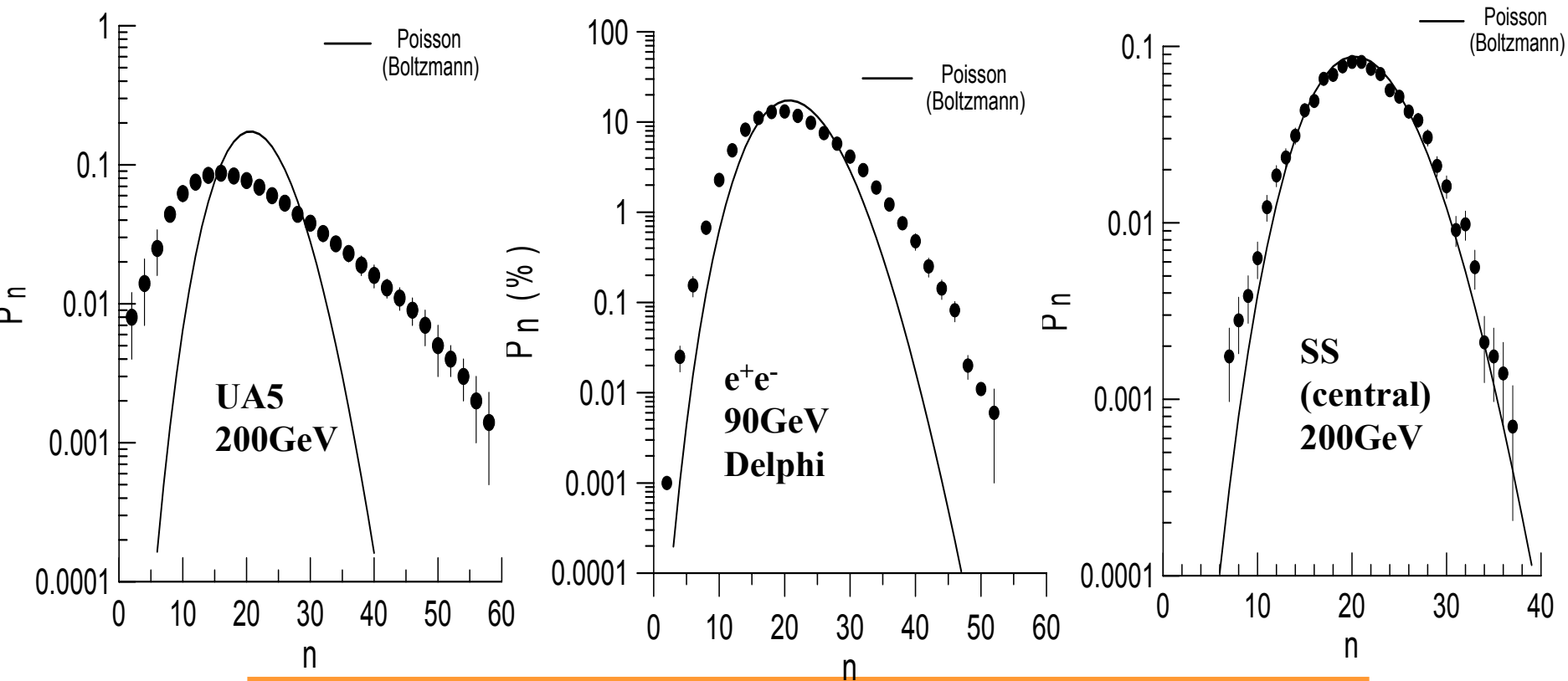
with $\gamma = \frac{k}{<\bar{n}>}$

$$\frac{1}{k} = D(\bar{n}) = \frac{\sigma^2(\bar{n})}{<\bar{n}>^2} = q - 1$$

(*) → *one expects: q=1+1/k what indeed is observed*

Multiplicity Distributions: (UA5, DELPHI, NA35)

Kodama et al..

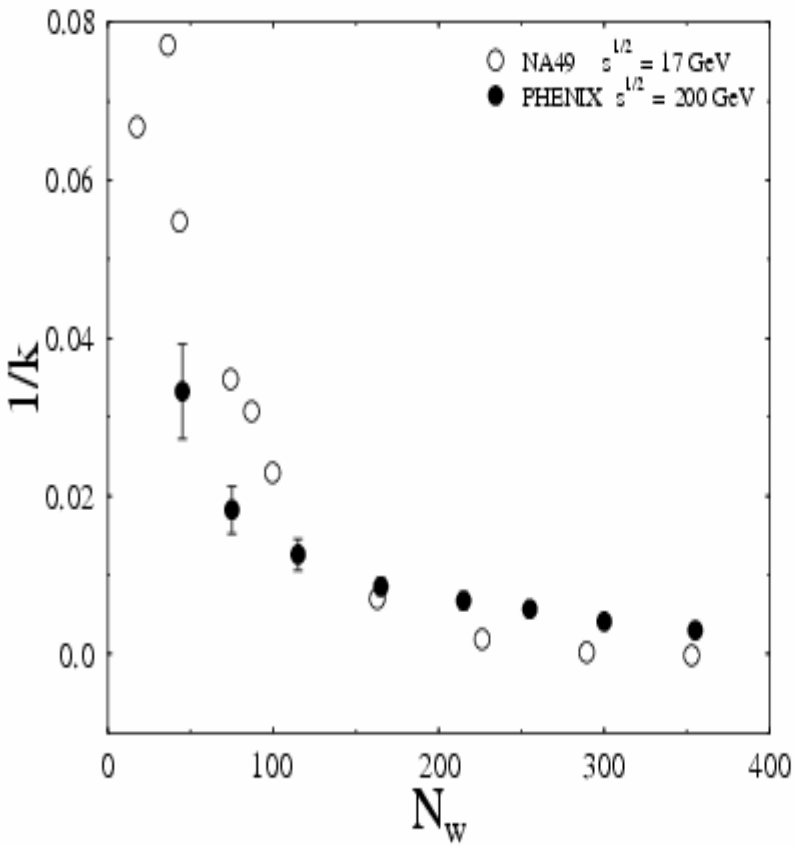


$\langle n \rangle =$	21.1;	21.2;	20.8
$D^2 = \langle n^2 \rangle - \langle n \rangle^2 =$	112.7;	41.4;	25.7

➔ ***Deviation from Poisson: 1/k***

$1/k = [D^2 - \langle n \rangle]/\langle n^2 \rangle =$ 0.21; 0.045; 0.011

Recent example from AA -(1) (RWW, APP B35 (2004) 819)



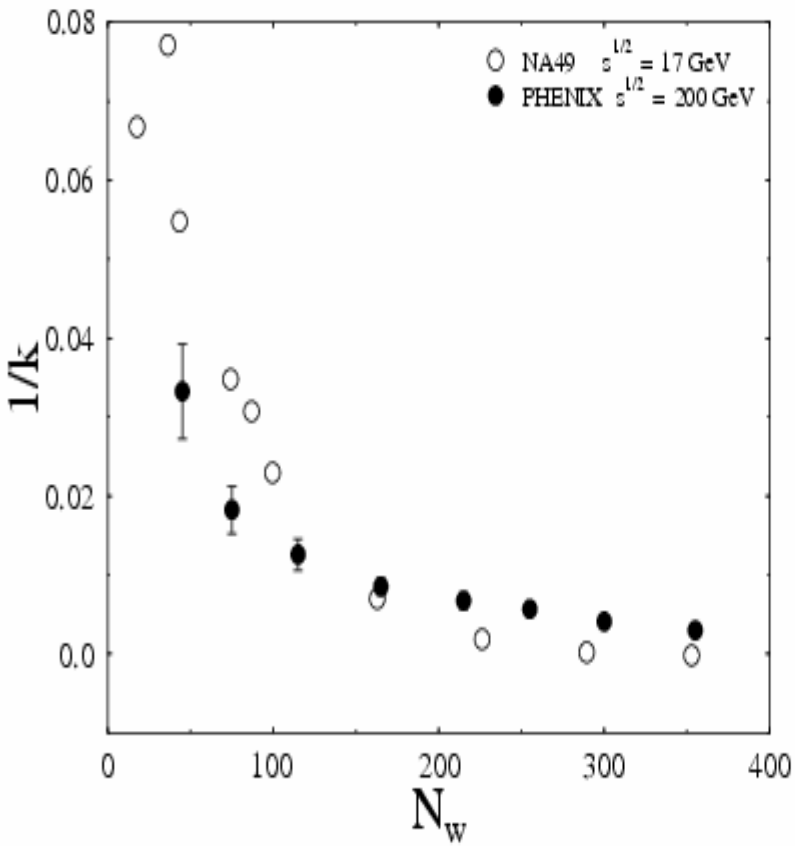
Dependence of the NBD parameter 1/k on the number of participants for NA49 and PHENIX data

With increasing centrality fluctuations of the multiplicity become weaker and the respective multiplicity distributions approach Poissonian form.

???

Perhaps: smaller $N_W \Rightarrow$ smaller volume of interaction $V \Rightarrow$ smaller total heat capacity $C \Rightarrow$ greater $q=1+1/C \Rightarrow$ greater $1/k = q-1$

Recent example from AA – (2) *(RWW, APP B35 (2004) 819)*



Dependence of the NBD parameter 1/k on the number of participants for NA49 and PHENIX data

In this case it can be shown that:

$$\frac{1}{k} = \frac{D^2(N)}{<N>^2} = R(q-1) \Rightarrow D(N) \approx \sqrt{R(q-1)}$$

$$R(q-1) \approx 0.33 \qquad (\leftarrow \text{ Wróblewski law })$$

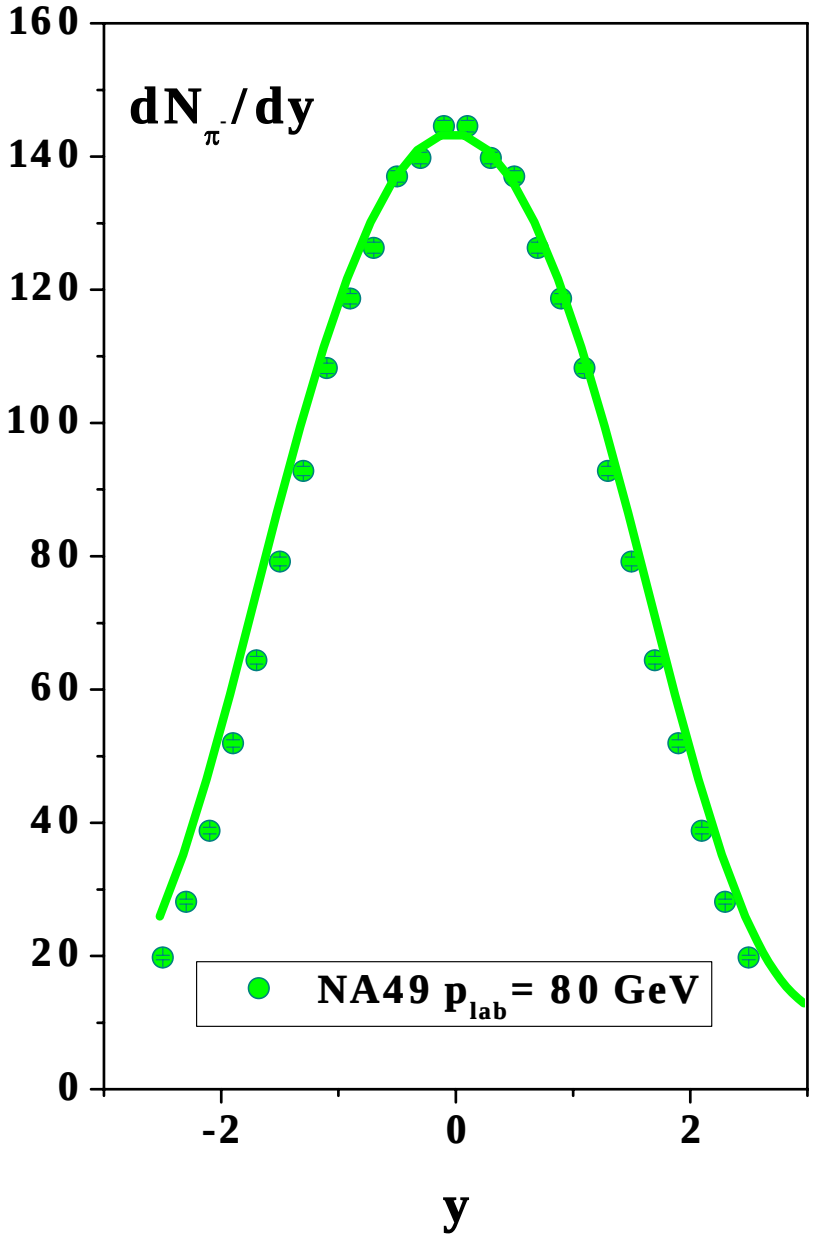
$$R = \frac{(\delta S)^2 / S^2}{(\delta E)^2 / E^2} \approx 0.56 \qquad (\leftarrow \text{for } p/e=1/3)$$

$\Rightarrow q \approx 1.59$ which apparently (over)saturates the limit imposed by Tsallis statistics: $q \leq 1.5$. For $q=1.5$ one has:

$0.33 \rightarrow 0.28$ (in WL)

or

$1/3 \rightarrow 0.23$ (in EoS)

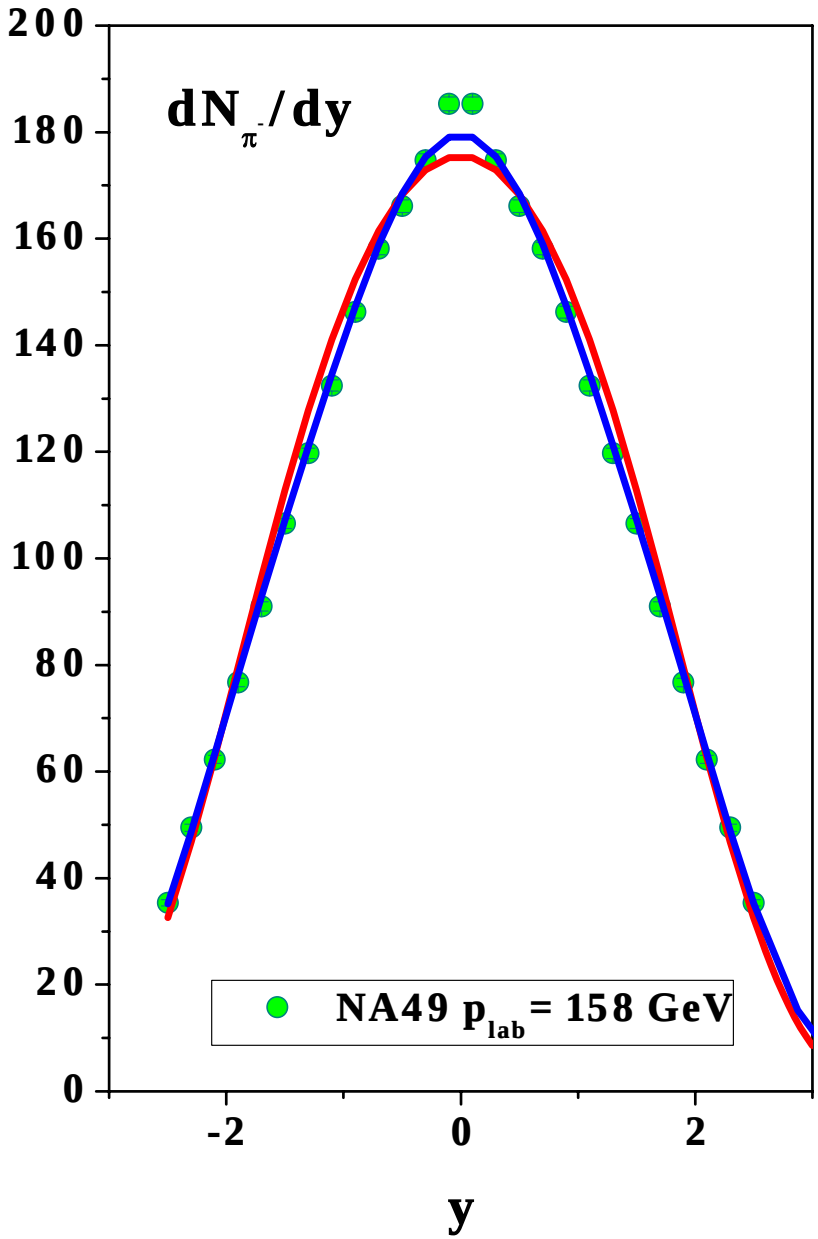


Example of use of MaxEnt method applied to some NA49 data for π^- production in PbPb collisions (centrality 0-7%) - (I) :

(*) the values of parameters used:

q=1.164

K=0.3



Example of use of MaxEnt method applied to some NA49 data for π^- production in PbPb collisions (centrality 0-7%) - (I) :

(*) the values of parameters used (red line)

q=1.2

K=0.33

(*) q=1, two sources of mass M=6.34 GeV located at |y|=0.83

➡ *this is example of adding new dynamical assumption*

Transverse momentu spectra from UA1 with TT=1/βT and qT equal to:

(0.134;1.095) for E=200 GeV

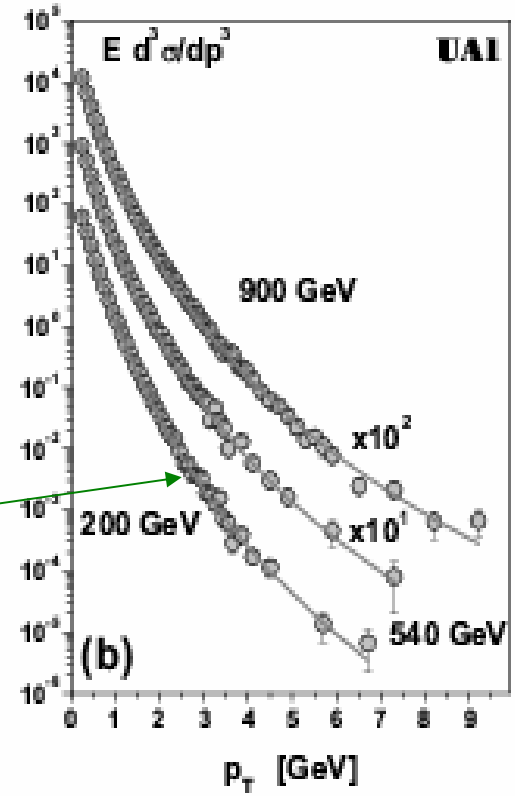
(0.135;1.105) 540

(0.14; 1.11) 900

Notice: ***qT < qL***

$$q = \frac{q_L T_L^2 + q_T T_T^2}{T^2} - \frac{T_L^2 + T_T^2}{T^2} + 1$$

⇨ $q \approx q_L$



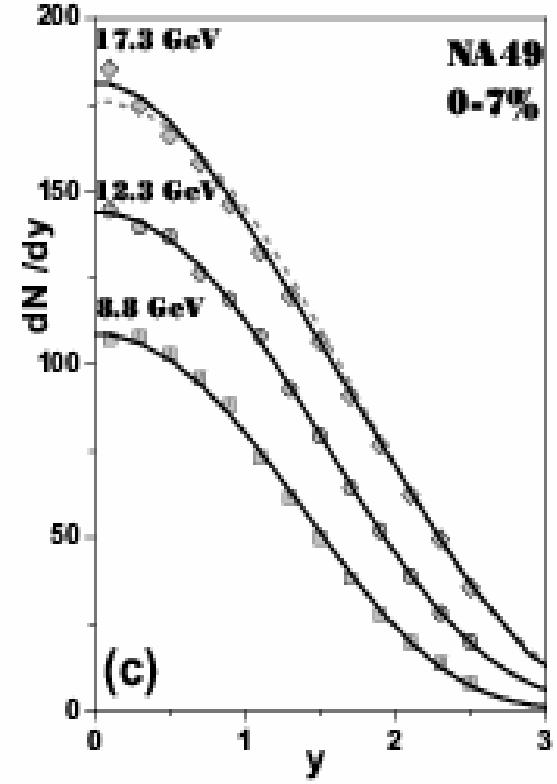
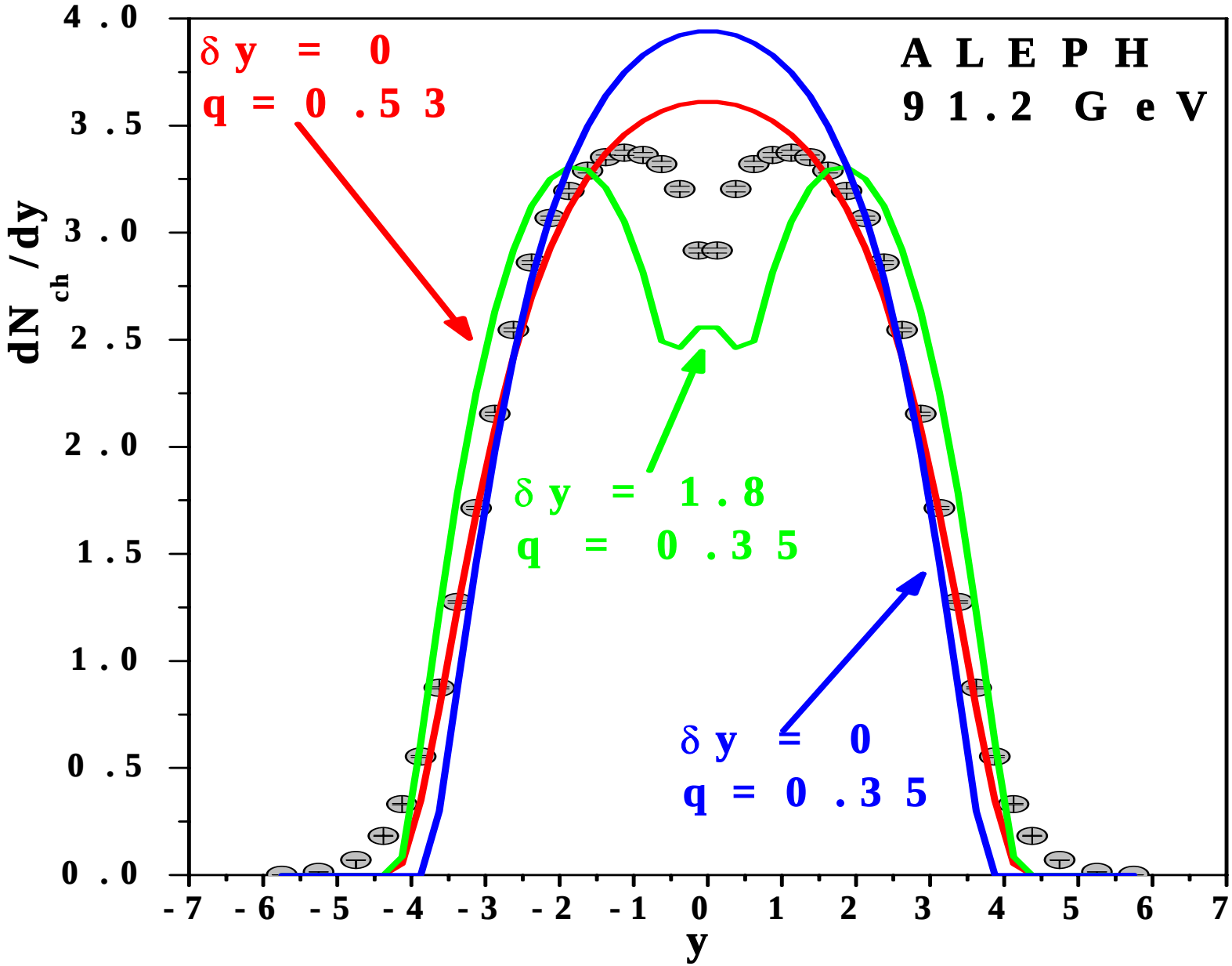
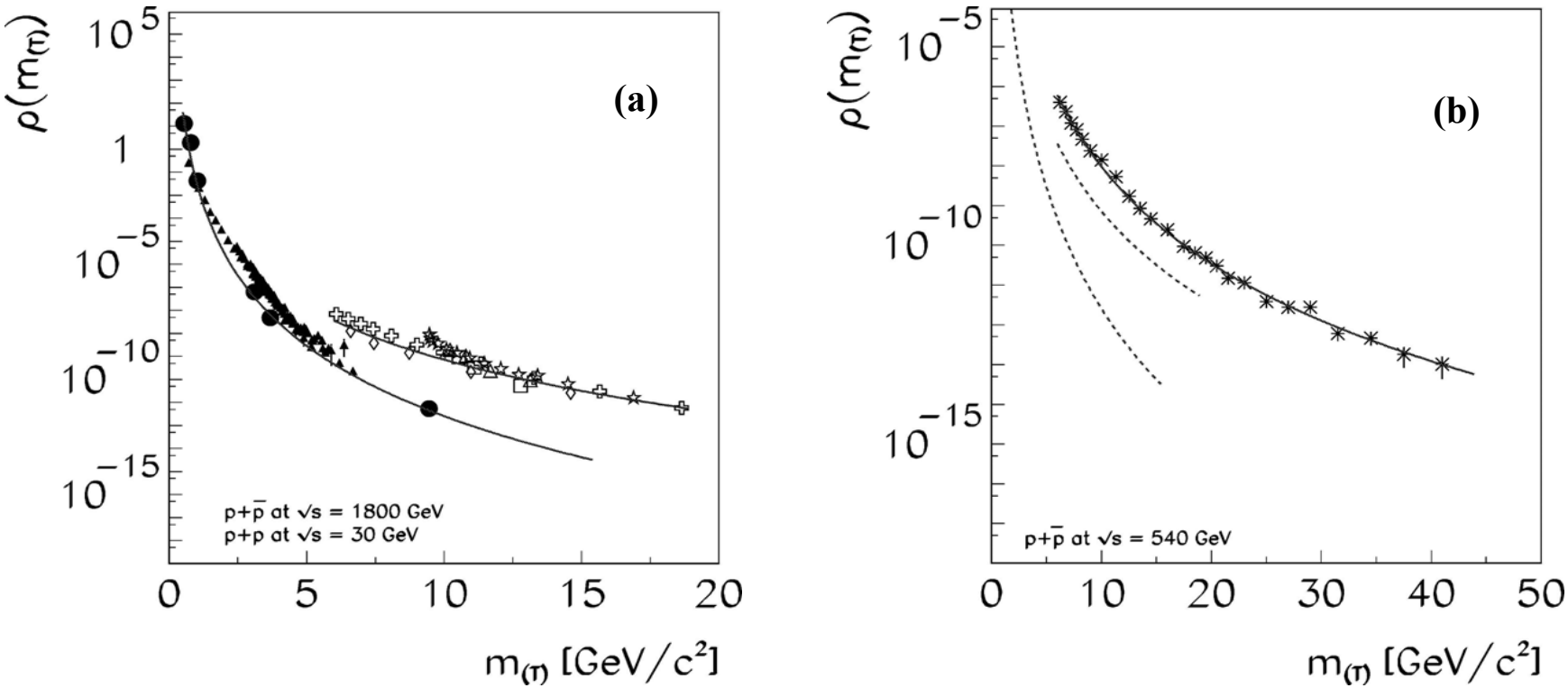


Fig. 1. Examples of nonextensive fits. (*a*) to rapidity spectra for charged pions produced in *pp* and $\bar{p}p$ collisions at different energies. (*b*) to p_T spectra from UA1 experiment. (*c*) to rapidity spectra for negatively charged pions produced in *PbPb* collisions at different energies (for the most central 0 – 7% interval of impact parameter of collision - NA49 data). The best fit for 17.3 GeV is for two-source case with $q = 1$ and $\Delta y = 0.83$. Total inelasticity in this case is $K = 0.58$.

NUWW, Physica A340 (2004) 467


δ y = 0
q = 0.53
ALEPH
91.2 GeV
δ y = 1.8
q = 0.35
δ y = 0
q = 0.35
dN_ch/dy
y
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
-7
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5
6
7

M.Gaździcki et al.., Phys. Lett. B517 (2001) 250 : Power law in hadron production (I)



(a) Mean multiplicity of neutral mesons (full dots) scaled to $\rho(m)=C\cdot m^{-P}$ and μ_T spectra of π^0 mesons (full triangles) scaled to $\rho(m_T)=C\cdot m_T^{-P}$ produced in pp interactions at $\sqrt{s}$=30 GeV; different quarkonia spectra at $\sqrt{s}$=1800GeV.

(b) The $\rho(m_T)=C\cdot m_T^{-P}$ spectra for pp at $\sqrt{s}$=540 GeV (solid line, dashed line show the corresponding fits to the results at 30 GeV and 1800 GeV).

P=7.7 (for quarkonia at 1800 GeV) to 10.1 (for neutral mesons at 30 GeV). In all cases considered the values of the constant C were the same (important!).

M.Gaździcki et al.., Phys. Lett. B517 (2001) 250 : Power law in hadron production (II)

(*) Authors: '...The Boltzmann function $\exp(-E^*/T)$ appearing in the standard statistical mechanics has to be substituted by a power-law function $(-E^*/T)^{-P}$.'

(*) What does it mean? Our proposition: look at the nonextensive statistics where $P \approx 1/(q-1)$ (or $q \approx 1+1/P = 1.1 - 1.13$) and treat this result as a new, strong imprint of nonextensivity present in multiparticle production processes.

(*) Can this result be explained in some dynamical way? Our answer: yes, provided we are willing to accept novel view of the hadron production process and treat it as formation of specific stochastic network (WW APPB 35(2004) 2141)

Summary (I)

(*) In many places one observes simple "exponential" or "exponential-like" behaviour of some selected distributions

(*) Usually regarded to signal some "thermal" behaviour they can also be considered as arising because insufficient information which given experiment is providing us with

(*) When treated by means of information theory methods (MaxEnt approach) the resultant formula are formally identical with those obtained by thermodynamical approach but their interpretation is different and they are valid even for systems which cannot be considered to be in thermal equilibrium.

(*) It means that statistical models based on this approach have more general applicability then naively expected.

Summary (II)

(*) Therefore: Statistical models of all kinds are widely used as source of some quick reference distributions. However, one must be aware of the fact that, because of such (interrelated) factors as:

- fluctuations of intensive thermodynamic parameters

- finite sizes of relevant regions of interaction/hadronization

- some special features of the „heath bath" involved in a given process

the use of only one parameter T in formulas of the type

$$\exp[-x/T]$$

is not enough and, instead, one should use two (... at least...) parameter formula

$$\exp_q[-x/T_0] = [1-(1-q)\,x/T_0]^{1/(1-q)}$$

with q accounting summarily for all factors mentioned above.

(*) In general, for small systems, microcanonical approach would be preferred (because in it one effectively accounts for all nonconventional features of the heat bath...) *(D.H.E.Gross, LNP 602)*

The end

Possible connection with networks ...(I)
(WW, APP B35(2004) 2141)

(*) Prompted by the observation of the power-like behaviour of the m_T spectra we have considered a possibility that :

they can be a reflection not so much of any special kind of equilibrium (resulting in some specific statistics) but rather of the formation process *resembling* the free network formation pattern discussed widely in the literature *(Barabasi et al.. RMP 74 (2002) 47; WW, APP B35 (2004) 871)*

(*) The power-law behaviour of spectra emerges naturally when:

- *hadrons are formed in process of $q\bar{q}$ linking themselves by gluons*
- *one identifies vertices in such network as ($q\bar{q}$) pairs and gluons as links*
- *one assumes that the observed m_T of hadrons reflects somehow the number of links **k** (i.e., gluons) in such network (α - diffusion parameter):*

$$m_T \approx k^{\alpha}$$

The distribution of number of links in the network has form:

$$P(k) \approx k^{-\gamma} \qquad (\text{with } \gamma \leq 5)$$

As result one gets:

$$P(m_T) \approx m_T^{-\beta}\ ; \quad \beta = 1+(\gamma-1)/\alpha$$

where $\gamma=1+\delta$ and δ is parameter defining the rate of growth of our network.

Possible connection with networks ...(II)

(*) The power-like distribution $P(k) \approx k^{-\gamma}$ *corresponds to large excitations where probability of connecting a new quark to the one already existing in the system depends on the number of the actual connections realised so far* $\Rightarrow$ *large number of connections results in large excitations* $\Rightarrow$ *large emission of gluons* $\Rightarrow$ *enhances chances of connection to such a quark.*

(*) If one assumes instead that the new quark attaches itself to the already existing one with equal probability (a case of small excitations, i.e., small p_T*) then one gets instead exponential distribution of links:*

$$P(k) \approx \exp[\, - k/\langle k\rangle \,]$$

which results in $P(m_T)$ *ranging from:*

$$P(m_T) \approx \exp[\, - m_T^2/\langle m_T^2\rangle \,] \quad \textit{for} \quad \alpha=1/2$$

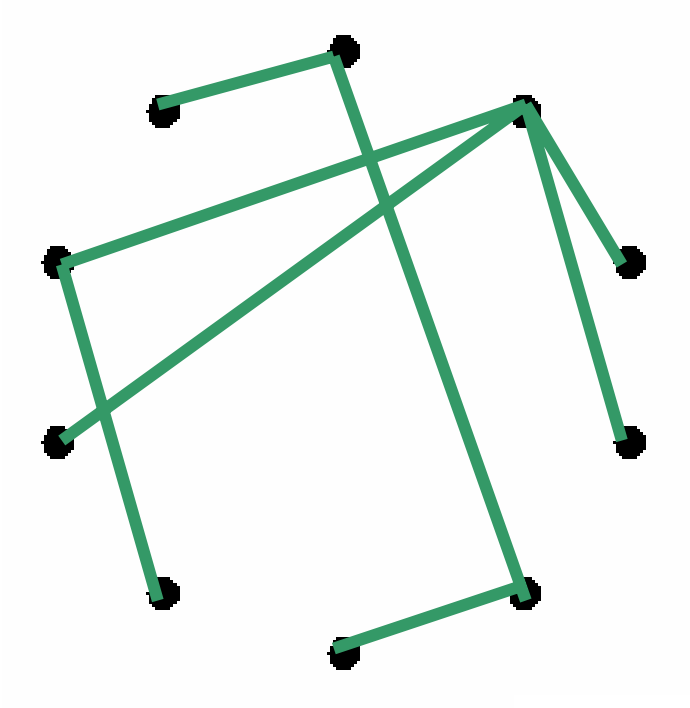
when the full fledged diffusion is allowed, to

$$P(m_T) \approx \exp[\, - m_T/\langle m_T\rangle \,] \quad \textit{for} \quad \alpha=1$$

where there is no diffusion (this would be the case of quarks located on the periphery of the hadronization region in which case they could interact only with interior quarks $\Rightarrow$ $m_T \approx k$*).*

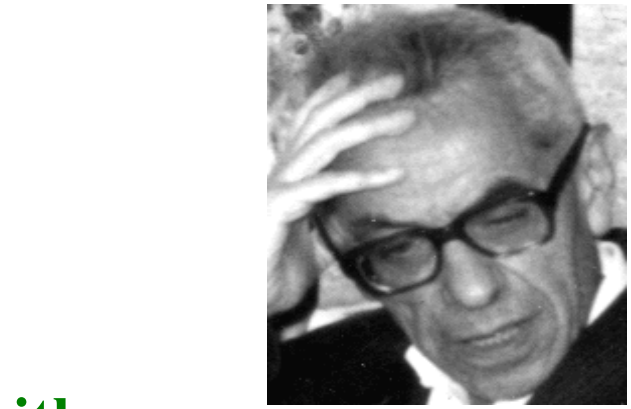
Insertion on networks - beginning

Erdös-Rényi model
(1960)



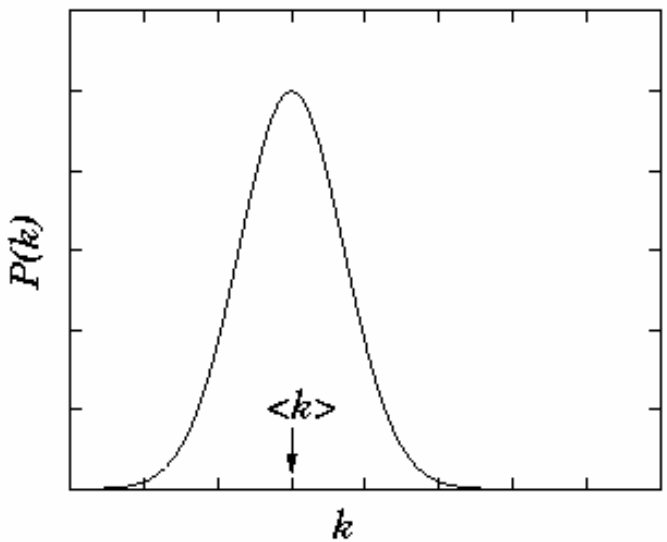
Connect with probability p

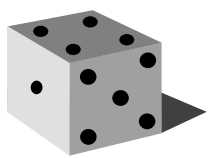
p=1/6
N=10
$\langle k \rangle \sim 1.5$



Pál Erdös
(1913-1996)

Poisson distribution

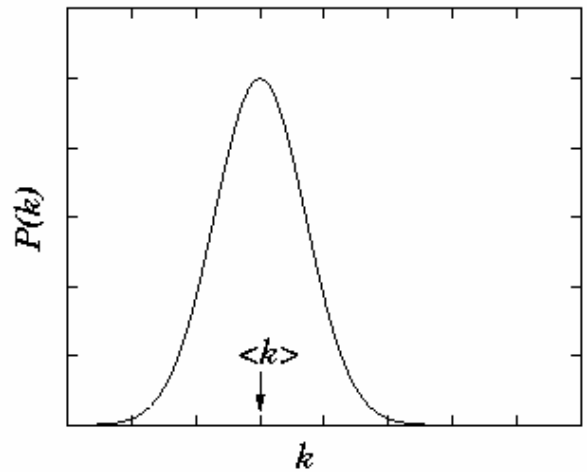




- Democratic

- Random

What did we expect?



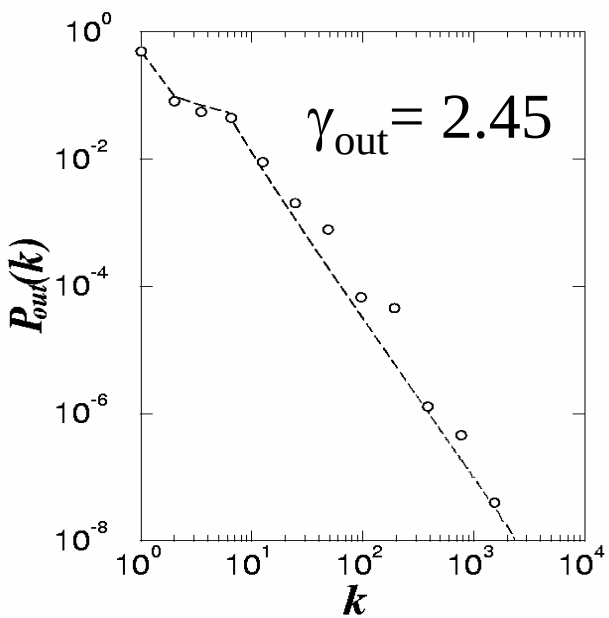
$\langle k \rangle \sim 6$

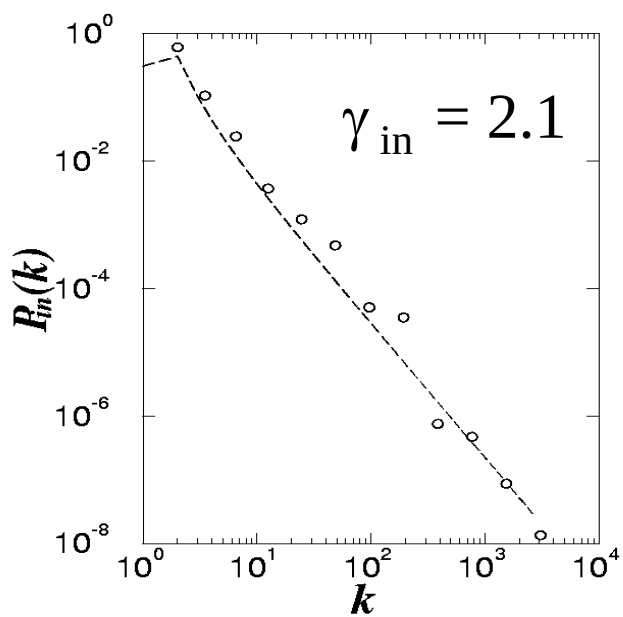
$P(k=500) \sim 10^{-99}$

$N_{WWW} \sim 10^{9}$

$\Rightarrow N(k=500) \sim 10^{-90}$

We find:





$P(k=500) \sim 10^{-6}$

$N_{WWW} \sim 10^{9}$

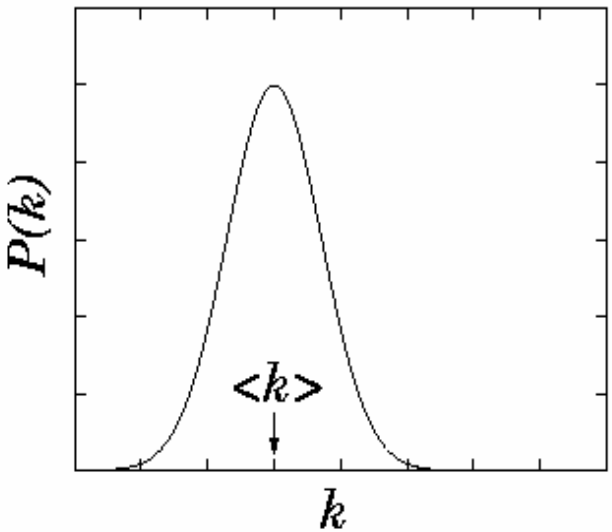
$\Rightarrow N(k=500) \sim 10^{3}$

$P_{out}(k) \sim k^{-\gamma out}$

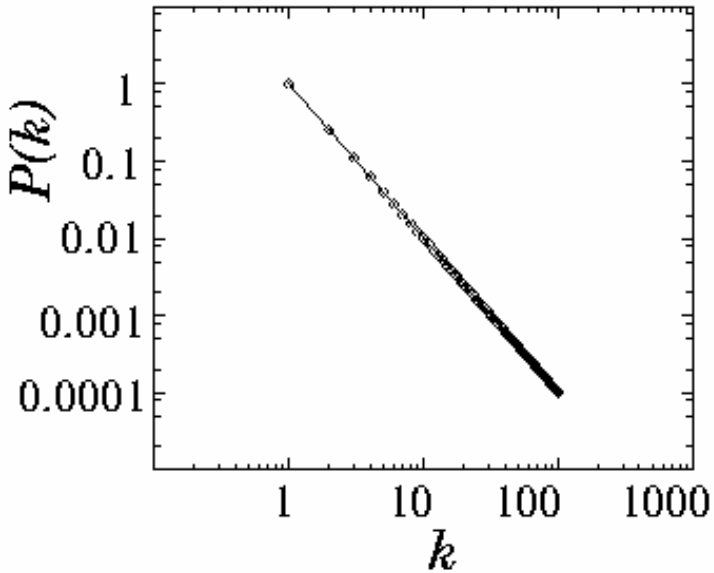
$P_{in}(k) \sim k^{-\gamma in}$

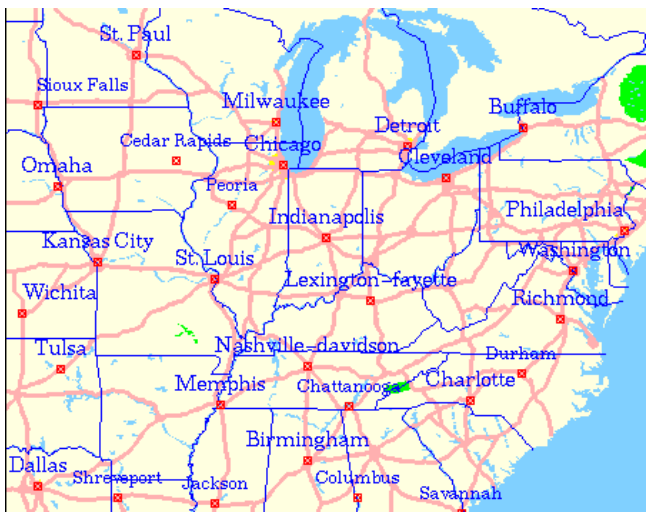
What does it mean?

Poisson distribution

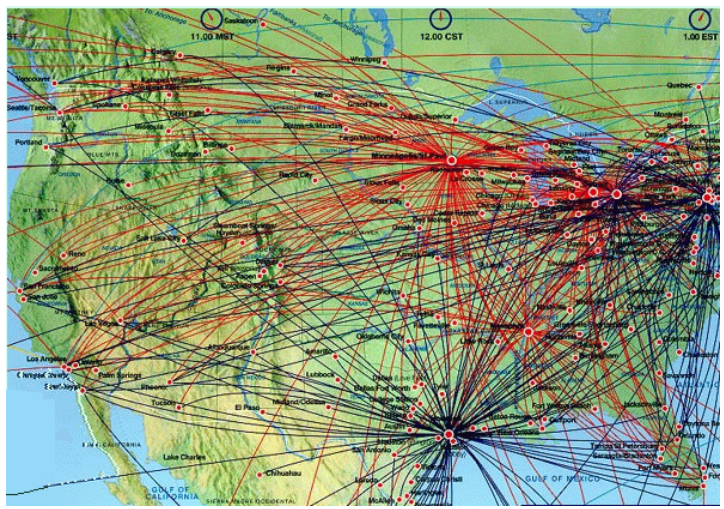


Power-law distribution

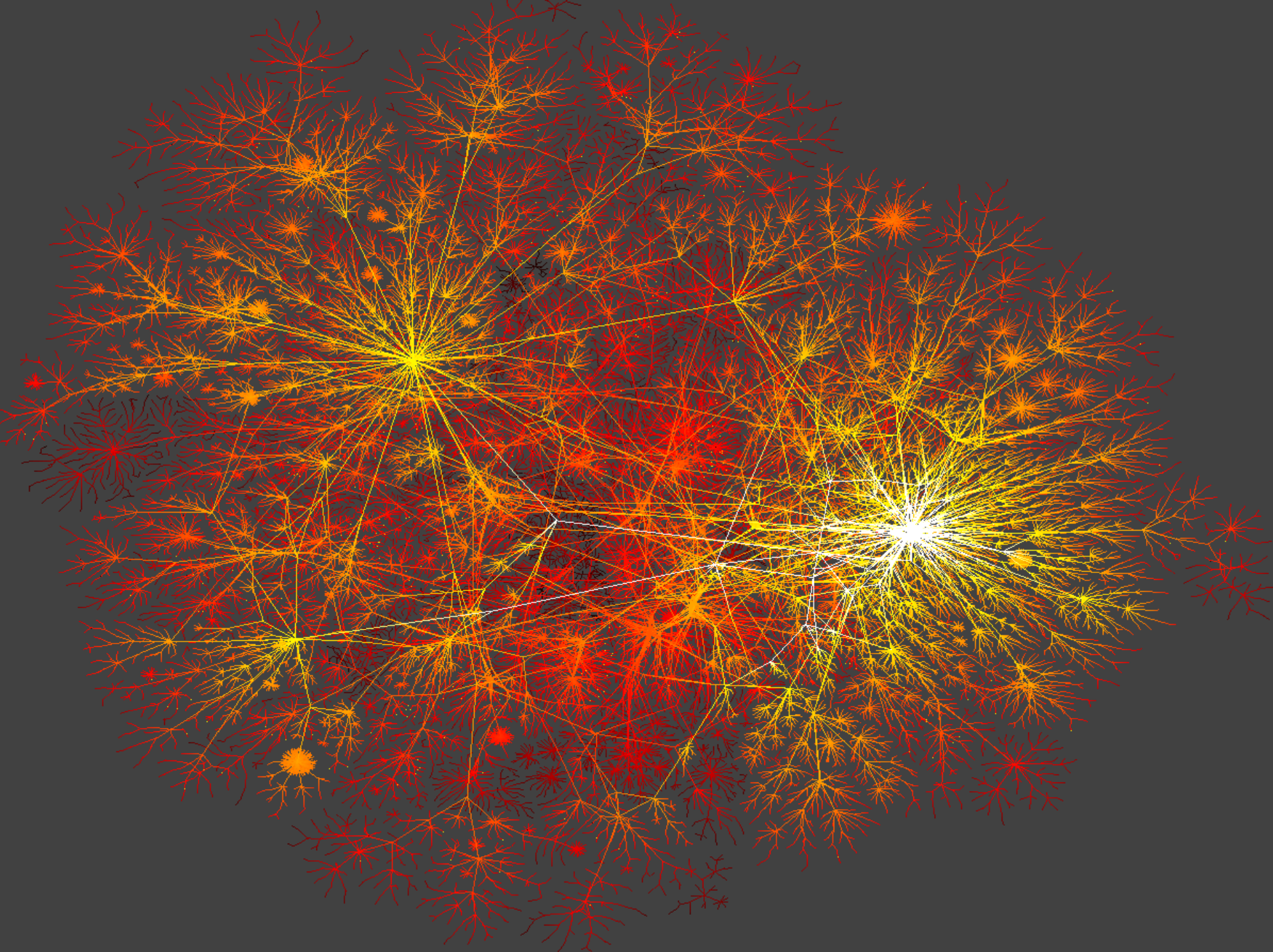




Exponential Network



Scale-free Network



Most real world networks have the same internal structure:

Scale-free networks

Why?

What does it mean?

Scale-free model

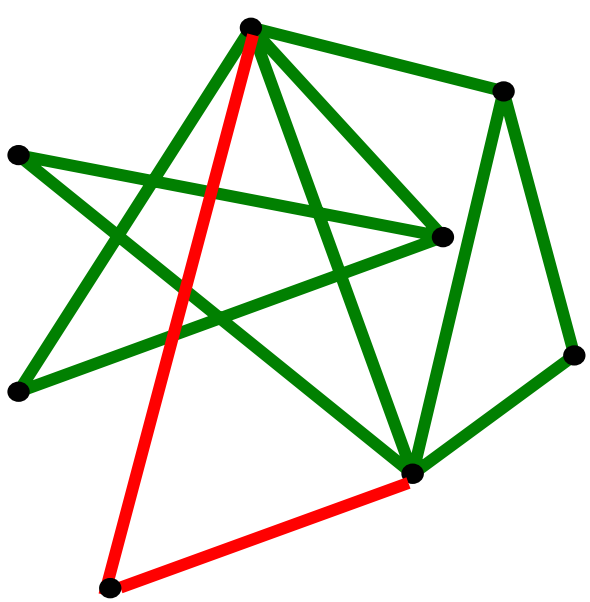
(1) GROWTH :

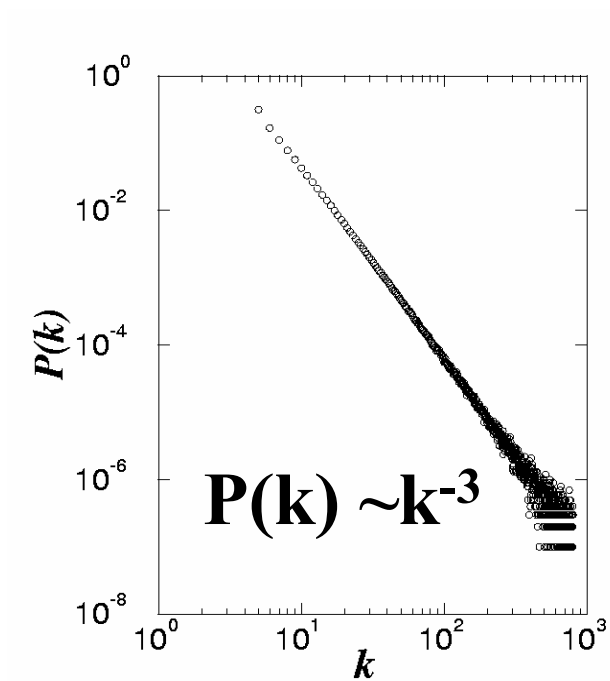
At every timestep we add a new node with *m* edges (connected to the nodes already present in the system).

(2) PREFERENTIAL ATTACHMENT :

The probability Π that a new node will be connected to node *i* depends on the connectivity k_i of that node

$$\Pi(k_i) = \frac{k_i}{\Sigma_j k_j}$$



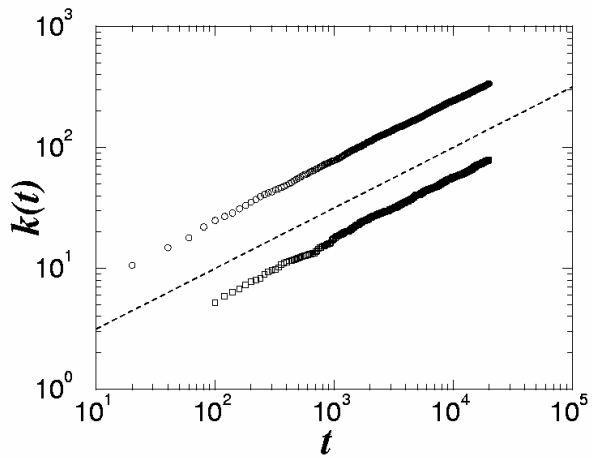


A.-L.Barabási, R. Albert, Science **286,** 509 (1999)

Mean Field Theory

$$\frac{\partial k_i}{\partial t} \propto \Pi(k_i) = A\frac{k_i}{\sum_j k_j} = \frac{k_i}{2t}$$, with initial condition $k_i(t_i) = m$

$$\boxed{k_i(t) = m\sqrt{\frac{t}{t_i}}}$$



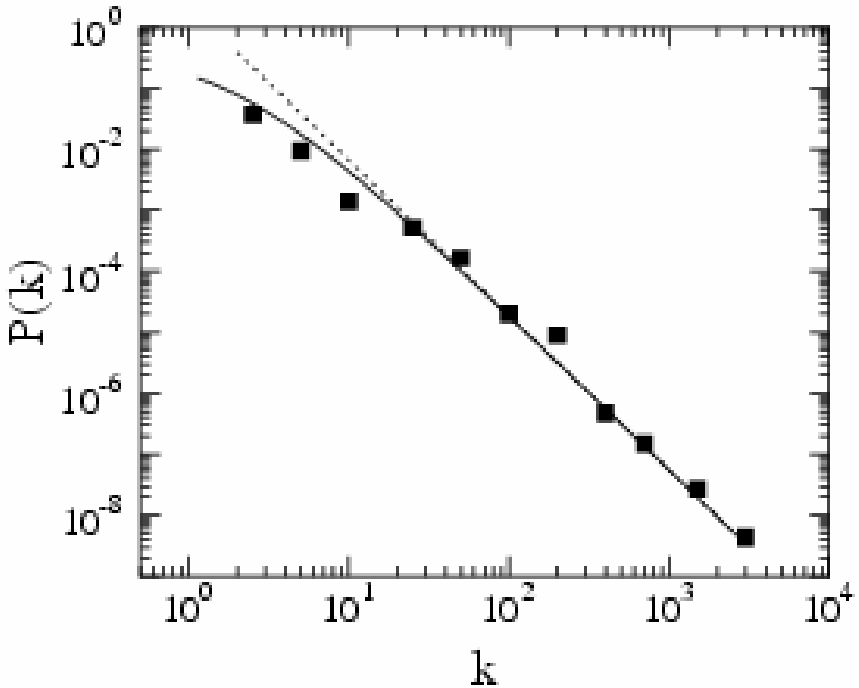
$$P(k_i(t) < k) = P_t(t_i > \frac{m^2 t}{k^2}) = 1 - P_t(t_i \le \frac{m^2 t}{k^2}) = 1 - \frac{m^2 t}{k^2(m_0 + t)}$$

$$\therefore P(k) = \frac{\partial P(k_i(t) < k)}{\partial k} = \frac{2m^2 t}{m_o + t}\frac{1}{k^3} \sim k^{-3}$$

$$\boxed{\gamma = 3}$$

A.-L.Barabási, R. Albert and H. Jeong, Physica A **272,** 173 (1999)

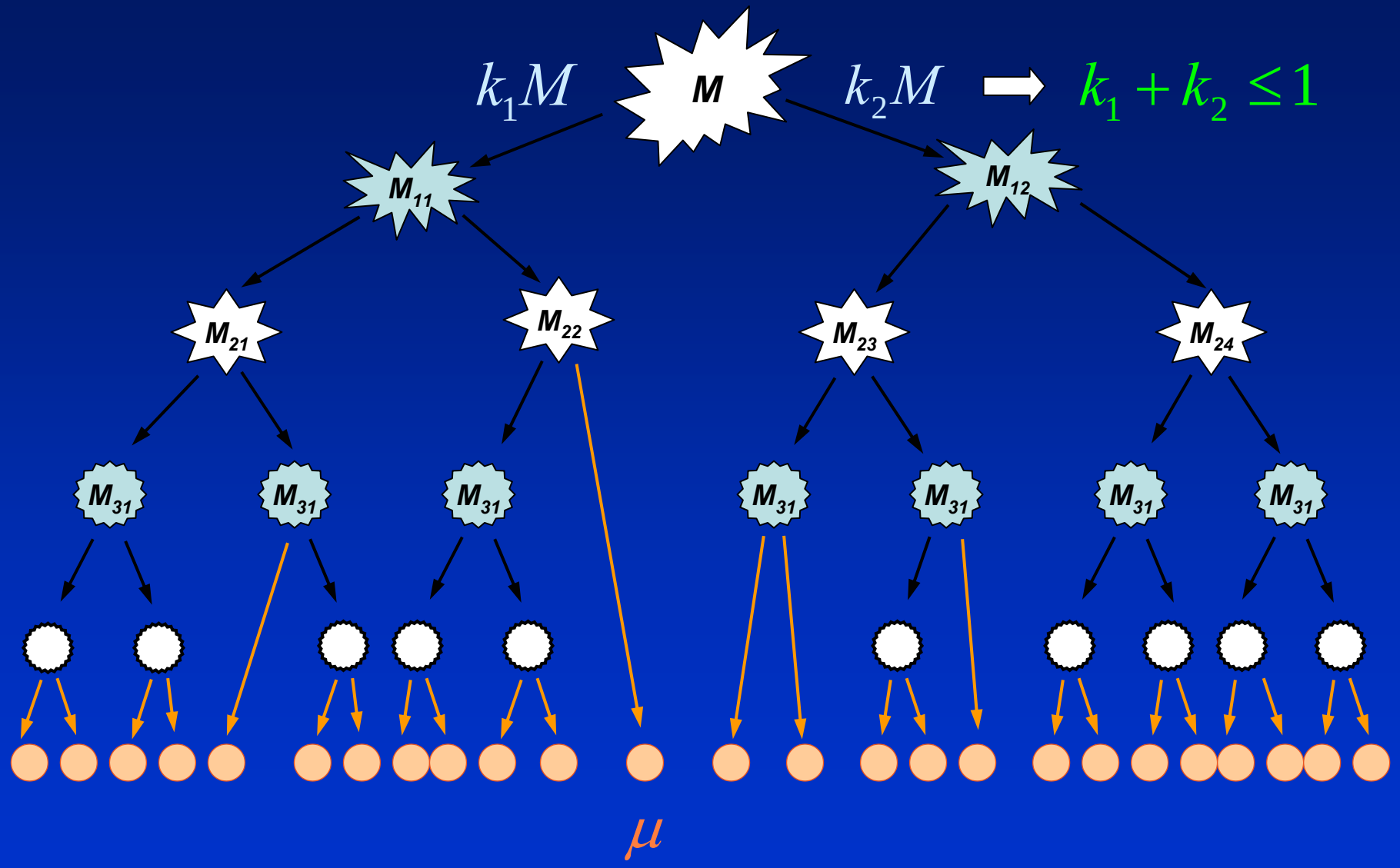
Networks from the Tsallis' point of view...(WW, APP B35 (2004) 871)



The probability distribution of connections in the WWW network fitted by Tsallis formula with q=1.65 and λ_0 =1.91. It reproduces the observed mean <k>= λ_0 /(2-q)=5.45 and leads to the asymptotic power-like distribution $\propto k^{-\gamma}$ with γ=q/(q-1) =2.54(dotted line).

Insertion on networks - end

Simple Cascade Model



Other posible interpretation of q-parameter (Kodama et al..)

(*) Proposition of yet another dynamical origin of power-laws *(Kodama et al.., cond-mat/0406732 and 36):* **supercorrelated systems ⇒ q-clusters**

(*) Motivated by observation *(Berges et al.., hep-ph/0403234)* ***that dynamics of quantum scalar fields exhibits a prethermalization behaviour : thermodynamical relations become valid long before the real thermal equilibrium is attained.***

(*) Possible realisation: strong correlations among some variables (leading to clustration):

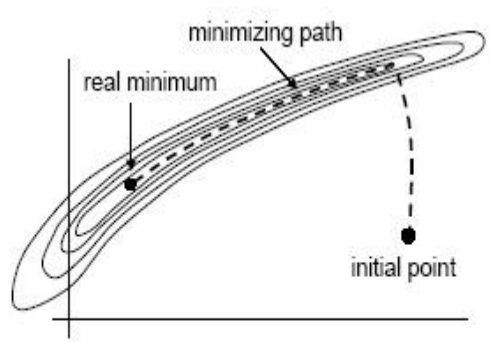


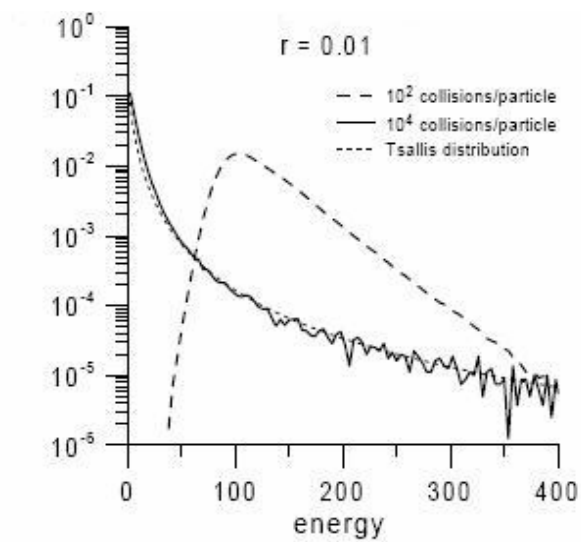
FIG. 1: Schematic illustration of the effect of a strong correlation among two variables for minimizing a function.

Example: system composed of N particles with strong correlation among any q of them and with dynamical evolution such that, for a given number of q-clusters, the configuration of the system tries to minimize the energy of the correlated subsystem (i.e., the system first generates correlations among particles minimizing the energy in the clusters) ⇒ power law distribution discussed before with the same q-parameter .

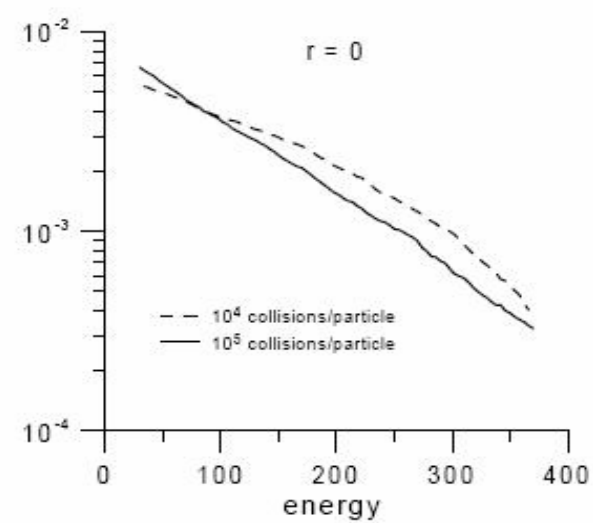
(Kodama et al.. cond-mat/046732)

Example how the existence of dynamical correlations leads to a preequilibrium state of the system:

(a)



(b)



Energy spectrum after a given number of collisions per particle, starting from a distribution peaked at E=125 GeV:

(a) correlated system $\Rightarrow$ non-Boltzmann distribution fitted by Tsallis distribution with β=0.39 GeV^{-1} and q=1.42

(b) uncorrelated system $\Rightarrow$ Boltzmann distribution (equilibration needs 10 times more steps now!)

Explanations of results from NUWW, hep-ph/0312136 (Physica A344(2004)568)

(a) Fit to pp data as before: q=1.05 – 1.33 going from E=20 GeV to E=1800GeV whereas 'partition temperature' T_q =1.76 GeV – 62.57 GeV

(b) PHOBOS most central data (on pseudorapidity distributions η) fitted with K_q=1 and (q;E)=(1.29; 19.6 GeV), (1.26; 130 GeV), (1.27; 200 GeV). The structure visible at the centre can be fitted only with more sophisticated approaches discussed before (cf. NA49 data)

(c) ALEPH data for e^+e^- annihilations at 91.2 GeV. Here:
- by definition K_q=1 only
- only q<1 (here: q=0.6) gives reasonably fit
- minimum at y=0 cannot be fitted in simplest approach

Possible explanation of q<1:
In this case temperature T does not reach an equilibrium state because now $T=T_0-(1-q)E$, instead of remaining constant: $T=T_0$, as is the case for q>1. We have a kind of dissipative transfer of energy from the region where T is higher (here: from q-jets to gluons and qq pairs) to observed hadrons.

NUWW, hep-ph/0312136 (Physica A344(2004))568)

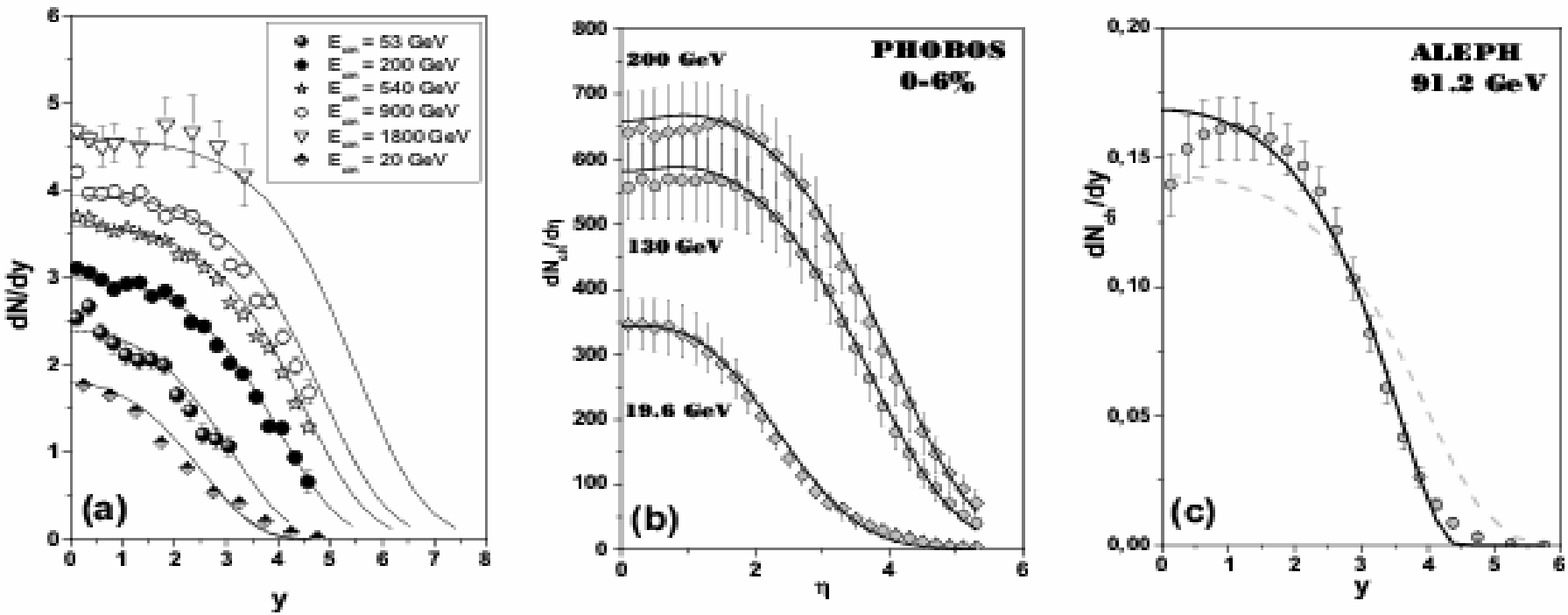


Fig. 1. Examples of: (*a*) rapidity spectra for charged pions produced in pp and $\bar{p}p$ collisions at different energies; (*b*) similar data obtained for the most central $Au + Au$ collisions; (*c*) rapidity spectra measured in e^+e^- annihilations at 91.2 GeV (dotted line is for $K_q = 1$ and $q = 1$ whereas full line is our fit with $K_q = 1$ and $q = 0.6$).

Examples of some special y-distributions

(*) Nonextensive spectra obtained for different parameter q practically coincide with extensive spectra produced by masses $M^*=M/(3-2q)$ ⇔ Fig. (a)

(*) Extensive spectra obtained for composition of smaller masses producing, respectively, smaller number of secondaries practically coincide if M/N=const ⇔ Fig. (b); otherwise differ drastically ⇔ Fig. (c)

(*) The p_T growing towards the centre of rapidity phse space ('minijets'?) has dramatic effect on spectra ⇔ Fig. (d) (q=1)

(*) The effect of momentum-dependent residual interactions (q=1):

$$p(y) = \frac{1}{Z}\exp\left[-\beta\mu_T \cosh y - \lambda\mu_T\,|\sinh y|\right] \qquad \int dy\,\mu_T\,|\sinh y|\,p(y) = \kappa\frac{|P|}{N}$$

⇔***Fig. (e)*** *(Schenke, Greiner, J.Phys. G30 (2004) 597)*

(*) Example of effect of two superimposed sources separated in rapidity by 2Δy with combined energies and masses equal M ⇔ Fig. (f)

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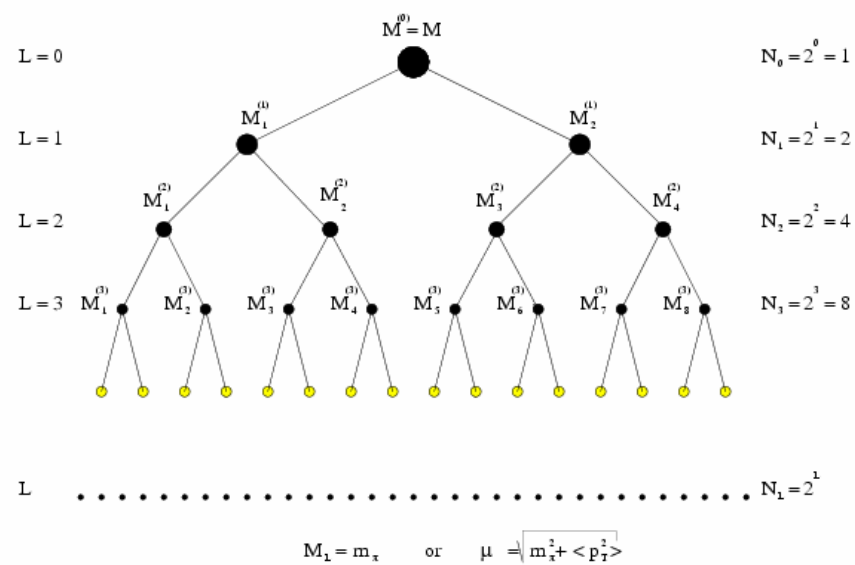
Explanations of results from NUWW, hep-ph/0312166 (Nukleonika 49 (2004) S19)

Confrontation of e^+e^- annihilation data with two different general schemes:

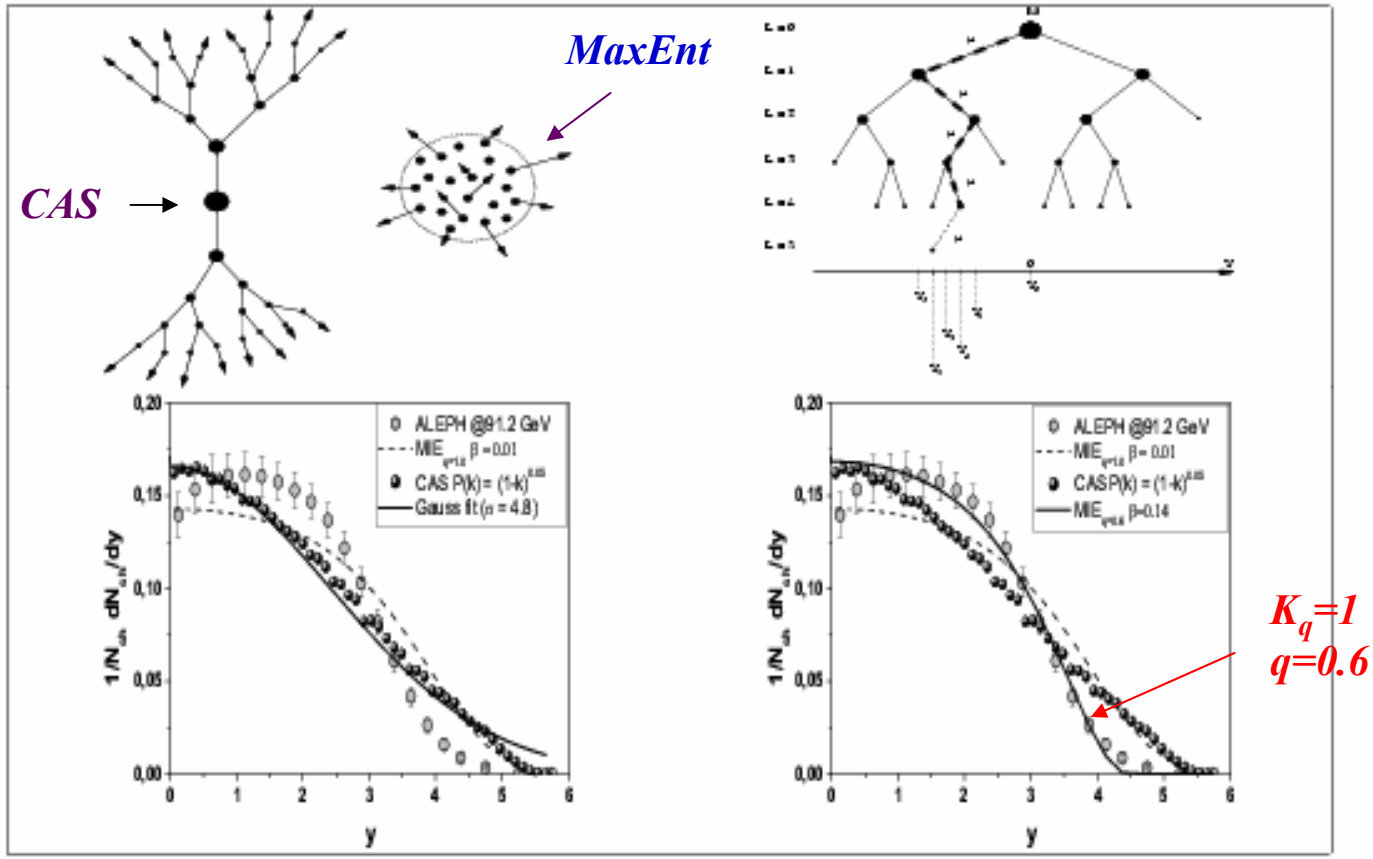
(*) general statistical approach: gives only distribution in the phase space once initial energy M and multiplicity N are given

(*) general cascade model: once initial energy M is given it provides both:
- the multiplicity N and
- distribution in the phase space

Scheme of the CAS model: it visualizes the possible fractal structure of hadronization process (hadronizing source) and encompasses a large variety of <N(s)>.



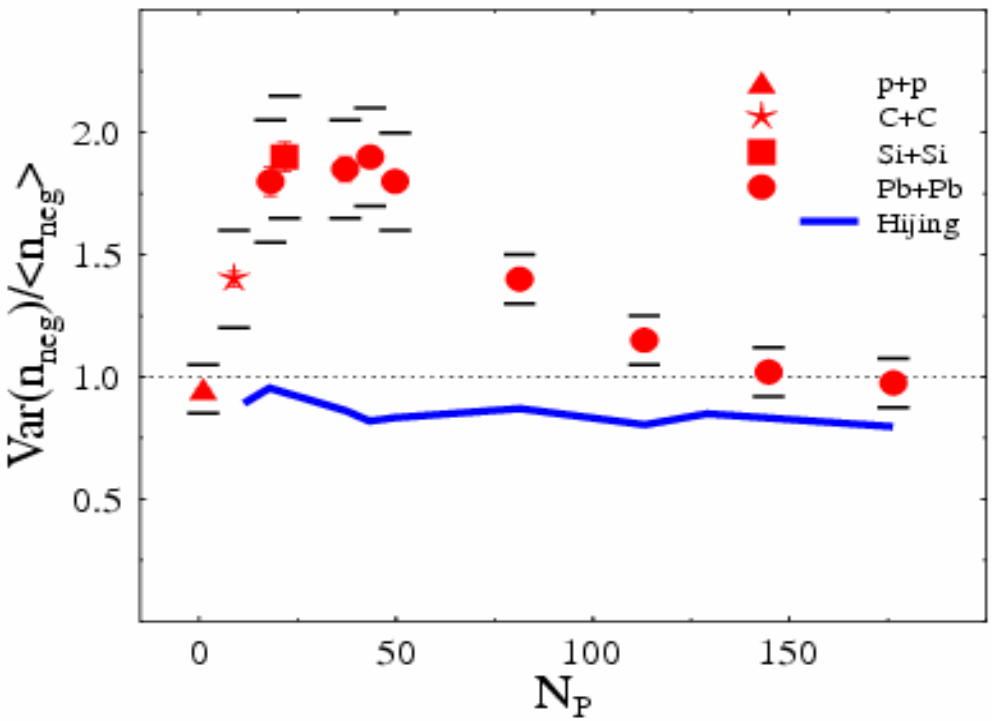
(UWW, PRD61 (1999) 034007



Upper-left panel: visualization of differences between sequential cascade hadronization (CAS) and instantaneous one described by MaxEnt. Upper-right panel: closer look at the way dN/dy arises in CAS (resembling random walk in y-space). Lower panels: results of calculations of dN/dy using CAS and MaxEnt and compared with e^+e^- annihilation data by ALEPH. Left panel - comparison with CAS model giving the same multiplicity (it can be extremely well approximated by gaussian fit) and with MaxEnt for $K_q = 1$ and $q = 1$ (it does not fit data). Right panel - gaussian fit has been replaced by MaxEnt fit with total inelasticity $K_q = 1$ and with varying q; for $q = 0.6$ we obtain quite good agreement with data.

(UWW, PRD61 (1999) 034007

Potentially very important result from AA collisions concerning fluctuations *(MRW, nucl-th/0407012)*



for AA collisions the usual superposition model does not work when applied to fluctuations (signal for the phase transition to Quark-Gluon-Plasma phase of matter?...)

Examples of some special y-distributions

(*) Nonextensive spectra obtained for different parameter q practically coincide with extensive spectra produced by masses $M^*=M/(3-2q)$ ⇔ Fig. (a)

(*) Extensive spectra obtained for composition of smaller masses producing, respectively, smaller number of secondaries practically coincide if M/N=const ⇔ Fig. (b); otherwise differ drastically ⇔ Fig. (c)

(*) The p_T growing towards the centre of rapidity phse space ('minijets'?) has dramatic effect on spectra ⇔ Fig. (d) (q=1)

(*) The effect of momentum-dependent residual interactions (q=1):

$$p(y) = \frac{1}{Z}\exp\left[-\beta\mu_T \cosh y - \lambda\mu_T\,|\sinh y|\right] \qquad \int dy\,\mu_T\,|\sinh y|\,p(y) = \kappa\frac{|P|}{N}$$

⇔***Fig. (e)*** *(Schenke, Greiner, J.Phys. G30 (2004) 597)*

(*) Example of effect of two superimposed sources separated in rapidity by 2Δy with combined energies and masses equal M ⇔ Fig. (f)

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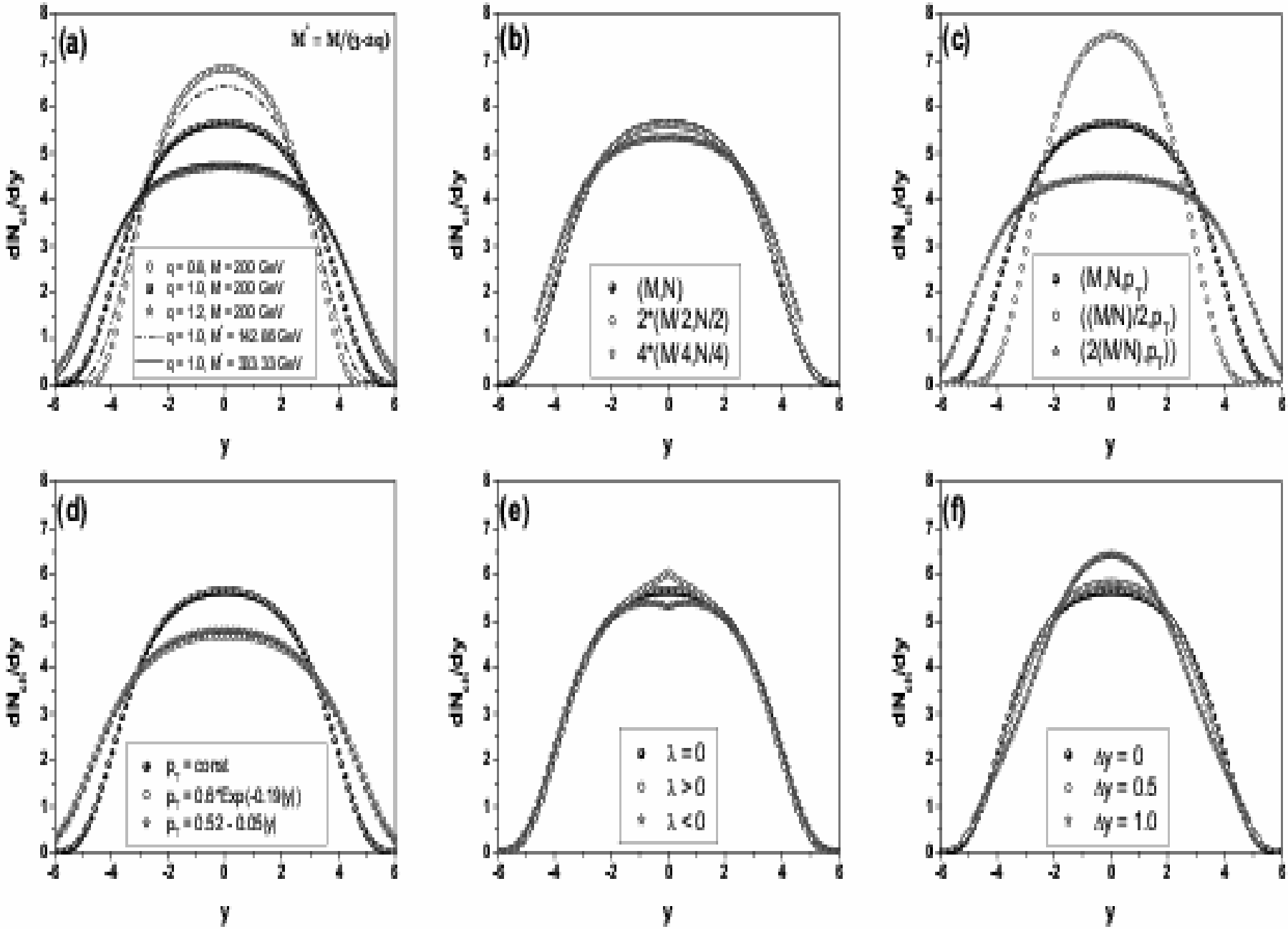
$M=200$ GeV; $N=60$; $\langle p_T \rangle = 0.4$ GeV/c



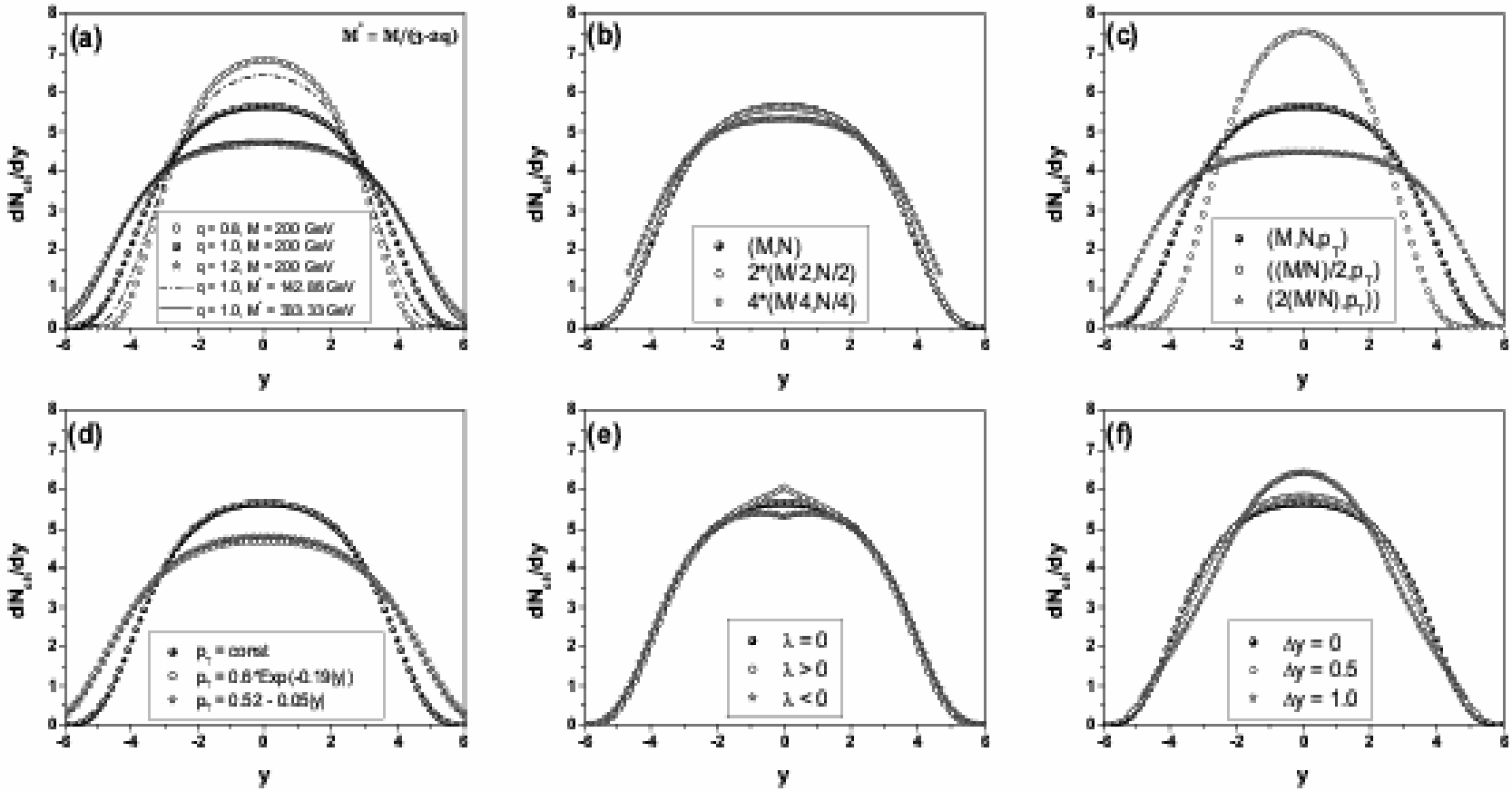


Fig. 2. Examples of different rapidity distributions ($W = 200$ GeV, $N = 60$, $\langle p_T \rangle = 0.4$ GeV/c). (a) spectra obtained for different parameter q practically coincide with extensive spectra produced by masses $M^* = M/(3-2q)$. For all remaining panels $q = 1$. (b) quasiscaling for spectra obtained for composition of smaller masses producing respectively smaller number of secondaries with M/N =const. (c) spectra with different values of M/N ratio differ drastically. (d) the p_T growing towards the center of rapidity phase space has dramatic effect on spectra. (e) the role of momentum dependent residual interaction is visualised. (f) the effect of two sources separated in rapidity by $2\Delta y$ with combined energies and masses equal M is visualised.