Indication for deconfinement at RHIC

Buda-Lund hydro in heavy ion physics

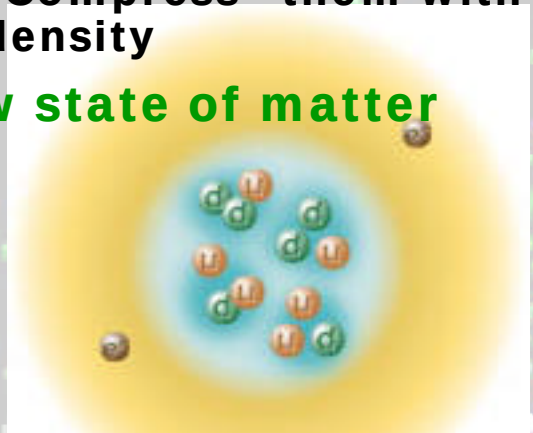
M. Csanád, T. Csörgõ, B. Lörstad and A. Ster

(Budapest & Lund)

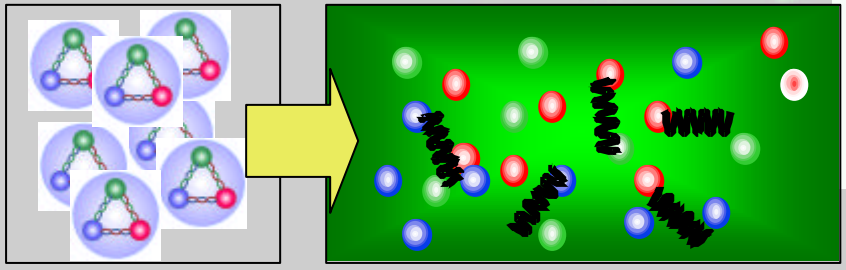
- **Buda-Lund hydro model**
  - **Group of hydro solutions**
  - **Calculating observables**
- **Buda-Lund fits to SPS and RHIC data**
- **Discussion of the results**
  - **Indication of deconfinement**
  - **Expansion rates**
  - **Comparison of result of models**
- **A resolution of the "RHIC HBT puzzle"**

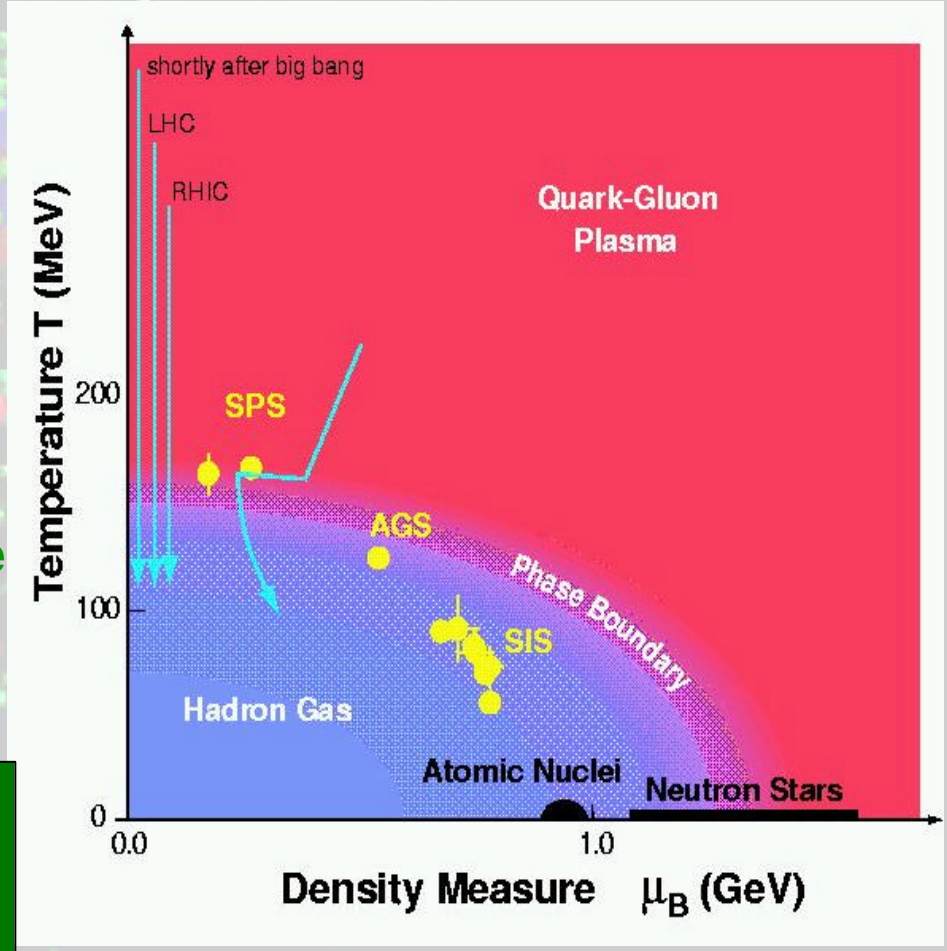
Phases of QCD Matter

- **Quark Gluon Plasma**
  - **"Ionize" nucleons with heat**
  - **"Compress" them with density**
- **New state of matter**



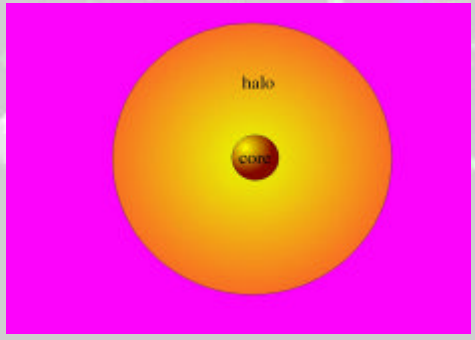
Z. Fodor and S.D. Katz:
T_c = 172 ± 3 MeV even at finite baryon density,
Cross over like transition.
(hep-lat/0106002)





Principles for Buda-Lund hydro model

- Analytic expressions for all the observables
- Symmetric, 3D expansion
- Local thermal equilibrium
- Goes back to known hydro solutions in nonrel limit
- Separation of the Core and the Halo
  - Core: hydrodynamic evolution
  - Halo: decay products of long-lived resonances



Nonrelativistic hydrodynamics

- **Equations of nonrelativistic hydro:**

$$\begin{aligned}\partial_t n + \nabla(n\mathbf{v}) &= 0\\ \partial_t \mathbf{v} + (\mathbf{v}\nabla)\mathbf{v} &= -(\nabla p)/(mn)\\ \partial_t \epsilon + \nabla(\epsilon\mathbf{v}) &= -p\nabla\mathbf{v}\end{aligned}$$

- **Not closed, EoS needed:**

$$\epsilon = \kappa p$$

$$p = nT$$

- **We use the following scaling variable:**

$$s = \frac{r_x^2}{X^2} + \frac{r_y^2}{Y^2} + \frac{r_z^2}{Z^2}$$

- **X, Y and Z are characteristic scales, depend on (proper-) time**

A nonrelativistic solution

- **A general group of scale-invariant solutions (hep-ph/0111139):**

$$
\begin{aligned}
n(t,\mathbf{r}) &= n_0 \frac{V_0}{V} \nu(s) \\
\mathbf{v}(t,\mathbf{r}) &= \left(\frac{\dot{X}}{X} r_x, \frac{\dot{Y}}{Y} r_y, \frac{\dot{Z}}{Z} r_z \right) \\
T(t,\mathbf{r}) &= T_0 \left(\frac{V_0}{V} \right)^{1/\kappa} \mathcal{T}(s) \\
\nu(s) &= \frac{1}{\mathcal{T}(s)} \exp\left(-\frac{T_i}{2T_0} \int_0^s \frac{du}{\mathcal{T}(u)} \right)
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{r}' &= \left(r_x \frac{X_0}{X}, r_y \frac{Y_0}{Y}, r_z \frac{Z_0}{Z} \right) \\
n(t,\mathbf{r}) &= n(t_0,\mathbf{r}') \left(\frac{X_0 Y_0 Z_0}{XYZ} \right) \\
v_x(t,\mathbf{r}) &= v_x(t_0,\mathbf{r}') \frac{\dot{X}}{\dot{X}_0}, \ldots \\
T(t,\mathbf{r}) &= T(t_0,\mathbf{r}') \left(\frac{X_0 Y_0 Z_0}{XYZ} \right)^{1/\kappa}
\end{aligned}
$$

- **This is a solution, if the scales fulfill:**

$$
X\ddot{X} = Y\ddot{Y} = Z\ddot{Z} = \frac{T_i}{m} \left(\frac{V_0}{V} \right)^{1/\kappa}
$$

- **$\mathcal{T}(s)$ is arbitrary, e.g. $\mathcal{T}$ constant Þ $\mathcal{V}$ gaussian, or:**

Buda-Lund

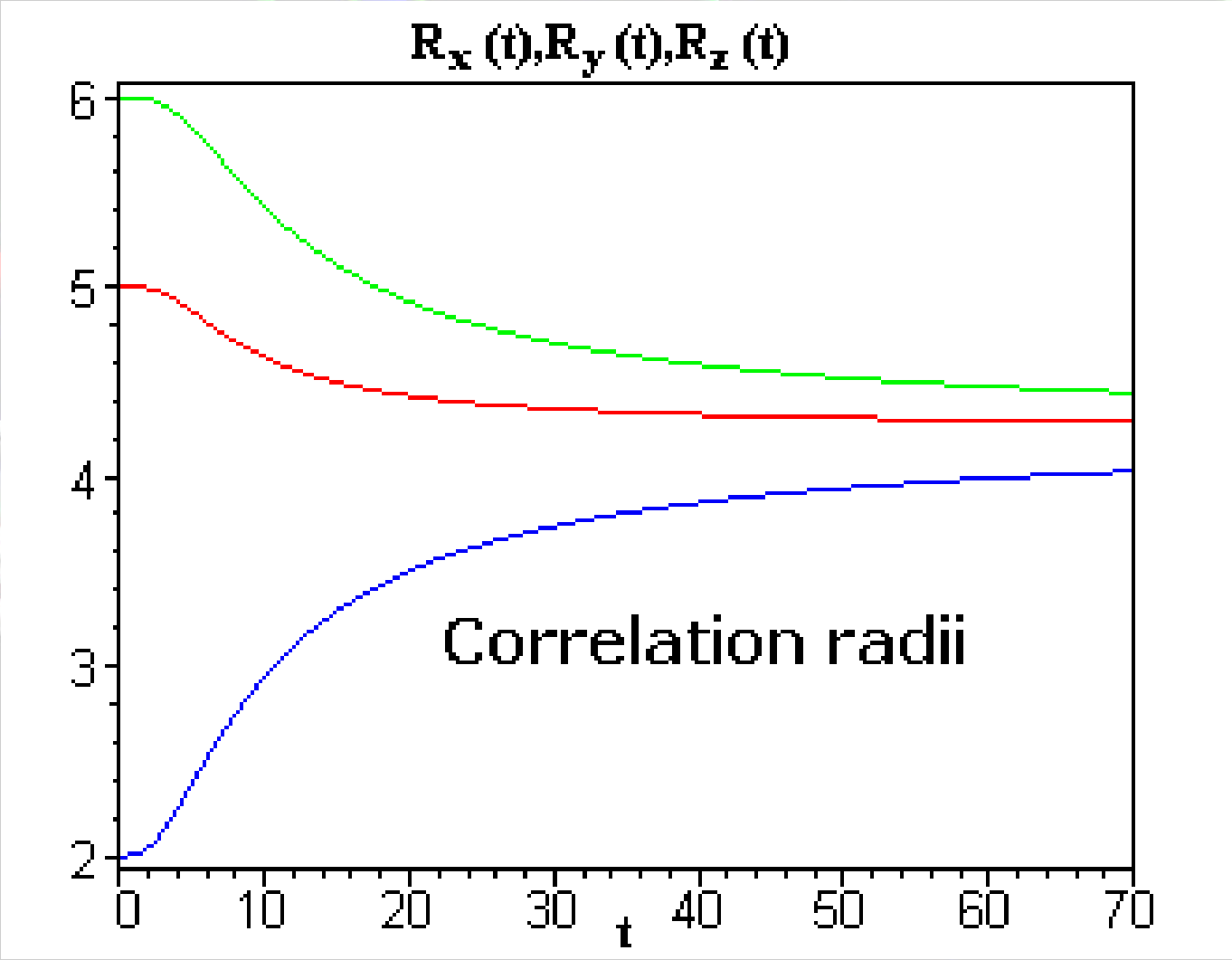
$$
\begin{aligned}
\mathcal{T}(s) &= \frac{1}{1+bs} \\
\nu(s) &= (1+bs) \exp\left[-\frac{T_i}{2T_0} (s + bs^2/2) \right]
\end{aligned}
$$

Bondorf-Zimanyi-Garpman

$$
\begin{aligned}
\mathcal{T}(s) &= (1-s)\,\Theta(1-s) \\
\nu(s) &= (1-s)^{\alpha}\,\Theta(1-s)
\end{aligned}
$$

Some numeric results from hydro

- **Propagate the hydro solution in time numerically:**



A relativistic solution

- **Relativistic hydro:**

$$\partial_\mu (n u^\mu) = 0$$
$$\partial_\mu T^{\mu\nu} = 0 \quad \text{with} \quad T^{\mu\nu} = (\varepsilon + p)\, u^\mu u^\nu - p g^{\mu\nu}$$

- **A general group of solutions (nucl-th/0306004):**

$$u^\mu = \frac{x^\mu}{\tau}$$
$$n(t, \mathbf{r}) = n_0 \left(\frac{\tau_0}{\tau}\right)^3 \mathcal{V}(s)$$
$$p(t, \mathbf{r}) = p_0 \left(\frac{\tau_0}{\tau}\right)^{3+3/\kappa}$$
$$T(t, \mathbf{r}) = T_0 \left(\frac{\tau_0}{\tau}\right)^{3/\kappa} \frac{1}{\mathcal{V}(s)}$$

- **Overcomes two shortcomings of Bjorken's solution:**
  - **Rapidity distribution**
  - **Transverse flow**
- **Hubble flow Þ lack of acceleration**

$$u^\mu \partial_\mu u_\nu = 0$$

The emission function

- **The phase-space distribution looks like**

  Maxwell-Boltzman and $\mathcal{T}$ = const. for sake of simplicity

$$f(\mathbf{r},\mathbf{p},t) = C\exp\left(-\frac{r_x^2}{2X(t)^2} - \frac{r_y^2}{2Y(t)^2} - \frac{r_z^2}{2Z(t)^2} - \frac{(\mathbf{p}-m\mathbf{v}(\mathbf{r},t))^2}{2mT(\mathbf{r},t)}\right)$$

  with the constant:

$$C = \frac{N}{V_0(4\pi^2 m T_0)^{3/2}}$$

- **Consider the collisionless Boltzmann-equation**

  Calculates the source of a given phase-space distribution:

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\nabla\right) f = \mathcal{S}(\mathbf{r},\ \mathbf{p},\ t)$$

- **Emission function in the simplest case:**

$$S(\mathbf{r},\ \mathbf{p},\ t) = C\exp\left(-\frac{r^2}{2R_0^2} - \frac{\left(\mathbf{p}-\frac{m}{\tau}\mathbf{r}\right)^2}{2mT_0}\right)\delta(t-t_0)$$

Observables from Buda-Lund hydro

- **Core-halo correction:**

$$S(x,p) = S_c(x,p) + S_h(x,p)$$

$$S_c(x,p) = \sqrt{\lambda_*} S(x,p)$$

- **One-particle spectrum with core-halo correction:**

$$\mathrm{N}_1(p) = \frac{1}{\sqrt{\lambda_*}} \int d^4x S_c(p,x)$$

- **Two-particle correlation function:**

$$C(Q,p) = 1 + \left| \frac{\tilde{S}(Q,p)}{\tilde{S}(0,p)} \right|^2 = 1 + \lambda_* \left| \frac{\tilde{S}_c(Q,p)}{\tilde{S}_c(0,p)} \right|^2$$

- **Flow coefficients:**

$$N_1(p) = \frac{d^3n}{dy p_t dp_t d\varphi} = \frac{d^2n}{2\pi dy p_t dp_t} \left[1 + 2 \sum_{n=1}^{\infty} v_n \cos(n\varphi) \right]$$

The generalized Buda-Lund model

- **The original model was developed for axial symmetry only Þ central collisions**
- **In the most general hydrodynamical form:**

  'Inspired by' nonrelativistic solutions

$$S_c(x,p)d^4x = \frac{g}{(2\pi)^3}\frac{p^\mu d^4\Sigma_\mu(x)}{\exp\left(\frac{p^\nu u_\nu(x)}{T(x)} - \frac{\mu(x)}{T(x)}\right) + s_q}$$

- **Have to assume special shapes:**
  - **Generalized Cooper-Frye prefactor:**

    $p^\mu d^4\Sigma_\mu(x) = p^\mu u_\mu(x) H(\tau) d^4x$ **and** $H(\tau) = \frac{1}{(2\pi\Delta\tau^2)^{1/2}}\exp\left(-\frac{(\tau-\tau_0)^2}{2\Delta\tau^2}\right)$
  - **Four-velocity distribution:**

    $u^\mu = (\gamma, \sinh\eta_x, \sinh\eta_y, \sinh\eta_z)$
  - **Temperature:**

    $\frac{1}{T(x)} = \frac{1}{T_0}\left(1 + \frac{T_0 - T_s}{T_s}s\right)\left(1 + \frac{T_0 - T_e}{T_e}\frac{(\tau-\tau_0)^2}{2\Delta\tau^2}\right)$
  - **Fugacity:**

    $\frac{\mu(x)}{T(x)} = \frac{\mu_0}{T_0} - s$

The saddlepoint approximation

- A good approximation for the product of a narrow Gaussian-like function and a broad distribution:

$$\int f(x)g(x)dx = f(x_0)\int g(x)dx = f(x_0)g(x_0)\int e^{-\frac{(x-x_0)^2}{\Delta x^2}}dx$$

$$\Delta x^{-2} = \partial_x^2 \left(-\ln g(x)\right)_{x_0}$$

- Exact for convolution of Gaussians, good for narrow distributions, where a parameter controls the width

- The saddlepoint can be computed from

$$\left.\frac{\mathrm{d}g}{\mathrm{d}x}\right|_{x_0} = 0$$

- This method can be generalized for more dimensions

The invariant momentum distribution

- **With the saddlepoint method, we get:**

$$\mathrm{N}_1(p) = \frac{g}{(2\pi)^3}\overline{E}\,\overline{V}\,\overline{C}\,\frac{1}{B(x_s,p)+s_q}$$

- **The 'average' energy and volume:**

$$\begin{aligned}\overline{E} &= p_\mu u^\mu(x_s)\\ \overline{V} &= \frac{(2\pi)^3}{\Delta\tau}\left[\det R^2_{\mu\nu}\right]^{1/2}\\ \overline{C} &= \frac{1}{\sqrt{\lambda_*}}\end{aligned}$$

with $R^{-2}_{\mu\nu} = \partial_\mu\partial_\nu\left[-\ln\frac{H(\tau)}{B(x,p)+s_q}\right]_s$

- **Here, we have to make a coordinate-transformation: System of expansion ⇒ system of observables**
  - **Direction of transverse momentum needed**
  - **Rotation due to angular momentum: tilt angle**

Correlation function and radii

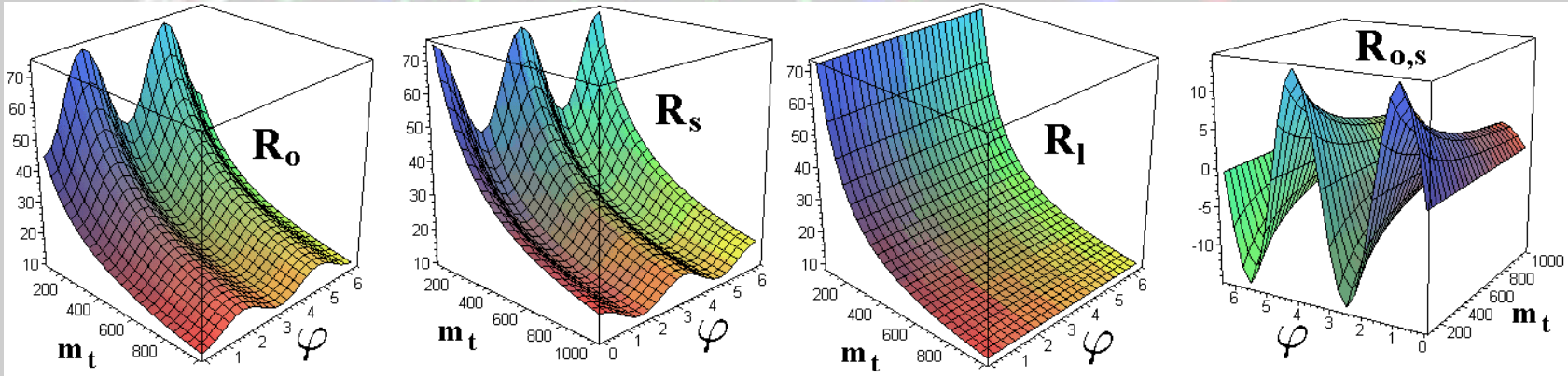
- **The correlation function:**

$$C(Q) = 1 + \lambda_* e^{-R^2_{\mu\nu} Q^\mu Q^\nu}$$

- **The radii are in the simplest case, and in the B-P system:**

$$\begin{aligned}
R_o^2 &= R_x'^2 \cos^2\varphi + R_y^2 \sin^2\varphi \\
R_s^2 &= R_x'^2 \sin^2\varphi + R_y^2 \cos^2\varphi \\
R_l^2 &= R_x^2 \sin^2\vartheta + R_z^2 \cos^2\vartheta - 2R_{x,z}^2 \sin\vartheta\cos\vartheta \\
2R_{s,o}^2 &= (R_y^2 - R_x'^2)\sin 2\varphi \\
2R_{l,o}^2 &= +(R_z^2 \sin 2\vartheta - R_x^2 \sin 2\vartheta + 2R_{x,z}\cos 2\vartheta)\cos\varphi \\
2R_{s,l}^2 &= -(R_z^2 \sin 2\vartheta - R_x^2 \sin 2\vartheta + 2R_{x,z}\cos 2\vartheta)\sin\varphi
\end{aligned}$$

- **Azimuthal depencence appears**



Some analytic results

- **Distribution widths**

$$\frac{1}{R_{i,i}^2} = \frac{B(x_s,p)}{B(x_s,p)+s_q}\left(\frac{1}{X_i^2}+\frac{1}{R_{T,i}^2}\right)$$

$$\frac{1}{R_{T,i}^2} = \frac{m_t}{T_0}\left(\frac{a^2}{X_i^2}+\frac{\dot{X}_i^2}{X_i^2}\right)$$

with $a^2 = \frac{T_0 - T_s}{T_s} = \left\langle \frac{\Delta T}{T} \right\rangle_r$

- **Slopes, effective temperatures**

$$T_{eff} = \frac{1}{2}\left(\frac{1}{T_{*,x}}+\frac{1}{T_{*,y}}\right)$$

$$T_{*,i} = T_0 + m_t \dot{X}_i^2 \frac{T_0}{T_0 + m_t a^2}$$

- **Flow coefficients**

$$v_{2n} = \frac{I_n(w)}{I_0(w)}$$

$$v_{2n+1} = 0$$

with $w = \frac{p_t^2}{4\overline{m}_t}\left(\frac{1}{T_{*,y}}-\frac{1}{T_{*,x}}\right)$

$$\overline{m}_t = m_t \cosh(\eta_s - y)$$

Buda-Lund fits to RHIC data

Calculation program packages:

Version 1.0: Calculation of IMD & Radii, linearized saddle-point equations
(NA22, NA44, NA49)

Version 1.4: Calculation for dN/dh added

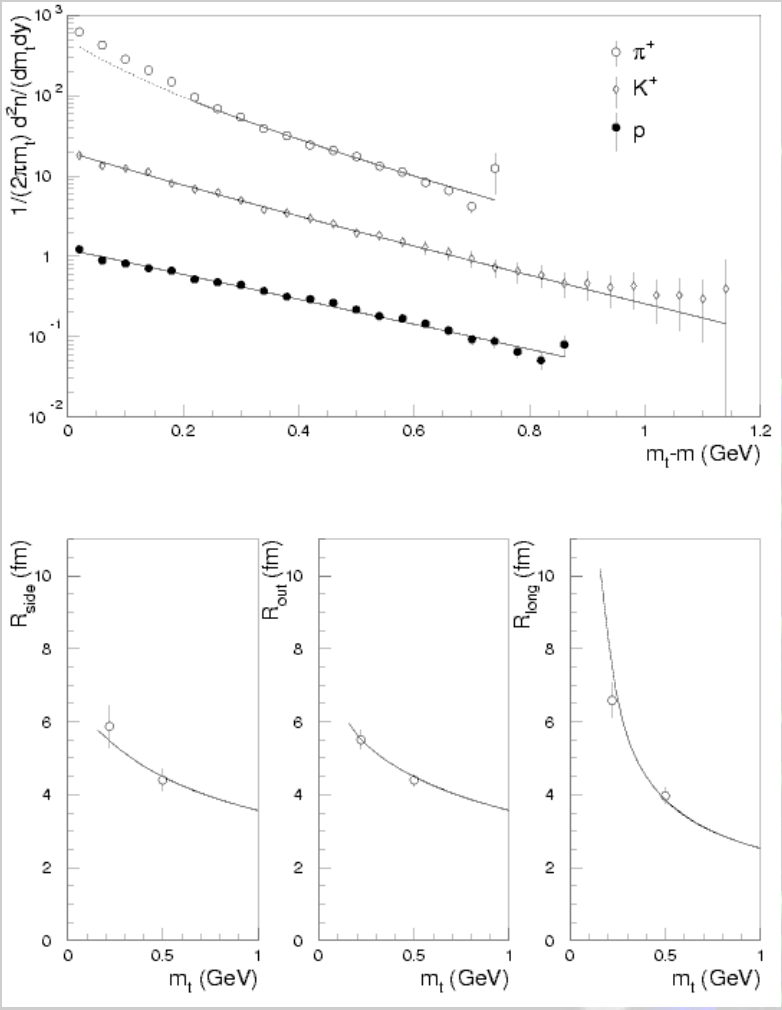
Version 1.5: Saddle point found exactly in direction h
(RHIC 200 and 130 GeV)
http://www.kfki.hu/~csorgo/budalund/

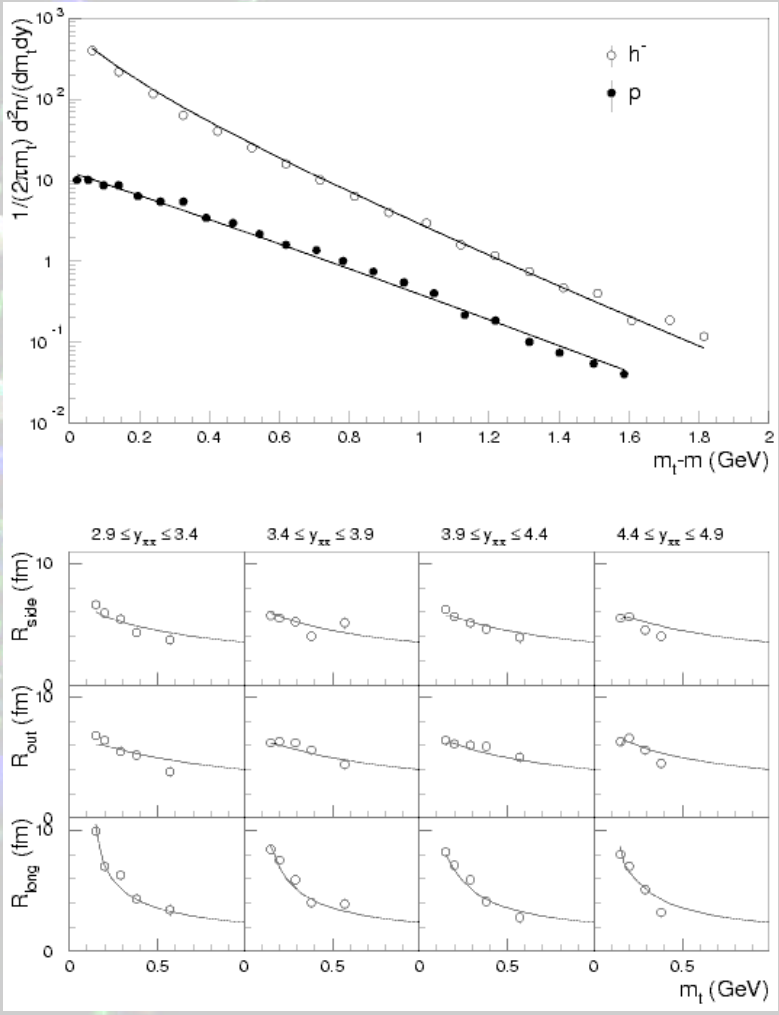
Version 2.0: Exact analytic calculation
(in progress)

Version 3.0: Relativistic, ellipsoidally symmetric model
(in progress)

All versions: Simultaneous fits to all the observables

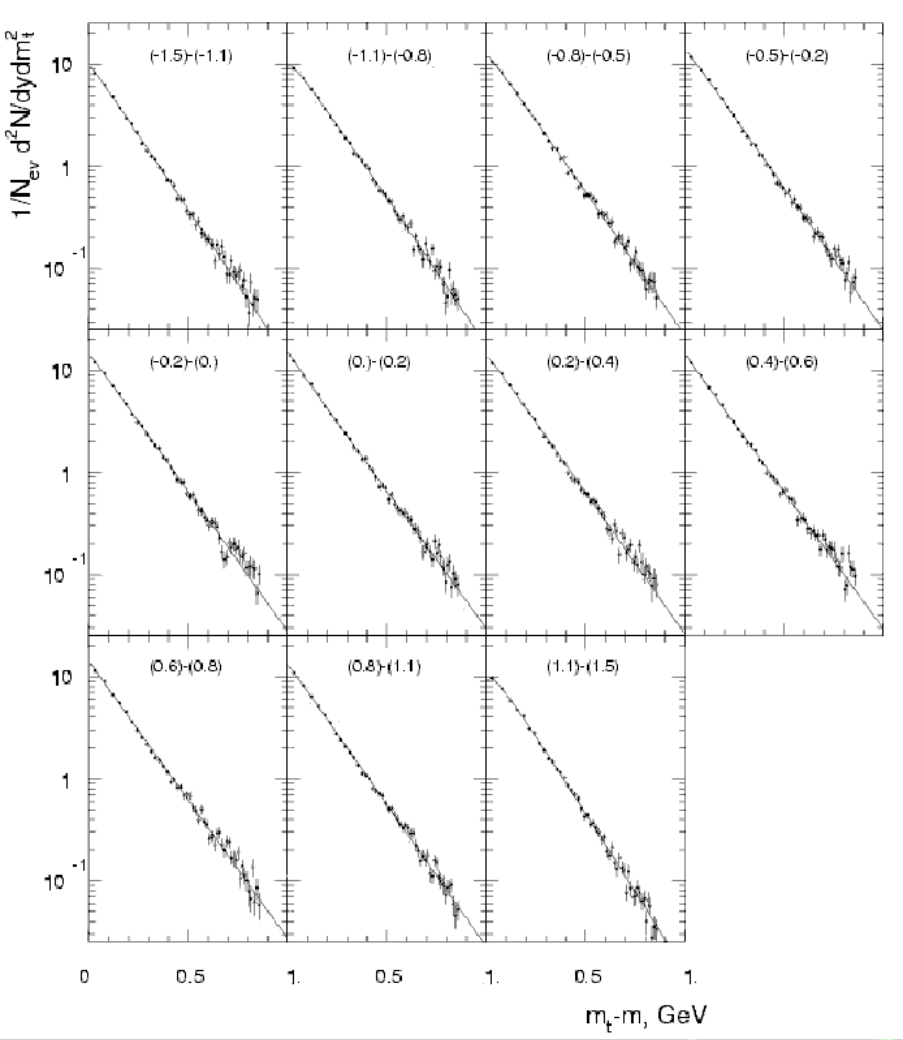
Buda-Lund fits to NA44/49 data

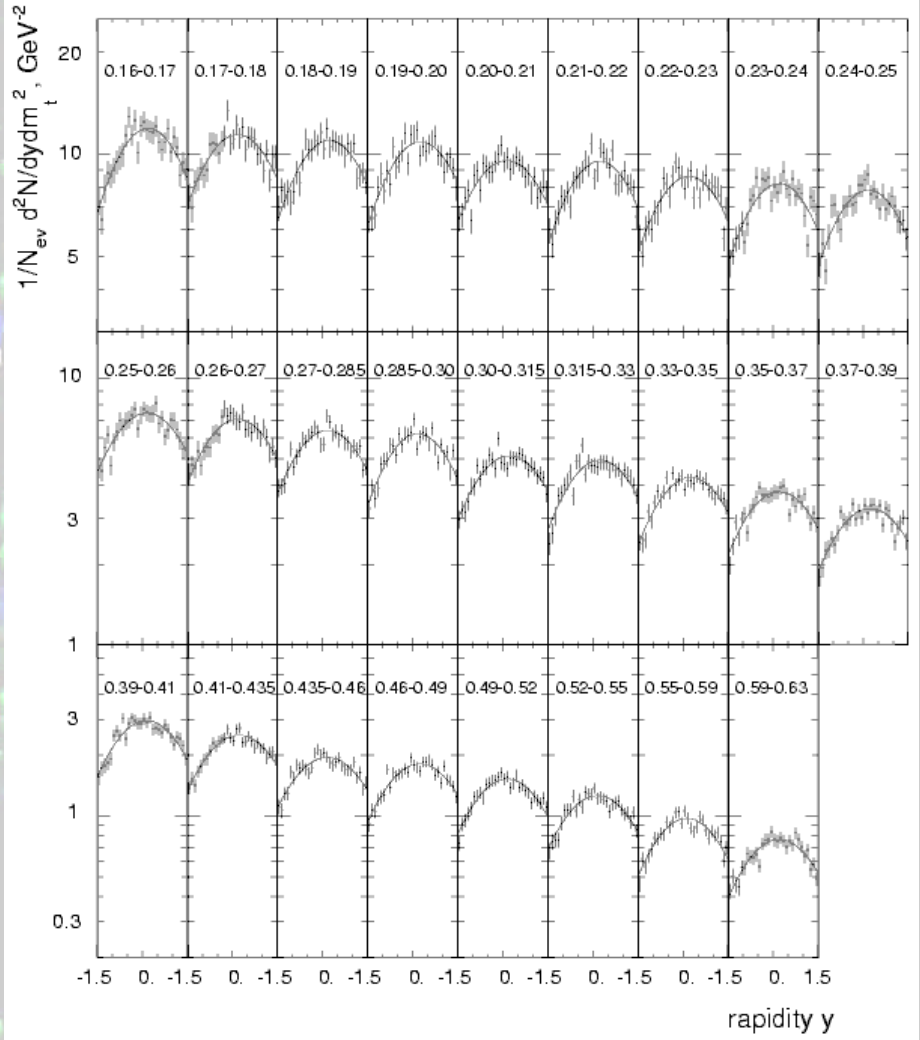




A. Ster, T. Cs, B. Lörstad, hep-ph/9907338

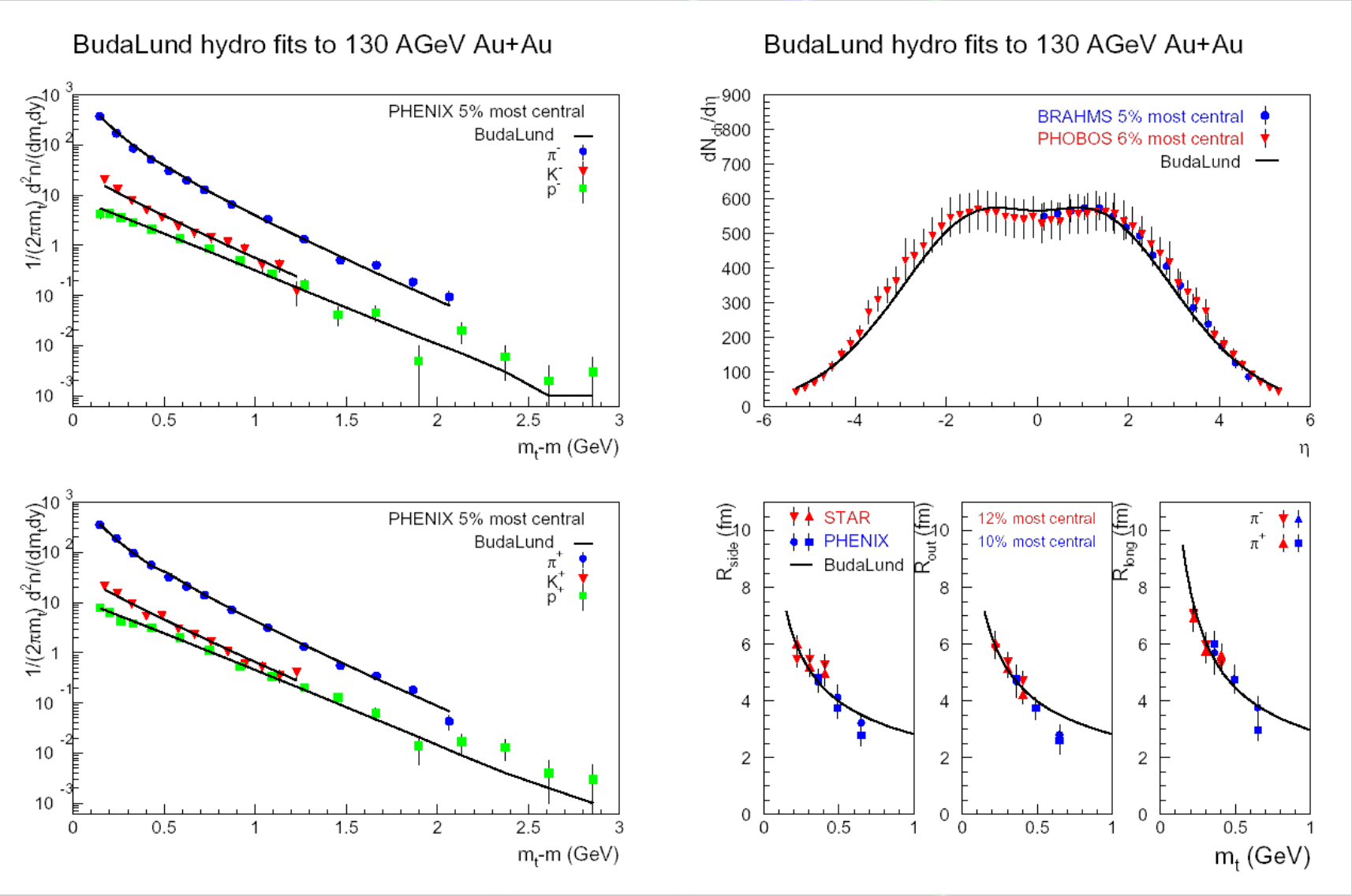
Buda-Lund fits to NA22 h + p data





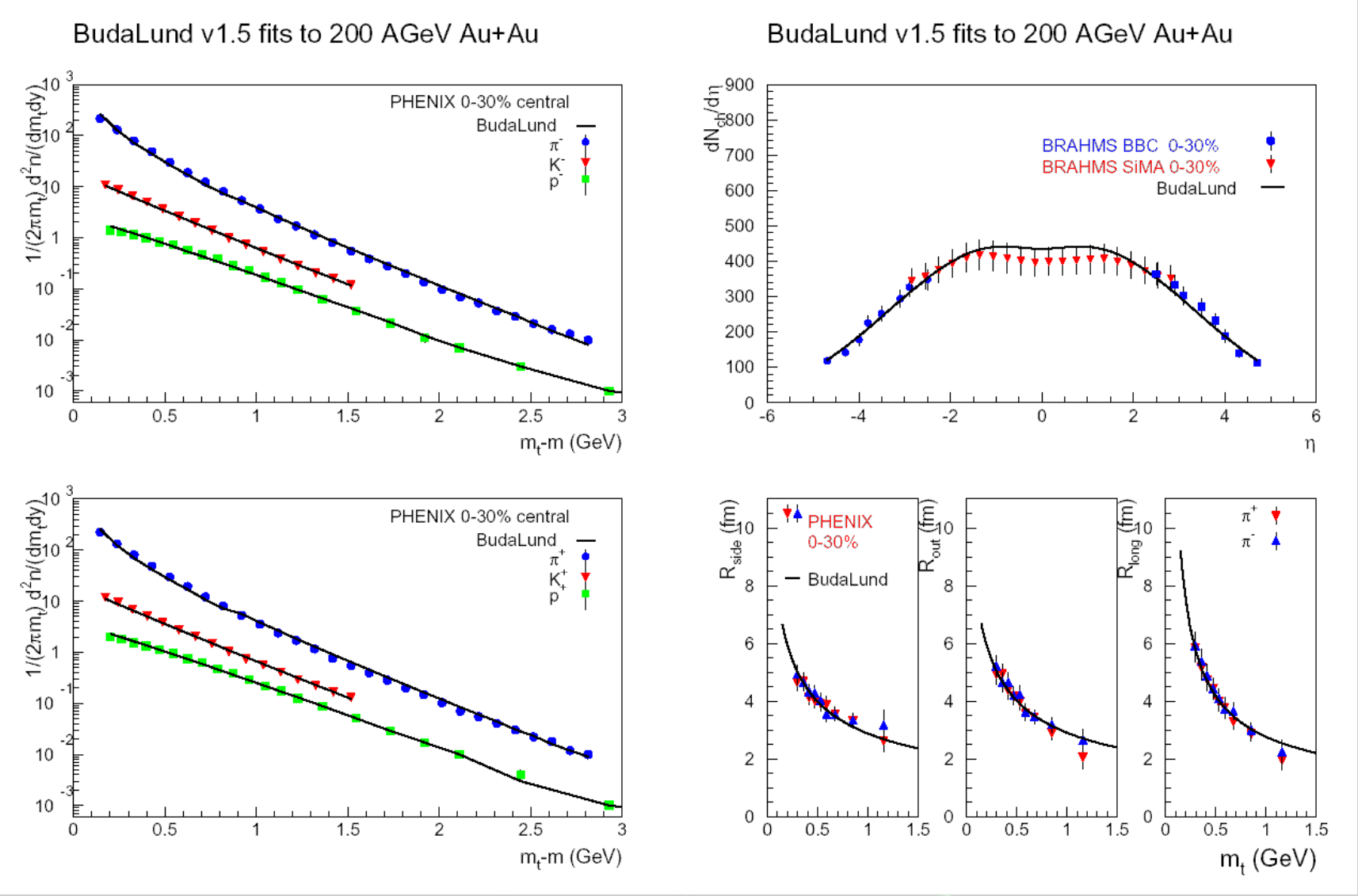
N. M. Agababyan et al, EHS/NA22 , PLB 422 (1998) 395
T. Csörgõ, hep-ph/0001233, Heavy Ion Phys. 15 (2002) 1-80

BudaLund fits to 130 GeV RHIC data



M. Csanád, T. Csörgõ, B. Lörstad, A. Ster, nucl-th/0311102, ISMD03

BudaLund fits to 200 GeV RHIC data



M. Csanád, T. Csörgõ, B. Lörstad, A. Ster, nucl-th/0403074, QM04

Fit results, comparing RHIC and SPS

BL at RHIC	Au+Au, 200 GeV	Au+Au, 130 GeV
T_0 [MeV]	196 ± 13	214 ± 7
T_e [MeV]	117 ± 12	102 ± 11
T_s [fm]	$T_0/2$ fixed	$T_0/2$ fixed
R_s [fm]	12.4 ± 1.6	8.6 ± 0.4

BL at SPS	Pb+Pb, NA44	Pb+Pb, NA49	h+p, NA22
T_0 [MeV]	145 ± 3	134 ± 3	140 ± 3
T_e [MeV]	76 ± 87	115 ± 5	-
T_s [fm]	134 ± 11	125 ± 5	82 ± 8

$$T_0(\text{RHIC}) > T_c \text{ by } 5\sigma$$

$$T_0(\text{RHIC}) > T_0(\text{SPS}) \text{ by } 5\sigma$$

$$\frac{T_0 - T_s}{T_s} \simeq 1 \text{ at RHIC}$$

$$\frac{T_0 - T_s}{T_s} \ll 1 \text{ at SPS Pb+Pb}$$

RHIC and the Universe

- Developed Hubble-flow at RHIC and in the Universe
- Universality of the Hubble expansion: $u = H r$

- Hubble constant of the Universe:
  - $H_0 = (71 \pm 7)$ km/sec/Mpc
- converted to SI units:
  - $H_0 = (2.3 \pm 0.2) \times 10^{-18}$ sec^{-1}

- Hubble constant at Au+Au collisions with 200 GeV
  - $H_{RHIC,1} = \langle u_t \rangle / R_G \simeq (3.8 \pm 0.5) \times 10^{22}$ sec^{-1}
  - $H_{RHIC,2} = 1/t_0 \simeq (5.1 \pm 0.1) \times 10^{22}$ sec^{-1}

- Ratio of expansion rates:
  - $H_{RHIC} / H_0 \simeq 2 \times 10^{40}$

- approx. the ratio of the ages of the objects
- without correction for inflation...

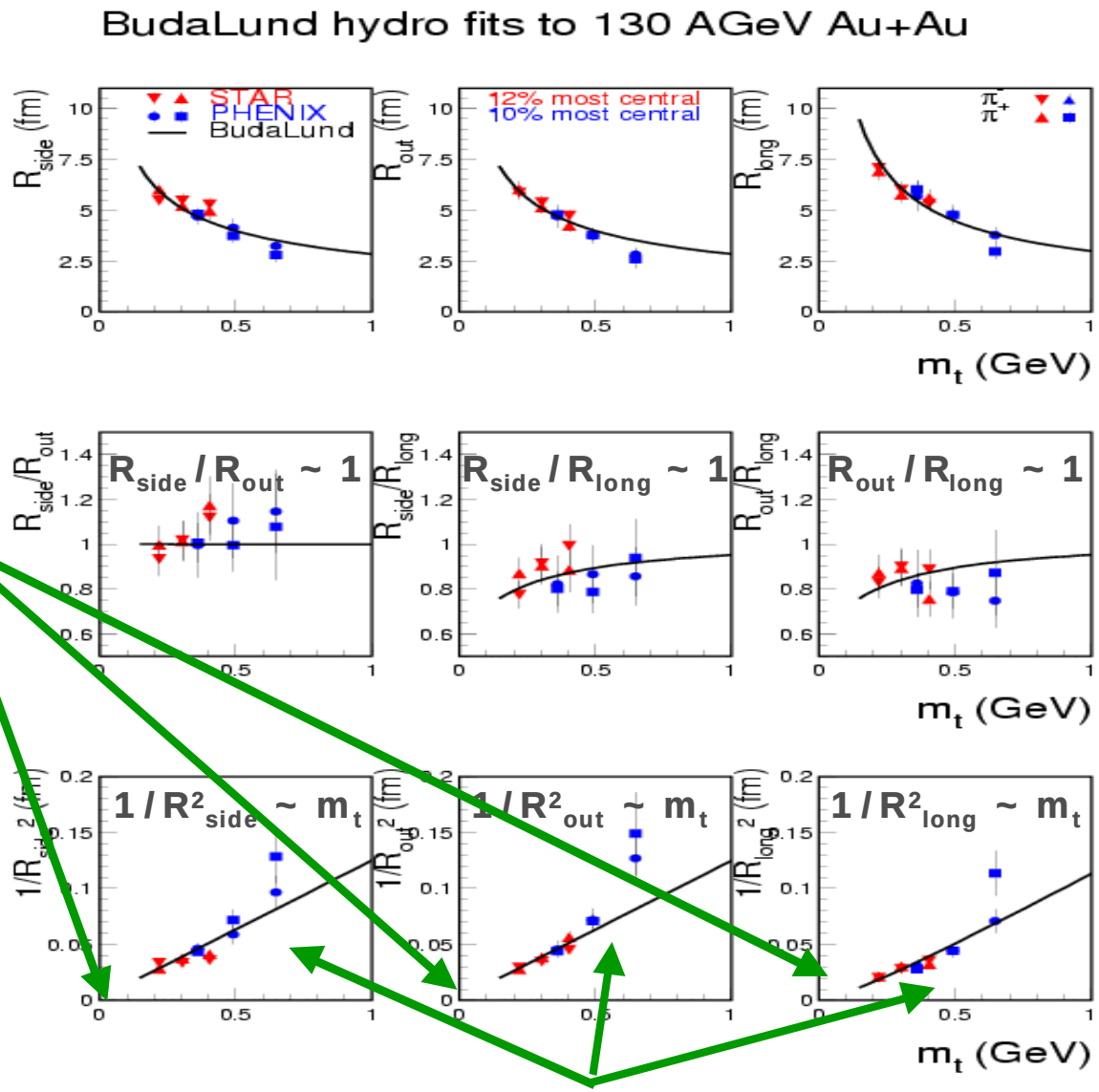
New ways of plotting the results

$1/R^2_{eff} = 1/R^2_{geom} + 1/R^2_{therm}$
and $1/R^2_{therm} \sim m_t$

intercept is nearly 0, indicating $1/R_G^2 \simeq 0$,

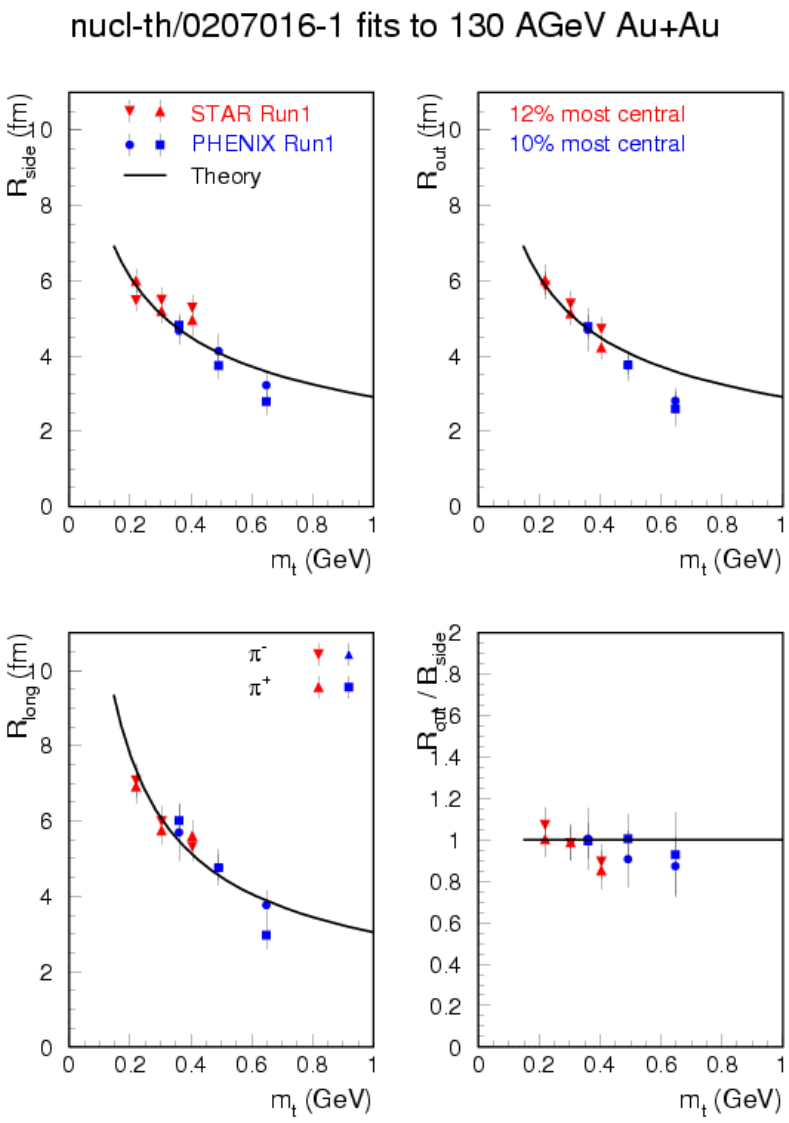
thus $\mu(x)/T(x)$ = const!

reason for success of thermal models @ RHIC!



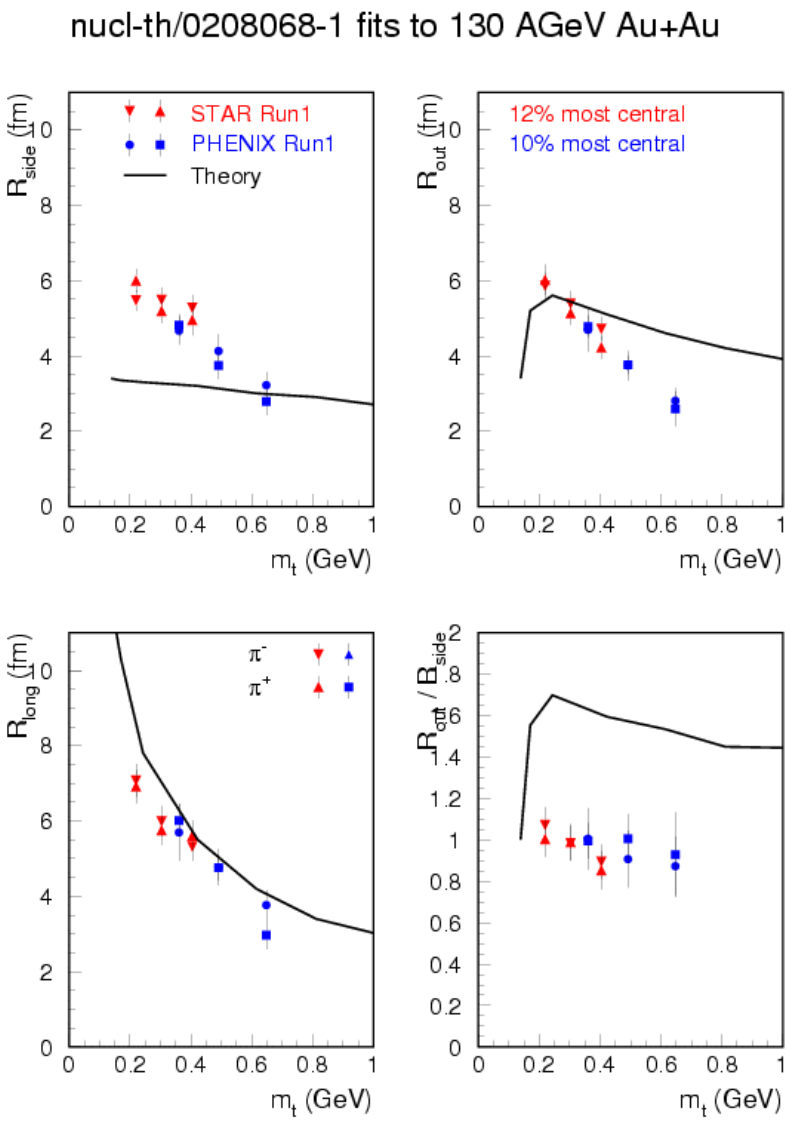
same slopes ~ fully developed, 3d Hubble flow

Comparison of results of models



Acceptable

Comparison of results of models



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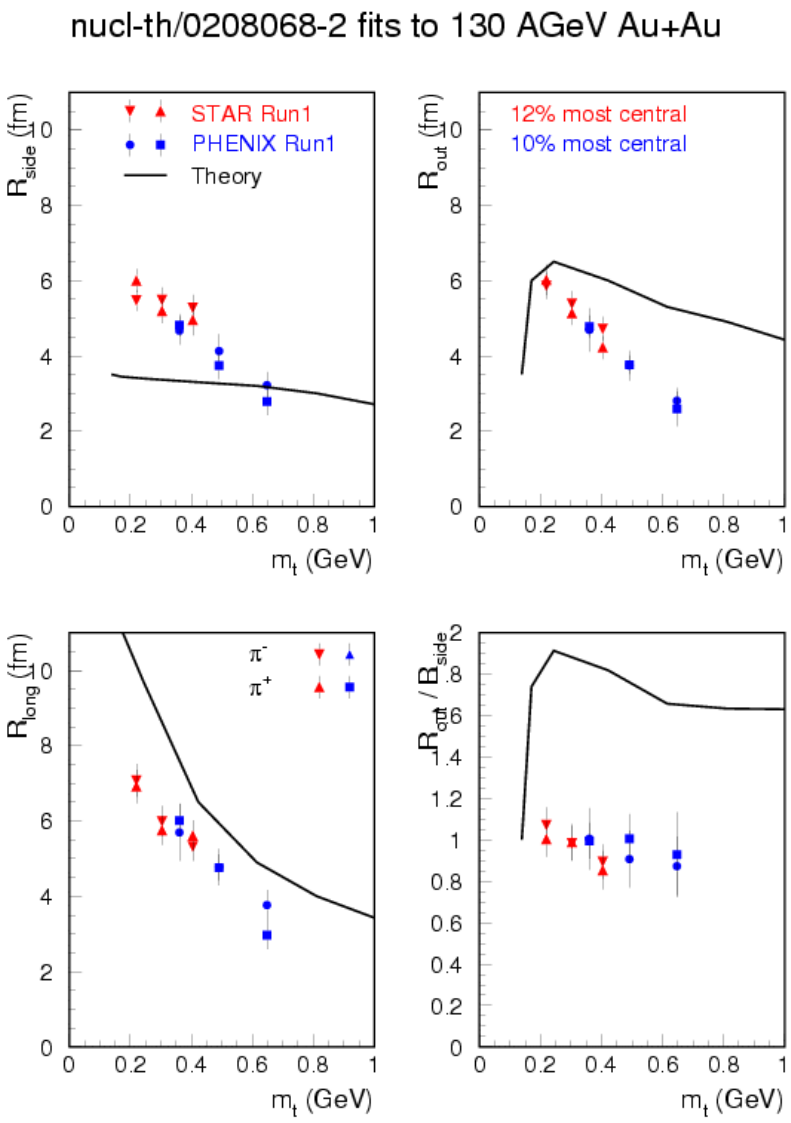

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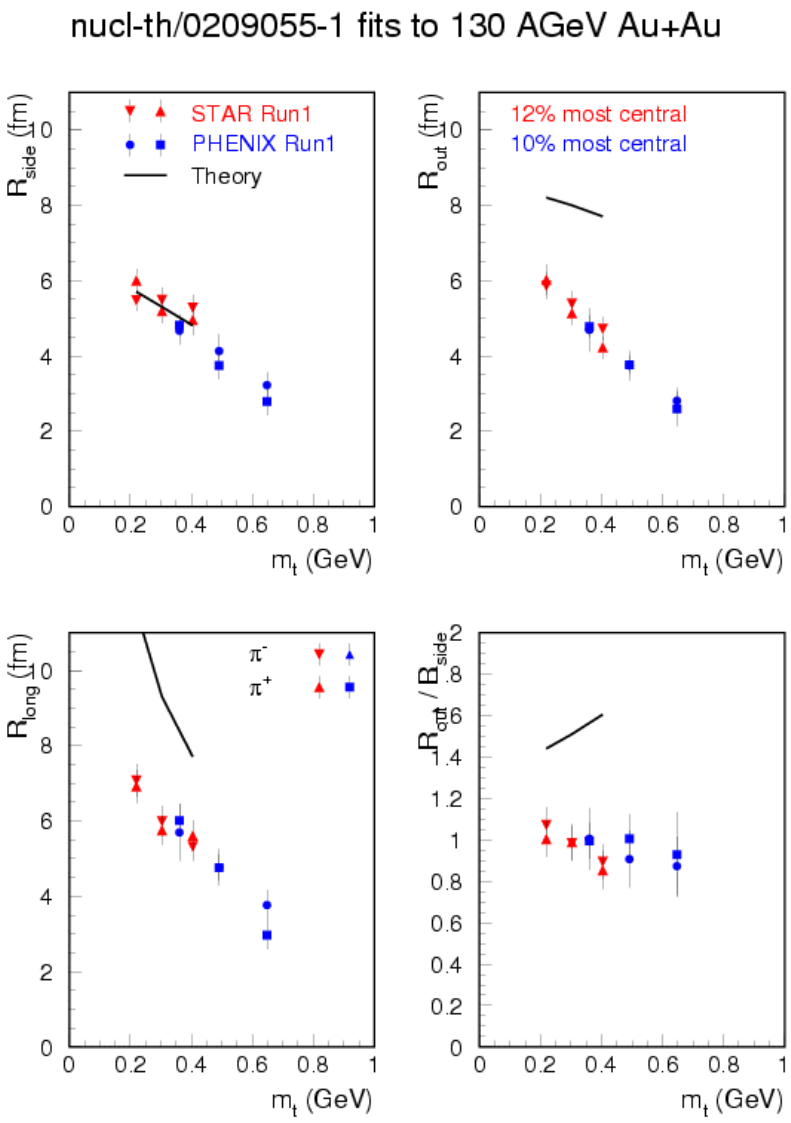

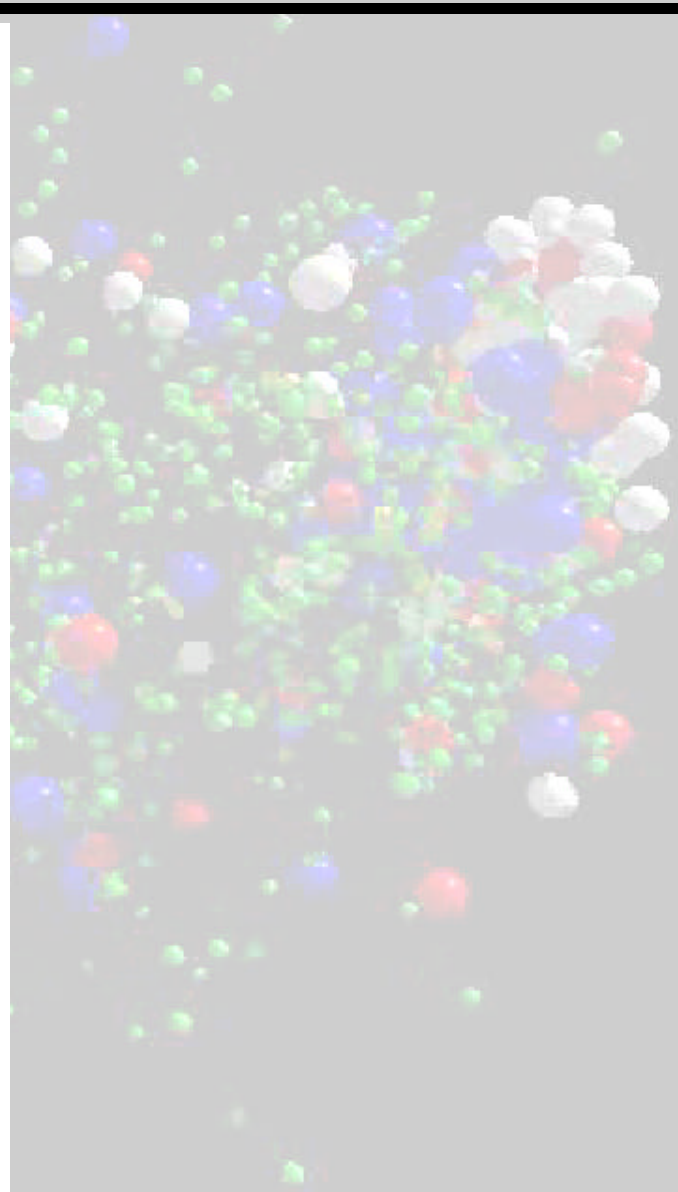

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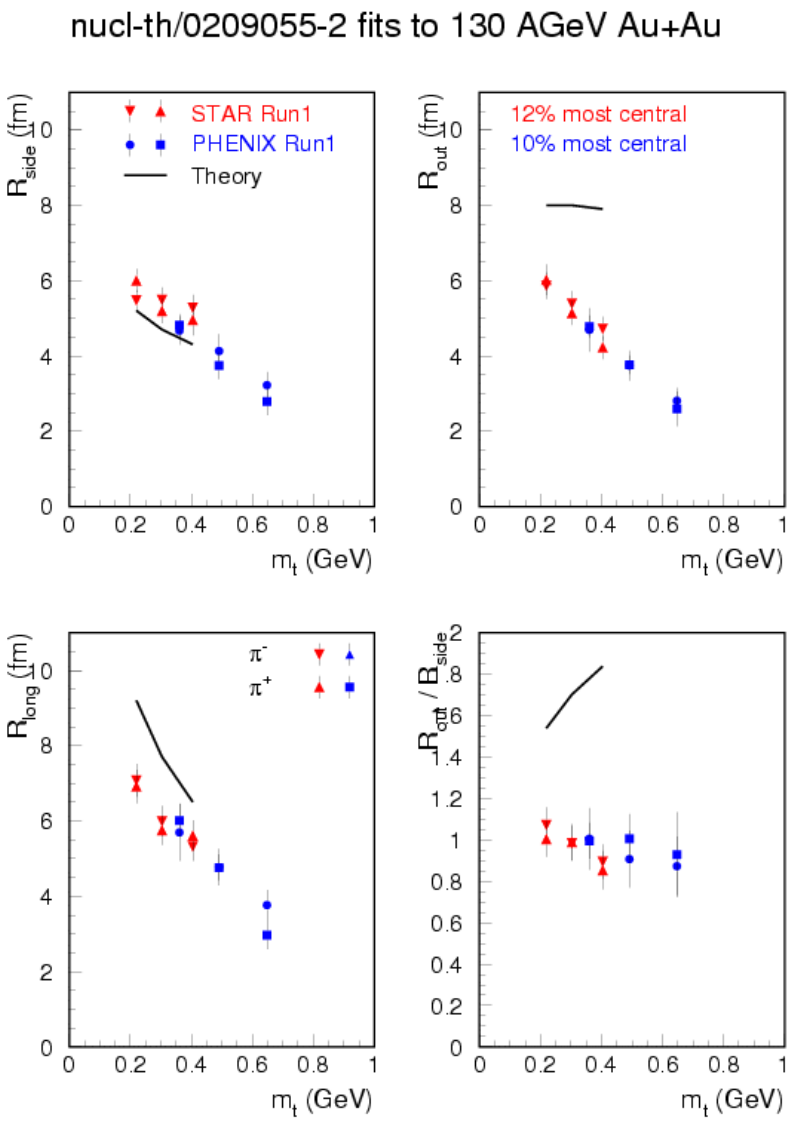

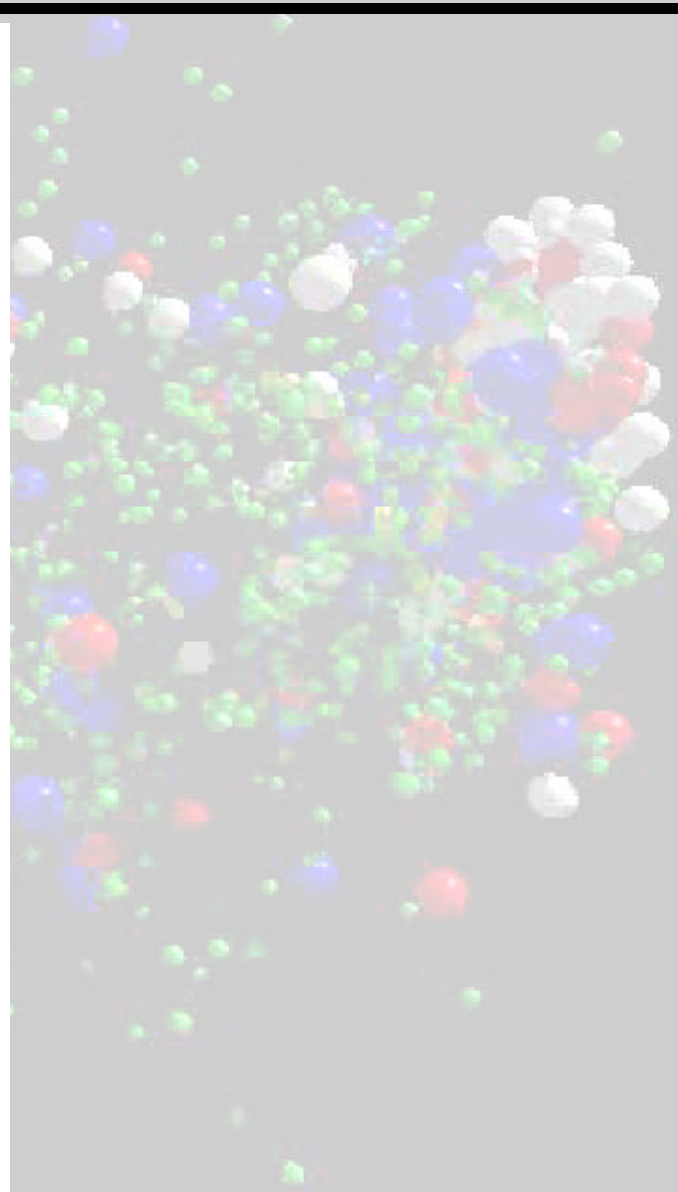

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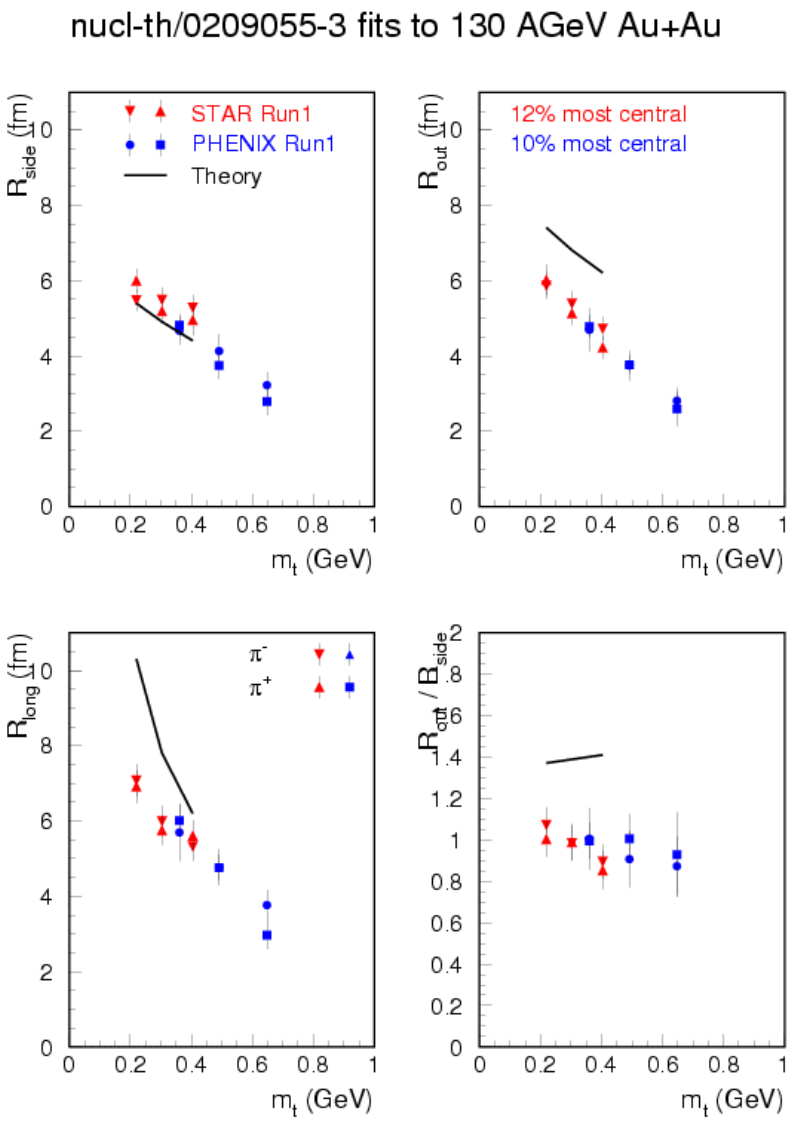

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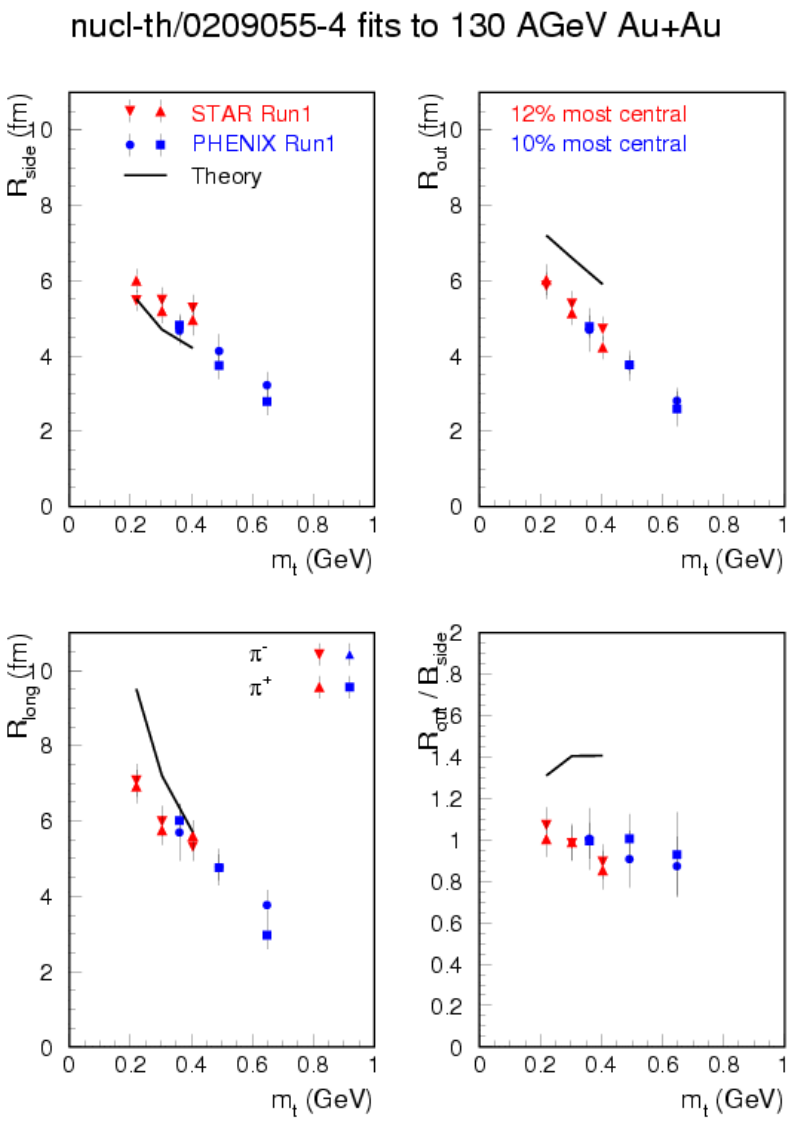

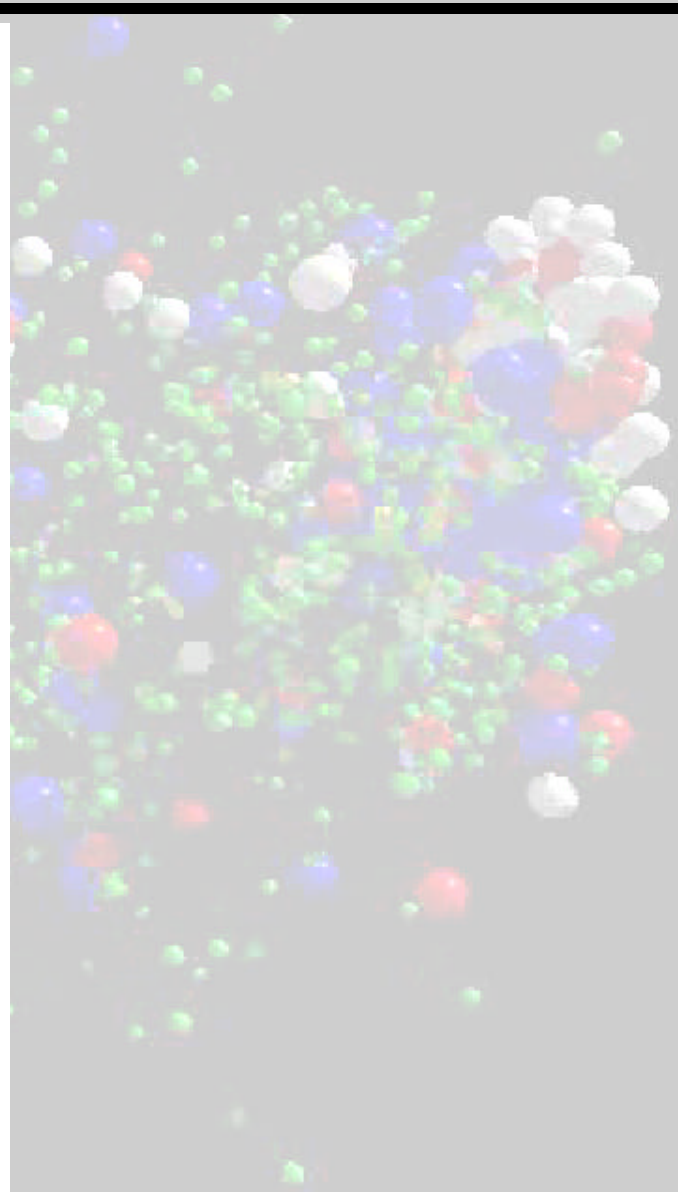

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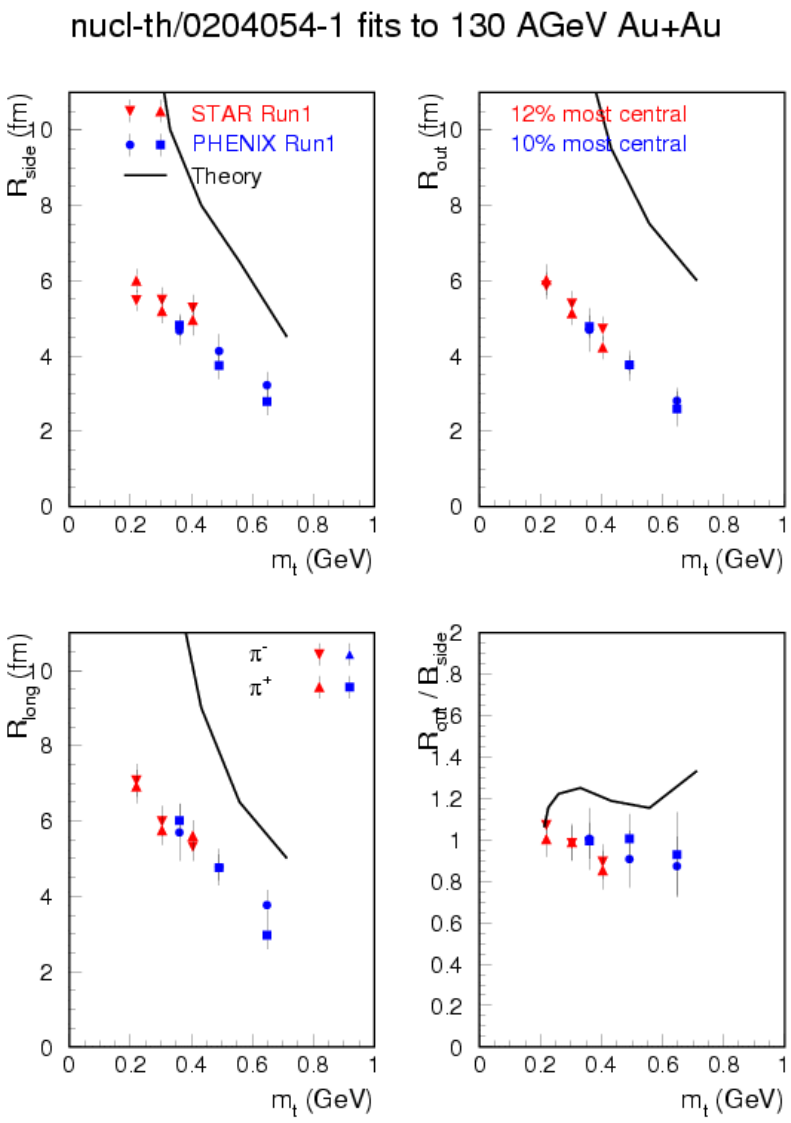
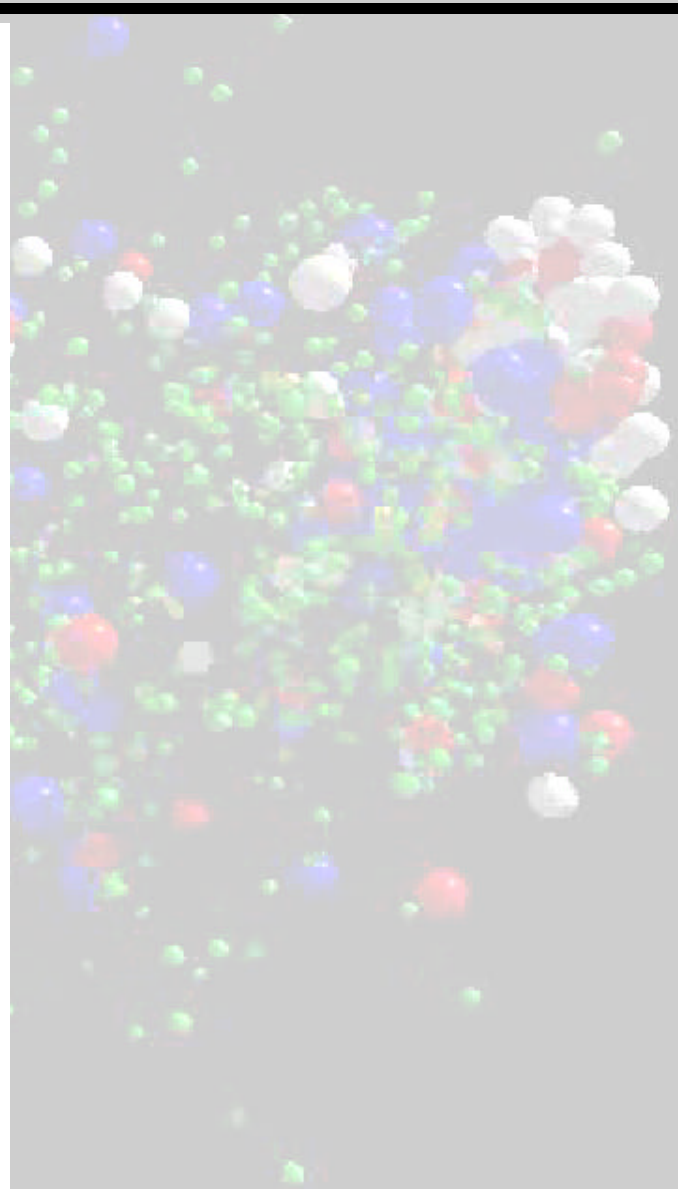
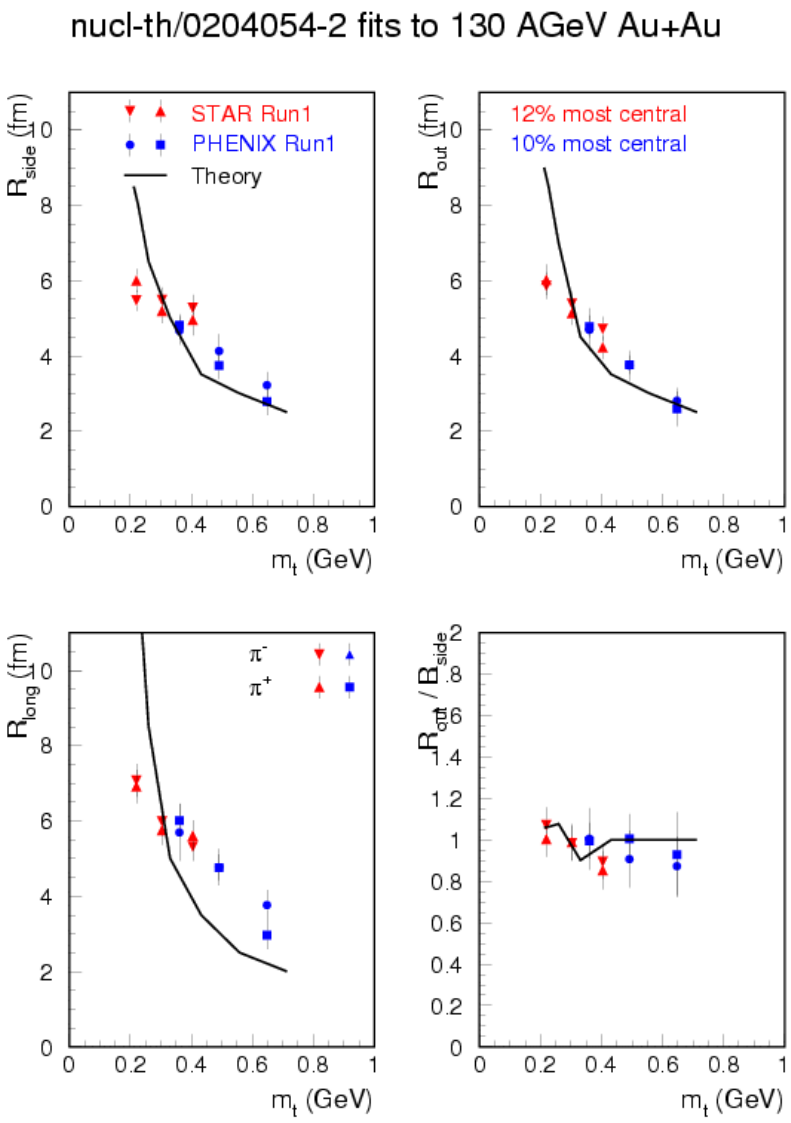


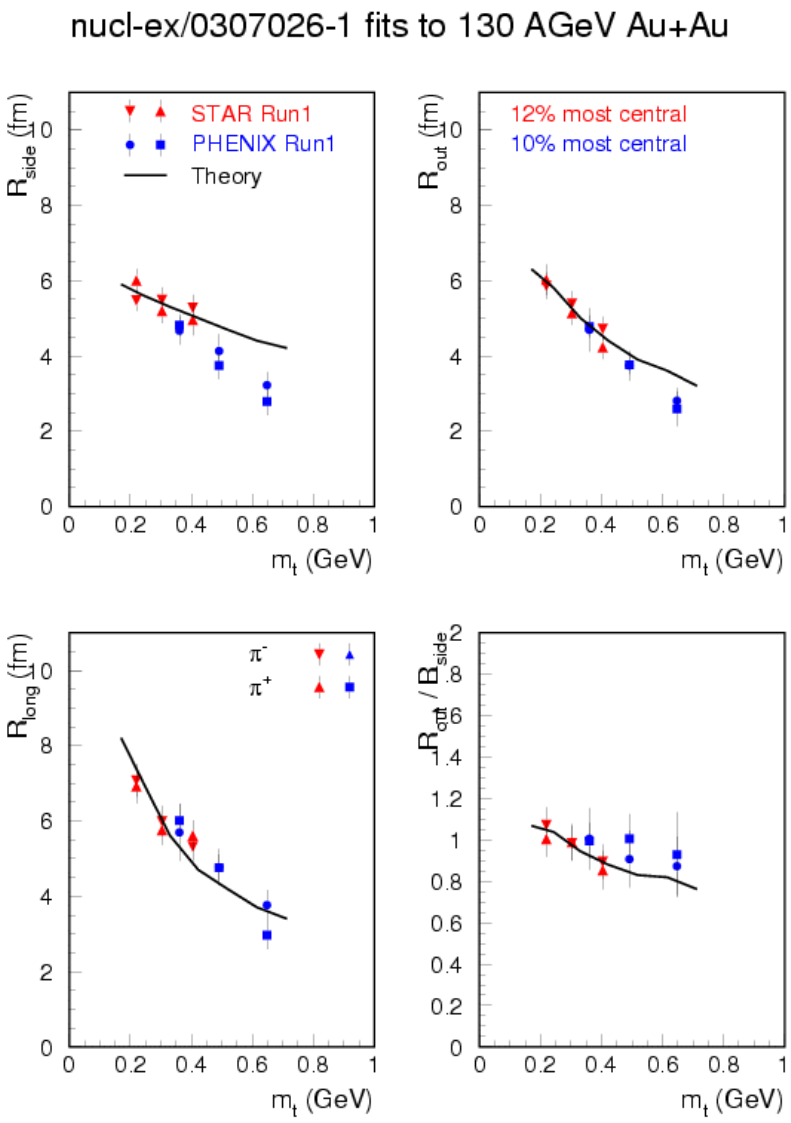


Comparison of results of models



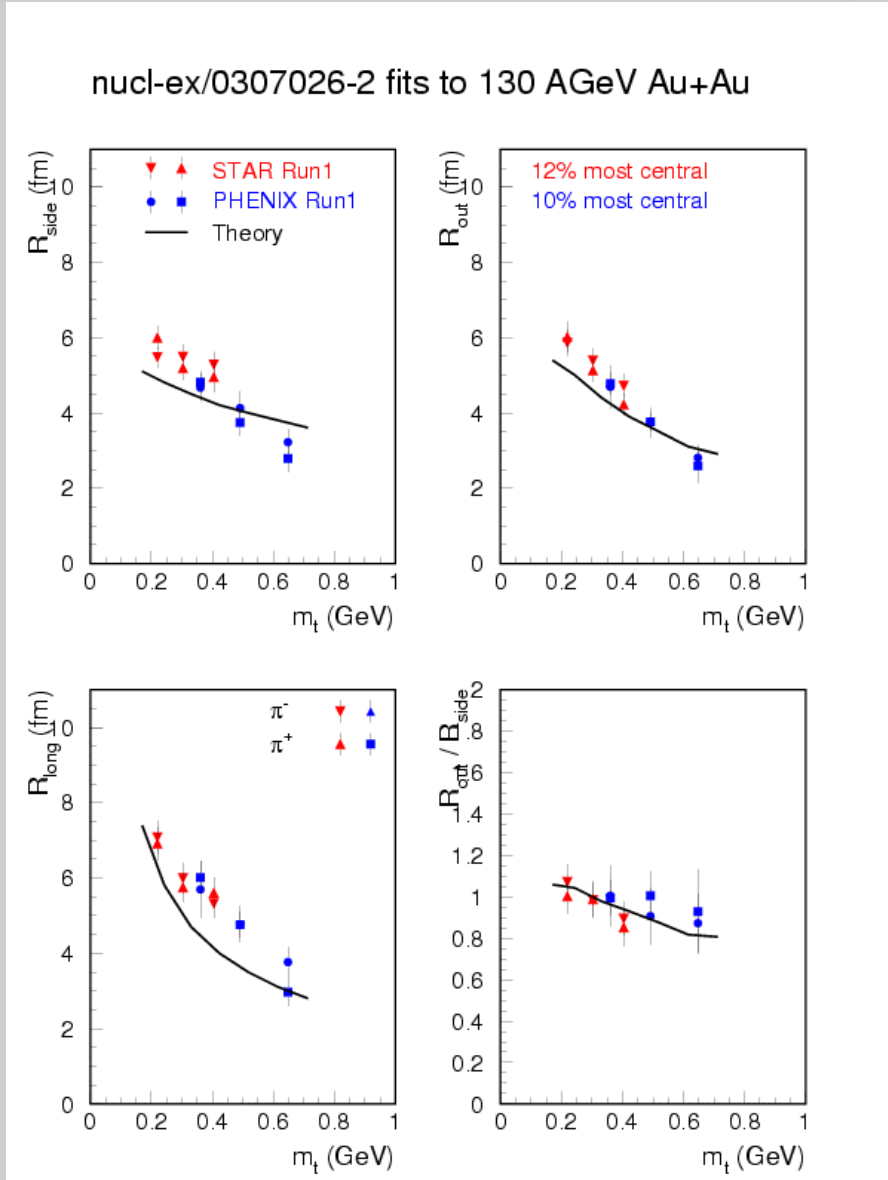
~Acceptable

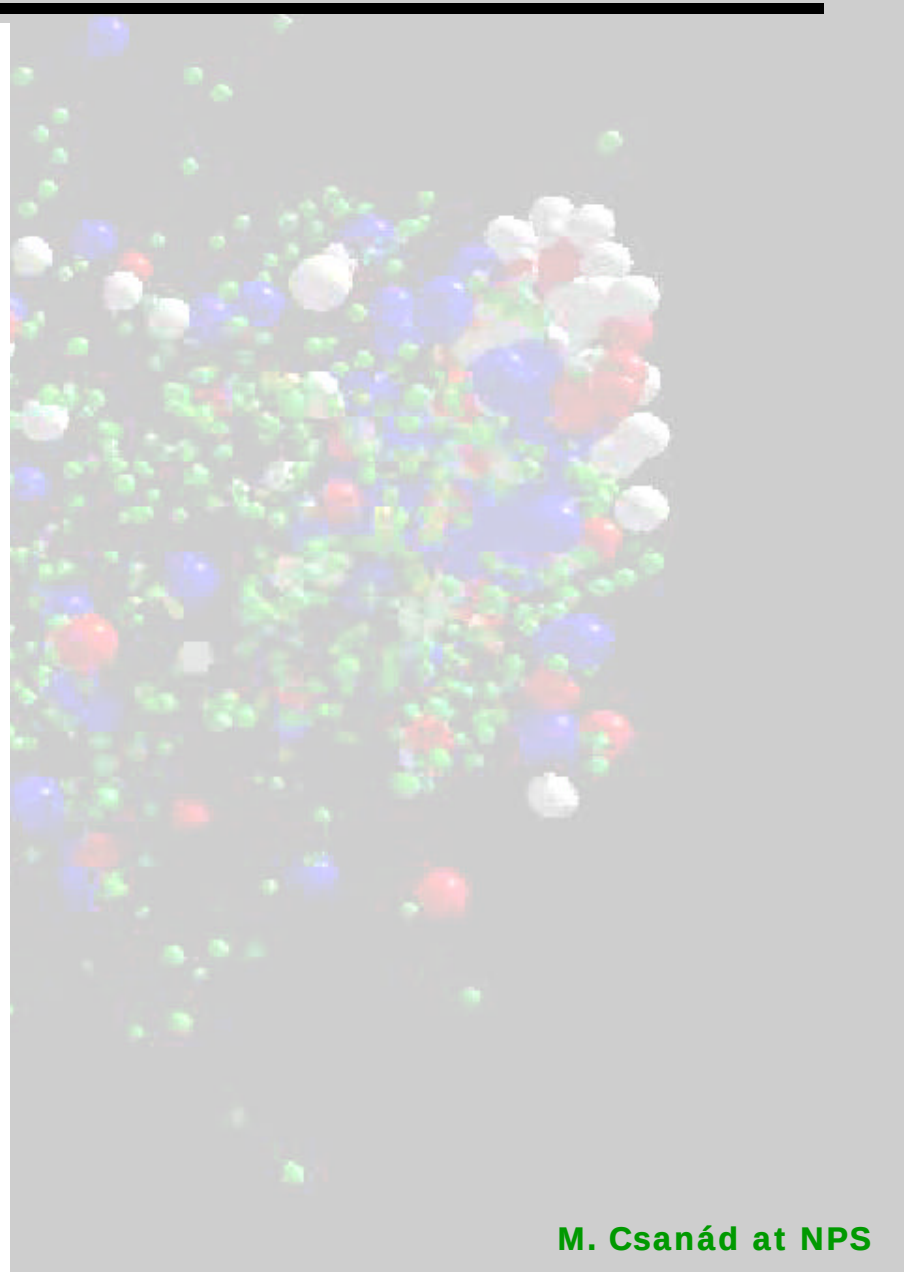
Comparison of results of models



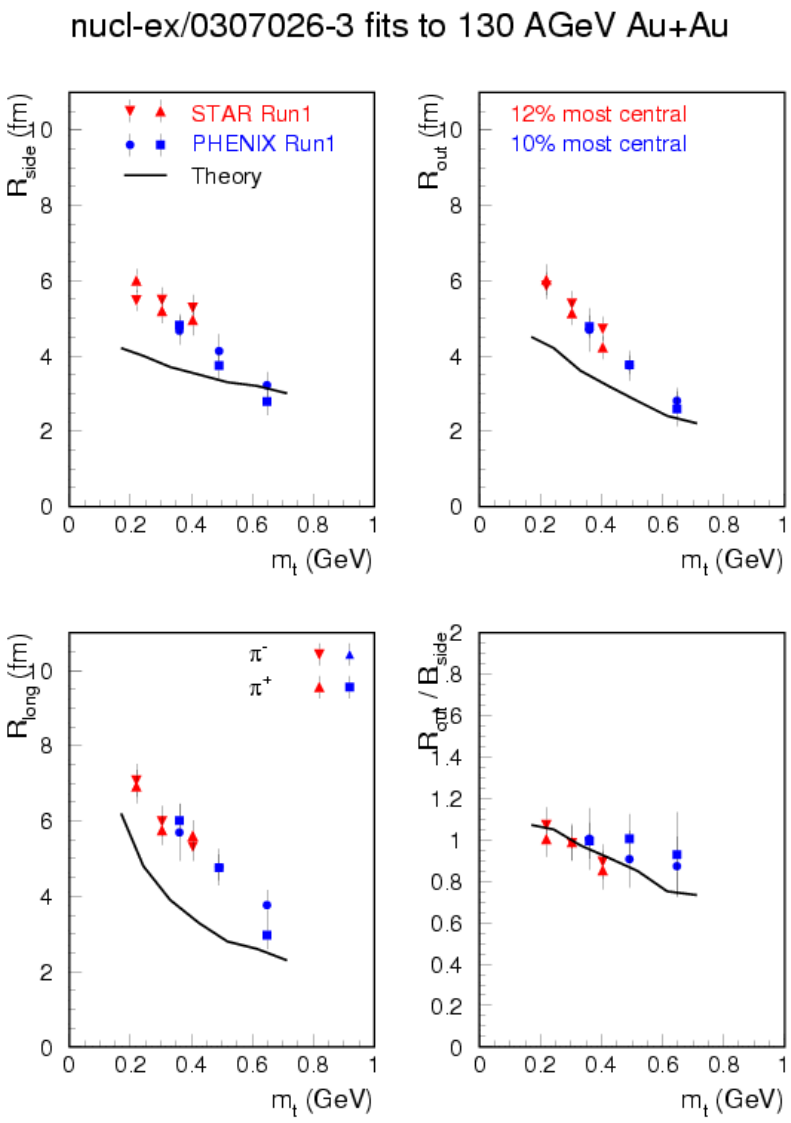
Acceptable

Comparison of results of models

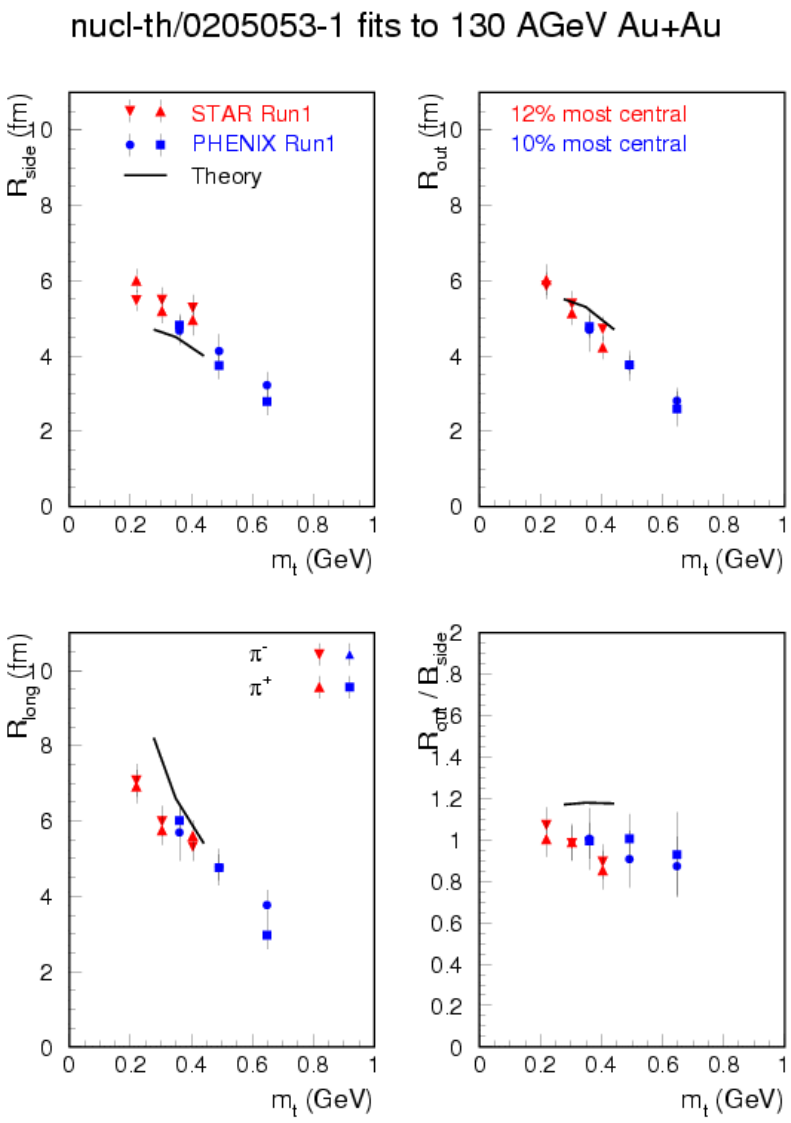




Comparison of results of models

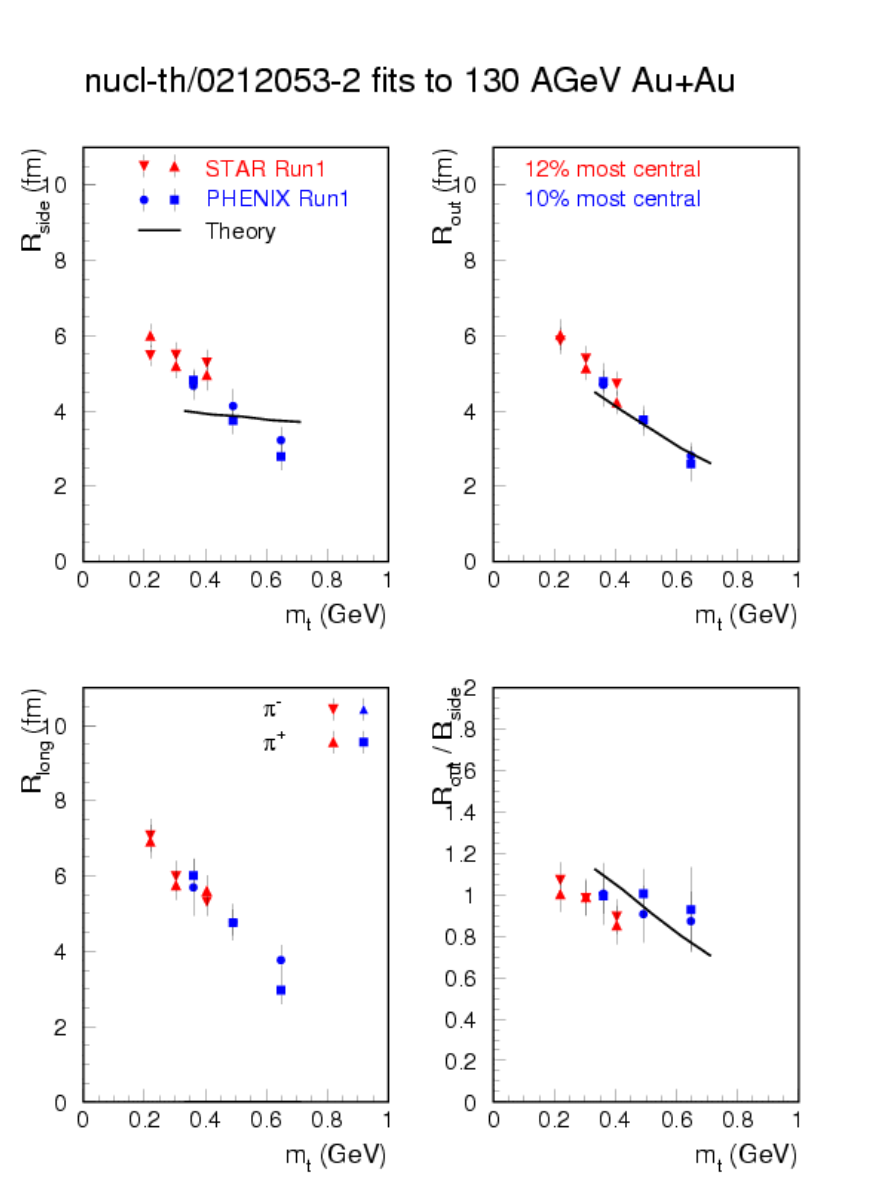


Comparison of results of models



~Acceptable

Comparison of results of models

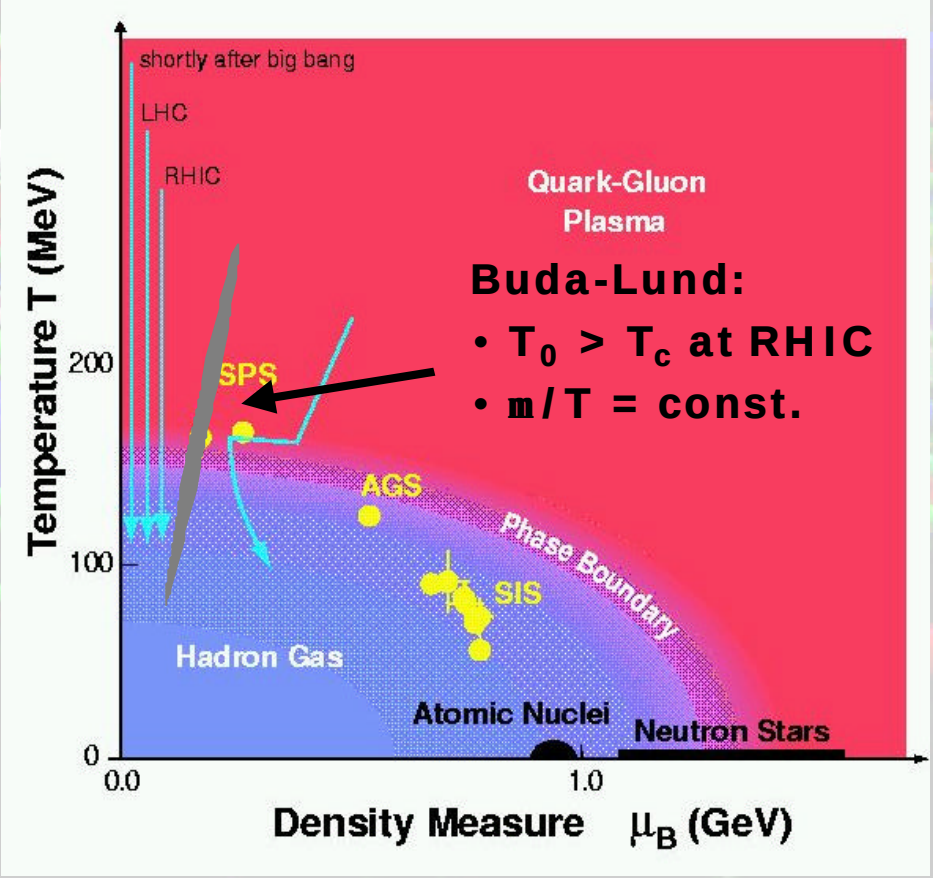


Comparison of results of models

- **Models with acceptable results:**
  - nucl-th/0204054 — Multiphase Trasport model (AMPT)
    Z. Lin, C. M. Ko, S. Pal
  - nucl-th/0205053 — Hadron cascade model
    T. Humanic
  - nucl-th/0207016 — Buda-Lund hydro model
    T. Csörgő. A. Ster
  - nucl-ex/0307026 — Blast wave model
    F. Retiére for STAR
- **Not shown here, but acceptable**
  - nucl-th/0208068 — 3D hydro model
    T. Hirano, T. Tsuda
  - hep-ph/0209054 — Cracow (single freeze-out, thermal) model
    W. Broniowski, A. Baran, W. Florkowski

A solution of the RHIC HBT puzzle

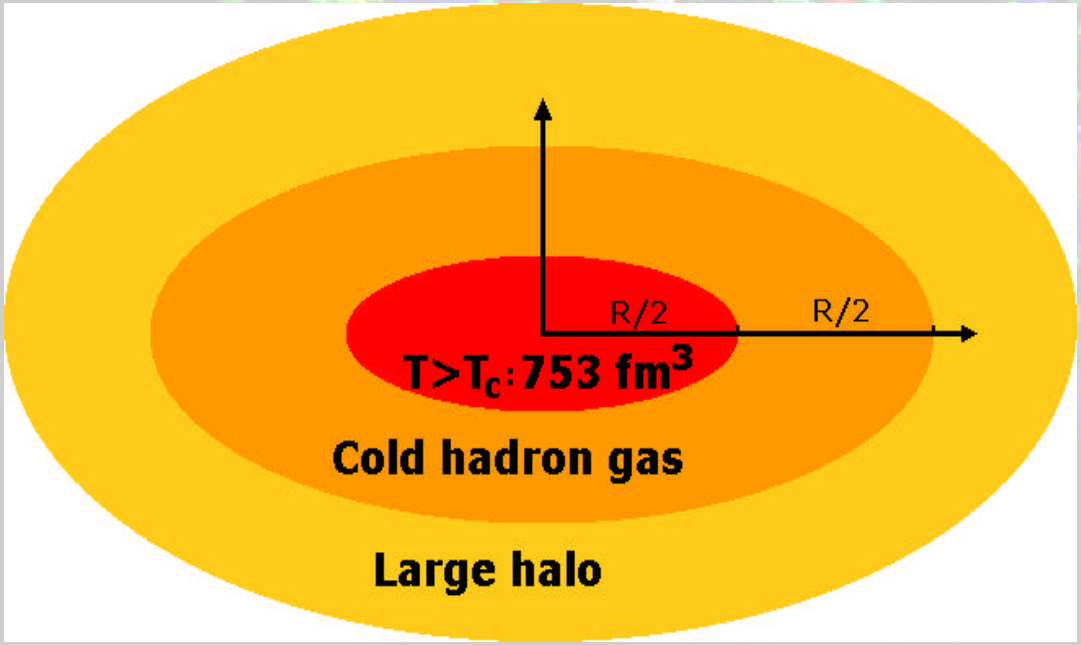
- Our answer: hot center, a fireball heated from inside
- If we assume a hot center, the difference between out and side vanishes

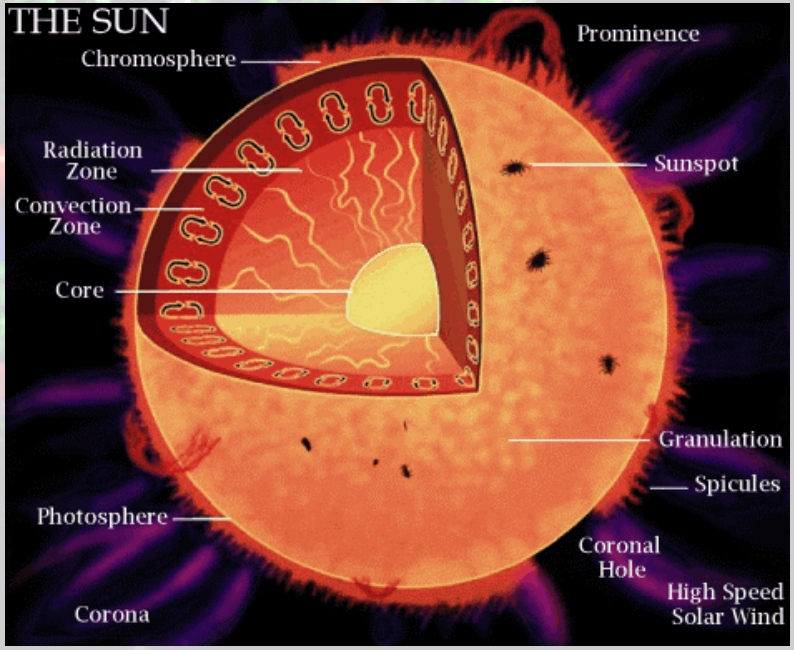


A useful analogy

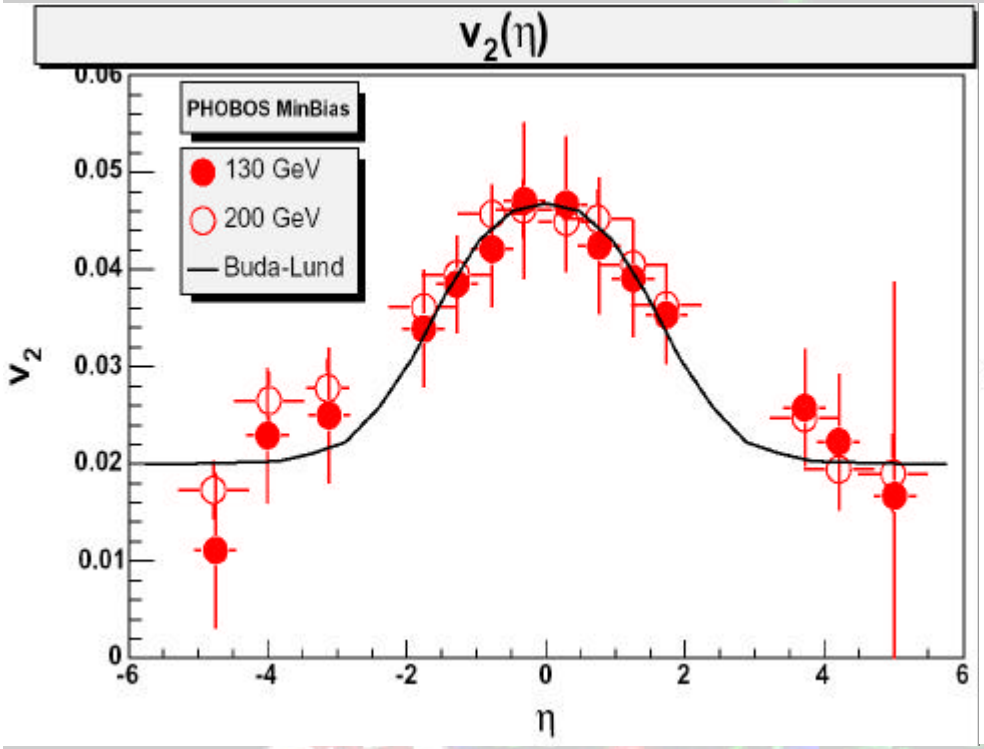
Fireball at RHIC Û our Sun

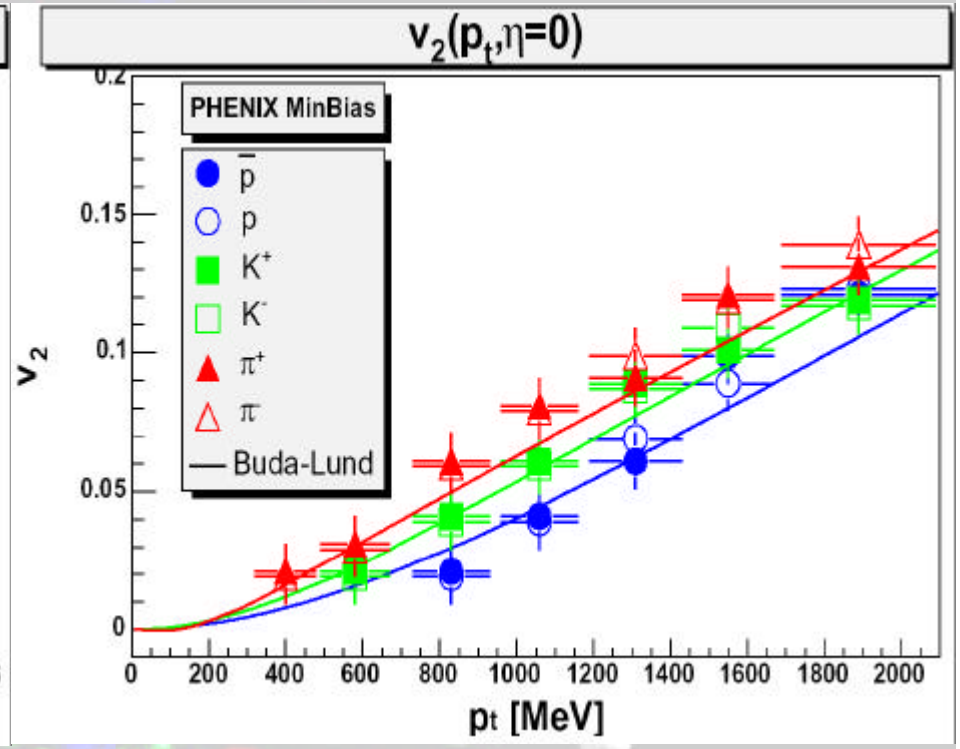
- Core Û Sun
- Halo Û Solar wind
- $T_{0,RHIC} \simeq 210$ MeV Û $T_{0,SUN} \simeq 16$ million K
- $T_{surface,RHIC} \simeq 100$ MeV Û $T_{surface,SUN} \simeq 6000$ K





Confirmation of hot center

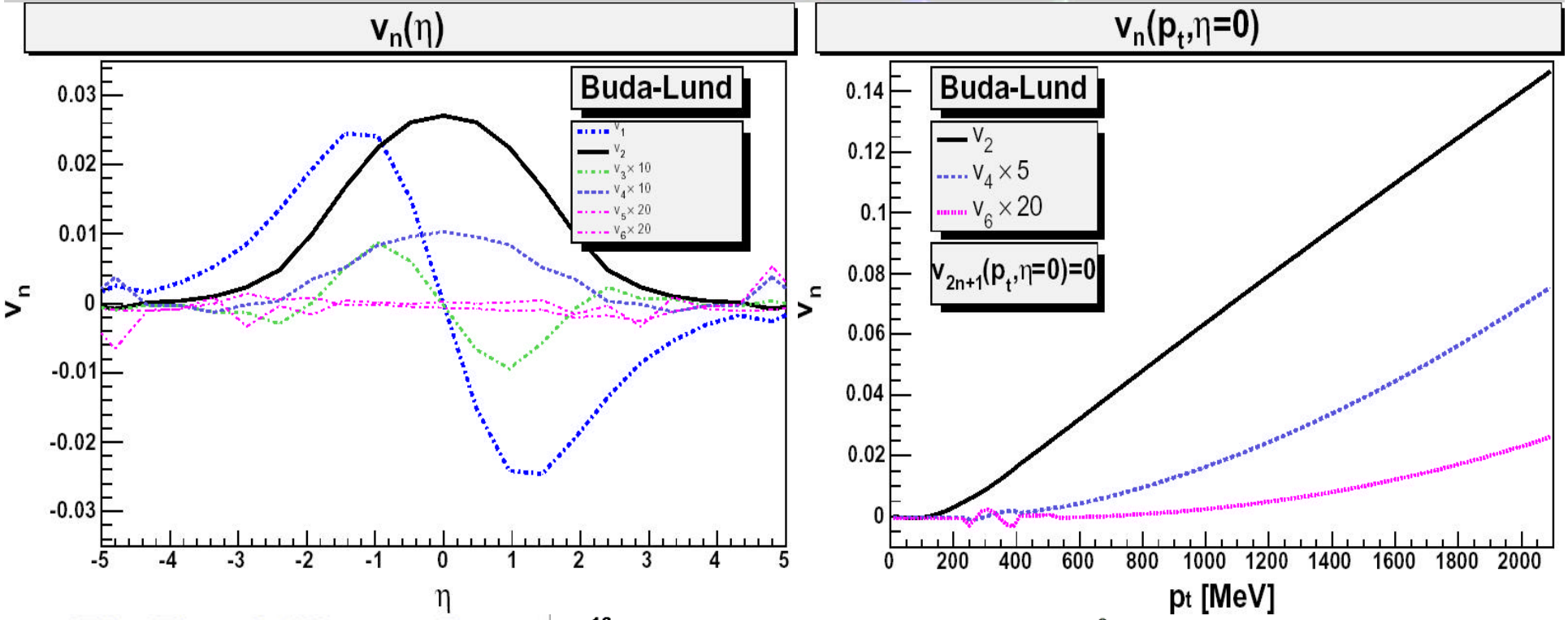


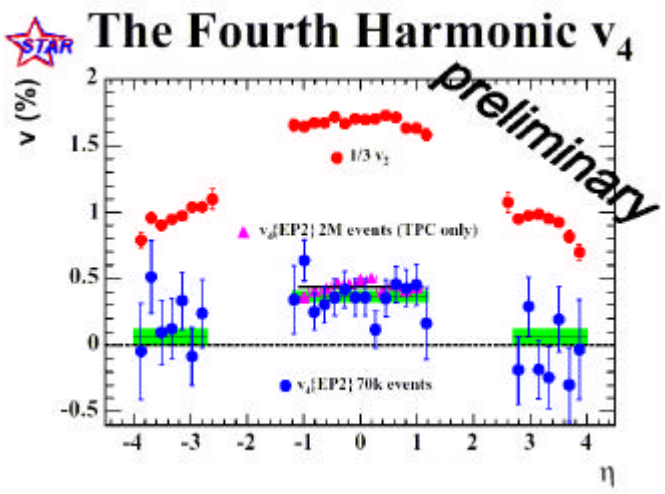


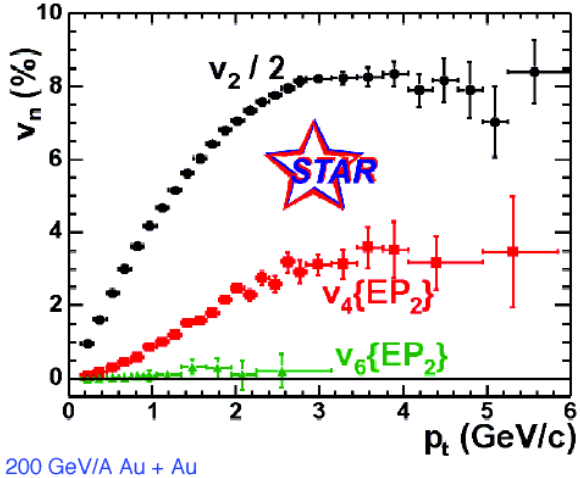
$$T_s = 105 \text{ MeV},\ T_0 = 210 \text{ MeV},\ \tau_0 = 7 \text{ fm/c},\ \vartheta = 0.06$$
$$\dot{X} = 0.57,\ \dot{Y} = 0.45,\ \dot{Z} = 2.4$$
$$X_f = 8.6 \text{ fm},\ Y_f = 10.5 \text{ fm},\ Z_f = 17.5 \text{ fm}$$

see nucl-th/0310040 and nucl-th/0403074

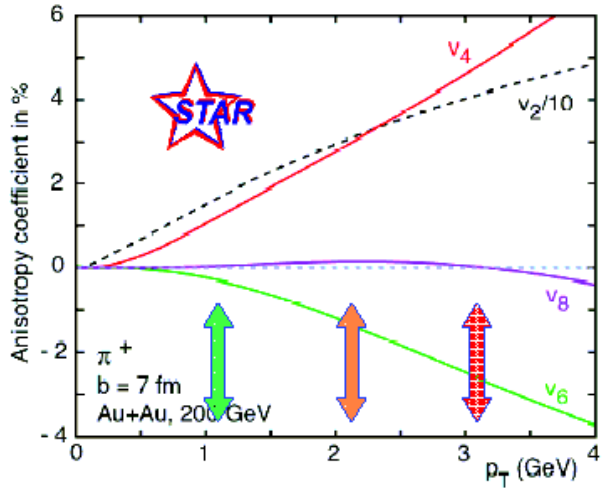
Higher harmonics







200 GeV/A Au + Au



Summary

The analysis of RHIC Au+Au data from BRAHMS, PHOBOS, PHENIX and STAR show:

- **succesful Buda-Lund hydro fits**

  (also at h+p and Pb+Pb at SPS)

- **indication for deconfinement at RHIC**

  ($T > T_c = 172$ MeV by 5s at RHIC, but not at SPS)

- **evidence for a developed 3d Hubble flow at RHIC**

Plans for the future

- Complete the fitting package for the relativistic calculations
- Fitting the new data
  - Anisotropic flow, higher order flows at STAR
  - Centrality and rapidity dependent elliptic flow
- Make prediction
  - J/Y yield
  - HBT for kaons
- Find the relativistic hydro solution that leads to our emission function

Thank you for your attention.