DERIVING ANGULAR MOMENTUM SUM RULES

THE GOOD, THE BAD AND THE UGLY

Elliot Leader

Imperial College London

work done in collaboration with

Ben Bakker, Vrije Universiteit, Amsterdam

and

Larry Trueman, BNL

Background to the study—or why did we bother to work like slaves for several months?

Shore and White's, at first sight, surprising claim about the axial anomaly.

Background to the study—or why did we bother to work like slaves for several months?

Shore and White's, at first sight, surprising claim about the axial anomaly.

Based on a classical paper of Jaffe and Manohar who stressed the subtleties and warned that 'a careful limiting procedure has to be introduced'.

Background to the study—or why did we bother to work like slaves for several months?

Shore and White's, at first sight, surprising claim about the axial anomaly.

Based on a classical paper of Jaffe and Manohar who stressed the subtleties and warned that 'a careful limiting procedure has to be introduced'.

Despite all the care, there are flaws. With the J-M result one cannot have a sum rule for a transversely polarized nucleon.

With the correct version one can!

OUTLINE OF TALK

The Ugly: The traditional way of deriving angular momentum sum rules. Its pitfalls and problems. Horrible infinities all over the place.

The Bad: Our improvement of the traditional approach. No infinities but the price is high in terms of complexity.

The Good: Larry Trueman's brilliant idea. All is beautiful and simple.

What is the aim???

We consider a nucleon with 4-momentum p^μ and covariant spin vector S corresponding to some specification of its spin state e.g. helicity, transversity or spin along the Z-axis i.e. a nucleon in state $|p, S\rangle$.

What is the aim???

We consider a nucleon with 4-momentum p^μ and covariant spin vector S corresponding to some specification of its spin state e.g. helicity, transversity or spin along the Z-axis i.e. a nucleon in state $|p, S\rangle$.

We require an expression for the expectation value of the angular momentum in this state i.e. for $\langle p, S|\boldsymbol{J}|p, S\rangle$

What is the aim???

We consider a nucleon with 4-momentum p^μ and covariant spin vector S corresponding to some specification of its spin state e.g. helicity, transversity or spin along the Z-axis i.e. a nucleon in state $|p, S\rangle$.

We require an expression for the expectation value of the angular momentum in this state i.e. for $\langle p, S|\boldsymbol{J}|p, S\rangle$

i.e. we require an expression in terms of p and S. This can then be used to relate the expectation value of $\boldsymbol{J}$ for the nucleon to the angular momentum carried by its constituents.

THE UGLY

The traditional approach: In every field theory there is an expression for the angular momentum density operator. The angular momentum operator $\boldsymbol{J}$ is then an integral over all space of this density.

THE UGLY

The traditional approach: In every field theory there is an expression for the angular momentum density operator. The angular momentum operator $\boldsymbol{J}$ is then an integral over all space of this density.

To understand the subtleties we need to recall Noether's famous theorem:

THE UGLY

The traditional approach: In every field theory there is an expression for the angular momentum density operator. The angular momentum operator $\boldsymbol{J}$ is then an integral over all space of this density.

To understand the subtleties we need to recall Noether's famous theorem:

For every continuous symmetry there is a conserved current and a conserved operator which generates the transformations of that symmetry.

THE UGLY

The traditional approach: In every field theory there is an expression for the angular momentum density operator. The angular momentum operator $\boldsymbol{J}$ is then an integral over all space of this density.

To understand the subtleties we need to recall Noether's famous theorem:

For every continuous symmetry there is a conserved current and a conserved operator which generates the transformations of that symmetry.

Thus invariance under time translations $\Rightarrow$ conservation of the energy operator (or Hamiltonian) P_0.

Invariance under spatial translations $\Rightarrow$ conservation of linear momentum $\boldsymbol{P}$

Then translations in space-time are generated as follows: For any local operator $F(x)$

$$F(x+a) = e^{iP.a}F(x)e^{-iP.a}$$

Thus

$$F(x) = e^{iP.x}F(0)e^{-iP.x}$$

Then translations in space-time are generated as follows: For any local operator $F(x)$

$$F(x+a) = e^{iP.a} F(x) e^{-iP.a}$$

Thus

$$F(x) = e^{iP.x} F(0) e^{-iP.x}$$

Danger! $G(x) = xF(x)$ seems like a reasonable local operator.

Then translations in space-time are generated as follows: For any local operator $F(x)$

$$F(x+a) = e^{iP.a}F(x)e^{-iP.a}$$

Thus

$$F(x) = e^{iP.x}F(0)e^{-iP.x}$$

Danger! $G(x) = xF(x)$ seems like a reasonable local operator.

But by the above:

$$G(x) = e^{iP.x}G(0)e^{-iP.x}$$

$\therefore G(x) = 0$ for ALL x.

Clearly absurd!

Typically the angular momentum density involves the energy-momentum tensor density $T^{\mu\nu}(x)$ in the form e.g.

$$\boldsymbol{J}_z = \boldsymbol{J}^3 = \int dV [xT^{02}(x) - yT^{01}(x)]$$

Typically the angular momentum density involves the energy-momentum tensor density $T^{\mu\nu}(x)$ in the form e.g.

$$\boldsymbol{J}_z = \boldsymbol{J}^3 = \int dV[xT^{02}(\boldsymbol{x}) - yT^{01}(\boldsymbol{x})]$$

Consider the expectation value of the first term

$$\langle p, \mathsf{S}| \int dV x T^{02}(\boldsymbol{x})|p, \mathsf{S}\rangle = \int dV x \langle p, \mathsf{S}|T^{02}(\boldsymbol{x})|p, \mathsf{S}\rangle$$

Typically the angular momentum density involves the energy-momentum tensor density $T^{\mu\nu}(x)$ in the form e.g.

$$\boldsymbol{J}_z = \boldsymbol{J}^3 = \int dV[xT^{02}(\boldsymbol{x}) - yT^{01}(\boldsymbol{x})]$$

Consider the expectation value of the first term

$$\langle p, \mathsf{S}| \int dV x T^{02}(\boldsymbol{x}) |p, \mathsf{S}\rangle = \int dV x \langle p, \mathsf{S}| T^{02}(\boldsymbol{x}) |p, \mathsf{S}\rangle$$

$$= \int dV x \langle p, \mathsf{S}| e^{i\boldsymbol{P}.\boldsymbol{x}} T^{02}(0) e^{-i\boldsymbol{P}.\boldsymbol{x}} |p, \mathsf{S}\rangle$$

Now the nucleon is in an eigenstate of momentum, so $\boldsymbol{P}$ acting on it just becomes $\boldsymbol{p}$. The numbers $e^{i\boldsymbol{p}.\boldsymbol{x}}e^{-i\boldsymbol{p}.\boldsymbol{x}}$ cancel out and we are left with:

$$\int dV x \langle p, \mathsf{S}|T^{02}(0)|p, \mathsf{S}\rangle$$

Now the nucleon is in an eigenstate of momentum, so $\boldsymbol{P}$ acting on it just becomes $\boldsymbol{p}$. The numbers $e^{i\boldsymbol{p}.\boldsymbol{x}}e^{-i\boldsymbol{p}.\boldsymbol{x}}$ cancel out and we are left with:

$$\int dV x \langle p, \mathsf{S} | T^{02}(0) | p, \mathsf{S} \rangle$$

The matrix element is independent of x so we are faced with $\int dV x = \infty$? or $= 0$? Totally ambiguous!

Now the nucleon is in an eigenstate of momentum, so $\boldsymbol{P}$ acting on it just becomes $\boldsymbol{p}$. The numbers $e^{i\boldsymbol{p}.\boldsymbol{x}}e^{-i\boldsymbol{p}.\boldsymbol{x}}$ cancel out and we are left with:

$$\int dVx\langle p, \mathsf{S}|T^{02}(0)|p, \mathsf{S}\rangle$$

The matrix element is independent of x so we are faced with $\int dVx = \infty$? or $= 0$? Totally ambiguous!

The problem is an old one: In ordinary QM plane wave states give infinities

Now the nucleon is in an eigenstate of momentum, so $\boldsymbol{P}$ acting on it just becomes $\boldsymbol{p}$. The numbers $e^{i\boldsymbol{p}.\boldsymbol{x}}e^{-i\boldsymbol{p}.\boldsymbol{x}}$ cancel out and we are left with:

$$\int dVx\langle p, \mathsf{S}|T^{02}(0)|p, \mathsf{S}\rangle$$

The matrix element is independent of x so we are faced with $\int dVx = \infty$? or $= 0$? Totally ambiguous!

The problem is an old one: In ordinary QM plane wave states give infinities

The solution is an old one: Build a wave packet, a superposition of physical plane wave states

In QM we use

$$\Psi_{\boldsymbol{p}_0}(\boldsymbol{x}) = \int d^3\boldsymbol{p}\,\psi(\boldsymbol{p}_0 - \boldsymbol{p})\,e^{i\boldsymbol{p}.\boldsymbol{x}}$$

where $\psi(\boldsymbol{p}_0 - \boldsymbol{p})$ is peaked at $\boldsymbol{p} = \boldsymbol{p}_0$

We then calculate some physical quantity and at the end take the limit of a very sharp wave packet

In QM we use

$$\Psi_{\boldsymbol{p}_0}(\boldsymbol{x}) = \int d^3\boldsymbol{p}\, \psi(\boldsymbol{p}_0 - \boldsymbol{p})\, e^{i\boldsymbol{p}.\boldsymbol{x}}$$

where $\psi(\boldsymbol{p}_0 - \boldsymbol{p})$ is peaked at $\boldsymbol{p} = \boldsymbol{p}_0$

We then calculate some physical quantity and at the end take the limit of a very sharp wave packet

In field theory we do essentially the same and build a physical wave packet state:

$$|\Psi(\boldsymbol{p}_0)\rangle = \int d^3\boldsymbol{p}\, \psi(\boldsymbol{p}_0 - \boldsymbol{p})\, |\boldsymbol{p}\rangle$$

In QM we use

$$\Psi_{\boldsymbol{p}_0}(\boldsymbol{x}) = \int d^3\boldsymbol{p}\, \psi(\boldsymbol{p}_0 - \boldsymbol{p})\, e^{i\boldsymbol{p}.\boldsymbol{x}}$$

where $\psi(\boldsymbol{p}_0 - \boldsymbol{p})$ is peaked at $\boldsymbol{p} = \boldsymbol{p}_0$

We then calculate some physical quantity and at the end take the limit of a very sharp wave packet

In field theory we do essentially the same and build a physical wave packet state:

$$|\Psi(\boldsymbol{p}_0)\rangle = \int d^3\boldsymbol{p}\, \psi(\boldsymbol{p}_0 - \boldsymbol{p})\, |\boldsymbol{p}\rangle$$

then an expectation value in the state $|\Psi(\boldsymbol{p}_0)\rangle$ will involve non-diagonal matrix elements

$$\langle \boldsymbol{p}' | \boldsymbol{J} | \boldsymbol{p} \rangle$$

What about the spin??? J-M use

$$|\Psi(\boldsymbol{p}_0, S)\rangle = \int d^3\boldsymbol{p}\, \psi(\boldsymbol{p}_0 - \boldsymbol{p})\, |\boldsymbol{p}, S\rangle$$

i.e. with a fixed S on both sides of the equation.

What about the spin??? J-M use

$$|\Psi(\boldsymbol{p}_0, S)\rangle = \int d^3\boldsymbol{p}\, \psi(\boldsymbol{p}_0 - \boldsymbol{p})\, |\boldsymbol{p}, S\rangle$$

i.e. with a fixed S on both sides of the equation.

They do this to simplify things so that the expectation value only involves

$$\langle \boldsymbol{p}', S | \boldsymbol{J} | \boldsymbol{p}, S \rangle$$

What about the spin??? J-M use

$$|\Psi(\boldsymbol{p}_0, S)\rangle = \int d^3\boldsymbol{p}\, \psi(\boldsymbol{p}_0 - \boldsymbol{p})\, |\boldsymbol{p}, S\rangle$$

i.e. with a fixed S on both sides of the equation.

They do this to simplify things so that the expectation value only involves

$$\langle \boldsymbol{p}', S | \boldsymbol{J} | \boldsymbol{p}, S \rangle$$

i.e. is at least diagonal in S—important for them because they try to write down the most general form for this matrix element

What about the spin??? J-M use

$$|\Psi(\boldsymbol{p}_0, S)\rangle = \int d^3\boldsymbol{p}\, \psi(\boldsymbol{p}_0 - \boldsymbol{p})\, |\boldsymbol{p}, S\rangle$$

i.e. with a fixed S on both sides of the equation.

They do this to simplify things so that the expectation value only involves

$$\langle \boldsymbol{p}', S| \boldsymbol{J} |\boldsymbol{p}, S\rangle$$

i.e. is at least diagonal in S—important for them because they try to write down the most general form for this matrix element

But this is incorrect. The wave packet is not physical. Recall that for a physical nucleon

$$p.S = 0$$

Thus if $\boldsymbol{p}$ is to vary freely in the wave packet integration S cannot remain fixed. — Point 1

Thus if $\boldsymbol{p}$ is to vary freely in the wave packet integration S cannot remain fixed. — Point 1

The second difficulty is the general form written down for the matrix element. The Lorentz structure assumed is not correct for non-diagonal matrix elements.

Thus if $\boldsymbol{p}$ is to vary freely in the wave packet integration S cannot remain fixed. — Point 1

The second difficulty is the general form written down for the matrix element. The Lorentz structure assumed is not correct for non-diagonal matrix elements.

To see this think of electromagnetic form factors:

$$\langle p', S | j^{\mu}_{em} | p, S \rangle$$

We cannot say: this transforms like a 4-vector, therefore we can express it terms of vectors built from p, p', S

Thus if $\boldsymbol{p}$ is to vary freely in the wave packet integration S cannot remain fixed. — Point 1

The second difficulty is the general form written down for the matrix element. The Lorentz structure assumed is not correct for non-diagonal matrix elements.

To see this think of electromagnetic form factors:

$$\langle p', S | j^{\mu}_{em} | p, S \rangle$$

We cannot say: this transforms like a 4-vector, therefore we can express it terms of vectors built from p, p', S

We have to first factor out the Dirac spinors

$$\bar{u}(p', S)[\gamma^{\mu} F_1 + \frac{i\sigma^{\mu\nu} q_{\nu}}{2m} F_2] u(p, S)$$

This is the second problem—-Point 2

THE BAD

Correcting the traditional approach

THE BAD

Correcting the traditional approach

Point 1: $\acute{a}$ la BLT, sandwich $\boldsymbol{J}$ between physical wave packet states

$$|\Psi(\boldsymbol{p}_0, \boldsymbol{s})\rangle = \int d^3\boldsymbol{p}\, \psi(\boldsymbol{p}_0 - \boldsymbol{p})\, |\boldsymbol{p}, \boldsymbol{s}\rangle$$

where $\boldsymbol{s}$ is the spin vector in the rest frame.

THE BAD

Correcting the traditional approach

Point 1: *á* la BLT, sandwich $\boldsymbol{J}$ between physical wave packet states

$$|\Psi(\boldsymbol{p}_0, \boldsymbol{s})\rangle = \int d^3\boldsymbol{p}\,\psi(\boldsymbol{p}_0 - \boldsymbol{p})\,|\boldsymbol{p}, \boldsymbol{s}\rangle$$

where $\boldsymbol{s}$ is the spin vector in the rest frame.

Note that the covariant spin vector, for spin quantized along the Z axis, is then

$$S^\mu = \left(\frac{\boldsymbol{p}.\boldsymbol{s}}{m}, \boldsymbol{s} + \frac{\boldsymbol{p}.\boldsymbol{s}}{m(p_0 + m)}\boldsymbol{p}\right)$$

Thus S varies as we integrate over $\boldsymbol{p}$

Point 2: Take care to utilize Dirac spinors when writing down general structure of

$$\langle \boldsymbol{p}', \boldsymbol{s}|\boldsymbol{J}|\boldsymbol{p}, \boldsymbol{s}\rangle$$

Point 2: Take care to utilize Dirac spinors when writing down general structure of

$$\langle \boldsymbol{p}', \boldsymbol{s}|\boldsymbol{J}|\boldsymbol{p}, \boldsymbol{s}\rangle$$

- Long, involved calculation!

Point 2: Take care to utilize Dirac spinors when writing down general structure of

$$\langle \boldsymbol{p}', \boldsymbol{s} | \boldsymbol{J} | \boldsymbol{p}, \boldsymbol{s} \rangle$$

• Long, involved calculation!

• Need to study narrow wave packet and limit as it approaches plane wave

Point 2: Take care to utilize Dirac spinors when writing down general structure of

$$\langle \boldsymbol{p}', \boldsymbol{s} | \boldsymbol{J} | \boldsymbol{p}, \boldsymbol{s} \rangle$$

•Long, involved calculation!

• Need to study narrow wave packet and limit as it approaches plane wave

Result: For general polarization state of nucleon BLT differs from J-M. Details later.

THE GOOD

Larry Trueman's brilliant idea

THE GOOD

Larry Trueman's brilliant idea

• It is simple.

• It is short

• It works for any spin. Previous methods only work for spin 1/2.

THE GOOD

Larry Trueman's brilliant idea

• It is simple.

• It is short

• It works for any spin. Previous methods only work for spin 1/2.

We know how rotations affect states. If $|\boldsymbol{p}, m\rangle$ is a state with momentum $\boldsymbol{p}$ and spin projection m in the rest frame of the particle, and if $\hat{R}_z(\beta)$ is the operator for a rotation β about OZ, then

$$\hat{R}_z(\beta)|\boldsymbol{p}, m\rangle = |\boldsymbol{R}_z(\beta)\boldsymbol{p}, m'\rangle\, D^s_{m'm}[R_z(\beta)]$$

But rotations are generated by the angular momentum operators! i.e.

$$\widehat{R}_z(\beta) = e^{-i\beta \boldsymbol{J}_z}$$

But rotations are generated by the angular momentum operators! i.e.

$$\hat{R}_z(\beta) = e^{-i\beta \boldsymbol{J}_z}$$

so that

$$\boldsymbol{J}_z = i \frac{d}{d\beta} \hat{R}_z(\beta) \Big|_{\beta=0}$$

But rotations are generated by the angular momentum operators! i.e.

$$\hat{R}_z(\beta) = e^{-i\beta \boldsymbol{J}_z}$$

so that

$$\boldsymbol{J}_z = i\frac{d}{d\beta}\hat{R}_z(\beta)\Big|_{\beta=0}$$

From the above we know what the matrix element of $\hat{R}_z(\beta)$ looks like. So we simply differentiate, multiply by i, and put $\beta = 0$.

One technical point: you have to know that the derivative of the rotation matrix for spin s at $\beta = 0$ is just the spin matrix for that spin. e.g. for spin 1/2 just $\sigma_z/2$.

COMPARISON OF RESULTS

For the expectation values we find, for any spin configuration (longitudinal, transverse etc) the remarkably simple result:

$$\langle\langle \boldsymbol{p}, \boldsymbol{s} | \boldsymbol{J}_i | \boldsymbol{p}, \boldsymbol{s} \rangle\rangle = \frac{1}{2} \boldsymbol{s}_i$$

COMPARISON OF RESULTS

For the expectation values we find, for any spin configuration (longitudinal, transverse etc) the remarkably simple result:

$$\langle\langle \boldsymbol{p}, \boldsymbol{s} | \boldsymbol{J}_i | \boldsymbol{p}, \boldsymbol{s} \rangle\rangle = \frac{1}{2} \boldsymbol{s}_i$$

Written in these variables the J-M result is:

$$\langle\langle \boldsymbol{p}, \boldsymbol{s} | \boldsymbol{J}_i | \boldsymbol{p}, \boldsymbol{s} \rangle\rangle_{JM} = \frac{1}{4mp_0} \left[(3p_0^2 - m^2) \boldsymbol{s}_i - \frac{3p_0 + m}{p_0 + m} (\boldsymbol{p}.\boldsymbol{s}) \boldsymbol{p}_i \right]$$

COMPARISON OF RESULTS

For the expectation values we find, for any spin configuration (longitudinal, transverse etc) the remarkably simple result:

$$\langle\langle \boldsymbol{p}, \boldsymbol{s} | \boldsymbol{J}_i | \boldsymbol{p}, \boldsymbol{s} \rangle\rangle = \frac{1}{2} \boldsymbol{s}_i$$

Written in these variables the J-M result is:

$$\langle\langle \boldsymbol{p}, \boldsymbol{s} | \boldsymbol{J}_i | \boldsymbol{p}, \boldsymbol{s} \rangle\rangle_{JM} = \frac{1}{4mp_0} \left[(3p_0^2 - m^2) \boldsymbol{s}_i - \frac{3p_0 + m}{p_0 + m} (\boldsymbol{p}.\boldsymbol{s}) \boldsymbol{p}_i \right]$$

These look completely different. But for a state of longitudinal polarization i.e when $\boldsymbol{s} = \widehat{\boldsymbol{p}}$ they agree!

COMPARISON OF RESULTS

For the expectation values we find, for any spin configuration (longitudinal, transverse etc) the remarkably simple result:

$$\langle\langle \boldsymbol{p}, \boldsymbol{s} | \boldsymbol{J}_i | \boldsymbol{p}, \boldsymbol{s} \rangle\rangle = \frac{1}{2} \boldsymbol{s}_i$$

Written in these variables the J-M result is:

$$\langle\langle \boldsymbol{p}, \boldsymbol{s} | \boldsymbol{J}_i | \boldsymbol{p}, \boldsymbol{s} \rangle\rangle_{JM} = \frac{1}{4mp_0} \left[(3p_0^2 - m^2)\boldsymbol{s}_i - \frac{3p_0 + m}{p_0 + m} (\boldsymbol{p}.\boldsymbol{s}) \boldsymbol{p}_i \right]$$

These look completely different. But for a state of longitudinal polarization i.e when $\boldsymbol{s} = \widehat{\boldsymbol{p}}$ they agree!

But for transverse spin they are crucially different.

This difference is critical for the purpose of deriving angular momentum sum rules, because these are derived for a fast moving nucleon i.e. for $p_0 \to \infty$.

This difference is critical for the purpose of deriving angular momentum sum rules, because these are derived for a fast moving nucleon i.e. for $p_0 \to \infty$.

For transverse spin i.e. for $\boldsymbol{s}$ perpendicular to $\boldsymbol{p}$ the J-M result gives:

$$\langle\langle \boldsymbol{p}, \boldsymbol{s} | \boldsymbol{J}_i | \boldsymbol{p}, \boldsymbol{s} \rangle\rangle_{JM} = \frac{1}{4mp_0} \left[(3p_0^2 - m^2) \boldsymbol{s}_i \right]$$

which $\to \infty$ as $p_0 \to \infty$, so no sum rule is possible.

SUM RULES

Expand nucleon state as superposition of n-parton Fock states.

$$|\boldsymbol{p}, m\rangle \simeq \sum_n \sum_{\{\sigma\}} \int d^3\boldsymbol{k}_1 \dots d^3\boldsymbol{k}_n$$
$$\psi_{\boldsymbol{p},m}(\boldsymbol{k}_1, \sigma_1, \dots \boldsymbol{k}_n, \sigma_n)$$
$$\delta^{(3)}(\boldsymbol{p} - \boldsymbol{k}_1 \dots - \boldsymbol{k}_n)|\boldsymbol{k}_1, \sigma_1, \dots \boldsymbol{k}_n, \sigma_n\rangle.$$

SUM RULES

Expand nucleon state as superposition of n-parton Fock states.

$$|\boldsymbol{p}, m\rangle \simeq \sum_n \sum_{\{\sigma\}} \int d^3\boldsymbol{k}_1 \ldots d^3\boldsymbol{k}_n \, \psi_{\boldsymbol{p},m}(\boldsymbol{k}_1, \sigma_1, \ldots \boldsymbol{k}_n, \sigma_n) \, \delta^{(3)}(\boldsymbol{p} - \boldsymbol{k}_1 \ldots - \boldsymbol{k}_n)|\boldsymbol{k}_1, \sigma_1, \ldots \boldsymbol{k}_n, \sigma_n\rangle.$$

There are two independent cases:

(a)Longitudinal polarization i.e. $\boldsymbol{s}$ along OZ. The sum rule for $\boldsymbol{J}_z$ yields the well known result

$$1/2 = 1/2\,\Delta\Sigma + \Delta G + \langle L_z^q \rangle + \langle L_z^G \rangle$$

(b) Transverse polarization i.e. $\boldsymbol{s} \perp \boldsymbol{p}$. The sum rule for $\boldsymbol{J}_x$ or $\boldsymbol{J}_y$ yields a a new sum rule

$$1/2 = 1/2 \sum_{q,\,\bar{q}} \int dx\, \Delta_T q(x) + \sum_{q,\,\bar{q},\,G} \langle L_{\boldsymbol{s}_T} \rangle$$

(b) Transverse polarization i.e. $\boldsymbol{s} \perp \boldsymbol{p}$. The sum rule for $\boldsymbol{J}_x$ or $\boldsymbol{J}_y$ yields a a new sum rule

$$1/2 = 1/2 \sum_{q,\,\bar{q}} \int dx\, \Delta_T q(x) + \sum_{q,\,\bar{q},\,G} \langle L_{\boldsymbol{s}_T} \rangle$$

Here $L_{\boldsymbol{s}_T}$ is the component of $\boldsymbol{L}$ along $\boldsymbol{s}_T$.

The structure functions $\Delta_T q(x) \equiv h_1^q(x)$ are known as the quark transversity or transverse spin distributions in the nucleon.

(b) Transverse polarization i.e. $\boldsymbol{s} \perp \boldsymbol{p}$. The sum rule for $\boldsymbol{J}_x$ or $\boldsymbol{J}_y$ yields a a new sum rule

$$1/2 = 1/2 \sum_{q,\,\bar{q}} \int dx\, \Delta_T q(x) + \sum_{q,\,\bar{q},\,G} \langle L_{\boldsymbol{s}_T} \rangle$$

Here $L_{\boldsymbol{s}_T}$ is the component of $\boldsymbol{L}$ along $\boldsymbol{s}_T$.

The structure functions $\Delta_T q(x) \equiv h_1^q(x)$ are known as the quark transversity or transverse spin distributions in the nucleon.

As mentioned no such parton model sum rule is possible with the J-M formula because, as $p \to \infty$, for $i = x, y$ the matrix elements diverge.

The structure functions $\Delta_T q(x) \equiv h_1^q(x)$ are most directly measured in doubly polarized Drell-Yan reactions

$$p(\boldsymbol{s}_T) + p(\boldsymbol{s}_T) \rightarrow l^+ + l^- + X$$

where the asymmetry is proportional to

$$\sum_f e_f^2 [\Delta_T q_f(x_1) \Delta_T \bar{q}_f(x_2) + (1 \leftrightarrow 2)].$$

The structure functions $\Delta_T q(x) \equiv h_1^q(x)$ are most directly measured in doubly polarized Drell-Yan reactions

$$p(\boldsymbol{s}_T) + p(\boldsymbol{s}_T) \to l^+ + l^- + X$$

where the asymmetry is proportional to

$$\sum_f e_f^2 [\Delta_T q_f(x_1) \Delta_T \bar{q}_f(x_2) + (1 \leftrightarrow 2)].$$

They can also be determined from the asymmetry in semi-inclusive hadronic interactions like

$$p + p(\boldsymbol{s}_T) \to H + X$$

where H is a detected hadron, typically a pion.

Also in SIDIS reactions with a transversely polarized target

$$\ell + p(\boldsymbol{s}_T) \to \ell + H + X.$$

The problem is that in these semi-inclusive reactions $\Delta_T q_f(x)$ always occurs multiplied by the largely unknown Collins fragmentation function. Moreover recent studies seem to indicate that in hadronic reactions the Collins asymmetry is largely washed out by phase effects.

SUMMARY

• In order to derive angular momentum sum rules you need an expression for the matrix elements of the angular momentum operators $\boldsymbol{J}$ in terms of the momentum $\boldsymbol{p}$ and spin $\boldsymbol{s}$ of the particle.

SUMMARY

• In order to derive angular momentum sum rules you need an expression for the matrix elements of the angular momentum operators $\boldsymbol{J}$ in terms of the momentum $\boldsymbol{p}$ and spin $\boldsymbol{s}$ of the particle.

• Such matrix elements are divergent and ambiguous in the traditional approach and are incorrect in some classic papers

SUMMARY

• In order to derive angular momentum sum rules you need an expression for the matrix elements of the angular momentum operators $\boldsymbol{J}$ in terms of the momentum $\boldsymbol{p}$ and spin $\boldsymbol{s}$ of the particle.

• Such matrix elements are divergent and ambiguous in the traditional approach and are incorrect in some classic papers

• This can be handled using wave packets but the calculations are long and unwieldy

SUMMARY

• In order to derive angular momentum sum rules you need an expression for the matrix elements of the angular momentum operators $\boldsymbol{J}$ in terms of the momentum $\boldsymbol{p}$ and spin $\boldsymbol{s}$ of the particle.

• Such matrix elements are divergent and ambiguous in the traditional approach and are incorrect in some classic papers

• This can be handled using wave packets but the calculations are long and unwieldy

• Using our knowledge of how states transform under rotations leads quickly and relatively painlessly to correct results

SUMMARY

• In order to derive angular momentum sum rules you need an expression for the matrix elements of the angular momentum operators $\boldsymbol{J}$ in terms of the momentum $\boldsymbol{p}$ and spin $\boldsymbol{s}$ of the particle.

• Such matrix elements are divergent and ambiguous in the traditional approach and are incorrect in some classic papers

• This can be handled using wave packets but the calculations are long and unwieldy

• Using our knowledge of how states transform under rotations leads quickly and relatively painlessly to correct results

• The great success of the correct approach is that it allows derivation of a sum rule also for transversely polarized nucleons