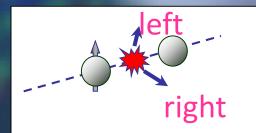
Spin-Momentum Correlations, <u>Aharonov-Bohm, and</u> **Color** Entanglement in Quantum Chromodynamics Christine A. Aidala University of Michigan



$$\psi(x)|P\rangle = e^{ig \int_x^{x'} ds_\mu A^\mu} \psi(x')|P\rangle$$

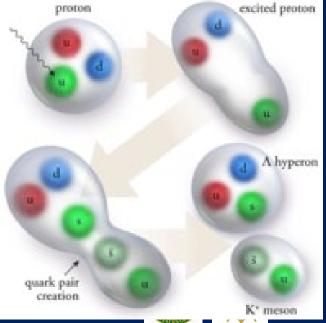
Colloquium University of Pavia January 30, 2020

Theory of strong nuclear interaction: Quantum Chromodynamics

- Fundamental field theory in hand since the early 1970s—BUT...
- Quark and gluon degrees of freedom in the theory cannot be observed or manipulated directly in experiment!

Color *confinement*—quarks and gluons are confined to color-neutral bound states

CLAS, PRL 113, 152004 (2014) PRL Editor's Choice Oct. 2014





How do we understand the visible matter in our universe in terms of the quark and gluon degrees of freedom of quantum chromodynamics?

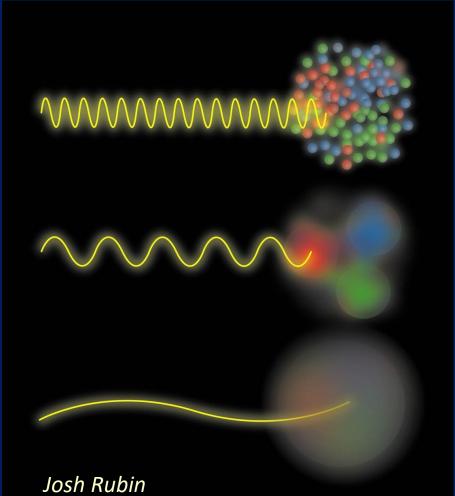
How can studying QCD systems teach us more about fundamental aspects of QCD as a theory?





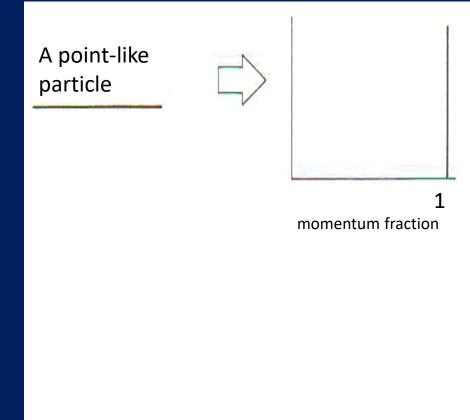
The proton as a "laboratory" for studying QCD

- Proton: simplest stable QCD bound state
- Different energy scales offer information on different aspects of proton internal structure





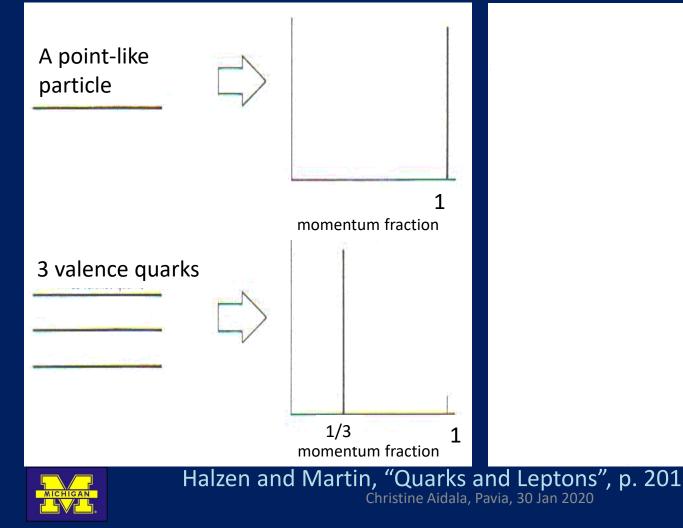




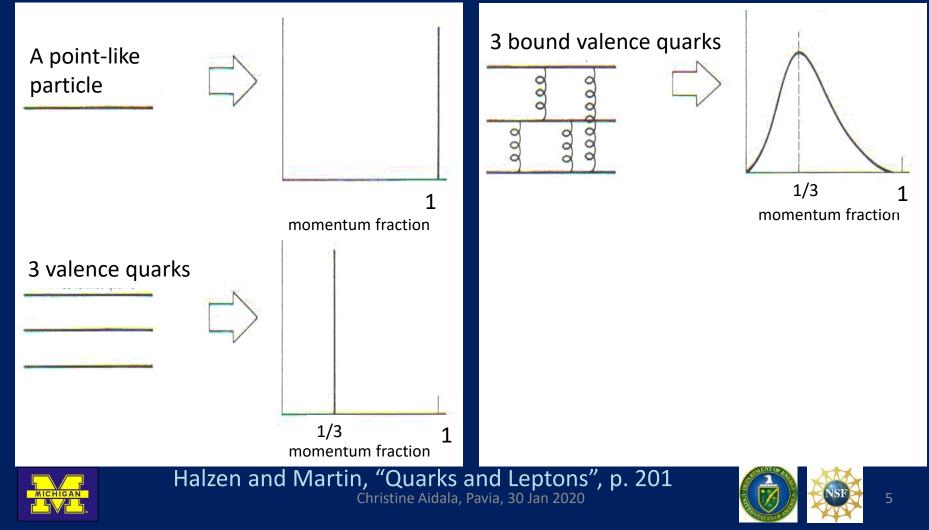


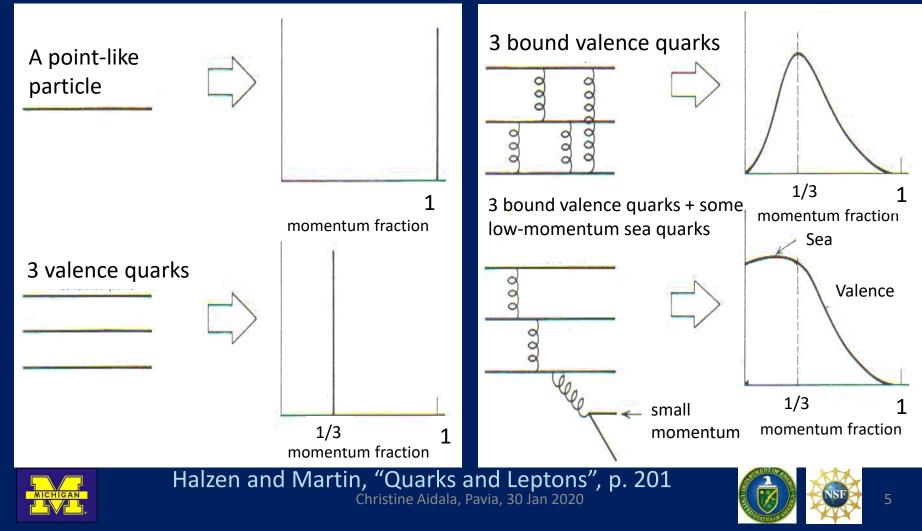
Halzen and Martin, "Quarks and Leptons", p. 201 Christine Aidala, Pavia, 30 Jan 2020





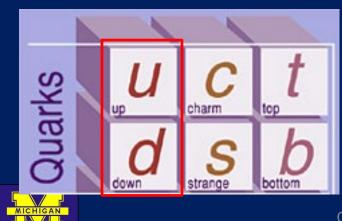


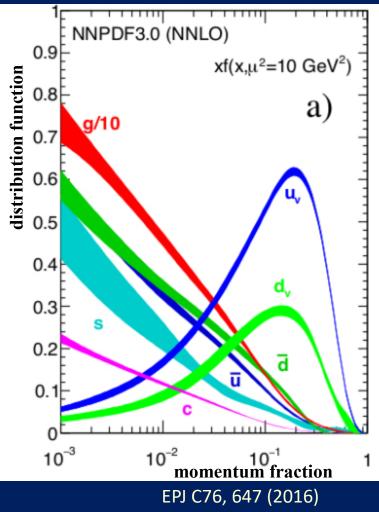




What have we learned in terms of this picture by now?

- Up and down quark "valence" distributions peaked ~1/3
- Lots of sea quarkantiquark pairs and even more gluons!



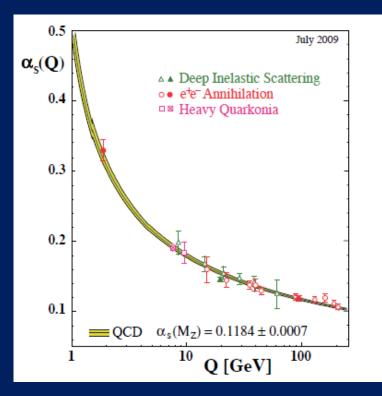




6

Perturbative QCD

- Take advantage of running of strong coupling constant with energy (*asymptotic freedom*)—weak coupling at high energies (short distances)
- Perturbative expansion as in quantum electrodynamics (but many more diagrams due to gluon self-coupling...)

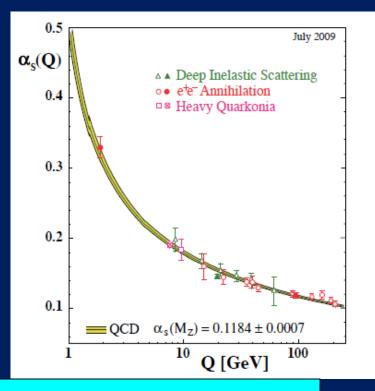






Perturbative QCD

- Take advantage of running of strong coupling constant with energy (*asymptotic freedom*)—weak coupling at high energies (short distances)
- Perturbative expansion as in quantum electrodynamics (but many more diagrams due to gluon self-coupling...)



Provides one rigorous way of relating the fundamental field theory to a variety of physical observables



Factorization and universality in perturbative QCD

- Systematically *factorize* short- and long-distance physics
 - Observable physical QCD processes always involve at least one "long-distance" scale of $\sim 10^{-15}$ m describing bound-state structure (confinement)
- Long-distance (i.e. not perturbatively calculable) functions describing structure need to be *universal*
 - Physically meaningful descriptions
 - Portable across calculations for many processes

Constrain functions describing proton structure by measuring scattering cross sections in many colliding systems over wide kinematic range and performing *simultaneous fits to world data*



Factorization and universality in perturbative QCD

- Systematically *factorize* short- and long-distance physics
 - Observable physical QCD processes always involve at least one "long-distance" scale of ~10⁻¹⁵ m describing boundstate structure (confinement)!
- Long-distance (i.e. not perturbat functions describing structure not
 - Physically meaningful description
 - Portable across calculations for magnetic structure

Note: Nonperturbative lattice QCD techniques have made tremendous progress toward *ab initio* calculations of proton structure in last ~7 years

Constrain functions describing proton structure by measuring scattering cross sections in many colliding systems over wide kinematic range and performing *simultaneous fits to world data*



What does the proton look like in terms of the quarks and gluons inside it?

- Position
- Momentum
- Spin
- Flavor
- Color

Vast majority of past five decades focused on *1-dimensional* momentum structure. Since 1990s starting to consider transverse components . . .





What does the proton look like in terms of the quarks and gluons inside it?

- Position
- Momentum
- Spin
- Flavor
- Color

Polarized protons first studied in 1980s. How angular momentum of quarks and gluons add up still not well understood!





What does the proton look like in terms of the quarks and gluons inside it?

- Position
- Momentum
- Spin
- Flavor

• Color

Good measurements of flavor distributions in valence region. Flavor structure for sea quarks still yielding surprises.





What does the proton look like in terms of the quarks and gluons inside it?

Theoretical and experimental concepts to describe and

access position only born in mid-1990s. Pioneering

measurements over past ~decade.

- Position
- Momentum
- Spin
- Flavor
- Color





What does the proton look like in terms of the quarks and gluons inside it?

- Position
- Momentum
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- Color

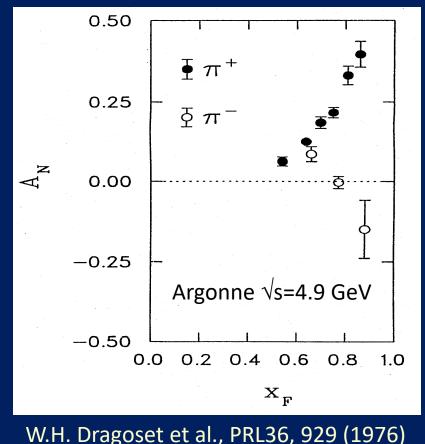
Accounted for theoretically from beginning of QCD, but more detailed, potentially observable effects of color flow have come to forefront in last decade . . .





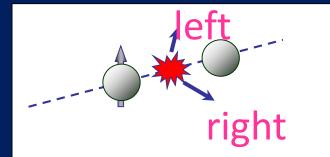
Spin-momentum correlations: 1976 discovery in p+p collisions

Argonne \sqrt{s} =4.9 GeV



Charged pions produced preferentially on one or the other side with respect to the transversely polarized beam direction—by up to 40%!!

Had to wait more than a decade for the birth of a new subfield in order to explore the possibilities . . .

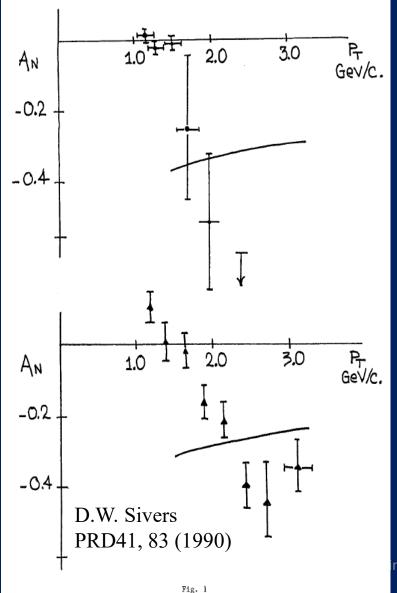


 $x_F = 2 p_{long} / \sqrt{s}$

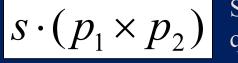


Transverse-momentum-dependent distributions and single-spin asymmetries

۲



1990: D.W. Sivers departs from traditional collinear factorization assumption in pQCD and proposes correlation between the *intrinsic transverse motion* of the quarks and gluons and the proton's spin



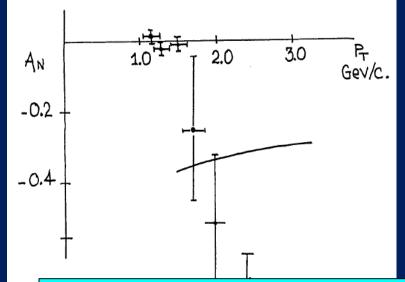
ne Aidala, Pavia, 30 Jan 2020





11

Transverse-momentum-dependent distributions and single-spin asymmetries



1990: D.W. Sivers departs from traditional collinear factorization assumption in pQCD and proposes correlation between the *intrinsic transverse motion* of the quarks and gluons and the proton's spin

First quark distribution function describing a spin-momentum correlation in the proton

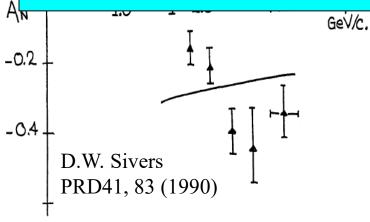


Fig. 1



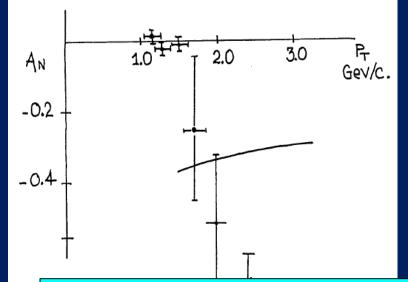
ne Aidala, Pavia, 30 Jan 2020

Spin and momenta of quarks and/or bound states



11

Transverse-momentum-dependent distributions and single-spin asymmetries



1990: D.W. Sivers departs from traditional collinear factorization assumption in pQCD and proposes correlation between the *intrinsic transverse motion* of the quarks and gluons and the proton's spin

First quark distribution function describing a spin-momentum correlation in the proton

New frontier! Quark **dynamics** inside QCD bound states, and in their formation process



ne Aidala, Pavia, 30 Jan 2020





11

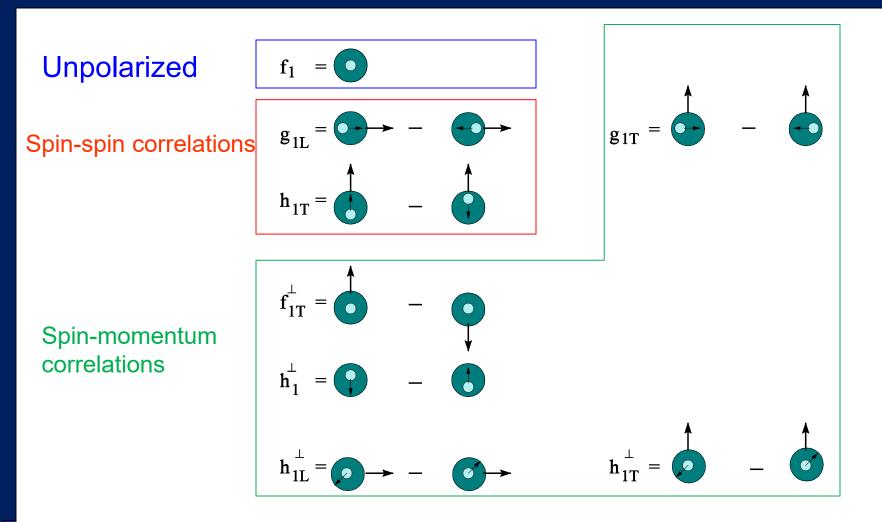
-0.2

-0.4

D.W. Sivers

PRD41, 83 (1990)

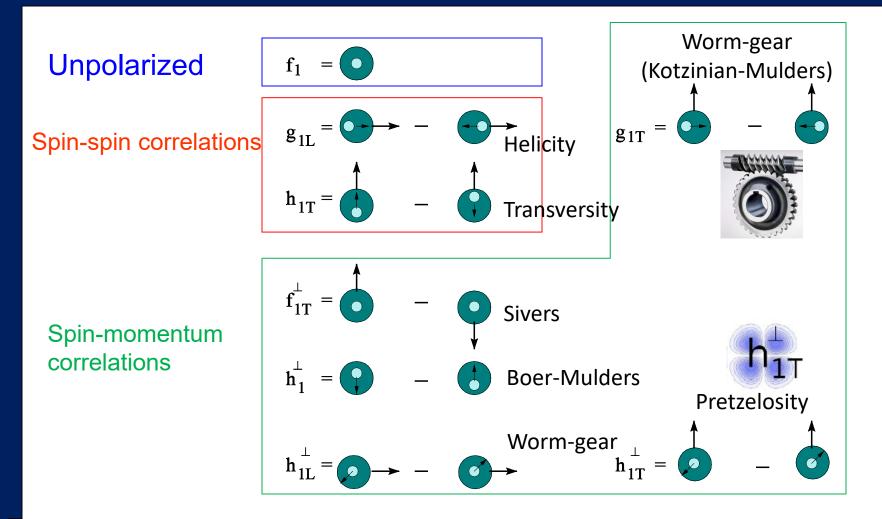
Spin-spin and spin-momentum correlations in QCD bound states







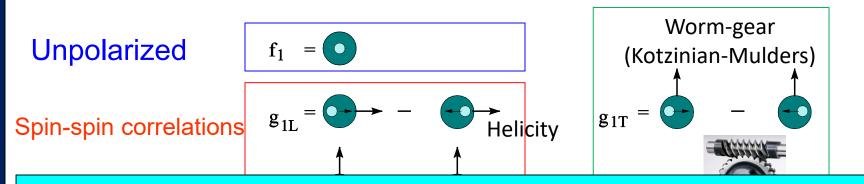
Spin-spin and spin-momentum correlations in QCD bound states







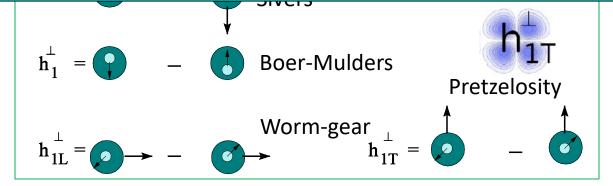
Spin-spin and spin-momentum correlations in QCD bound states



Lots of evidence from deep-inelastic lepton-nucleon scattering experiments over past ~ 15 years that many of these correlations

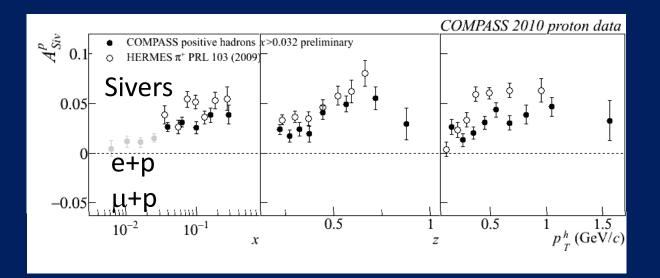
are nonzero in nature!

Spin-momentum correlations





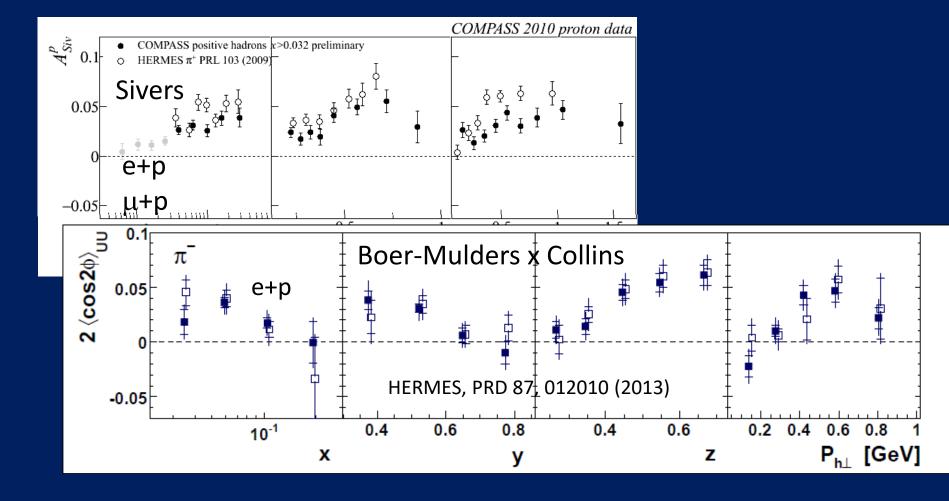






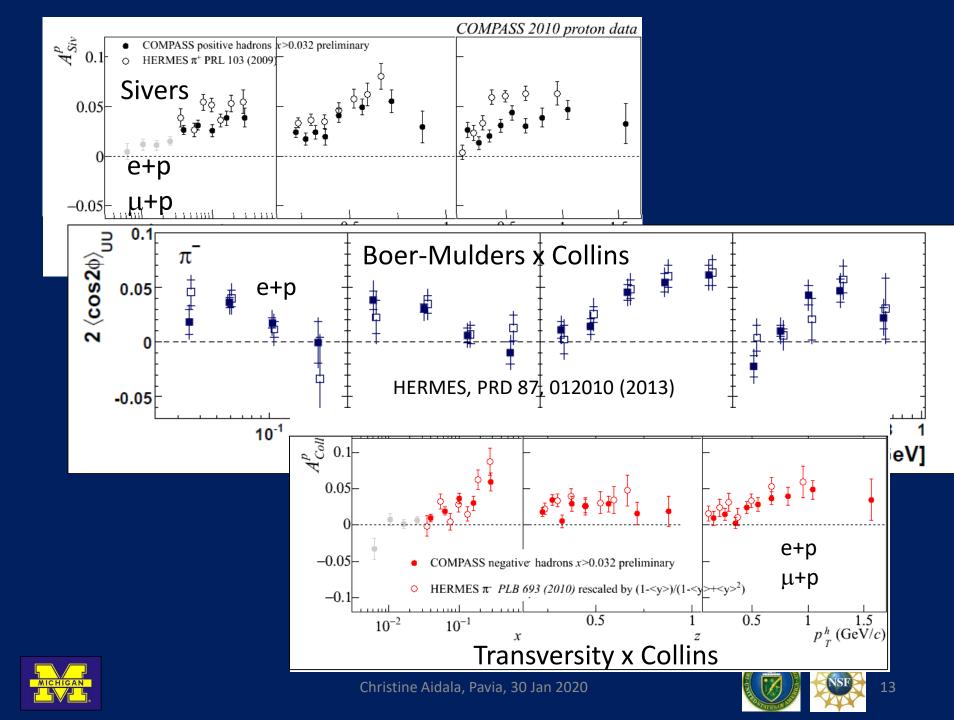


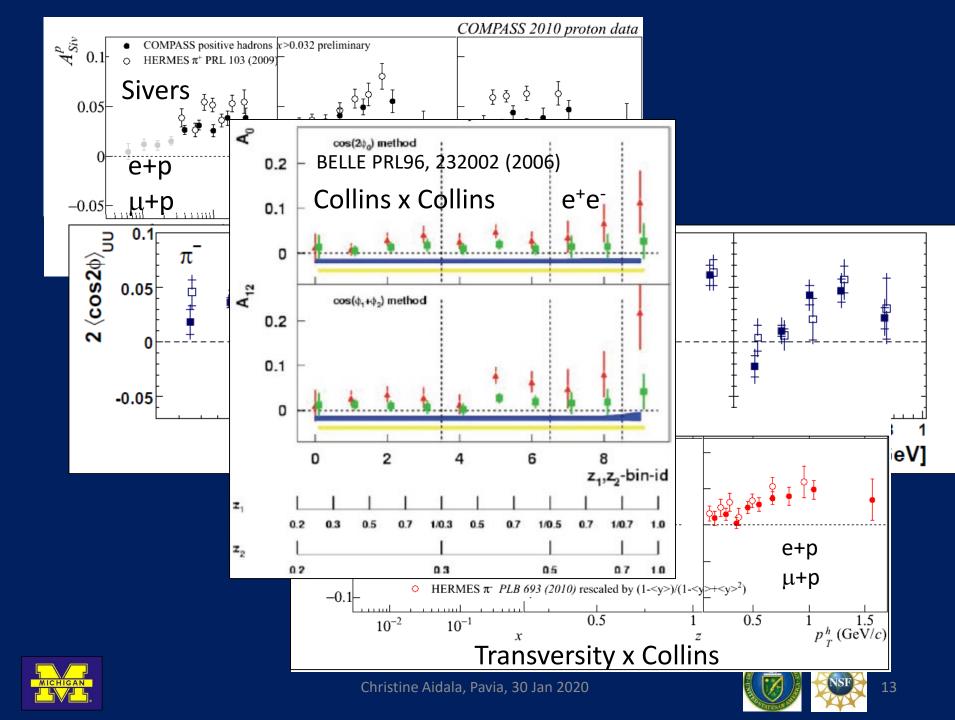




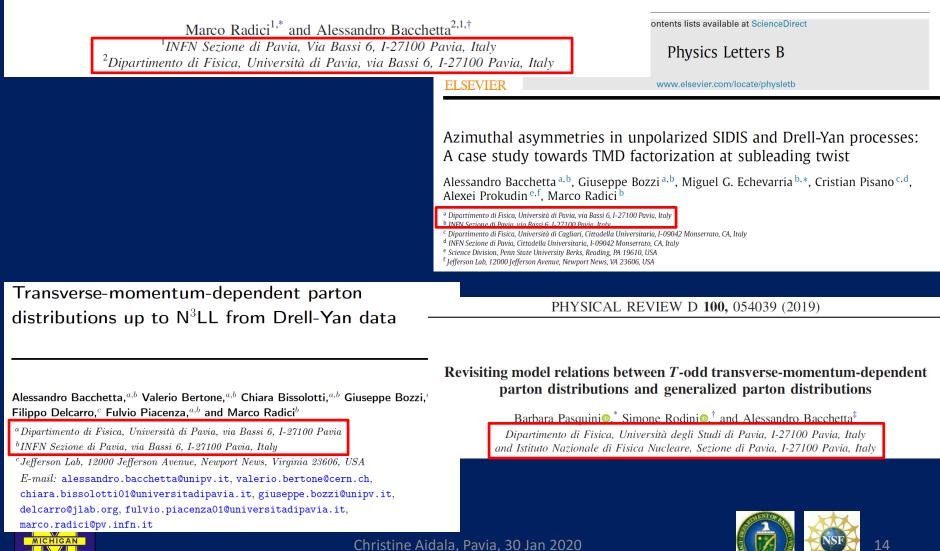






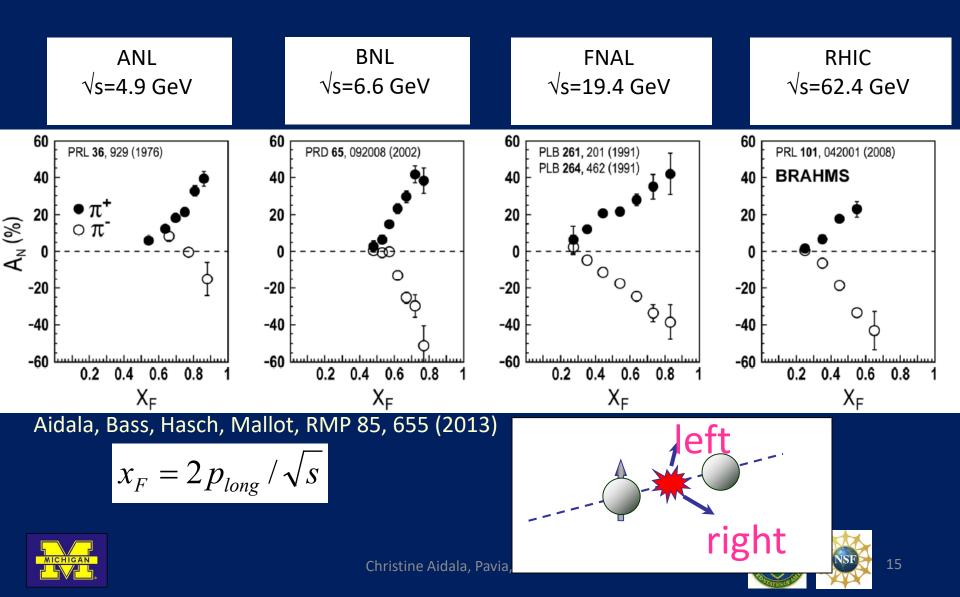


First Extraction of Transversity from a Global Analysis of Electron-Proton and Proton-Proton Data

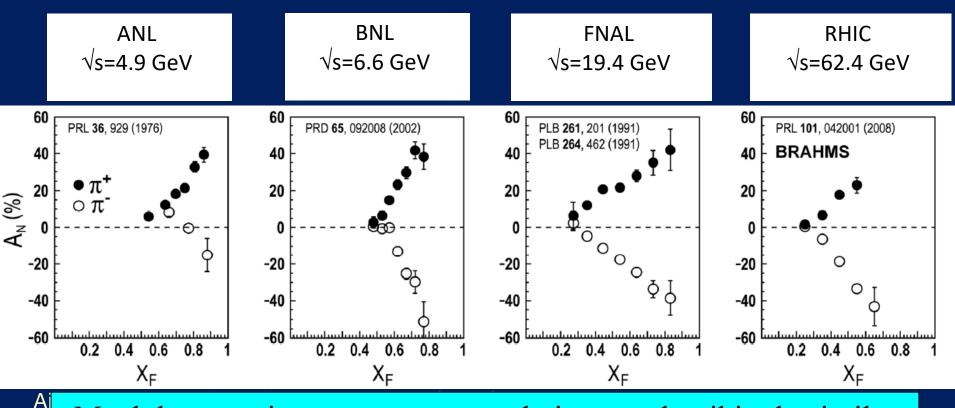


Physics Letters B 797 (2019) 134850

But what about proton-proton collisions?



But what about proton-proton collisions?



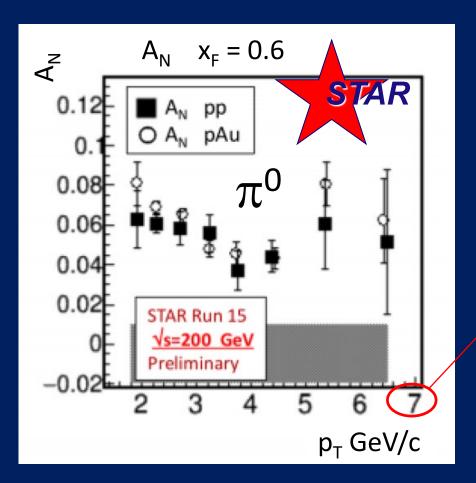
Much larger spin-momentum correlations, and strikingly similar effects across energies!

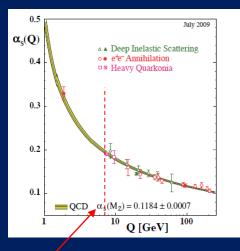




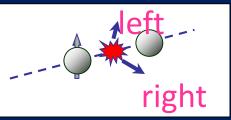


Single-spin asymmetries in transversely polarized proton-proton collisions





Effects persist to kinematic regimes where perturbative QCD techniques clearly apply







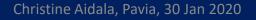
proton-proton \rightarrow pion + X: Challenging to interpret

• Always huge effects!

 But in p+p → pion +X don't have enough information to separate initial-state (proton structure) from final-state (pion formation) effects

• Need to think more carefully . . .







Different symmetry properties for different spin-momentum correlations

• Some transverse-momentum-dependent quark distribution functions odd under a parity- and time-reversal (PT) transformation







Different symmetry properties for different spin-momentum correlations

- Some transverse-momentum-dependent quark distribution functions odd under a parity- and time-reversal (PT) transformation
- In 1993, after original 1990 paper by D.W. Sivers, J.C. Collins claimed such functions must vanish





Different symmetry properties for different spin-momentum correlations

- Some transverse-momentum-dependent quark distribution functions odd under a parity- and time-reversal (PT) transformation
- In 1993, after original 1990 paper by D.W. Sivers, J.C. Collins claimed such functions must vanish
- Only realized in 2002 by Brodsky, Hwang, and Schmidt that could be nonvanishing if *phase interference effects due to color interactions* present





<u>Modified universality</u> of PT-odd correlations: Color in action!

Deep-inelastic lepton-nucleon scattering: Final-state color exchange

incoming scattered incoming proton electron proton electron produced remnant positron scattering scattering scattered antiquark quark guark scattering quark produced proton proton incoming incoming electron remnant remnant proton proton

Opposite sign for PT-odd spin-momentum correlations in the proton measured in these two processes: process-dependent! (Collins 2002)

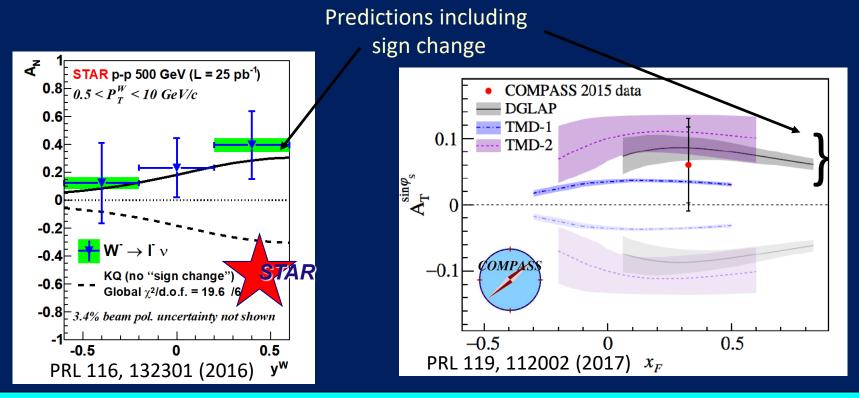




Quark-antiquark annihilation to

leptons: Initial-state color exchange

Modified universality: Initial experimental hints

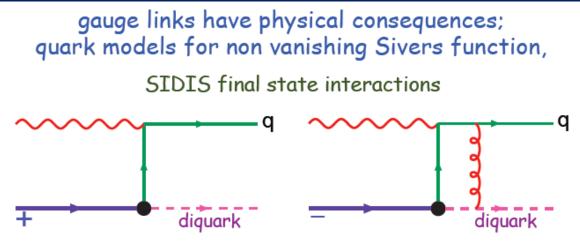


First measurements by STAR at the Relativistic Heavy Ion Collider and COMPASS at CERN suggestive of predicted sign change in colorannihilation processes compared to quark knock-out by a lepton. More statistics forthcoming . . .



Modified universality requires full QCD: Gauge-invariant quantum field theory

We have ignored here the subtleties needed to make this a gauge invariant definition: an appropriate path ordered exponential of the gluon field is needed [18].



Brodsky, Hwang, Schmidt, PL B530 (2002) 99 - Collins, PL B536 (2002) 43

An earlier proof that the Sivers asymmetry vanishes because of time-reversal invariance is invalidated by the path-ordered exponential of the gluon field in the operator definition of parton densities. Instead, the time-reversal argument shows that the Sivers asymmetry is reversed in sign in hadron-induced hard processes (e.g., Drell-Yan), thereby violating naive universality of parton densities. Previous phenomenology with time-reversal-odd parton densities is therefore validated.

$$[f_{1T}^{q\perp}]_{\text{SIDIS}} = -[f_{1T}^{q\perp}]_{\text{DY}}$$

From 1993 claim by J.C. Collins that such processes must vanish

Slide from M. Anselmino, Transversity 2014



21

Physical consequences of a gauge-invariant quantum theory: Aharonov-Bohm (1959)

Wikipedia:

"The Aharonov–Bohm effect is important conceptually because it bears on three issues apparent in the recasting of (Maxwell's) classical electromagnetic theory as a gauge theory, which before the advent of quantum mechanics could be argued to be a mathematical reformulation with no physical consequences. The Aharonov–Bohm thought experiments and their experimental realization imply that the issues were not just philosophical.

The three issues are:

- whether potentials are "physical" or just a convenient tool for calculating force fields;
- whether action principles are fundamental;
- the principle of locality."





Physical consequences of a gauge-invariant quantum theory: Aharonov-Bohm (1959)

Physics Today, September 2009 : The Aharonov–Bohm effects: Variations on a subtle theme, by Herman Batelaan and Akira Tonomura.

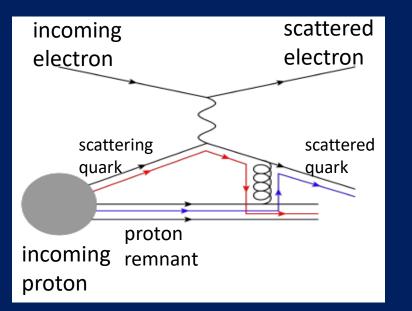
"Aharonov stresses that the arguments that led to the prediction of the various electromagnetic AB effects apply equally well to any other gauge-invariant quantum theory. In the standard model of particle physics, the strong and weak nuclear interactions are also described by gauge-invariant theories. So one may expect that particle-physics experimenters will be looking for new AB effects in new domains."



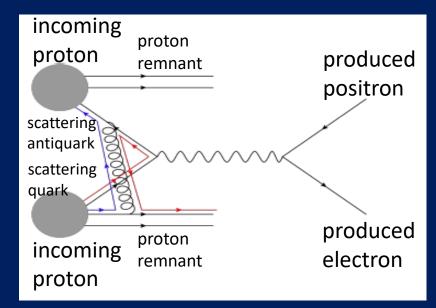


Physical consequences of a gauge-invariant quantum theory: an Aharonov-Bohm effect in QCD!

Deep-inelastic lepton-nucleon scattering: Final-state color exchange



Quark-antiquark annihilation to leptons: Initial-state color exchange



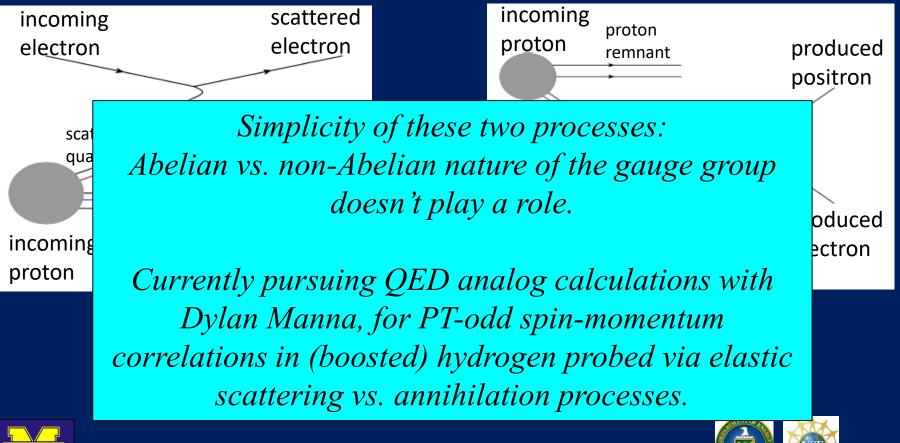
See e.g. Pijlman, hep-ph/0604226 or Sivers, arXiv:1109.2521

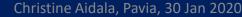




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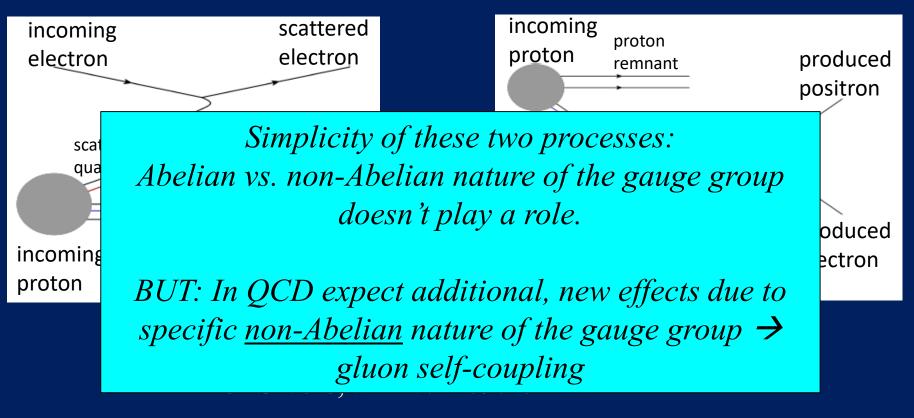






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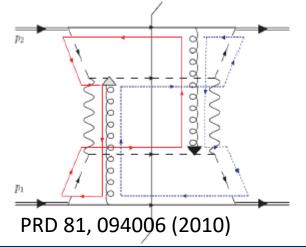




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QCD Aharonov-Bohm effect: Color entanglement

- 2010: T.C. Rogers and P. Mulders predict *color entanglement* in processes involving proton-proton production of QCD bound states if quark transverse momentum taken into account
- Quarks become correlated *across* the two colliding protons
 - Novel QCD state!
- Consequence of QCD specifically as a *non-Abelian* gauge theory



$$p + p \rightarrow h_1 + h_2 + X$$

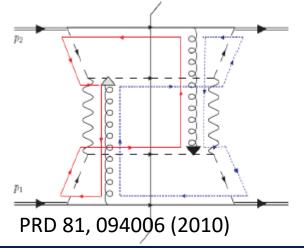
Color flow can't be described as flow in the two gluons separately. Requires presence of both.





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25

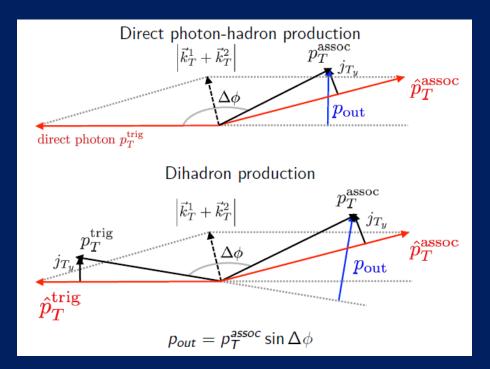
Huge transverse spin asymmetries in p+p a color entanglement effect??



Searching for evidence of color entanglement at RHIC

- Need observable sensitive to a nonperturbative momentum scale
 - Nearly back-to-back particle production
- Need 2 initial QCD bound states
 - color exchange between a scattering quark and remnant of other proton
- And at least 1 final QCD bound state
 - exchange between scattered quark and either remnant

→ In p+p collisions, measure out-ofplane momentum component in nearly back-to-back photon-hadron and hadron-hadron production



PHENIX Collaboration: PRD95, 072002 (2017) PRD98, 072004 (2018) PRC99, 044912 (2019)



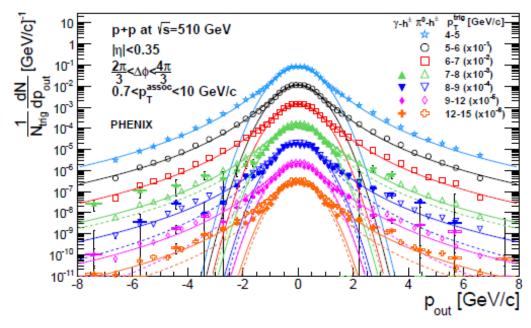
Christine Aidala, Pavia, 30 Jan 2020

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Out-of-plane momentum component distributions



- Gaussian near 0 nonperturbative transverse momentum
- Power-law at large
 p_{out}—kicks from hard
 (perturbative) gluon
 radiation
- Different colors → different bins in hard interaction scale



PRD95, 072002 (2017)

Curves are fits to Gaussian and Kaplan functions, *not* calculations





Look at evolution of nonperturbative transverse momentum widths with hard scale (Q^2)

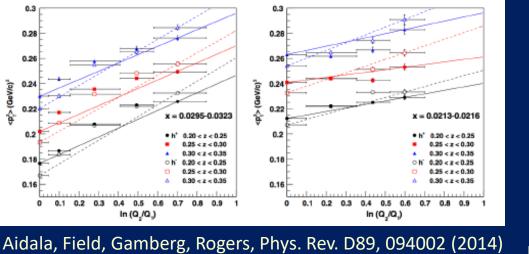
- Proof of factorization (i.e. no entanglement) directly predicts that nonperturbative transverse momentum widths *increase* with hard scattering energy scale
 - Increased phase space for gluon radiation

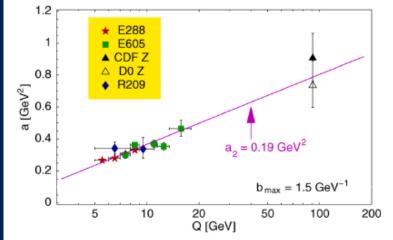




Look at evolution of nonperturbative transverse momentum widths with hard scale (Q^2)

- Proof of factorization (i.e. no entanglement) directly predicts that nonperturbative transverse momentum widths *increase* with hard scattering energy scale
 - Increased phase space for gluon radiation
- Confirmed experimentally in deep-inelastic lepton-nucleon scattering (left) and quark-antiquark annihilation to leptons (right)





Konychev + Nadolsky, Phys. Lett. B633, 710 (2006)

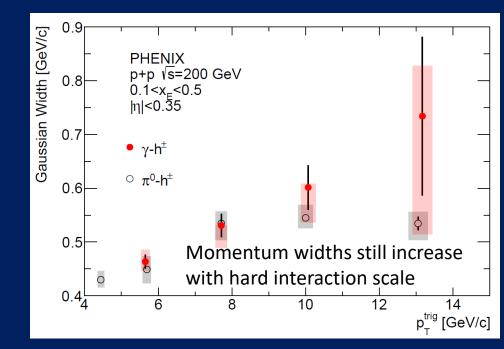






So far see **qualitatively** similar trend where factorization predicted to be broken

- Phenomenological calculations assuming factorization holds recently started here at Pavia
 - Search for *quantitative* deviations . . .
- Goal is to study factorization breaking and non-Abelian phenomena in a controlled way



Don't reconstruct jets, so use x_E as a proxy for fraction of jet momentum carried by hadron:

$$x_E \equiv -\frac{p_T^{\text{trig}} \cdot p_T^{\text{assoc}}}{|p_T^{\text{trig}}|^2} = -\frac{|p_T^{\text{assoc}}|}{|p_T^{\text{trig}}|} \cos \Delta \phi$$





Christine Aidala, Pavia, 30 Jan 2020



So far see qualitatively similar trend where factorization predicted to be broken

Phenomenological calculations assuming factorization holds

[GeV/c] p+p √s=200 GeV In the meantime, performing follow-up measurements at LHCb at the Large Hadron Collider:

0.9

- *Z-jet correlations, which provide better handle on* quark and gluon kinematics
- As a control, similarly constructed measurements of quark-antiquark annihilation to dimuons, where no color entanglement is predicted

Discussions of other potential observables ongoing . . .





GeV/c1

A cyclical process



Synthesis





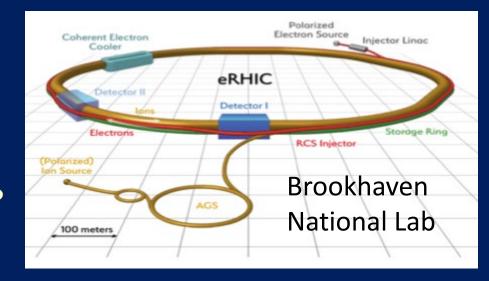


Next-generation QCD facility: The Electron-Ion Collider

Key science questions:

- *How does a nucleon acquire mass?*
- How does the spin of the nucleon arise from its elementary quark and gluon constituents?
- What are the emergent properties of dense systems of gluons?

Project approval ("Critical Decision 0") and site selection at Brookhaven National Lab announced Jan 9, 2020!







Next-generation QCD facility: The Electron-Ion Collider

Key science questions:

- *How does a nucleon acquire mass?*
- How does the spin of the nucleon arise from its elementary quark and gluon constituents?

· What are the emercont

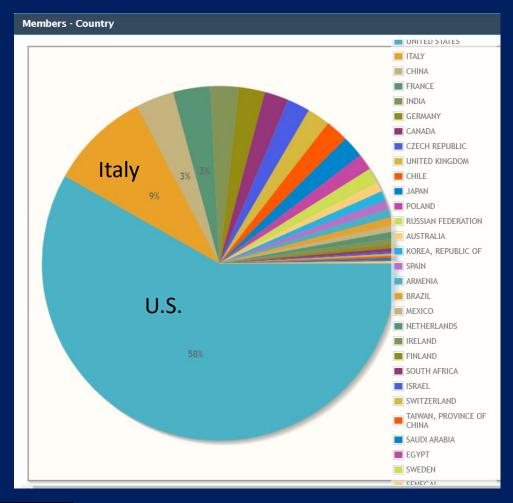
Project approval ("Critical Decision 0") and site selection at Brookhaven National Lab announced Jan 9, 2020!



Electron-Ion Collider User Group: Currently 1007 members from 207 institutions in 30 *countries* www.eicug.org



Next-generation QCD facility: The Electron-Ion Collider



- Italy has largest
 number of member
 scientists in EIC User
 Group after the U.S.
- Leadership roles by Marco Radici, Barbara Pasquini, and Alessandro Bacchetta here at Pavia

www.eicug.org



Christine Aidala, Pavia, 30 Jan 2020



• Early years of a new era of research in quantum chromodynamics!







Summary

- Early years of a new era of research in quantum chromodynamics!
- Gradually shifting to think about QCD systems in new ways, focusing on topics/ideas/concepts that have long been familiar to the world of condensed matter, atomic, and optical physics
 - All sorts of correlations within systems
 - Quantum mechanical phase interference effects
 - Quantum entangled systems





Summary

- Early years of a new era of research in quantum chromodynamics!
- Gradually shifting to think about QCD systems in new ways, focusing on topics/ideas/concepts that have long been familiar to the world of condensed matter, atomic, and optical physics
 - All sorts of correlations within systems
 - Quantum mechanical phase interference effects
 - Quantum entangled systems

Will be exciting to continue testing and exploring these ideas and phenomena at existing facilities as well as at the future Electron-Ion Collider . . .





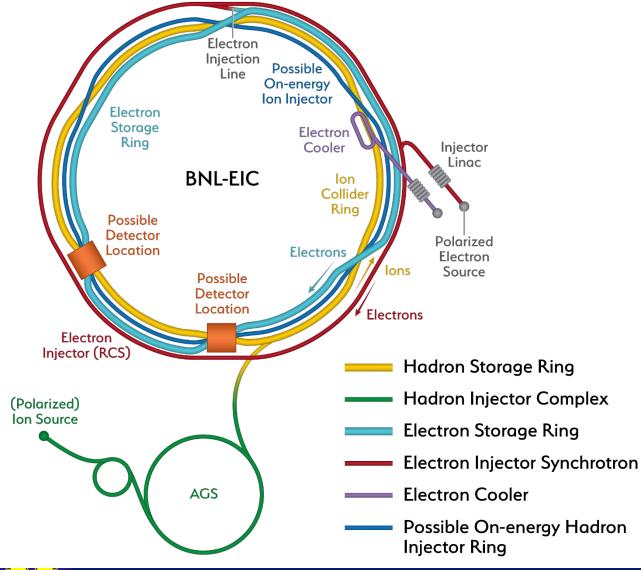








How RHIC is transformed into an EIC

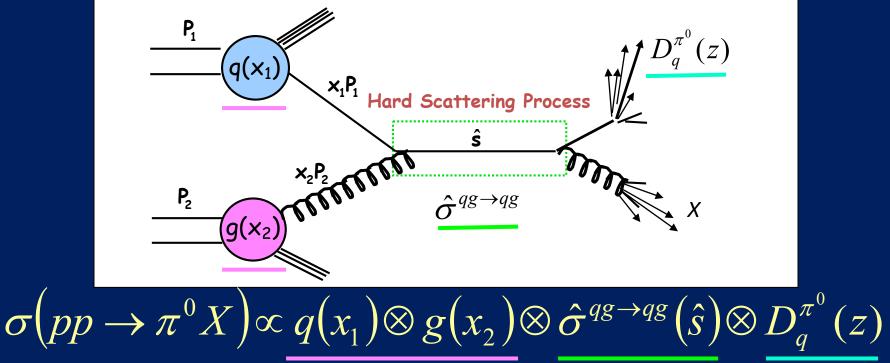




Christine Aidala, Pavia, 30 Jan 2020



Parton distribution functions in perturbative QCD calculations of observables



High-energy processes have predictable rates given:

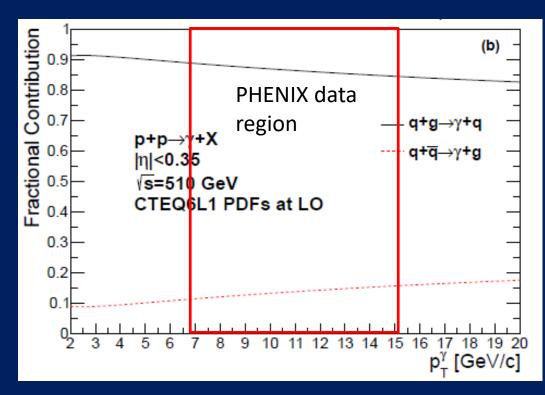
- Partonic hard scattering rates (calculable in pQCD)
- Parton distribution functions (experiment or lattice)
- Fragmentation functions (experiment or lattice)

Universal nonperturbative factors





Partonic process contributions for direct photon production



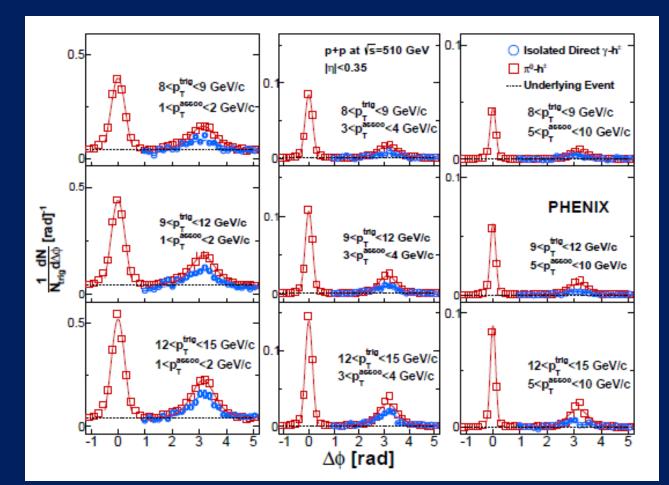
Quark-gluon Compton scattering still dominates at NLO -PLB140, 87 (1984)

PHENIX Collab., Phys. Rev. D95, 072002 (2017). Calculation by T. Kaufmann





Two-particle correlation distributions show expected jet-like structure







Christine Aidala, Pavia, 30 Jan 2020



Magnetic and electric A-B effects; Type-I and Type-II A-B effects

Box 1. Types and duals

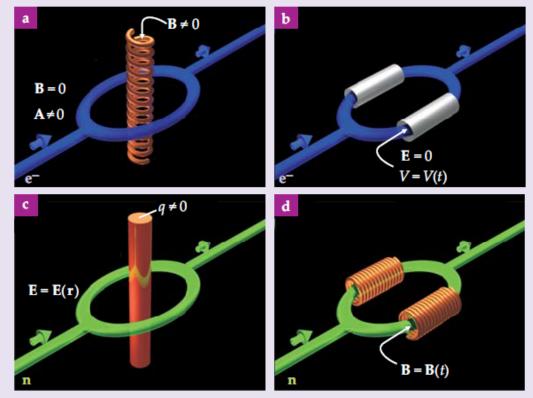
Physics Today, September 2009

The original magnetic and electric Aharonov–Bohm effects (panels a and b) are type I effects in the sense that in an ideal experiment, the electron sees no **B** or **E** fields, though it does traverse different potentials **A** and *V*. In their respective dual effects—the Aharonov–Casher effect (panel c) and the so-called neutronscalar AB effect (panel d)—polarized neutrons (neutral particles

with magnetic dipole moments) replace unpolarized electrons, and electrostatic configurations change places with solenoids.¹⁰ In panel c, a neutron interferometer encloses a line of charge, and in panel d, neutrons pass through pulsed solenoids. These duals are classified as type II effects because the neutron must traverse a nonvanishing **E** or **B** field.

In either case, to acquire an AB phase shift, the electron or neutron must pass through a region of nonzero electromagnetic potential. That quantum mechanical result seems to elevate the status of the potentials to a physical reality absent from classical electromagnetism. Yakir Aharonov has pointed out that the potentials do overdetermine the experimental outcome; the phase shift need only be known modulo 2π . An alternative view is that the original magnetic AB effect shows electromagnetic fields acting nonlocally.¹

For type II effects, the wavepackets can plow straight through force fields, and forces are allowed in the interaction. But the AB interpretation requires that the emerging wavepackets not be deflected or delayed in any way. Quantum mechanical descriptions generally circumvent the notion of forces. But one can use here an operational definition of forces that might be mimicking an AB effect: If the interaction has produced no deflection or delay, there were no forces.





Exploring the role of color interactions in QCD

- Process-dependent sign change for PT-odd TMD functions and TMD-factorization breaking prediction both due to color flow in hadronic interactions
- Renewed/increasing interest in color interactions in recent years! Various motivations. Some examples of recent papers (not by any means comprehensive!)...



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Further discussions of color entanglement

- A. Schaefer + J. Zhou PRD90, 094012 (2014) "Color entanglement for gamma-jet in polarized p+A collisions"
 - "...the new gluon distribution function $G_4(x, k_T)$ generated by color entanglement"
 - Entanglement "can be seen not as a nuisance, but as a chance to explore the nontrivial interplay of color flow in local non-Abelian gauge theories"
- J. Zhou PRD96, 114001 (2017) "Color entanglement like effect in collinear twist-3 factorization"





Quarkonium suppression in p+A; Collective behavior in high-multiplicity p+p

- Ma, Venugopalan, Watanabe, Zhang PRC97, 014909 (2018) – "Psi(2S) versus J/Psi suppression in protonnucleus collisions from factorization violating soft color exchanges"
- Ortiz Velasquez, Christiansen, Cuautle Flores, Maldonado Cervantes, Paic PRL 111, 042001 (2013) – "Color reconnection and flowlike patterns in pp collisions"
- Ortiz, Palomo arXiv:1809.01744 "Probing color reconnection with underlying event observables at the LHC energies"





"Color coherence" in e+e-, p(bar)p

- "Color coherence" ideas about increased soft radiation between colorconnected partons/remnants go back to e+e- measurements in the 1980s, e.g.
 - TPC/2g Collaboration, "Comparison of the Particle Flow in q-qbar-g and qqbar-γ Events in e+e- Annihilation", Phys. Rev. Lett. 57, 945 (1986)
 - MARK2 Collaboration, "Comparison of the particle flow in Three-Jet and radiative Two-Jet Events from e+e- Annihilation at E_{c.m.} = 29 GeV", Phys. Rev. Lett. 57, 1398 (1986)
 - OPAL Collaboration, "A study of coherence of soft gluons in hadron jets", Phys. Lett. B247, 617 (1990)
 - L3 Collaboration, "Evidence for gluon interference in hadronic Z decays", Phys. Lett. B353, 145 (1995)
- In 3-jet events in hadronic collisions, color coherence predicts that gluon radiation leading to lowest- p_T jet more likely to be in plane defined by emitting hard-scattered parton, i.e. "second" jet, and beam remnant, with stronger effects expected when second jet is closer to beam rapidity.

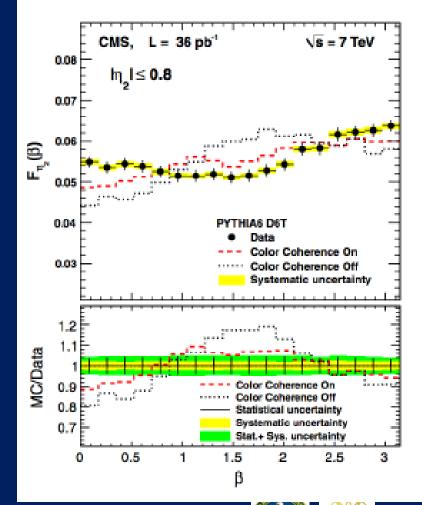






"Color coherence" in e+e-, p(bar)p

- D0, CDF, CMS have all published evidence for "color coherence effects"
 - CDF: PRD50, 5562 (1994) "Evidence for color coherence in pp collisions at sqrt(s) = 1.8 TeV"
 - D0: PLB414, 419 (1997) –
 "Color coherent radiation in multijet events from pp collisions at sqrt(s) = 1.8 TeV"
 - CMS: EPJ C74, 2901 (2014) –
 "Probing color coherence effects in pp collisions at sqrt(s) = 7 TeV"







"Color coherence" in e+e-, p(bar)p

- ATLAS NPB918, 257 (2017) "High-E_T isolated-photon plus jets production in pp collisions at sqrt(s) = 8 TeV with the ATLAS detector"
 - Measured isolated photon+(1, 2, or 3) jets enhancements in QCD radiation "observed around the leading jet with respect to the photon in the directions towards the beams"





Using color correlations to reduce background in beyond-the-SM searches

- Gallicchio + Schwartz PRL 105, 022001 (2010) –
 "Seeing in Color: Jet Superstructure"
 - "the radiation on each end of a color dipole is being pulled towards the other end of the dipole"
 - Define "jet pull" observable based on color connection

ideas

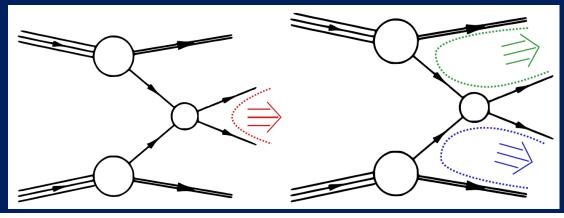




FIG. 1: Possible color connections for signal $(pp \to H \to b\bar{b})$ and for background $(pp \to g \to b\bar{b})$.



Using color correlations to reduce background in beyond-the-SM searches

- ATLAS proof-of-principle measurement using Gallicchio-Schwartz proposal: PLB 750, 475 (2015) – "Measurement of colour flow with the jet pull angle in ttbar events using the ATLAS detector at sqrt(s) = 8 TeV"
 - "The jet pull angle is found to correctly characterise the W boson as a colour singlet"



