Hot and dense QCD on the lattice

- Introduction:

  the QCD phase diagram, screening,
  strong coupling and perturbation theory

- Bulk thermodynamics

  T_c and the equation of state

- Hadronic fluctuations

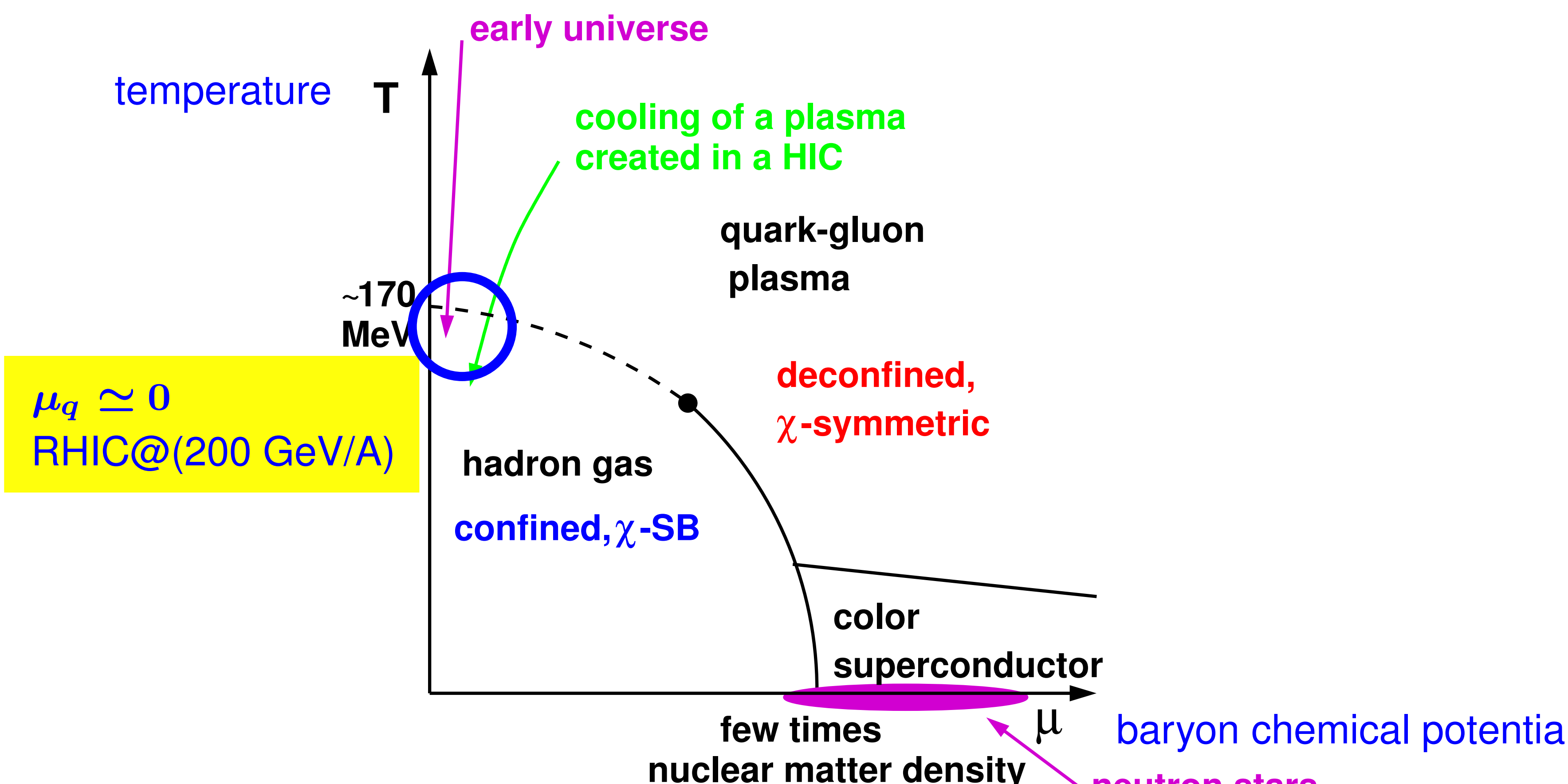
  quark number and charge fluctuations

- The chiral critical point

  ..where is it?

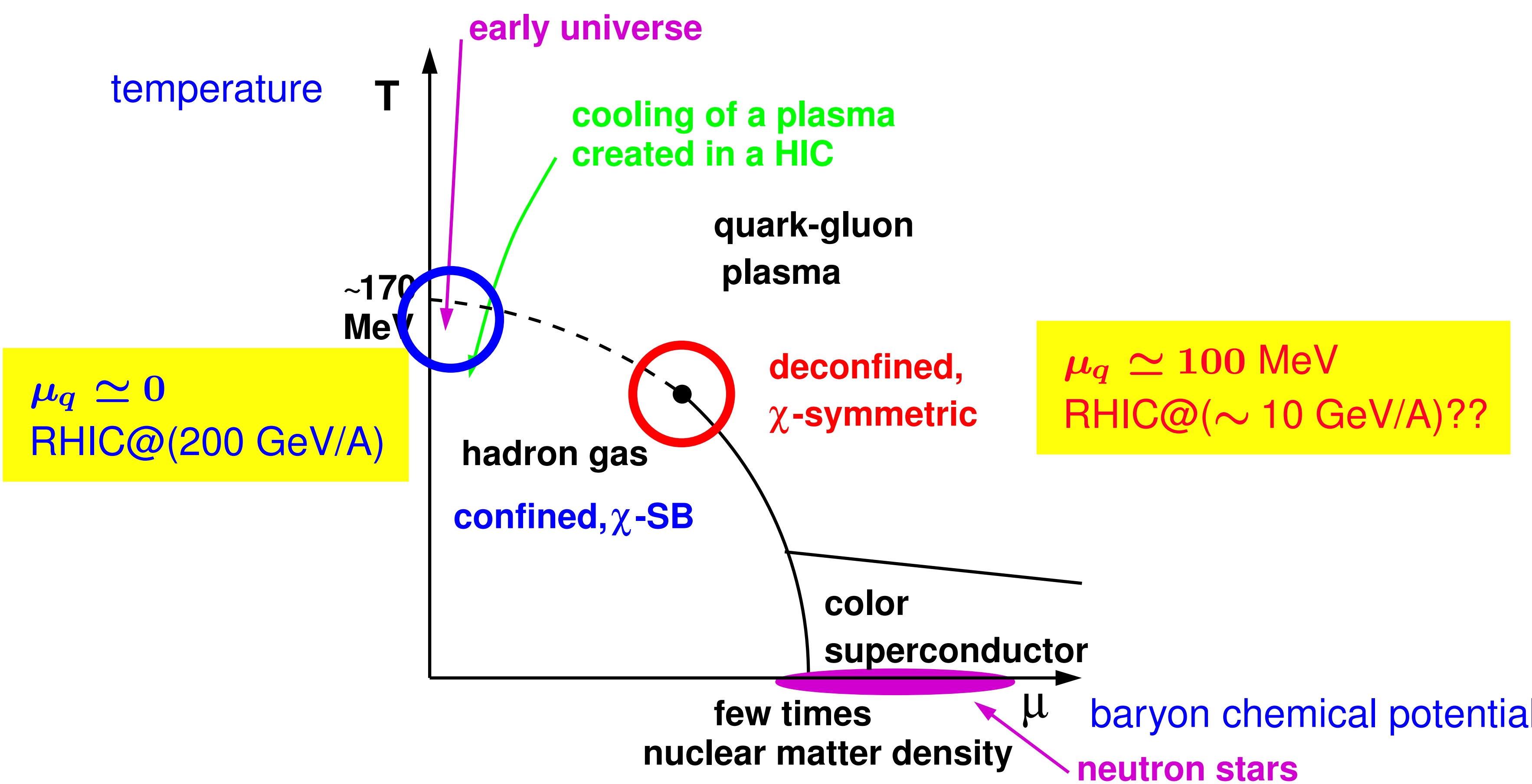
- Conclusions

Phase diagram of strongly interacting matter



Phase diagram of strongly interacting matter

RHIC at low energy ⇔ LGT at non zero chemical potential



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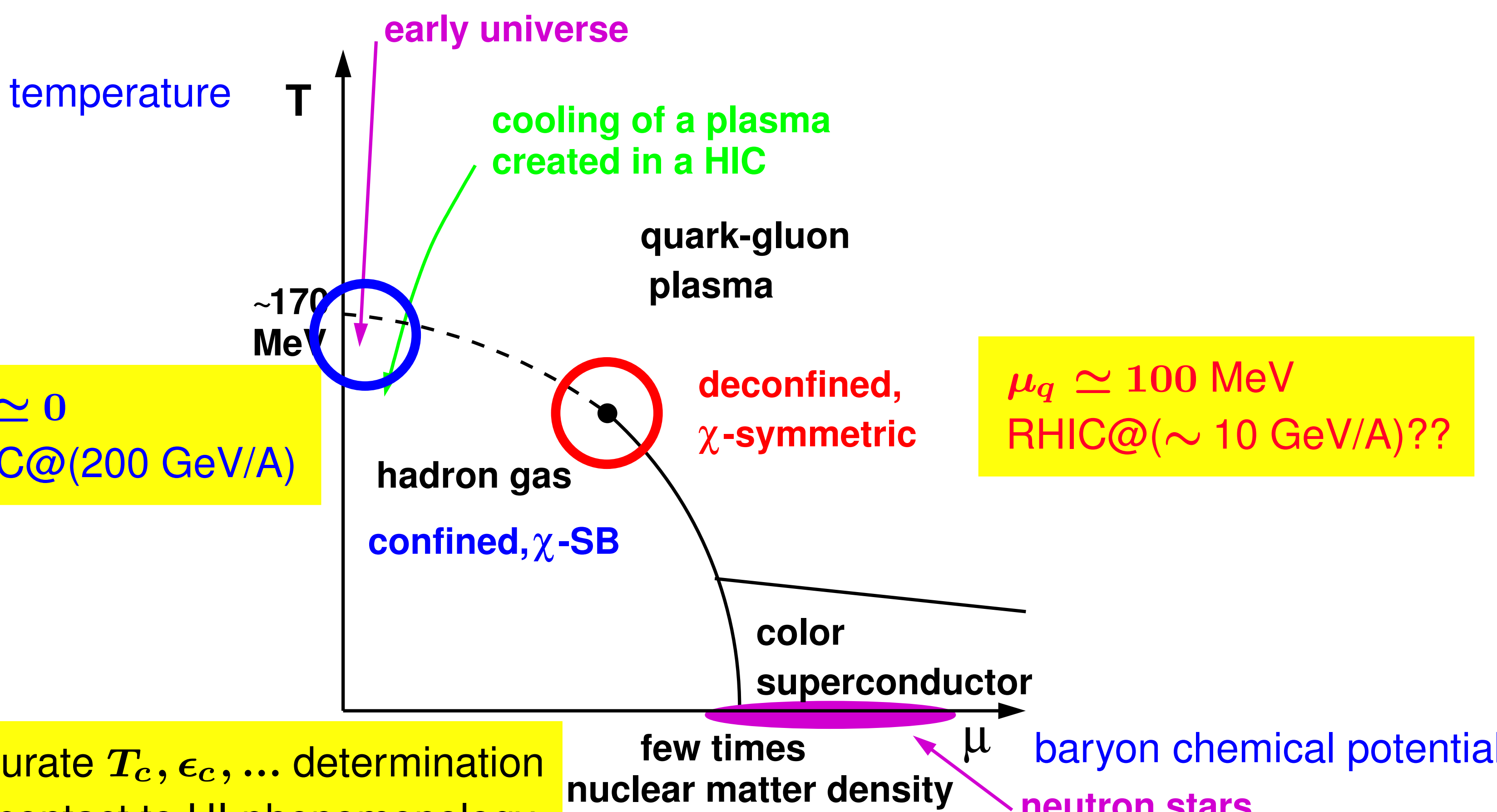


$\mu_q \simeq 0$
RHIC@(200 GeV/A)

$\mu_q \simeq 100$ MeV
RHIC@($\sim$ 10 GeV/A)??

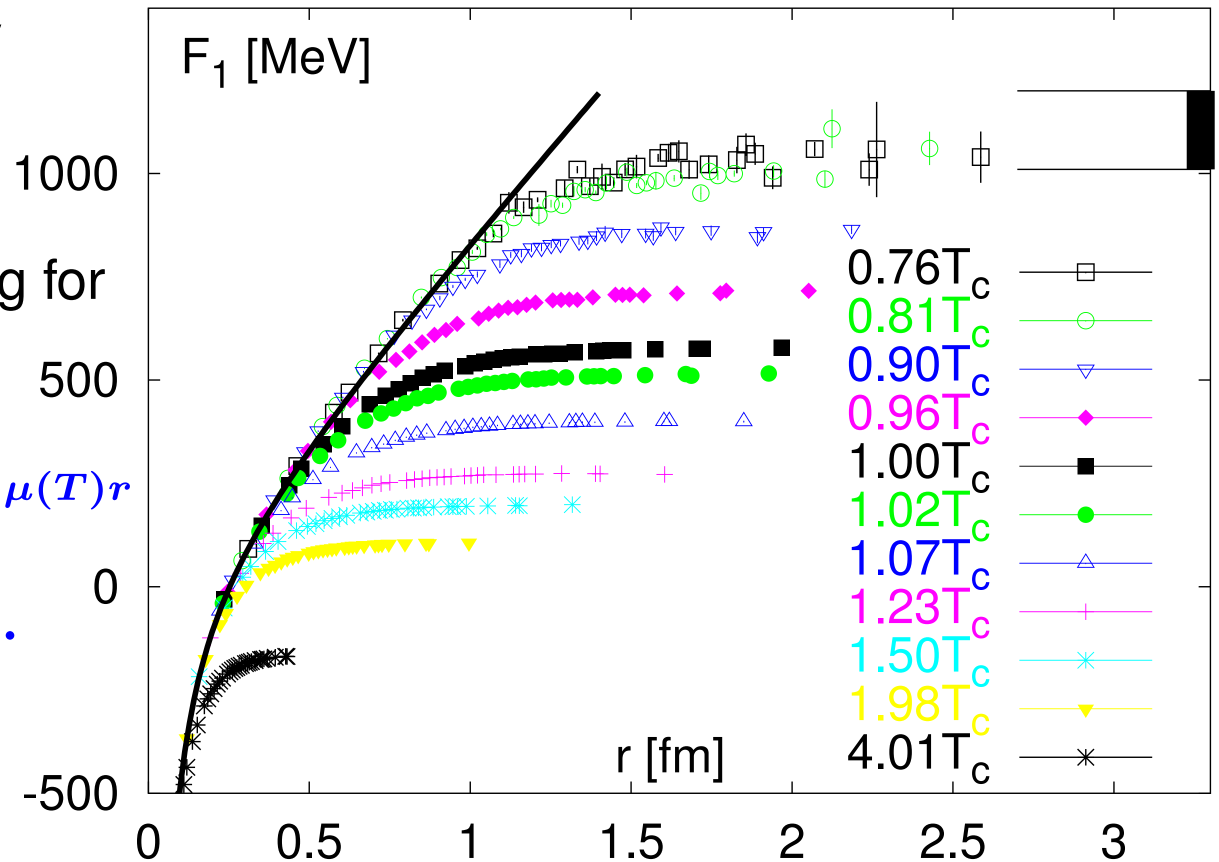
want accurate $T_c, \epsilon_c, \ldots$ determination
to make contact to HI-phenomenology

Screening of heavy quark free energies – remnant of confinement above T_c –

pure gauge: O.Kaczmarek, FK, P. Petreczky, F. Zantow, PRD70 (2005) 074505
2-flavor QCD: O.Kaczmarek, F. Zantow, Phys. Rev. D71 (2005) 114510

- singlet free energy
- $T \simeq T_c$: screening for $r \gtrsim 0.5$fm

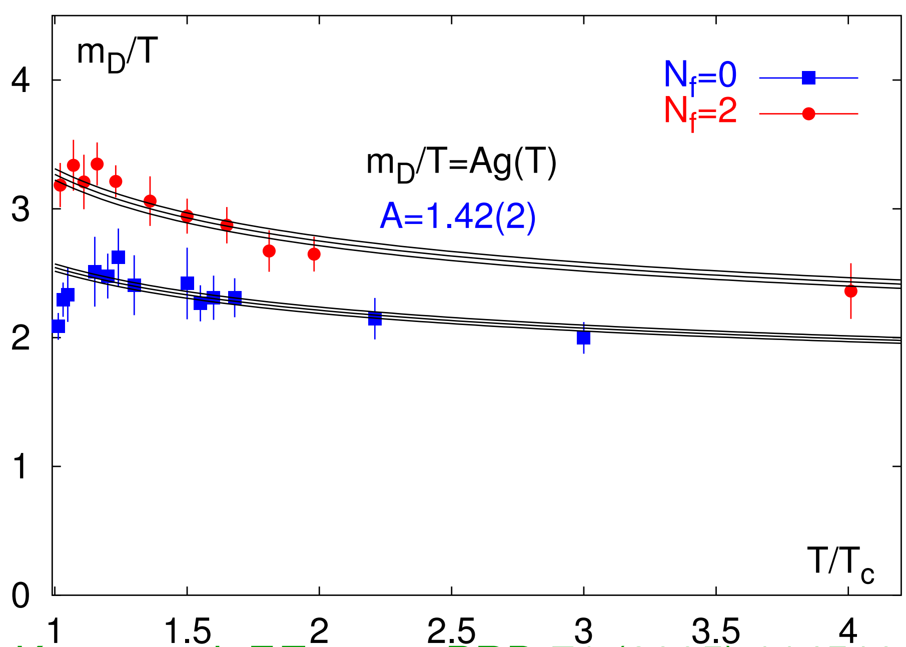
$$F_1(r,T) \sim \frac{\alpha(T)}{r} e^{-\mu(T)r} + const.$$



- $F_1(r,T)$ follows linear rise of $V_{\bar{q}q}(r, T=0) = -\frac{4\alpha(r,T=0)}{3r} + \sigma r$ for $T \lesssim 1.5T_c$, $r \lesssim 0.3$ fm

Non-perturbative Debye screening

- leading order perturbation theory: $m_D = g(T)T\sqrt{1+\frac{n_f}{6}}$
- $T_c < T \lesssim 10T_c$: non-perturbative effects are well represented by an "A-factor": $m_D \equiv Ag(T)T,\ A \simeq 1.5$
- perturbative limit is reached very slowly (logarithms at work!!)



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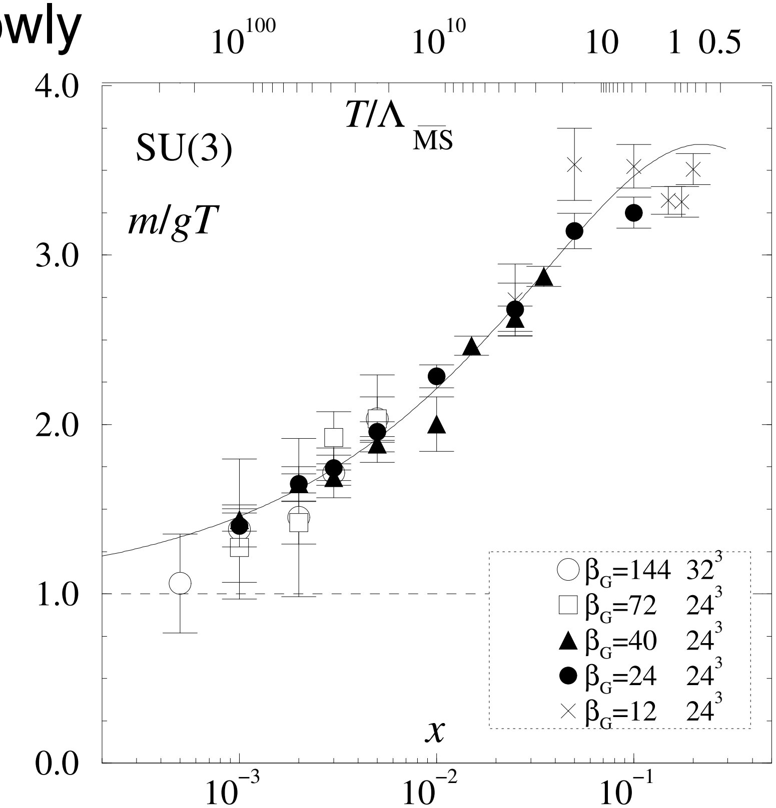

K.Kajantie et al, PRL 79 (1997) 3130

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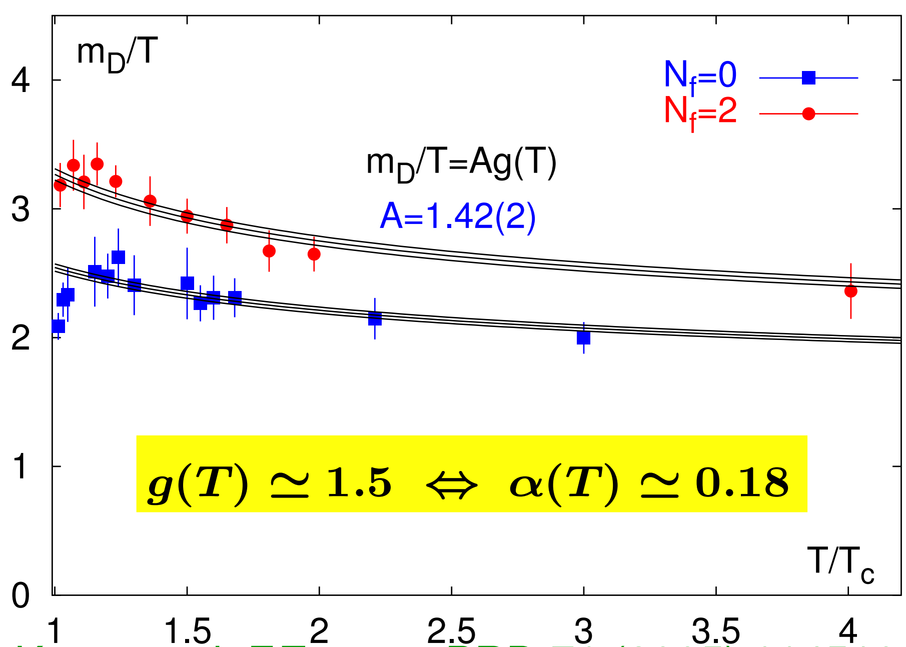

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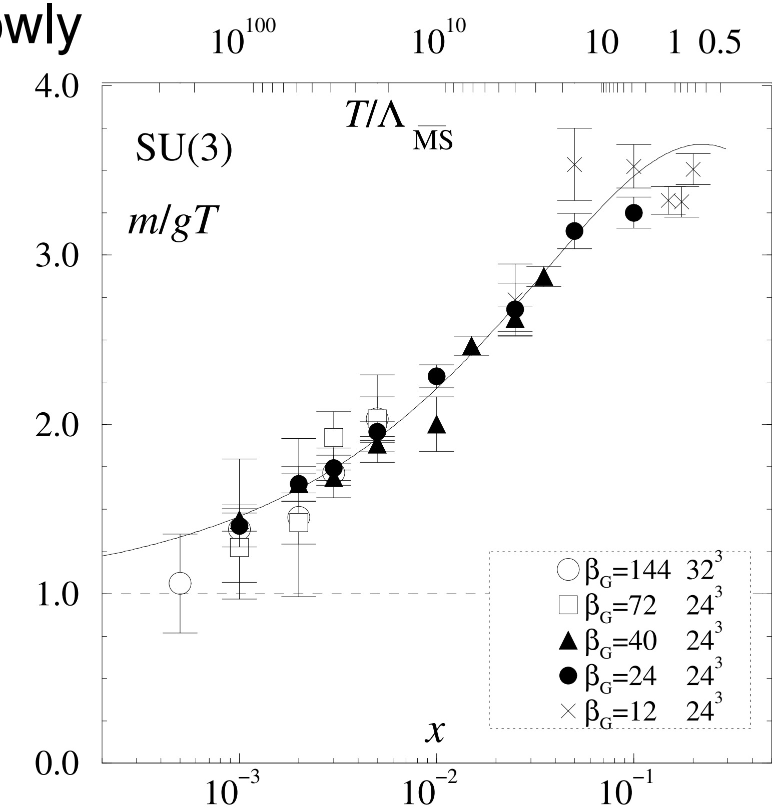

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Singlet free energy and asymptotic freedom

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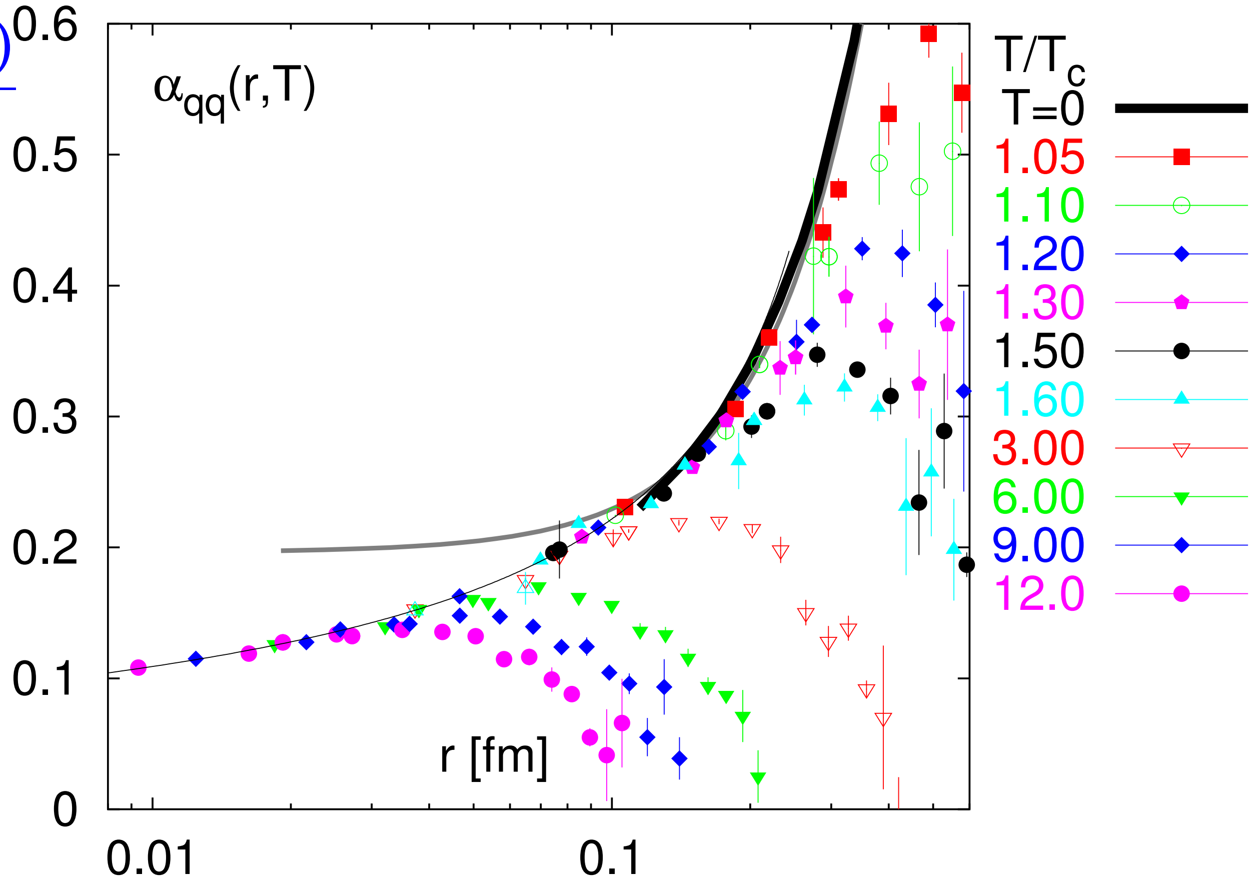
- singlet free energy defines a running coupling:

$$\alpha_{\rm eff} = \frac{3r^2}{4}\,\frac{{\rm d}F_1(r,T)}{{\rm d}r}$$



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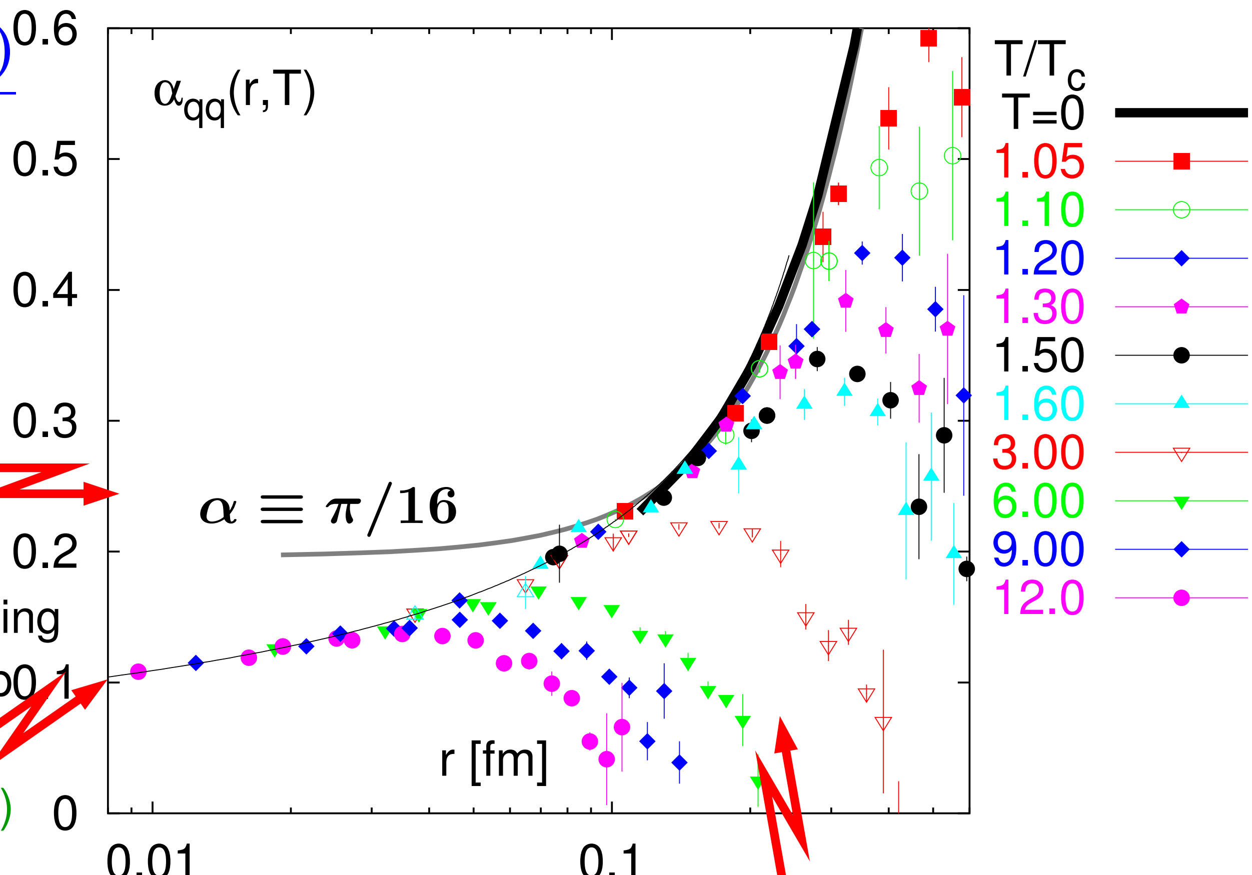

large distance: constant Coulomb term (string model)

short distance: running coupling $\alpha(r)$ from $(T=0)$, 3-loop
(S. Necco, R. Sommer, Nucl. Phys. B622 (2002) 328)

T-dependence starts in non-perturbative regime for $T \lesssim 3\ T_c$

- short distance physics ⇔ vacuum physics

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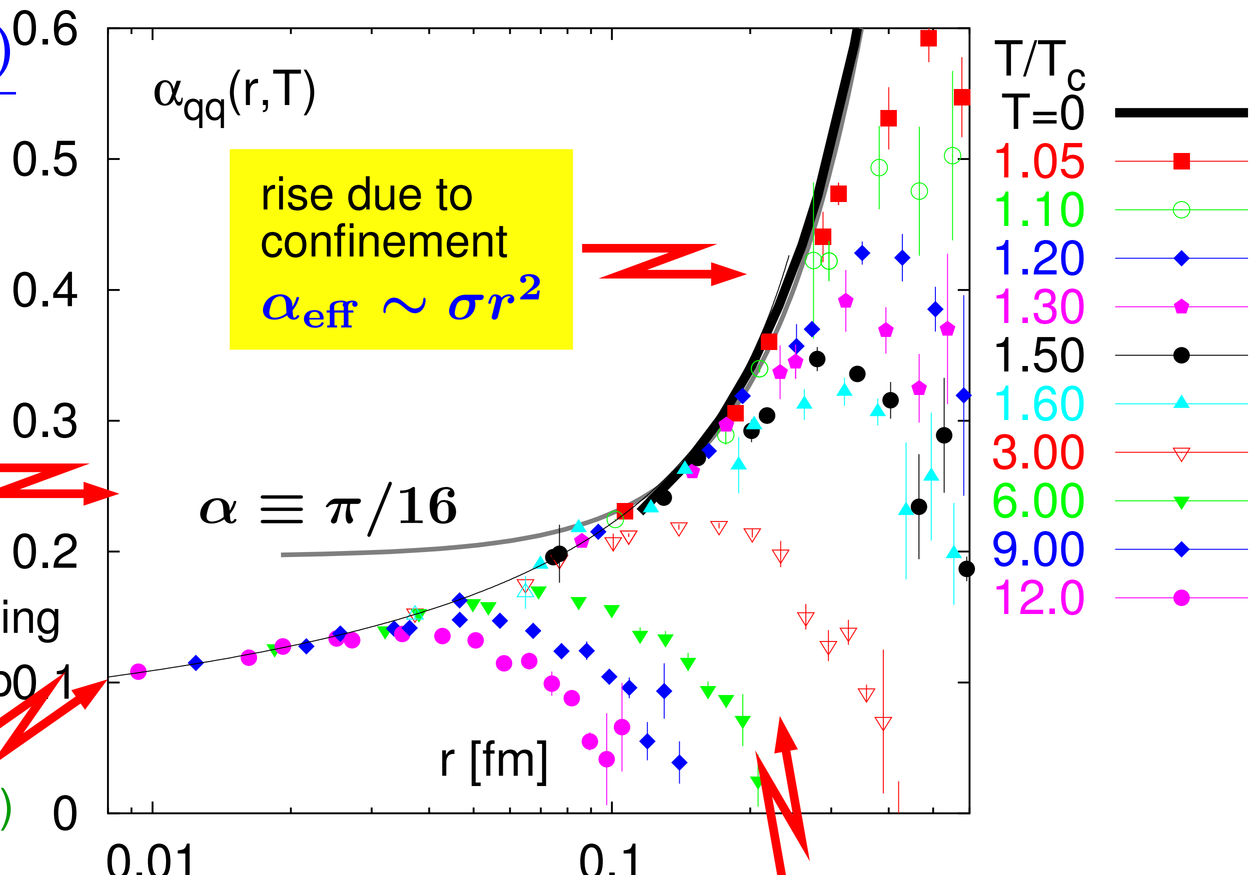

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The spatial string tension

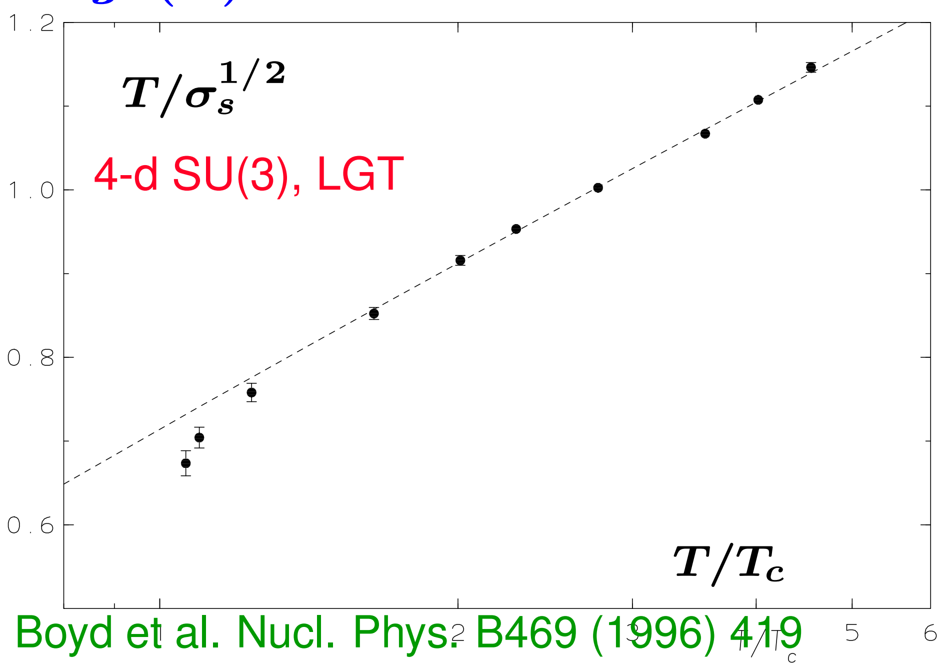
- Non-perturbative, vanishes in high-T perturbation theory:

$$\sqrt{\sigma_s} = -\lim_{R_x,R_y\to\infty} \ln \frac{W(R_x,R_y)}{R_x R_y}$$

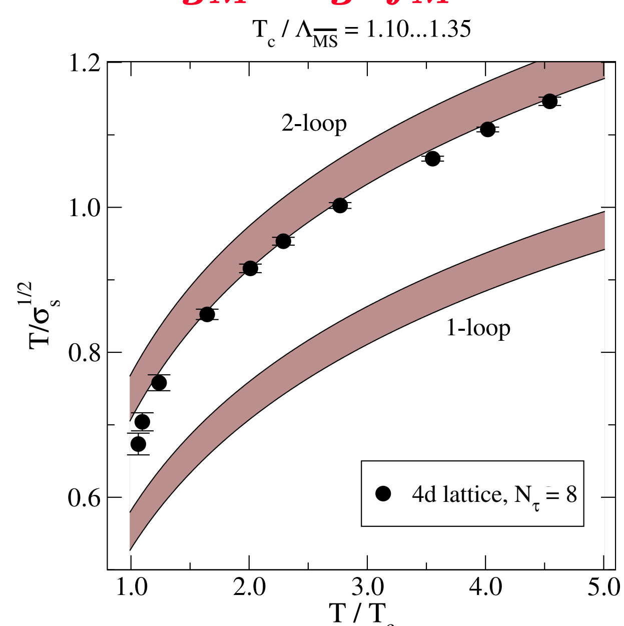
- $\frac{\sqrt{\sigma_s}}{g^2(T)T} = c_M f_M(g(T))\ ,\ c_M = 0.553(1)$

c_M: 3-d SU(3), LGT

$g_M \equiv g^2 f_M$: dim. red. pert. th.



G. Boyd et al. Nucl. Phys. B469 (1996) 419



M. Laine, Y. Schröder, JHEP 0503 (2005) 067

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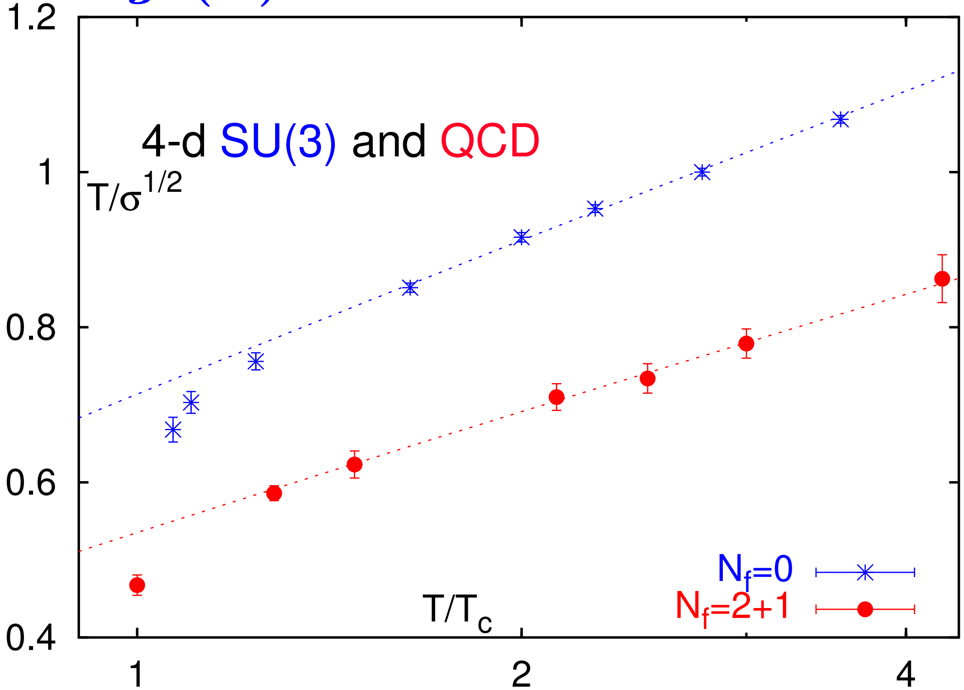


dimensional reduction works for $T \gtrsim 2T_c$

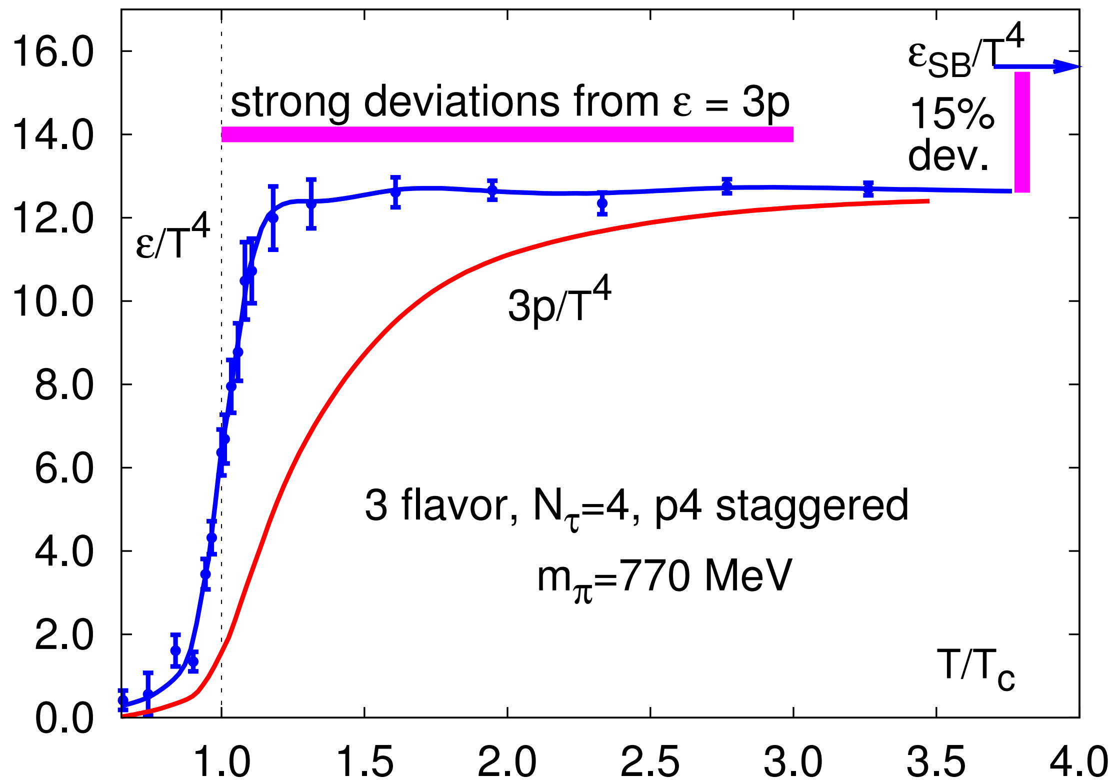
- c_M (almost) flavor independent
- $g^2(T)$ shows 2-loop running

$c = 0.566(13)$ [SU(3)]
$c = 0.587(41)$ [QCD]

RBC-Bielefeld, preliminary

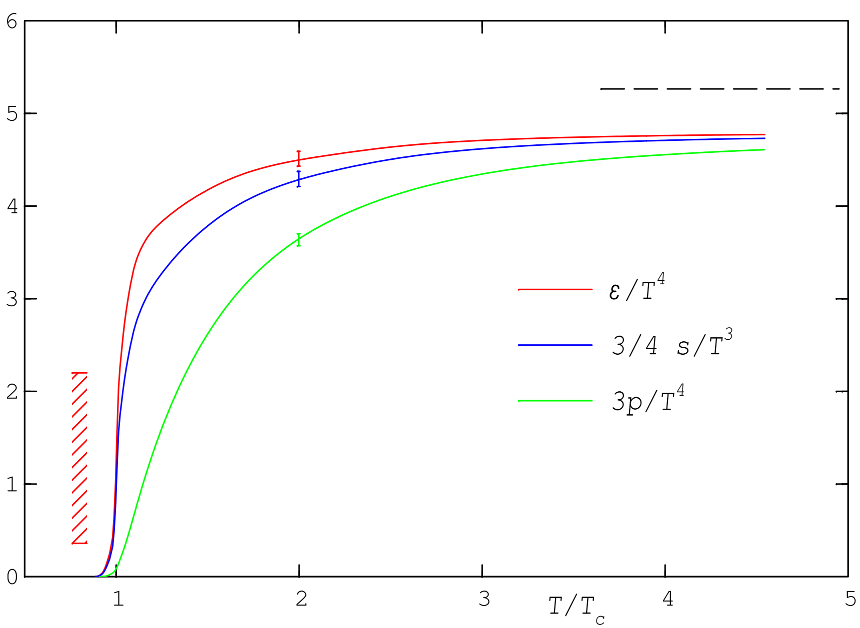
QCD equation of state

- two features of EoS are central in the ongoing discussion of the non-perturbative structure of QCD at high temperature
  - strong deviations from ideal gas behavior ($\epsilon = 3p$) for $T_c \leq T \sim 3T_c$
  - deviations from Stefan-Boltzmann limit persist even at high T



SU(3) Thermodynamics - revisited: EoS

- SU(3) EoS deviates from ideal gas by about 15% at $4T_c$
- slow approach to the high temperature limit
- consistent with logarithmic running of the coupling (cf. 4d vs. 3d)



$\sim$ 15% deviations from ideal gas

NOTE: $p, \ \epsilon, \ s$ normalized to be zero at $T = 0$;

non-pertubative vacuum properties show up at high-T

SU(3) Thermodynamics - revisited: $\langle G^2 \rangle_T$

- SU(3) EoS deviates from ideal gas by about 15% at $4T_c$
- slow approach to the high temperature limit
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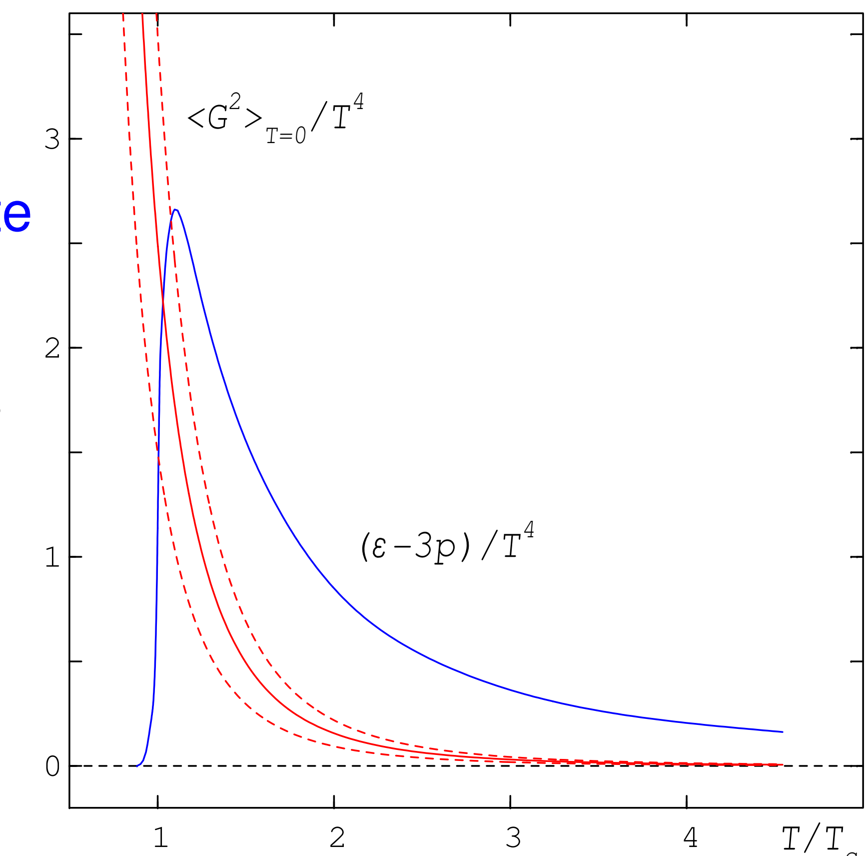
$T = 0$: non-vanishing gluon condensate

$$\epsilon - 3p = \langle G^2 \rangle_{T=0} - \langle G^2 \rangle_T$$

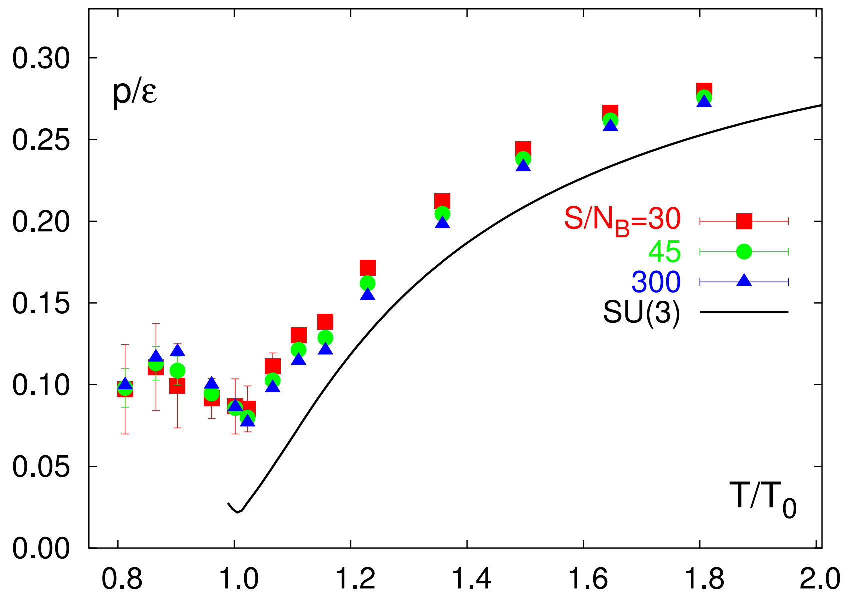
non-pertubative vacuum properties show up at high-T

NOTE: AdS/CFT would give

$$\epsilon - 3p \equiv 0$$



Isentropic Equation of State: p/ϵ



EoS for 2-flavor QCD at fixed S/N_B and EoS for SU(3) gauge theory

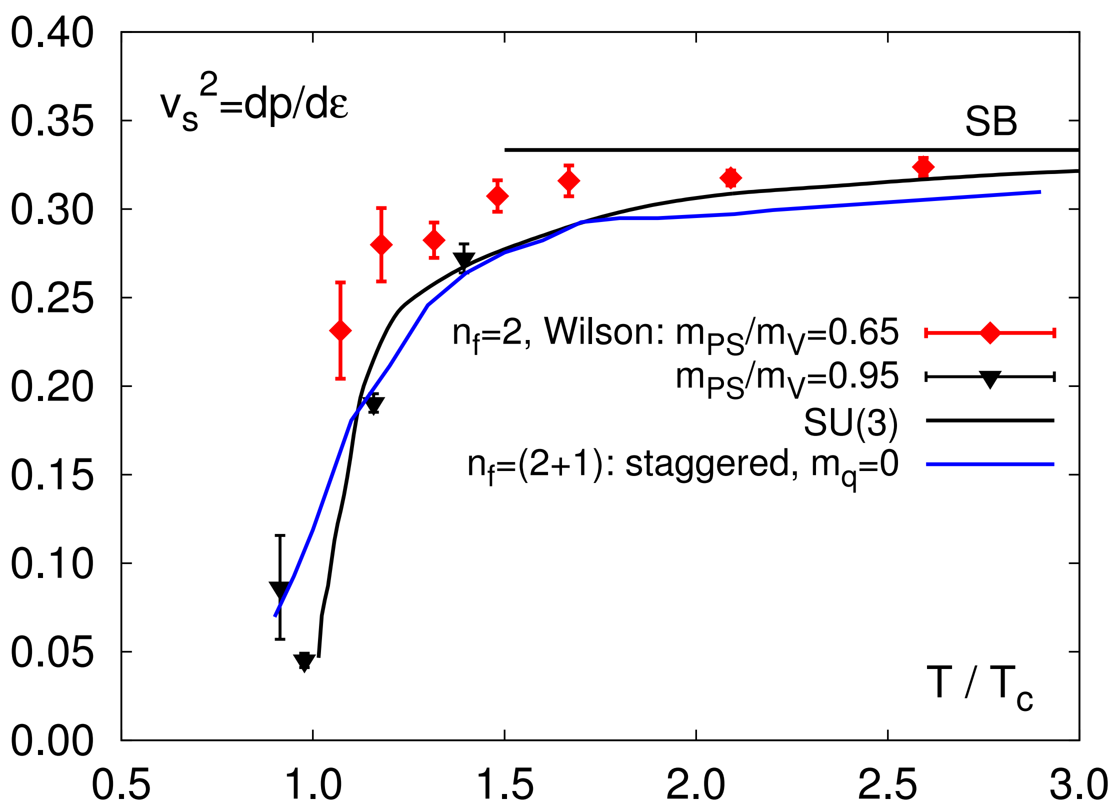
S. Ejiri et al., PRD 73 (2006) 054506

- p/ϵ vs. ϵ shows almost no dependence on S/N_B, *i.e.* μ_q/T
- p/ϵ has only weak dependence on n_f (cut-off effects??)
- phenomenological EoS for $T_0 \lesssim T \lesssim 2T_0$

$$\frac{p}{\epsilon} = \frac{1}{3}\left(1 - \frac{1.2}{1+0.5\epsilon}\right) \ , \ \left(\frac{p}{\epsilon}\right)_{min} \simeq 0.075$$

Velocity of sound

- steep EoS:
  rapid change of energy density; slow change of pressure
  $\Rightarrow$ reduced velocity of sound $\Rightarrow$ more time for equilibration



pure gauge theory:
G. Boyd et al.,
NP B469 1996

$\boldsymbol{n_f = 2}$:
A. Ali Khan et al.,
PR D64 2001

$\boldsymbol{n_f = (2+1)}$:
Y. Aoki et al.,
hep-lat/0510084
$(\boldsymbol{TV^{1/3} = 2})$

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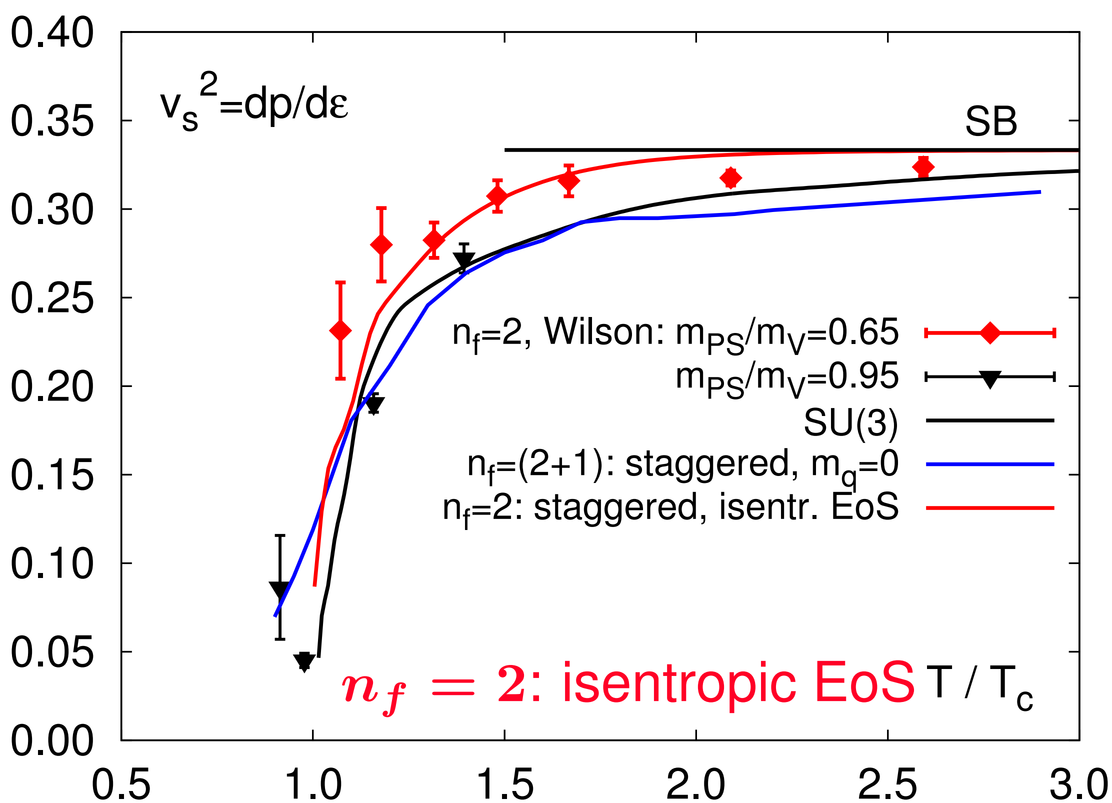

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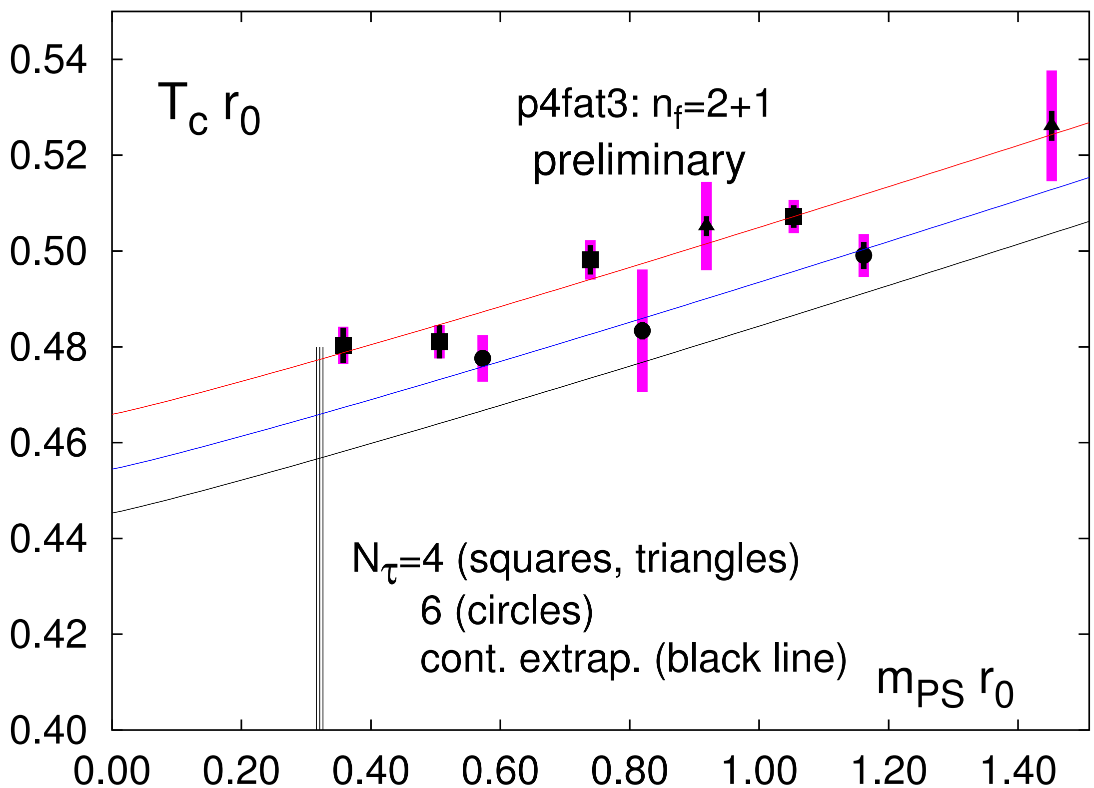
A new determination of the transition temperature in QCD

- calculation of transition temperature with almost physical quark masses and different lattice cut-off values

$\Rightarrow$ extrapolation to physical limit ($m_\pi = 135$ MeV) and continuum limit ($a \longrightarrow 0$)



RIKEN-BNL-Columbia-Bielefeld collaboration, in preparation.

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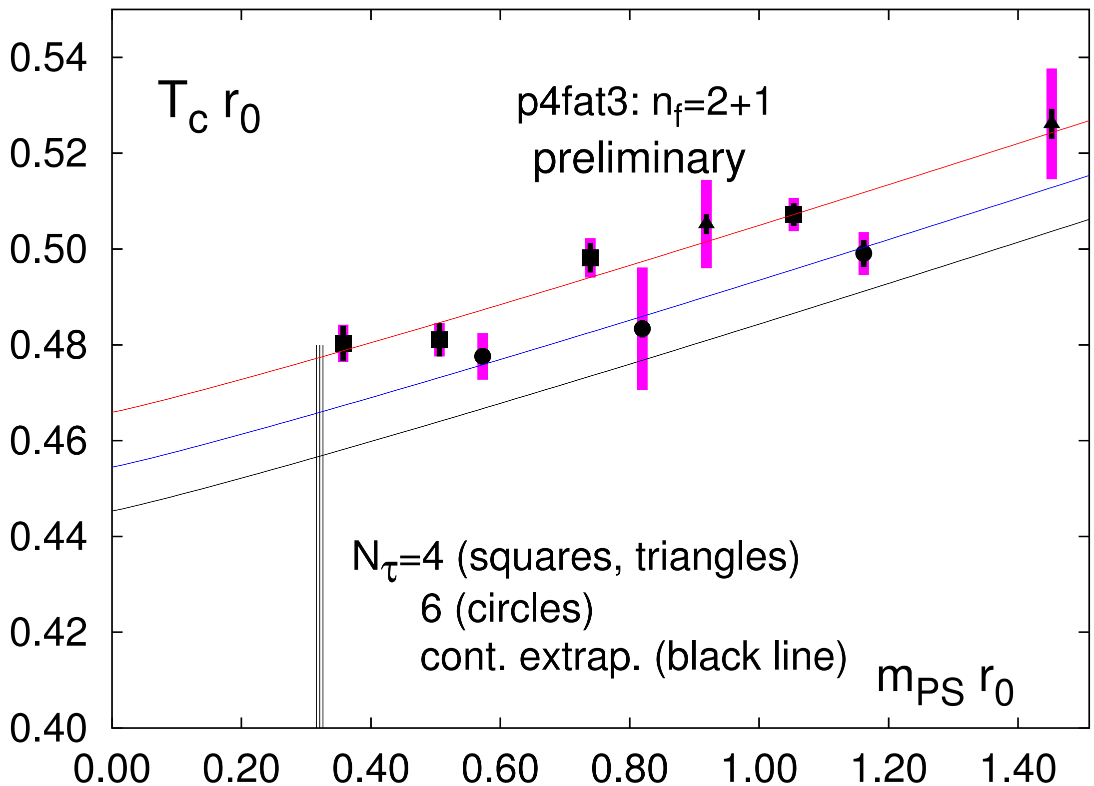

$\sqrt{\sigma} \simeq 465$ MeV

$r_0 = 0.469(7)$ fm

$\Downarrow$

$T_0 \simeq 192(5)(4)$ MeV

preliminary

RIKEN-BNL-Columbia-Bielefeld collaboration, in preparation.

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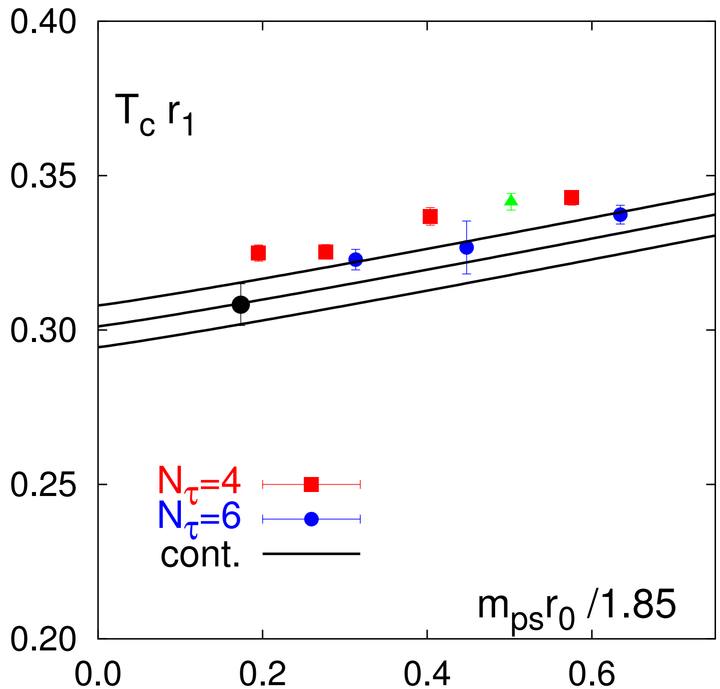
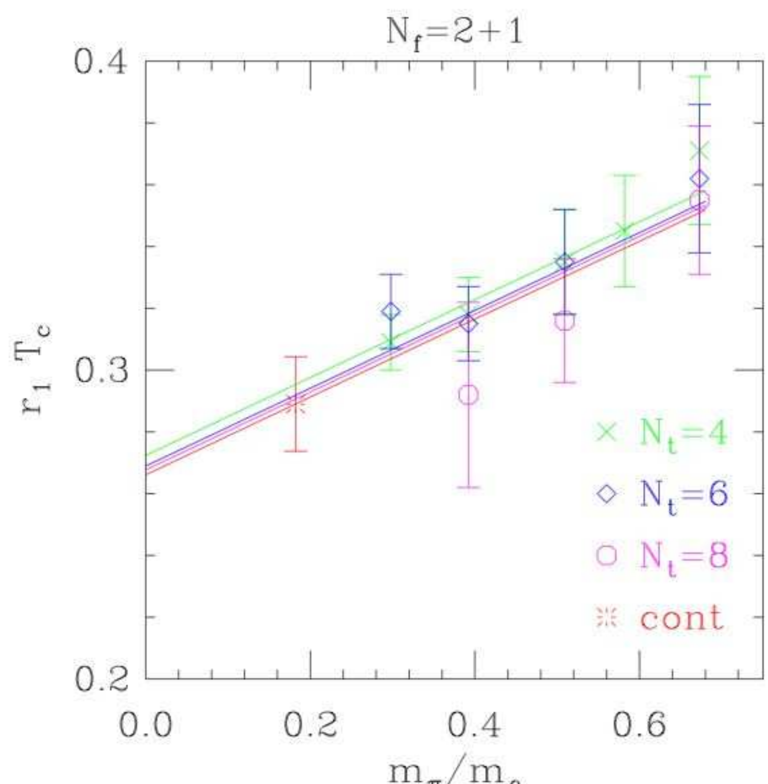


RIKEN-BNL-Columbia-Bielefeld



MILC, PRD71 (2005) 034504
(figure unpublished)

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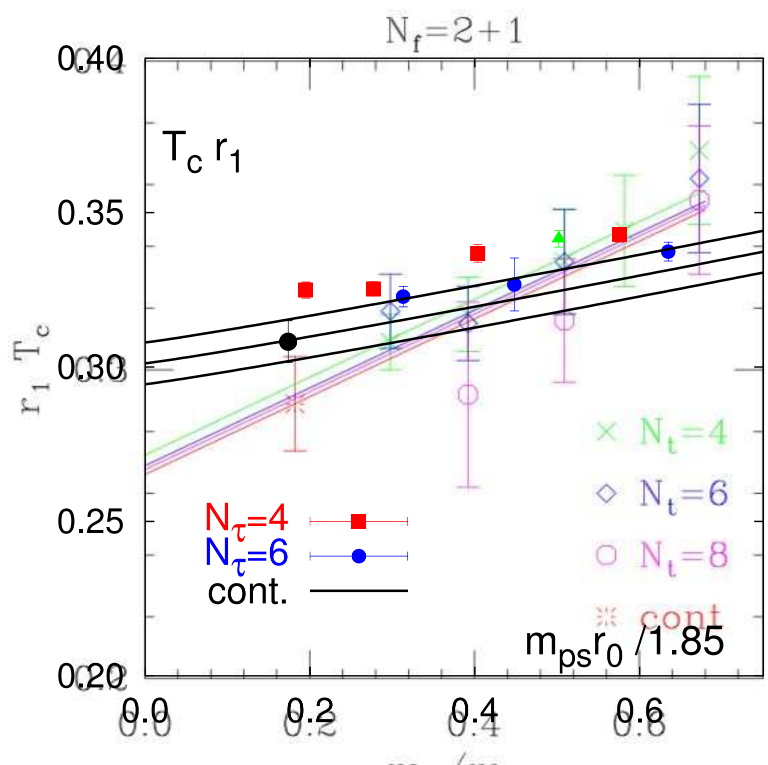

RIKEN-BNL-Columbia-Bielefeld

MILC, PRD71 (2005) 034504
(figure unpublished)

Bulk thermodynamics with non-vanishing chemical potential

$$Z(V,T,\mu) = \int \mathcal{D}\mathcal{A}\mathcal{D}\psi\mathcal{D}\bar{\psi}\ \mathrm{e}^{-S_E(V,T,\mu)}$$
$$= \int \mathcal{D}\mathcal{A}\ \left[det\ M(\mu)\right]^f\ \mathrm{e}^{-S_G(V,T)}$$

$\Uparrow$complex fermion determinant;

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ways to circumvent this problem:

- reweighting: works well on small lattices; requires exact evaluation of $detM$
  Z. Fodor, S.D. Katz, JHEP 0203 (2002) 014
- Taylor expansion around $\mu = 0$: works well for small μ;
  C. R. Allton et al. (Bielefeld-Swansea), Phys. Rev. D66 (2002) 074507
  R.V. Gavai, S. Gupta, Phys. Rev. D68 (2003) 034506
- imaginary chemical potential: works well for small μ; requires analytic continuation
  Ph. deForcrand, O. Philipsen, Nucl. Phys. B642 (2002) 290
  M. D'Elia and M.P. Lombardo, Phys. Rev. D64 (2003) 014505

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- reweighting: larger lattices; smaller quark mass;
  Z. Fodor, S.D. Katz, JHEP 0404 (2004) 050
- Taylor expansion: higher orders; larger volumes;
  C. R. Allton et al., Phys. Rev. D71 (2005) 054508
  R.V. Gavai, S. Gupta, Phys. Rev. D71 (2005) 114014
- imaginary chemical potential: improved algorithms
  O. Philipsen, hep-lat/0510077 (Lattice 2005)

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searches for the CCP:

μ_B sensitive to V (and m_q)
$\mu_B \sim 360$ MeV

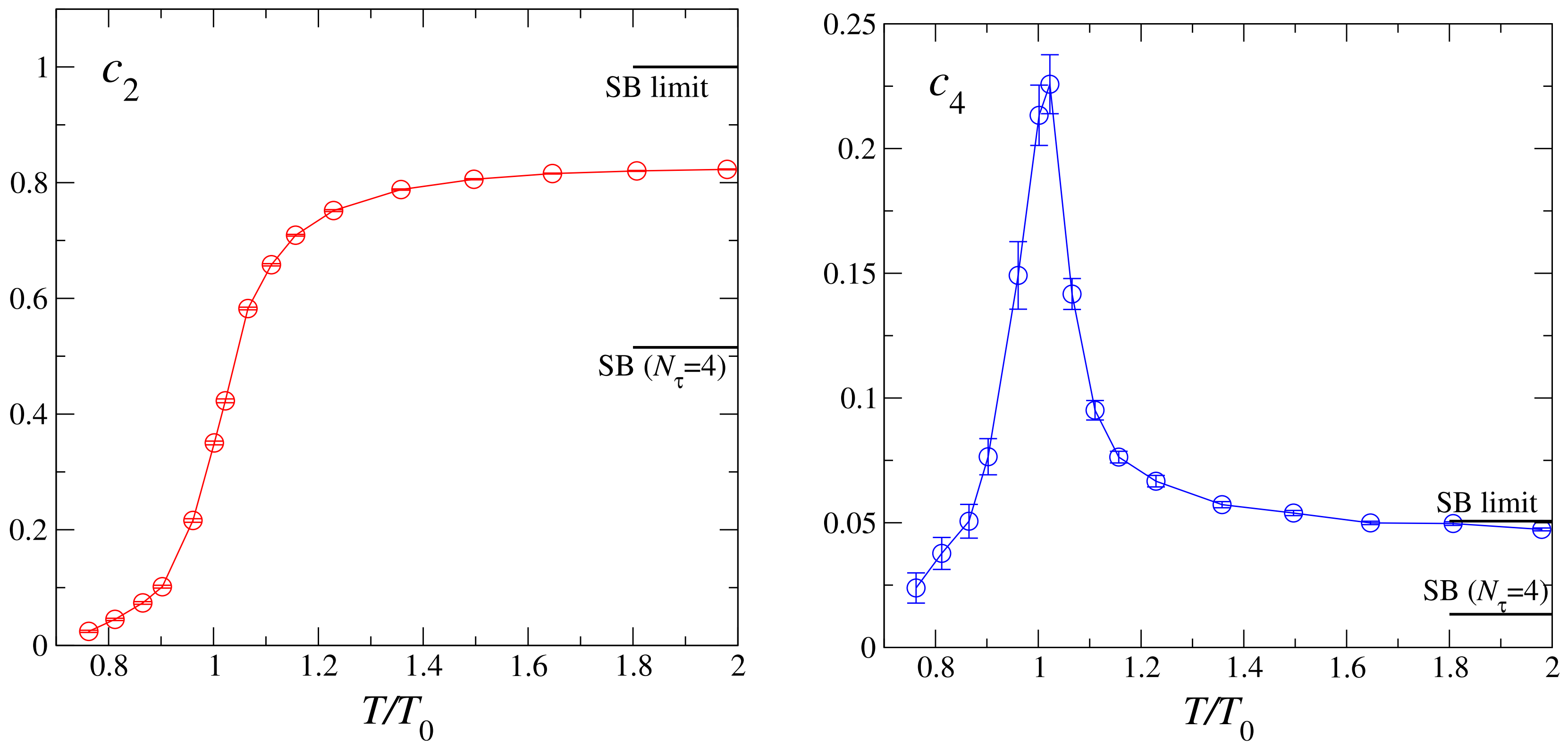
no clear-cut evidence
$\mu_B \sim 180$ MeV

no evidence

Bulk thermodynamics for small μ_q/T on $16^3 \times 4$ lattices

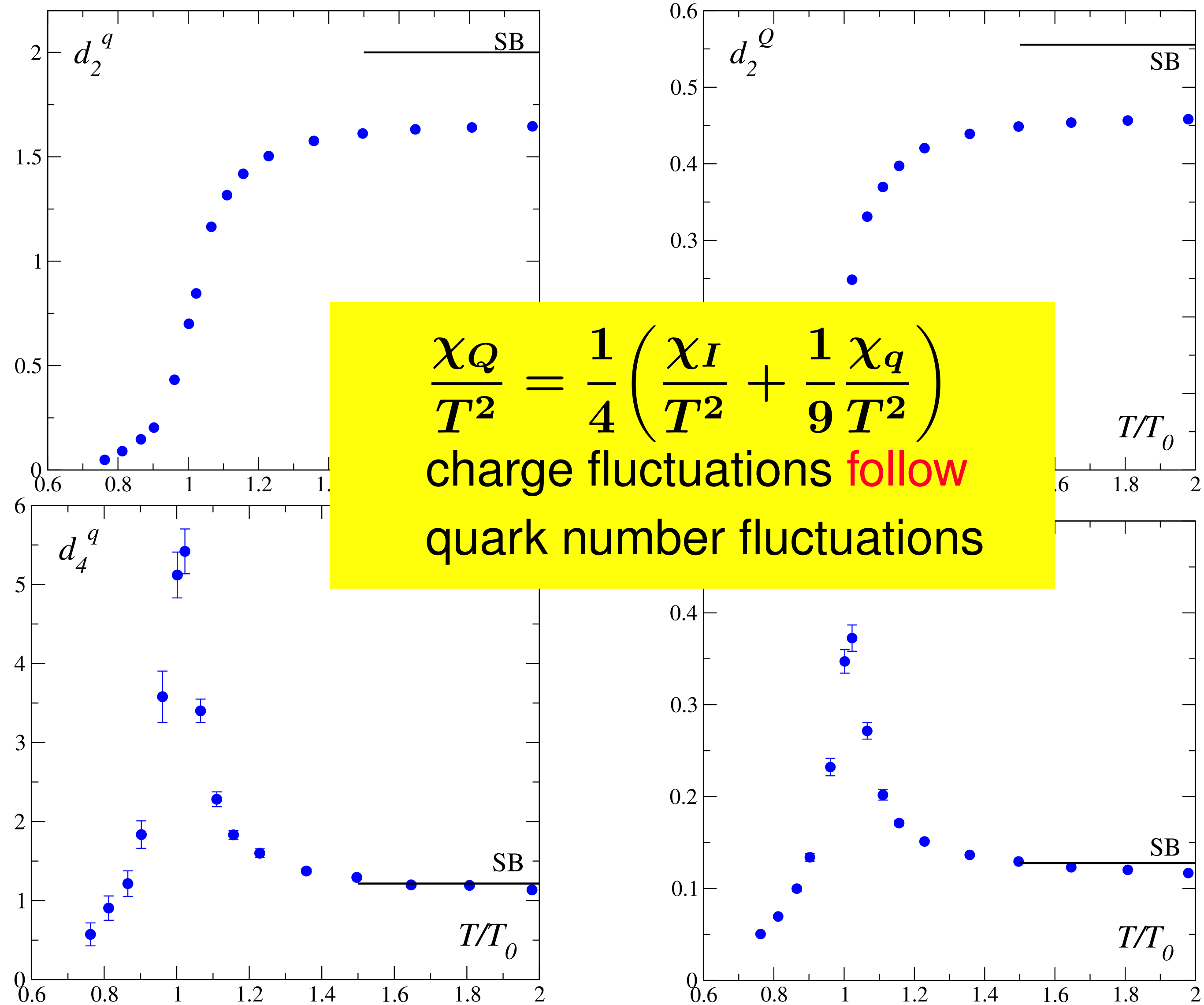
- Taylor expansion of pressure up to $\mathcal{O}\left((\mu_q/T)^4\right)$

$$\frac{p}{T^4} = \frac{1}{VT^3}\ln \mathcal{Z} = \sum_{n=0}^{\infty} c_n(T)\left(\frac{\mu_q}{T}\right)^n \simeq c_0 + c_2\left(\frac{\mu_q}{T}\right)^2 + c_4\left(\frac{\mu_q}{T}\right)^4 + \mathcal{O}((\mu_q/T)^6)$$



Quark number and charge fluctuations at $\mu_B = 0$; 2-flavor QCD ($m_\pi \simeq 770\ MeV$)

C. Allton et al. (Bielefeld-Swansea), PRD71 (2005) 054508



$$\frac{\chi_Q}{T^2} = \frac{1}{4}\left(\frac{\chi_I}{T^2} + \frac{1}{9}\frac{\chi_q}{T^2}\right)$$

charge fluctuations follow quark number fluctuations

monotonic increase; close to ideal gas value for $T \gtrsim 1.5 T_c$

develops cusp at T_c

reaches ideal gas value for $T \gtrsim 1.5 T_c$

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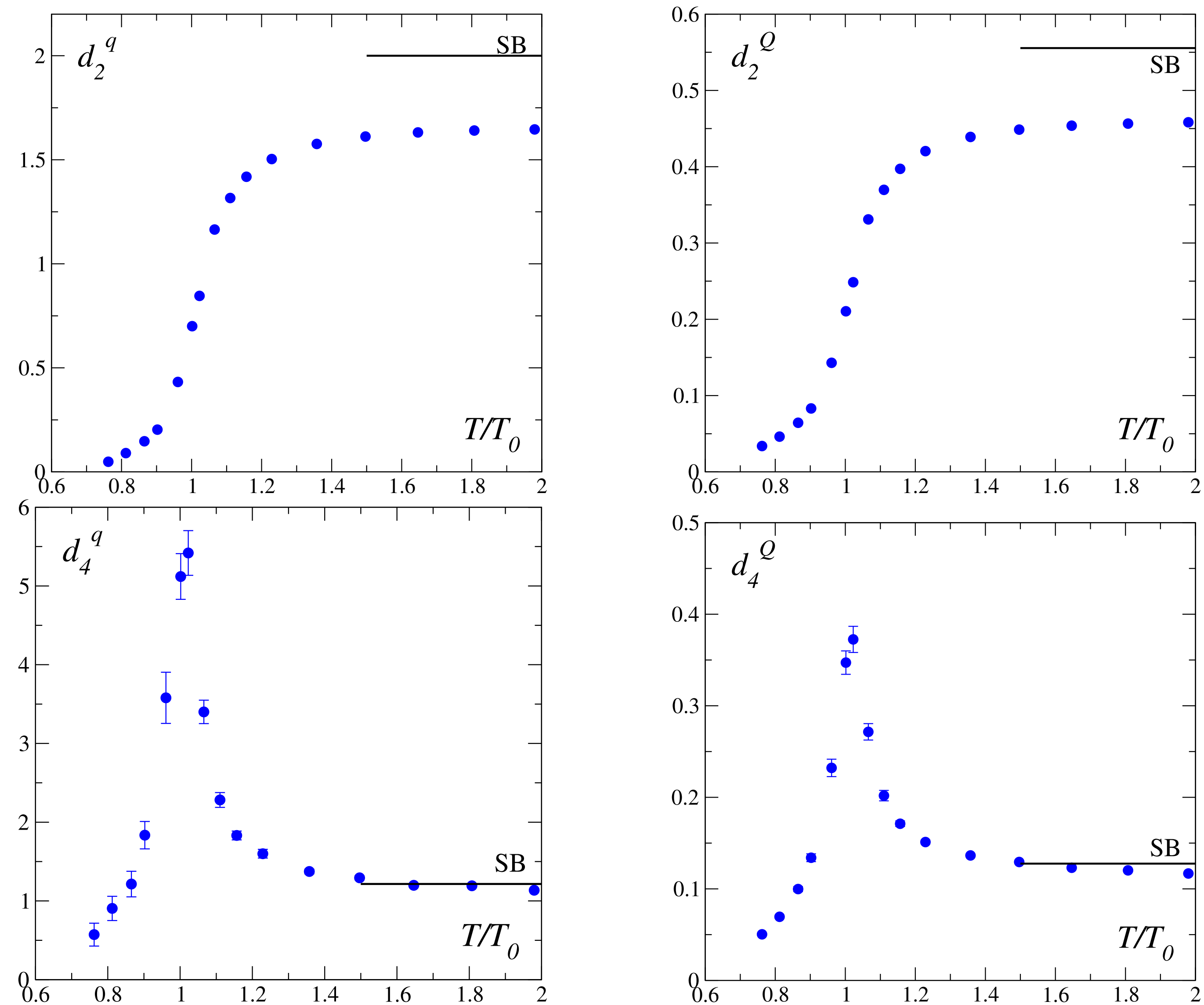

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close to ideal gas value for $T \gtrsim 1.5 T_c$

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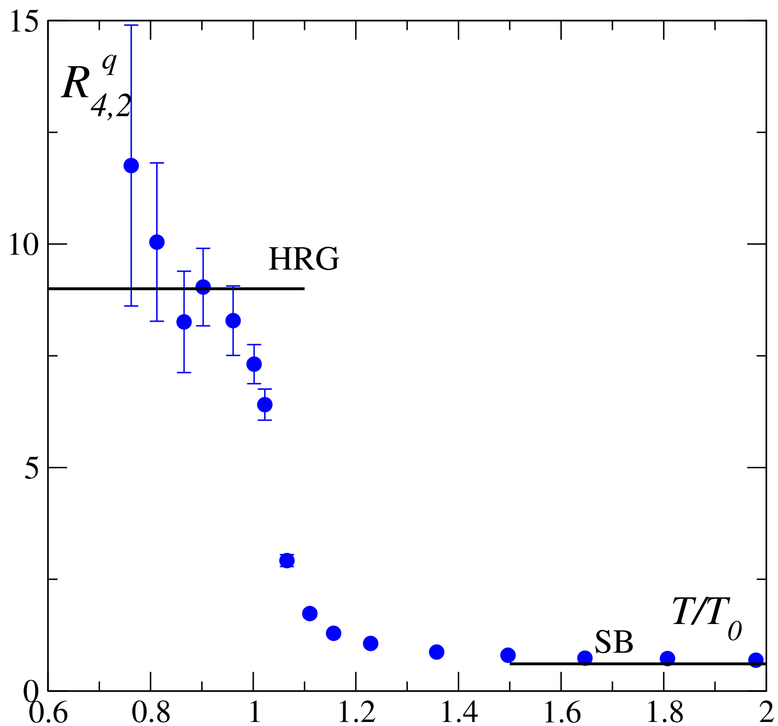
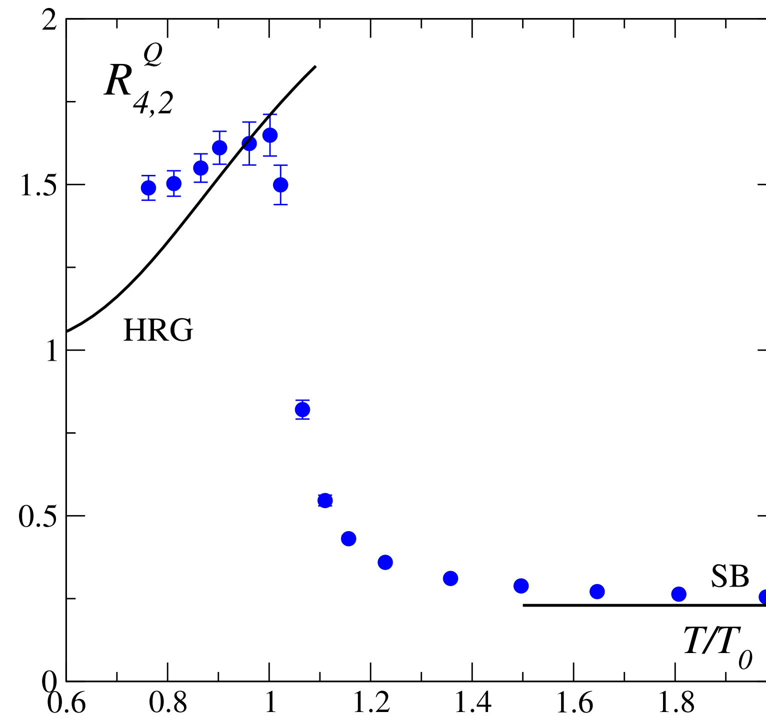
Cumulant ratios

- ratios of cumulants reflect carriers of baryon number and charge

$$R^x_{4,2} = d^x_4/d^x_2 \quad , \quad x = q,\ Q$$

$$R^q_{4,2} = \begin{cases} 9 & ,\mathrm{HRG} \\ \frac{6}{\pi^2} + \mathcal{O}(g^3) & ,\mathrm{high}-T \end{cases} \qquad R^Q_{4,2} = \begin{cases} 1 & ,\mathrm{HRG}, T \to 0 \\ \frac{34}{15\pi^2} + \mathcal{O}(g^3) & ,\mathrm{high}-T \end{cases}$$

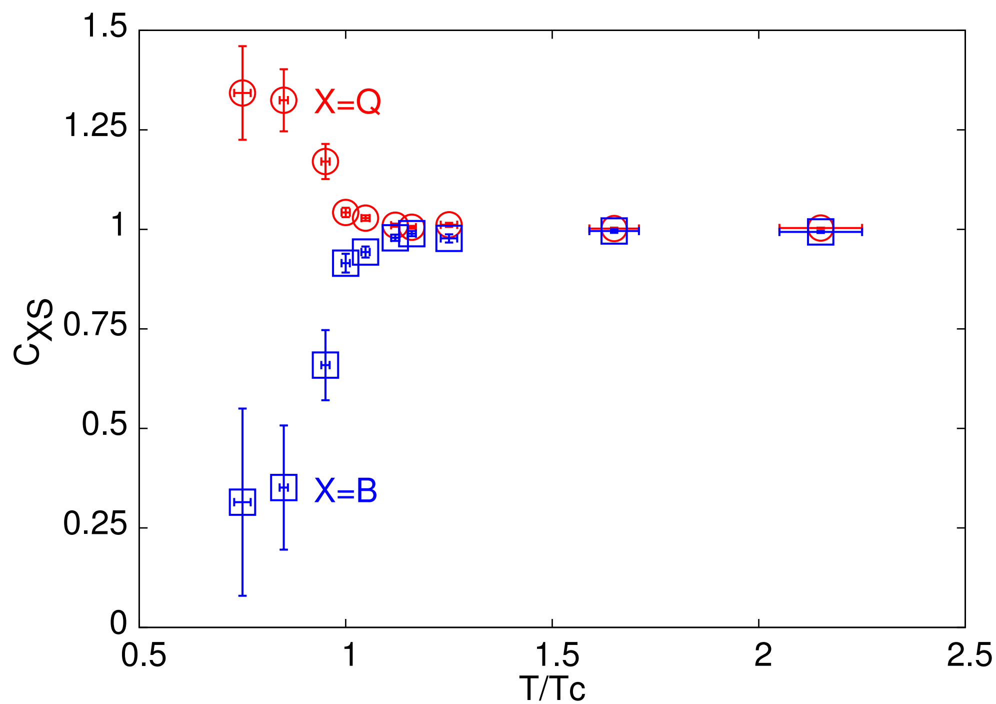




Off-diagonal correlations

- similar: correlations between eg. strangeness and baryon number sensitive to "carriers of quantum numbers"

$$\chi_{XS} \equiv \langle XS \rangle - \langle X \rangle \langle S \rangle \quad , \quad X = B,\, Q$$



R.V. Gavai, S. Gupta, PRD 73 (2006) 014004

inspired by: V. Koch, A. Majumder, J. Randrup, PRL 95 (2005) 182301

Lattice results on the QCD critical point

- Where is the critical point?

Lattice results on the QCD critical point

- attempts to determine the QCD critical point
  - reweighting around $\mu_q = 0$
    Z. Fodor, S. Katz, JHEP 0203 (2002) 014 and 0404 (2004) 050
  - Taylor expansion around $\mu_q = 0$
    C. R. Allton et al., Phys. Rev. D71 (2005) 054508
    R.V. Gavai, S. Gupta, Phys. Rev. D71 (2005) 114014
- results in a nutshell
  - FK: $\mu_B \sim 360$ MeV
  - GG: $\mu_B \sim 180$ MeV
  - WE: so far, no clear-cut evidence for the chiral critical point

Bulk thermodynamics with non-vanishing chemical potential

$$Z(V,T,\mu) = \int \mathcal{D}\mathcal{A}\mathcal{D}\psi\mathcal{D}\bar{\psi}\ \mathrm{e}^{-S_E(V,T,\mu)}$$

$$= \int \mathcal{D}\mathcal{A}\ \left[det\ M(\mu)\right]^f\ \mathrm{e}^{-S_G(V,T)}$$

⇑complex fermion determinant;

⇓Taylor expansion;

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln Z(V,T,\mu)$$

$$\equiv \sum_{n=0}^{\infty} c_n(T) \left(\frac{\mu}{T}\right)^n$$

$$= c_0 + c_2 \left(\frac{\mu}{T}\right)^2 + c_4 \left(\frac{\mu}{T}\right)^4 + \mathcal{O}((\mu/T)^6)$$

$$\mu = 0 \quad \Rightarrow \quad \frac{p}{T^4} \equiv c_0(T)$$

Bulk thermodynamics for small μ_q/T on $16^3 \times 4$ lattices

- Taylor expansion of pressure up to $\mathcal{O}\left((\mu_q/T)^6\right)$

$$\frac{p}{T^4} = \frac{1}{VT^3}\ln \mathcal{Z} = \sum_{n=0}^{\infty} c_n(T)\left(\frac{\mu_q}{T}\right)^n \simeq c_0 + c_2\left(\frac{\mu_q}{T}\right)^2 + c_4\left(\frac{\mu_q}{T}\right)^4 + c_6\left(\frac{\mu_q}{T}\right)^6$$

quark number density
$$\frac{n_q}{T^3} = 2c_2\frac{\mu_q}{T} + 4c_4\left(\frac{\mu_q}{T}\right)^3 + 6c_6\left(\frac{\mu_q}{T}\right)^5$$

quark number susceptibility
$$\frac{\chi_q}{T^2} = 2c_2 + 12c_4\left(\frac{\mu_q}{T}\right)^2 + 30c_6\left(\frac{\mu_q}{T}\right)^4$$

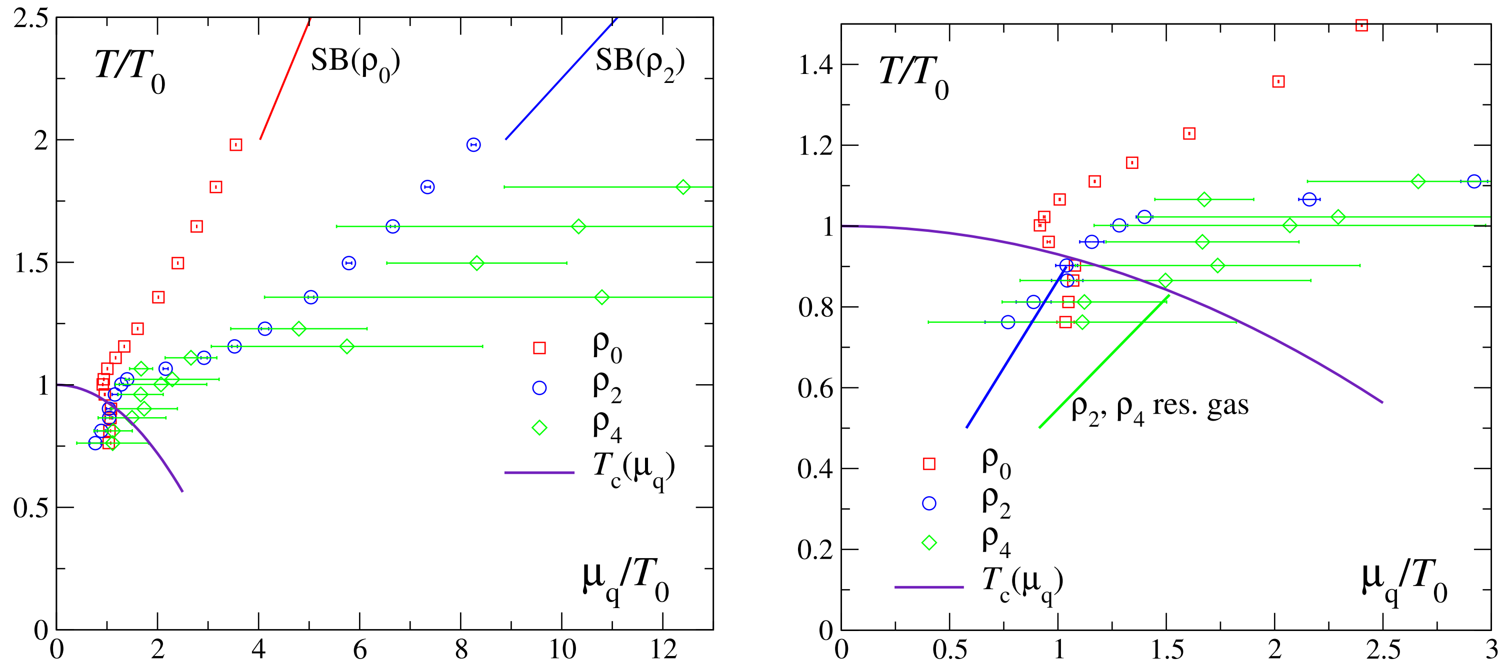
an estimator for the radius of convergence

$$\left(\frac{\mu_q}{T}\right)_{crit} = \lim_{n\to\infty}\left|\frac{c_{2n}}{c_{2n+2}}\right|^{1/2}$$

$c_n > 0$ for all n;
singularity for real μ

Radius of convergence:
lattice estimates vs. resonance gas

- Taylor expansion $\Rightarrow$ estimates for radius of convergence $\rho_{2n} = \sqrt{\left|\frac{c_{2n}}{c_{2n+2}}\right|}$



$T < T_0$: $\rho_n \simeq 1.0$ for all n $\Rightarrow$ $\mu_B^{crit} \simeq 500$ MeV

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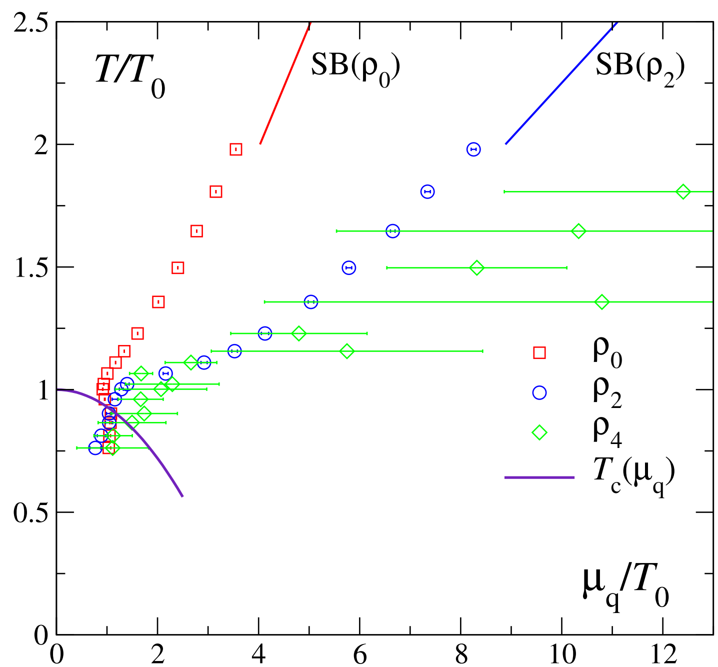

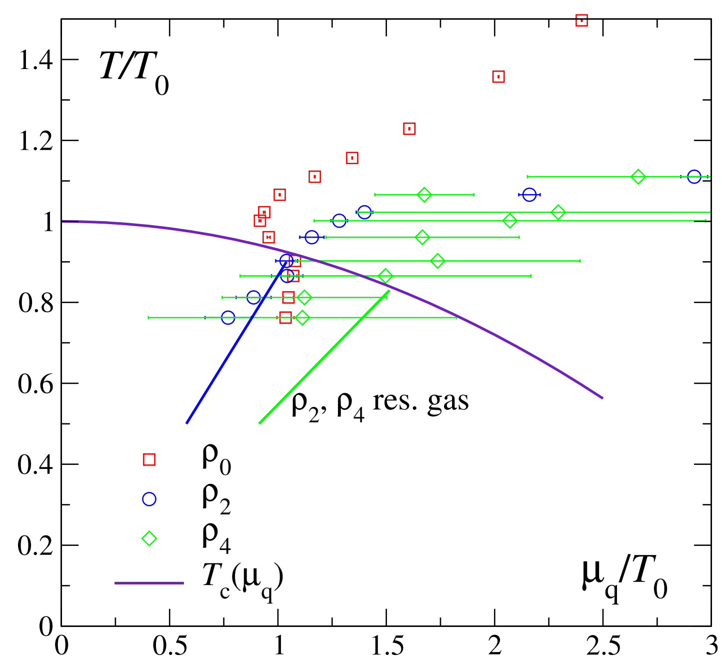

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HRG analytic, LGT consistent with HRG $\Rightarrow$ infinite radius of convergence not yet ruled out

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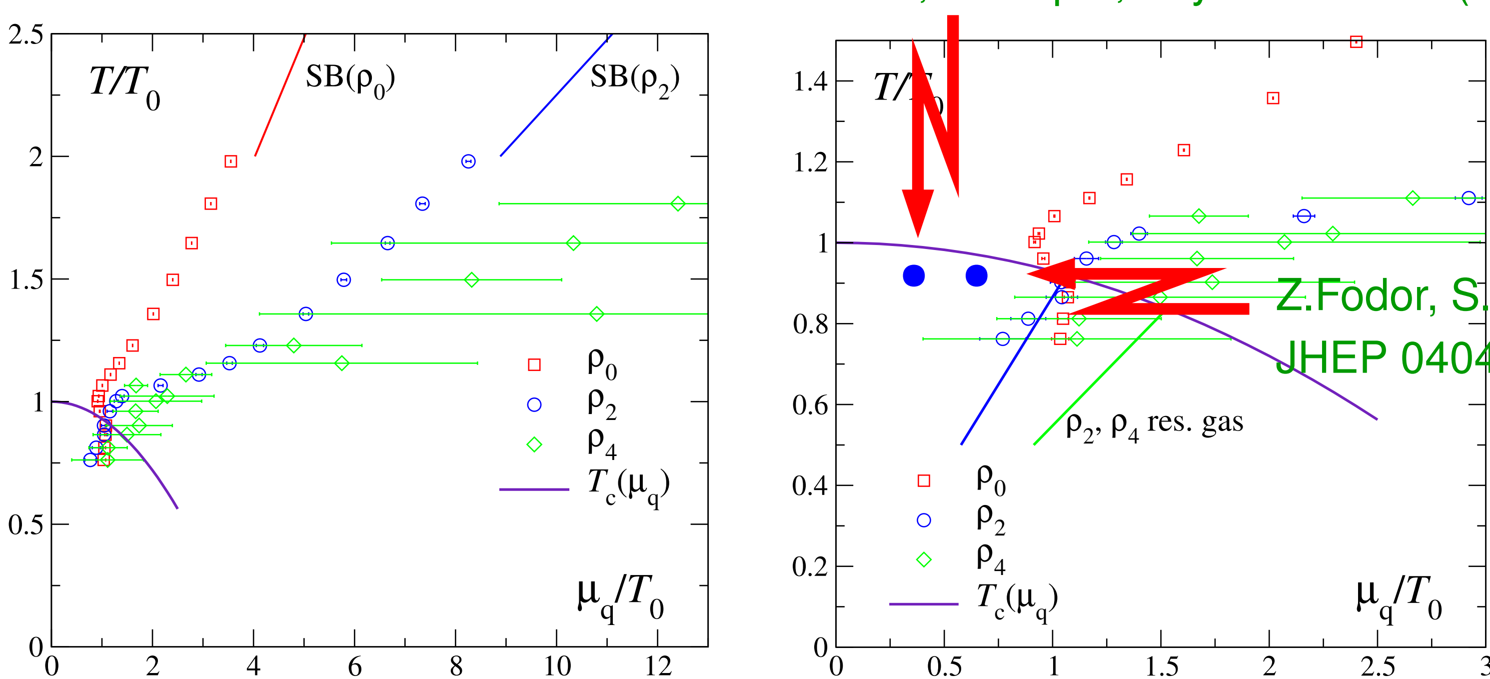
R.V. Gavai, S. Gupta, Phys. Rev. D71 (2005) 114014



Z.Fodor, S.D.Katz
JHEP 0404 (2004) 050

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Conclusions

- Bulk thermodynamics is currently under intensive study:
  T_c has increased over earlier estimates: $T_c = 192(5)(4)$ MeV
  the EoS shows little quark mass and cut-off dependence
  for $T \geq T_c$: $\epsilon_c/T_c = 6 \pm 2$ still a valid estimate.
- The last word on the QCD phase diagram is not yet spoken:
  universal properties of the transition in 2-flavor QCD still have to be established;
  the location of the chiral critical point still is uncertain.
- Fluctuations of baryon number, electric charge, strangeness confirm that quasi-particles with quantum numbers of quarks are the dominant degrees of freedom in the QGP for $T \gtrsim 1.5 T_c$;
  bulk thermodynamics is consistent with a logarithmic running of the coupling for $T \gtrsim 2T_c$.