Gluon Saturation and Color Glass Condensate

François Gelis

CEA / DSM / SPhT

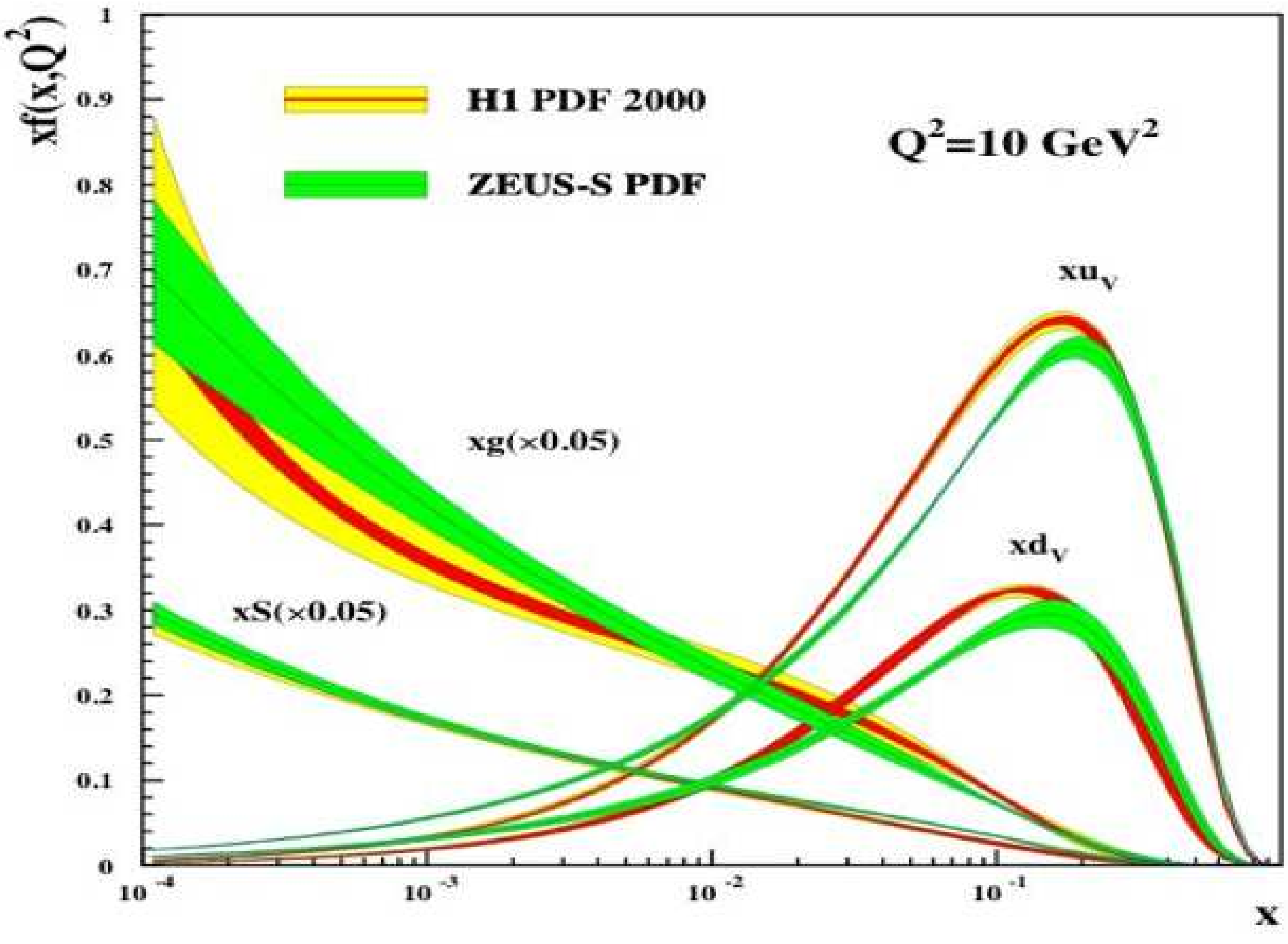
Outline

- Nucleons at high energy
- Parton evolution with energy – gluon saturation
- Color Glass Condensate
- What is the present evidence?
- Pros and cons of pp, pA, AA colliders vs eA colliders

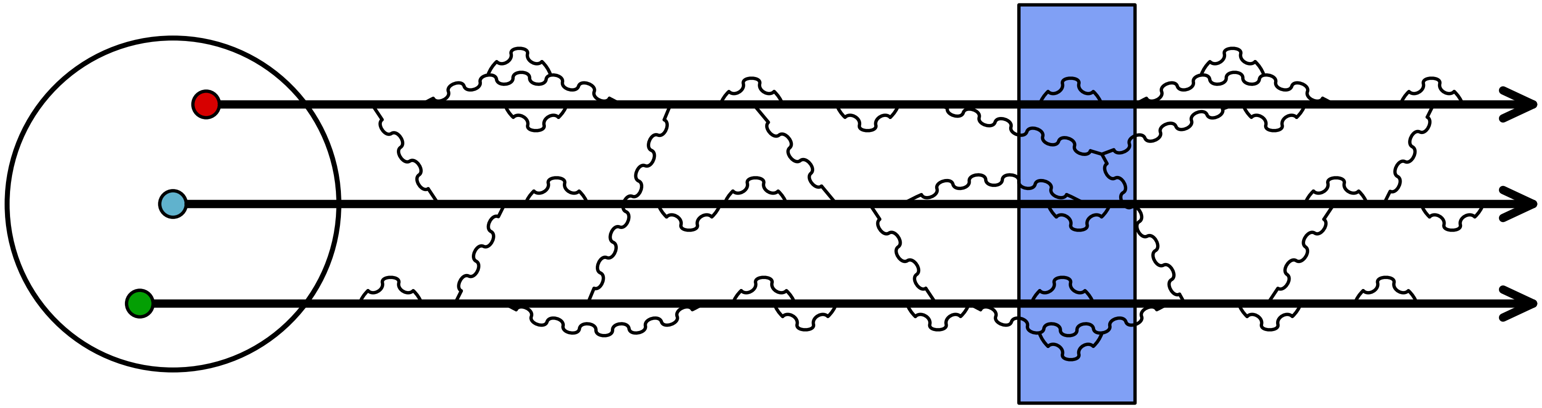
See also : related talks in the “QCD at zero temperature” session

Nucleons at high energy

Parton distributions in a proton

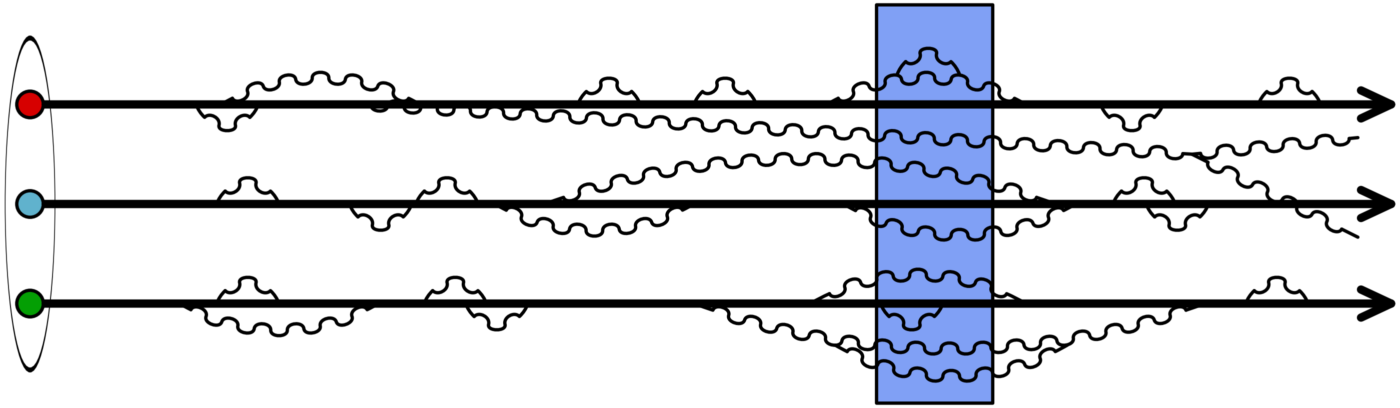


Nucleon at rest



- Very complicated non-perturbative object...
- Contains fluctuations at all space-time scales smaller than its own size
- Only the fluctuations that are longer lived than the external probe participate in the interaction process
- The only role of short lived fluctuations is to renormalize the masses and couplings
- Interactions are very complicated if the constituents of the nucleon have a non trivial dynamics over time-scales comparable to those of the probe

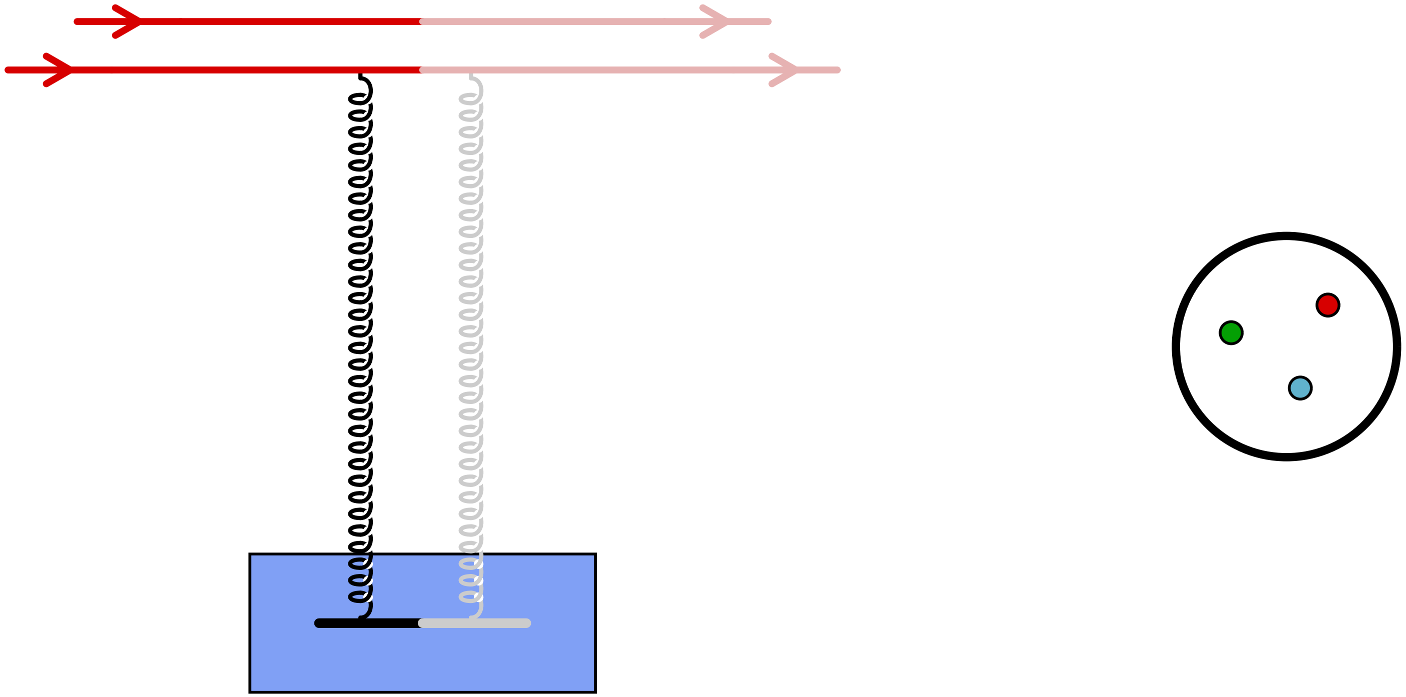
Nucleon at high energy



- Dilation of all internal time-scales of the nucleon
- Interactions among constituents now take place over time-scales that are longer than the characteristic time-scale of the probe
  - ▷ the constituents behave as if they were free
- Many fluctuations live long enough to be seen by the probe. The nucleon appears denser at high energy (it contains more gluons)
- Pre-existing fluctuations are totally frozen over the time-scale of the probe, and act as static sources of new partons

Parton evolution – saturation

Parton evolution

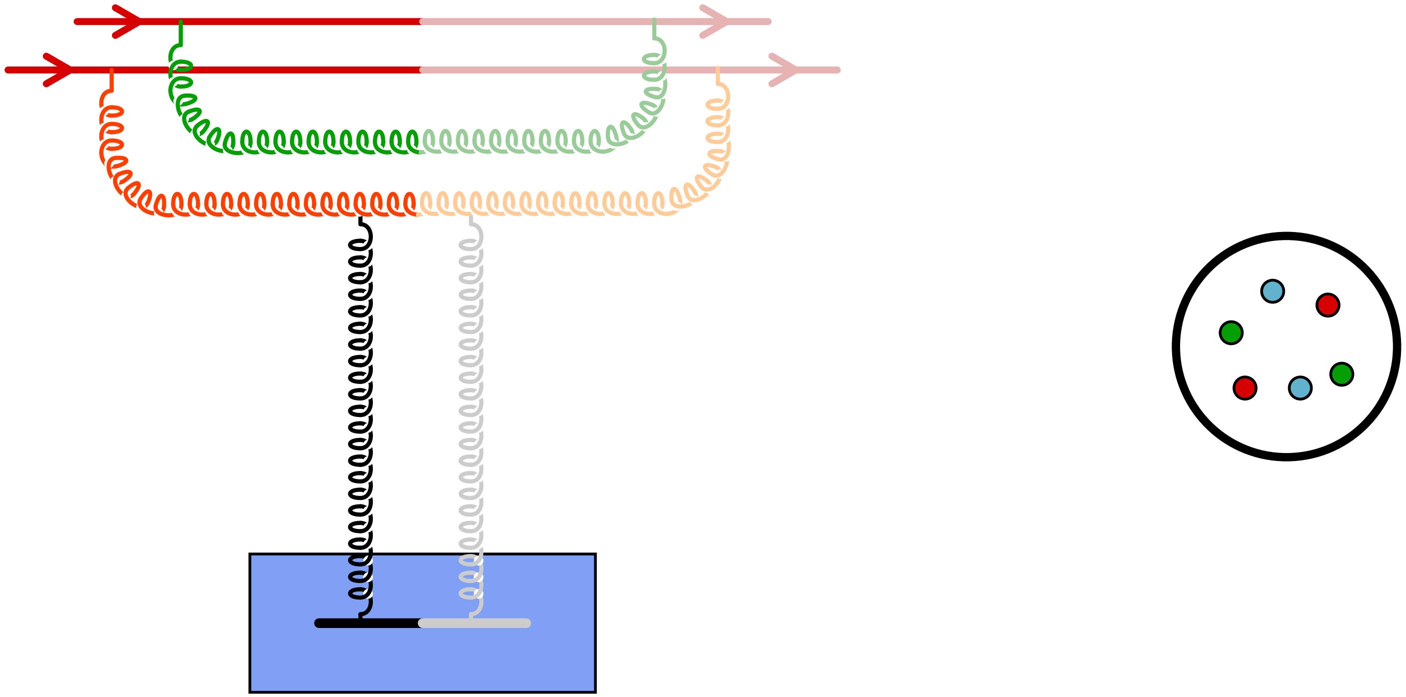


▷ assume that the projectile is big, e.g. a nucleus, and has many valence quarks (only two are represented)

▷ on the contrary, consider a small probe, with few partons

▷ at low energy, only valence quarks are present in the hadron wave function

Parton evolution

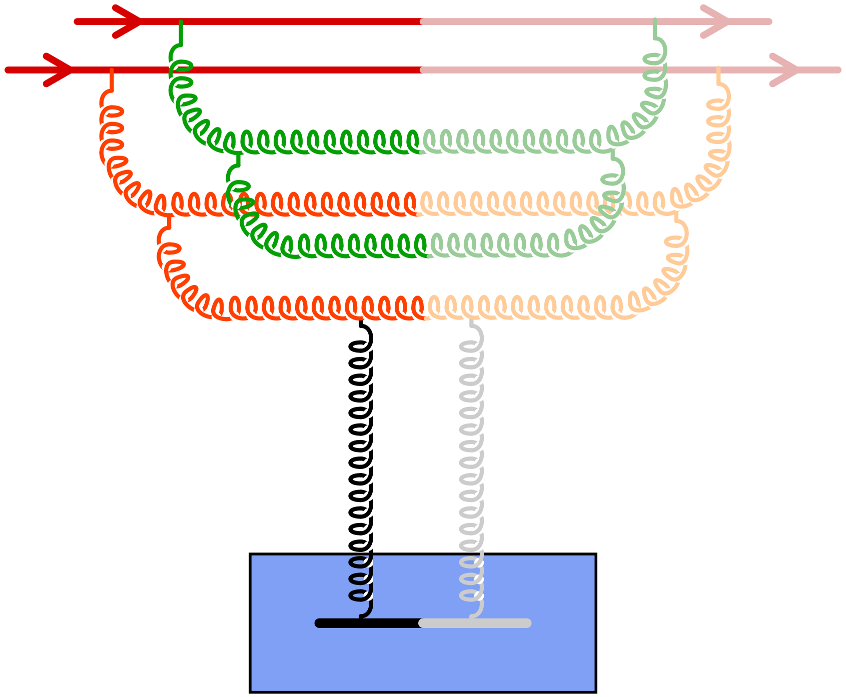


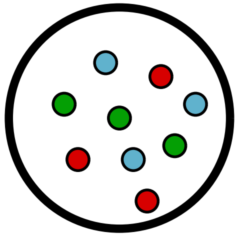
▷ when energy increases, new partons are emitted

▷ the emission probability is $\alpha_s \int \frac{dx}{x} \sim \alpha_s \ln(\frac{1}{x})$, with x the longitudinal momentum fraction of the gluon

▷ at small-x (i.e. high energy), these logs need to be resummed

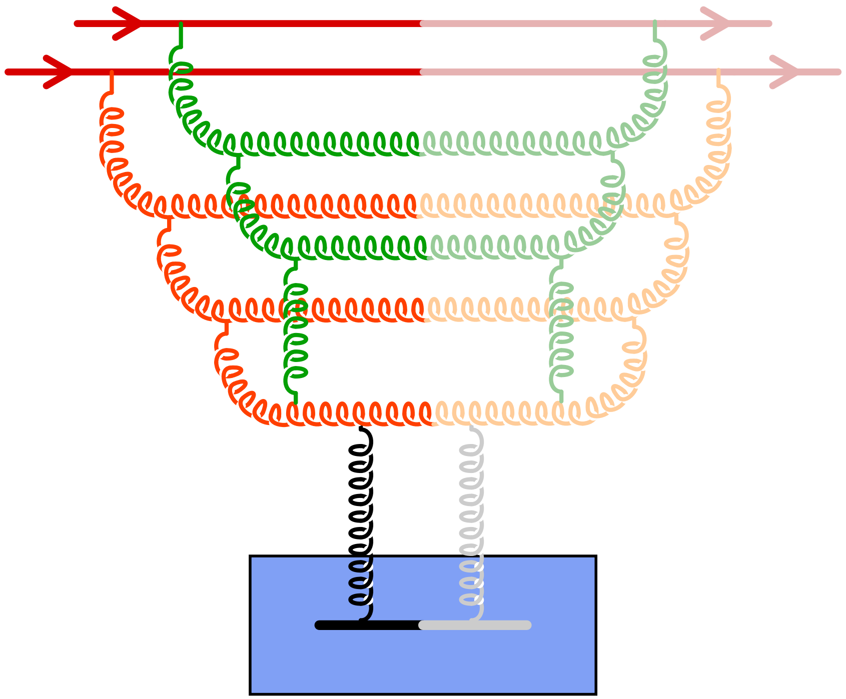
Parton evolution

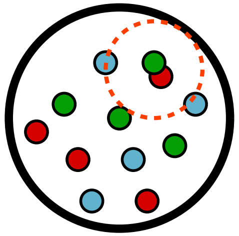




▷ as long as the density of constituents remains small, the evolution is linear: the number of partons produced at a given step is proportional to the number of partons at the previous step (BFKL)

Parton evolution





▷ eventually, the partons start overlapping in phase-space

▷ parton recombination becomes favorable

▷ after this point, the evolution is non-linear:
the number of partons created at a given step depends non-linearly on the number of partons present previously

Saturation criterion

Gribov, Levin, Ryskin (1983)

- Number of gluons per unit area:

$$\rho \sim \frac{xG_A(x,Q^2)}{\pi R_A^2}$$

- Recombination cross-section:

$$\sigma_{gg\to g} \sim \frac{\alpha_s}{Q^2}$$

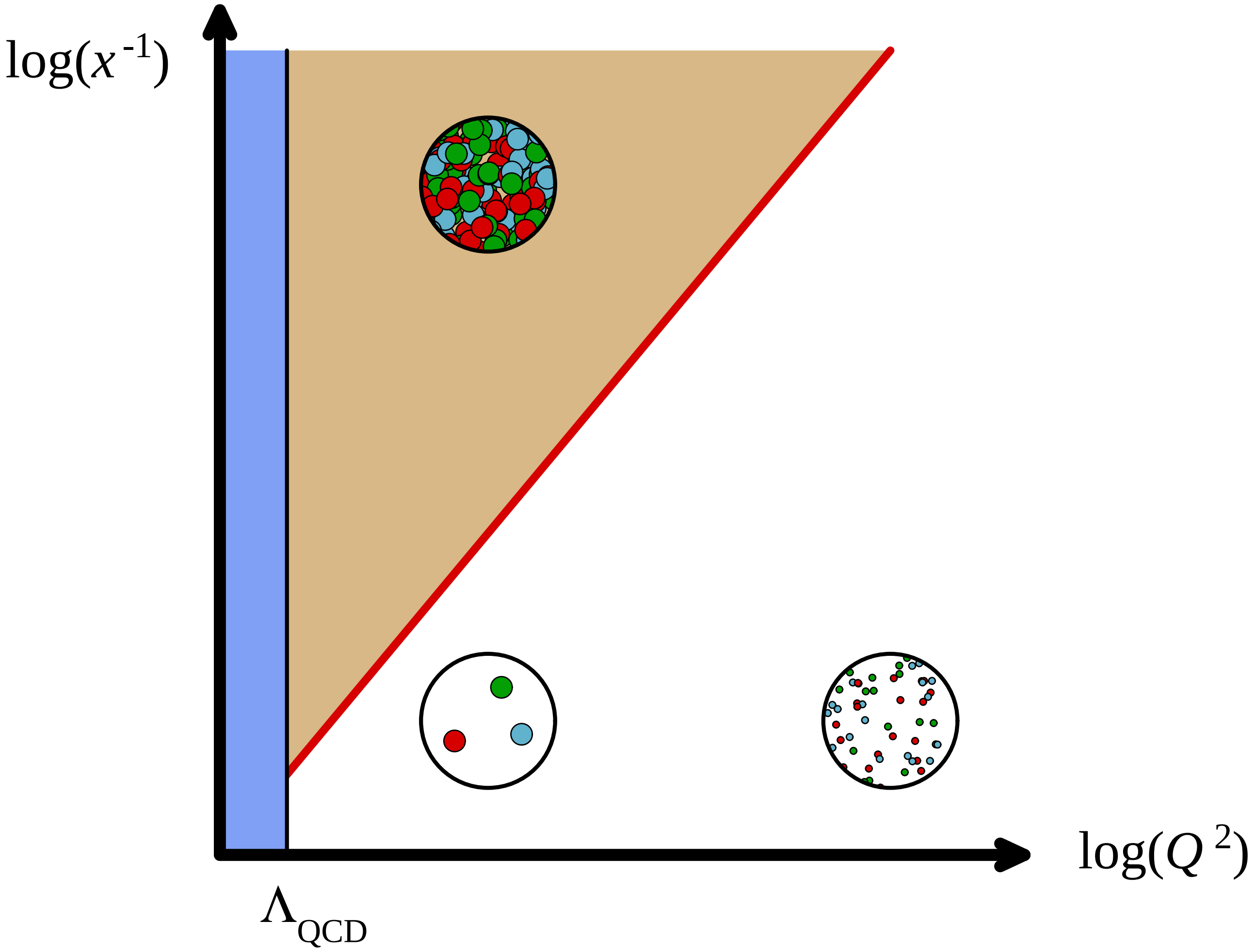
- Recombination happens if $\rho\sigma_{gg\to g} \gtrsim 1$, i.e. $Q^2 \lesssim Q_s^2$, with:

$$Q_s^2 \sim \frac{\alpha_s xG_A(x,Q_s^2)}{\pi R_A^2} \sim A^{1/3}\frac{1}{x^{0.3}}$$

- At saturation, the phase-space density is:

$$\frac{dN_g}{d^2\vec{\boldsymbol{x}}_\perp d^2\vec{\boldsymbol{p}}_\perp} \sim \frac{\rho}{Q^2} \sim \frac{1}{\alpha_s}$$

Saturation domain



Color Glass Condensate

Degrees of freedom and their interplay

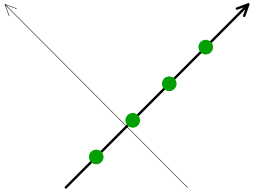
McLerran, Venugopalan (1994), Iancu, Leonidov, McLerran (2001)

- Small-x modes have a large occupation number
  ▷ they are described by a classical color field A^μ,
  that obeys Yang-Mills's equation:

$$[D_\nu, F^{\nu\mu}] = J^\mu$$

- The source term J^μ comes from the faster partons. The large-x modes, slowed down by time dilation, are described as frozen color sources ρ. Hence :

$$J^\mu = \delta^{\mu +}\delta(x^-)\rho(\vec{\boldsymbol{x}}_\perp)$$

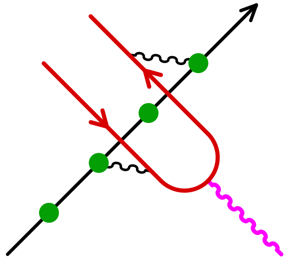


- The color sources ρ are random, and described by a distribution functional $W_Y[\rho]$, where $Y \equiv \ln(1/x)$ defines the frontier between "small-x" and "large-x"

e-A collisions

- In order to study electron-hadron collisions , solve the classical Yang-Mills equations in the presence of the following current :

$$J^\mu \equiv \delta^{\mu+}\delta(x^-)\,\rho_1(\vec{\boldsymbol{x}}_\perp)$$



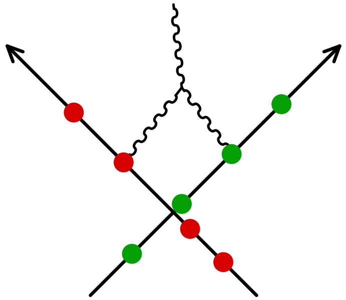
- Compute the observable $\mathcal{O}$ of interest – e.g. the transition amplitude between a γ^* and a state made of quarks and gluons – in the background field created by a configuration of the source ρ_1

- Average over the source ρ_1

$$\langle\mathcal{O}\rangle = \int [D\rho_1]\; W_{Y_1}[\rho_1]\; \mathcal{O}[\rho_1]$$

Purely hadronic collisions

- In order to study the collisions of two hadrons, solve the classical Yang-Mills equations in the presence of the following current :



$$J^\mu \equiv \delta^{\mu +}\delta(x^-)\,\rho_1(\vec{\boldsymbol{x}}_\perp) + \delta^{\mu -}\delta(x^+)\,\rho_2(\vec{\boldsymbol{x}}_\perp)$$

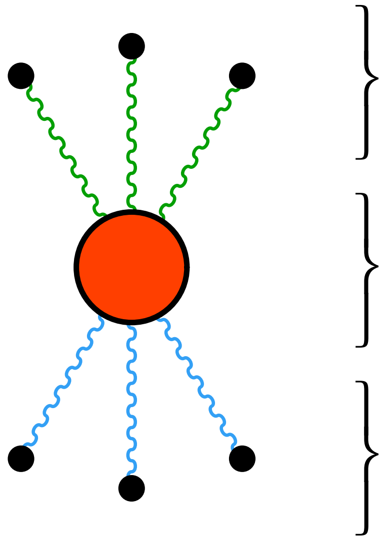
- Compute the observable $\mathcal{O}$ of interest in the background field created by a configuration of the sources ρ_1, ρ_2. Note : the sources are of order $1/g$ ▷ this is a very non-linear problem

- Average over the sources ρ_1, ρ_2

$$\langle \mathcal{O} \rangle = \int [D\rho_1]\,[D\rho_2]\;W_{Y_1}[\rho_1]\;W_{Y_2}[\rho_2]\;\mathcal{O}[\rho_1,\rho_2]$$

Factorization ?

- Anatomy of a typical calculation :



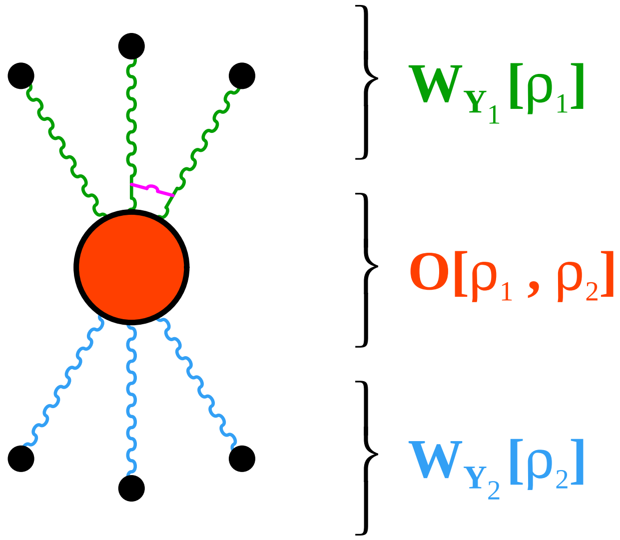
$W_{Y_1}[\rho_1]$

$O[\rho_1\,,\,\rho_2]$

$W_{Y_2}[\rho_2]$

Factorization ?

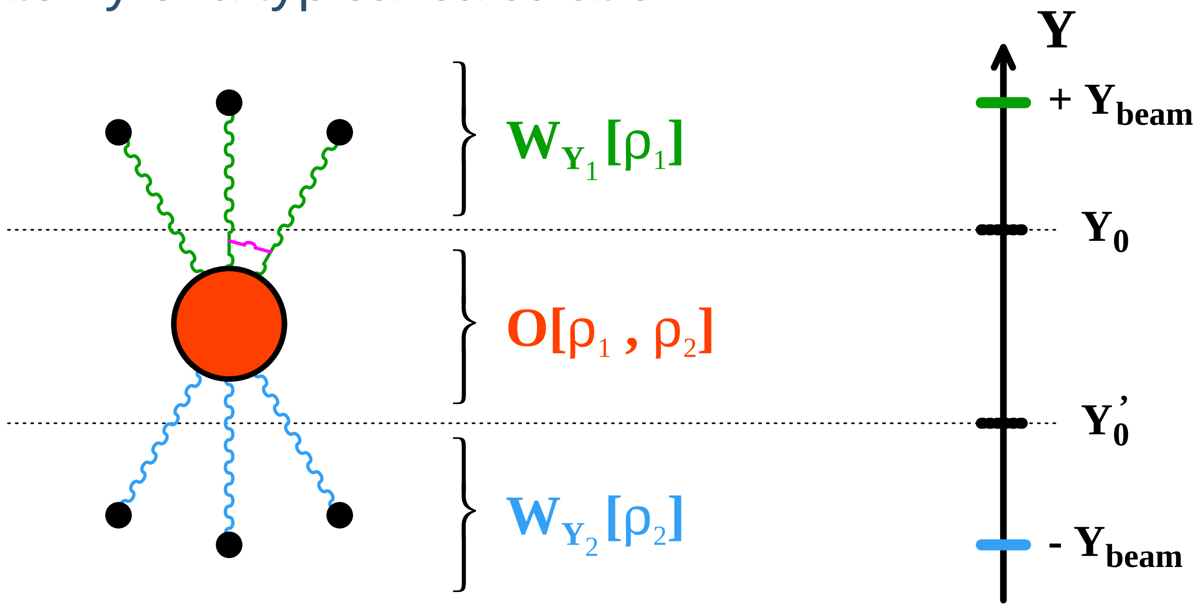
- Anatomy of a typical calculation :



- When the observable $\mathcal{O}[\rho_1, \rho_2]$ is corrected by an extra gluon, one typically gets divergences of the form $\alpha_s \int dY$
  ▷ one would like to be able to absorb these divergences into the Y dependence of the source densities $W_Y[\rho_{1,2}]$

Factorization ?

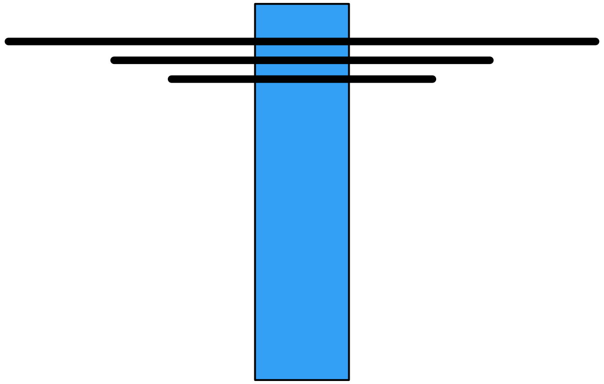
- Anatomy of a typical calculation :



- When the observable $\mathcal{O}[\rho_1,\rho_2]$ is corrected by an extra gluon, one typically gets divergences of the form $\alpha_s\int dY$
  ▷ one would like to be able to absorb these divergences into the Y dependence of the source densities $W_Y[\rho_{1,2}]$
- Equivalently, if one puts some arbitrary frontier Y_0 between the “observable” and the “source distribution”, the dependence on Y_0 should cancel between the two factors

How far must we evolve the sources?

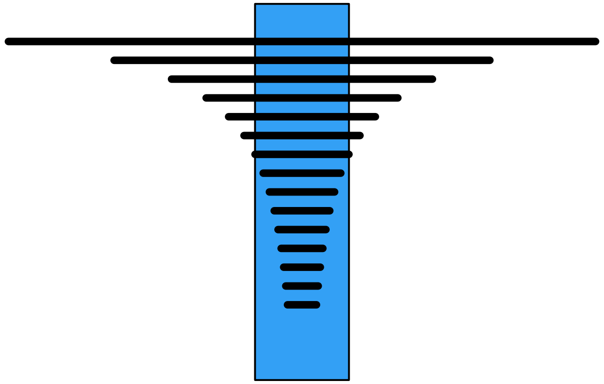
- The value of Y at which the distribution $W_Y[\rho]$ must be evaluated plays the same role as the factorization scale μ^2 in perturbative calculations based on collinear factorization
  - By choosing it appropriately, one can resum all the leading logs in $\left(\alpha_s \ln(s/\Lambda^2)\right)^n$ at no additional cost
  - In principle, the sensitivity to this parameter should decrease when higher orders are included
- All the projectiles fluctuations that are longer lived than the probe must be treated as sources



Not enough...

How far must we evolve the sources?

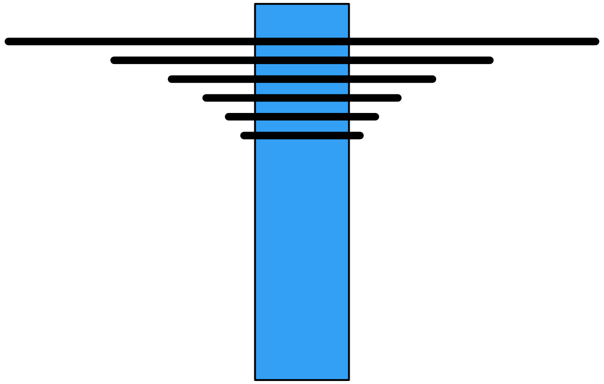
- The value of Y at which the distribution $W_Y[\rho]$ must be evaluated plays the same role as the factorization scale μ^2 in perturbative calculations based on collinear factorization
  - By choosing it appropriately, one can resum all the leading logs in $\left(\alpha_s \ln(s/\Lambda^2)\right)^n$ at no additional cost
  - In principle, the sensitivity to this parameter should decrease when higher orders are included
- All the projectiles fluctuations that are longer lived than the probe must be treated as sources



Too much...

How far must we evolve the sources?

- The value of Y at which the distribution $W_Y[\rho]$ must be evaluated plays the same role as the factorization scale μ^2 in perturbative calculations based on collinear factorization
  - By choosing it appropriately, one can resum all the leading logs in $\left(\alpha_s \ln(s/\Lambda^2)\right)^n$ at no additional cost
  - In principle, the sensitivity to this parameter should decrease when higher orders are included
- All the projectiles fluctuations that are longer lived than the probe must be treated as sources



Optimum

JIMWLK equation

- The evolution of $W_Y[\rho]$ is governed by a functional equation (Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner) :

$$\frac{\partial W_Y[\rho]}{\partial Y} = \mathcal{H}[\rho]\;\; W_Y[\rho]$$

- This leads to the following evolution for the scattering amplitude $\boldsymbol{S}(\vec{\boldsymbol{x}}_\perp,\vec{\boldsymbol{y}}_\perp)$ of a dipole on a nucleus :

$$\frac{\partial\left\langle\boldsymbol{S}(\vec{\boldsymbol{x}}_\perp,\vec{\boldsymbol{y}}_\perp)\right\rangle}{\partial Y} = -\frac{\alpha_s N_c}{2\pi^2}\int d^2\vec{\boldsymbol{z}}_\perp\;\frac{(\vec{\boldsymbol{x}}_\perp-\vec{\boldsymbol{y}}_\perp)^2}{(\vec{\boldsymbol{x}}_\perp-\vec{\boldsymbol{z}}_\perp)^2(\vec{\boldsymbol{y}}_\perp-\vec{\boldsymbol{z}}_\perp)^2}$$
$$\times\Big\{\left\langle\boldsymbol{S}(\vec{\boldsymbol{x}}_\perp,\vec{\boldsymbol{y}}_\perp)\right\rangle-\left\langle\boldsymbol{S}(\vec{\boldsymbol{x}}_\perp,\vec{\boldsymbol{z}}_\perp)\boldsymbol{S}(\vec{\boldsymbol{z}}_\perp,\vec{\boldsymbol{y}}_\perp)\right\rangle\Big\}$$

- By doing the approximation $\langle\boldsymbol{S}\boldsymbol{S}\rangle\approx\langle\boldsymbol{S}\rangle\,\langle\boldsymbol{S}\rangle$, one obtains the Balitsky-Kovchegov equation, which is a closed equation for $\langle\boldsymbol{S}\rangle$
- The BK equation is a good approximation for large N_c and large A

Balitsky-Kovchegov equation

- The Balitsky-Kovchegov has two fixed points :

$\boldsymbol{S} = 1$	unstable	(weakly interacting limit)
$\boldsymbol{S} = 0$	stable	(black disc limit)

- The attractive fixed point at $\boldsymbol{S} = 0$ means that the Balitsky-Kovchegov equation preserves the unitarity of the dipole scattering amplitude

- Note : if one writes $\langle \boldsymbol{S} \rangle \equiv 1 - \langle \boldsymbol{T} \rangle$, and linearizes the equation in $\langle \boldsymbol{T} \rangle$, one gets the BFKL equation

  There is no stable fixed point anymore, and the solutions run away exponentially as x decreases

Current status – observables

- In a few situations, the calculation of the observable $\mathcal{O}$ in the background created by the color sources is “elementary” :
  - When one of the projectiles has a perturbative wave-function, e.g. the virtual photon in DIS
  - When one of the hadronic projectiles has a low density of color sources, e.g. in proton-nucleus collisions, that can be treated as a perturbative expansion parameter
- In the generic case of two projectiles with large densities of color sources – e.g. in nucleus-nucleus collisions –, the problem must be handled numerically
  - Single inclusive gluon spectrum at LO
    Krasnitz, Nara, Venugopalan (1999 – 2001), Lappi (2003)
  - Single inclusive quark spectrum at LO
    FG, Kajantie, Lappi (2005)
  - General formalism for studying particle production
    FG, Venugopalan (2006)

What is the present evidence ?

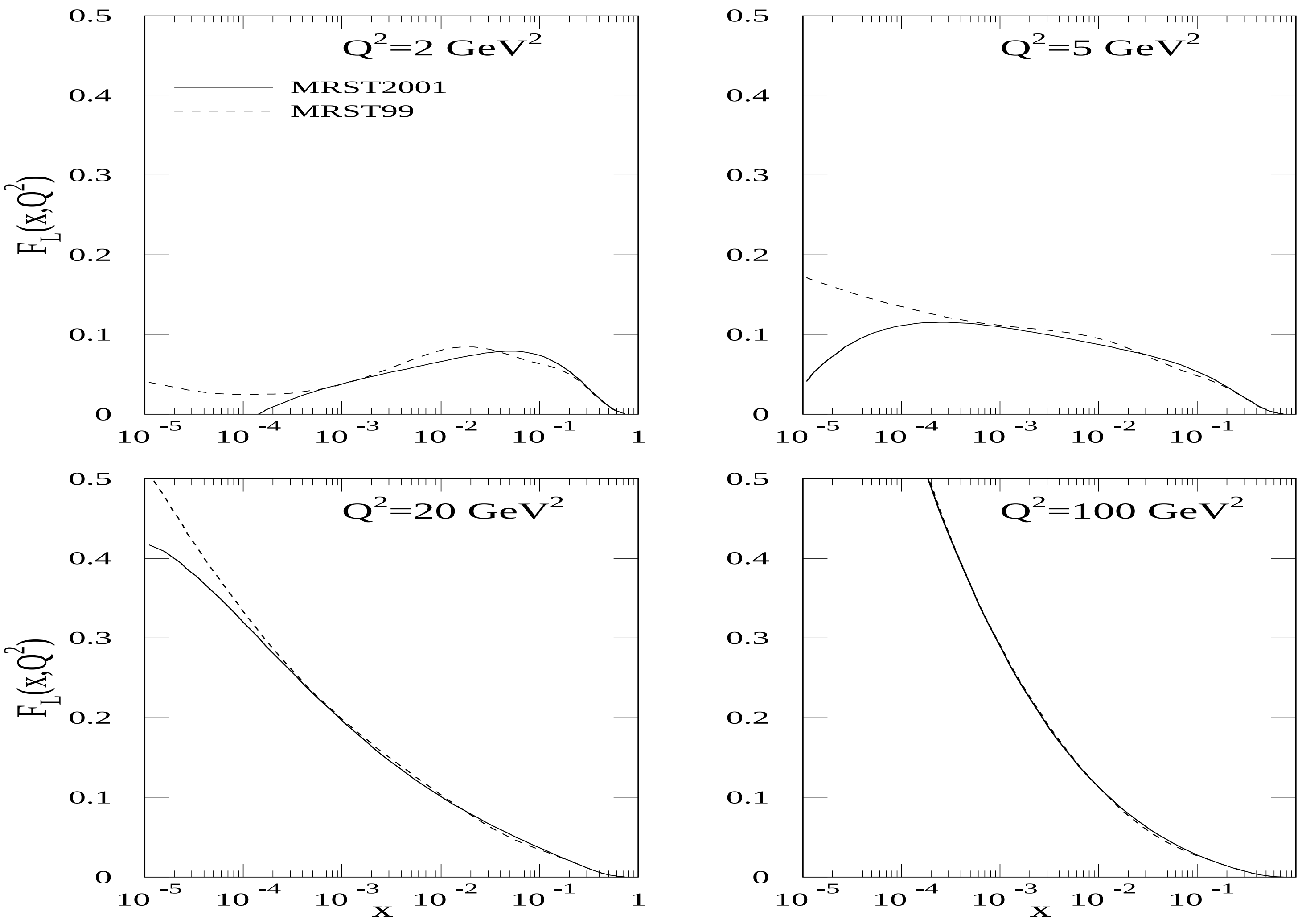
Problems of the conventional approach

- The conventional DGLAP (leading twist) analysis of DIS at NLO encounters some difficulties at low Q^2 :
  - The fit tends to favor a gluon distribution which is negative in some regions of x
  - Note : this by itself may or may not be a problem, because $g(x,Q^2)$ at NLO is scheme-dependent and not necessarily positive definite

- But, when one takes the gluon distribution obtained from the fit in order to calculate F_L, one obtains a negative structure function in some regions, which cannot be true

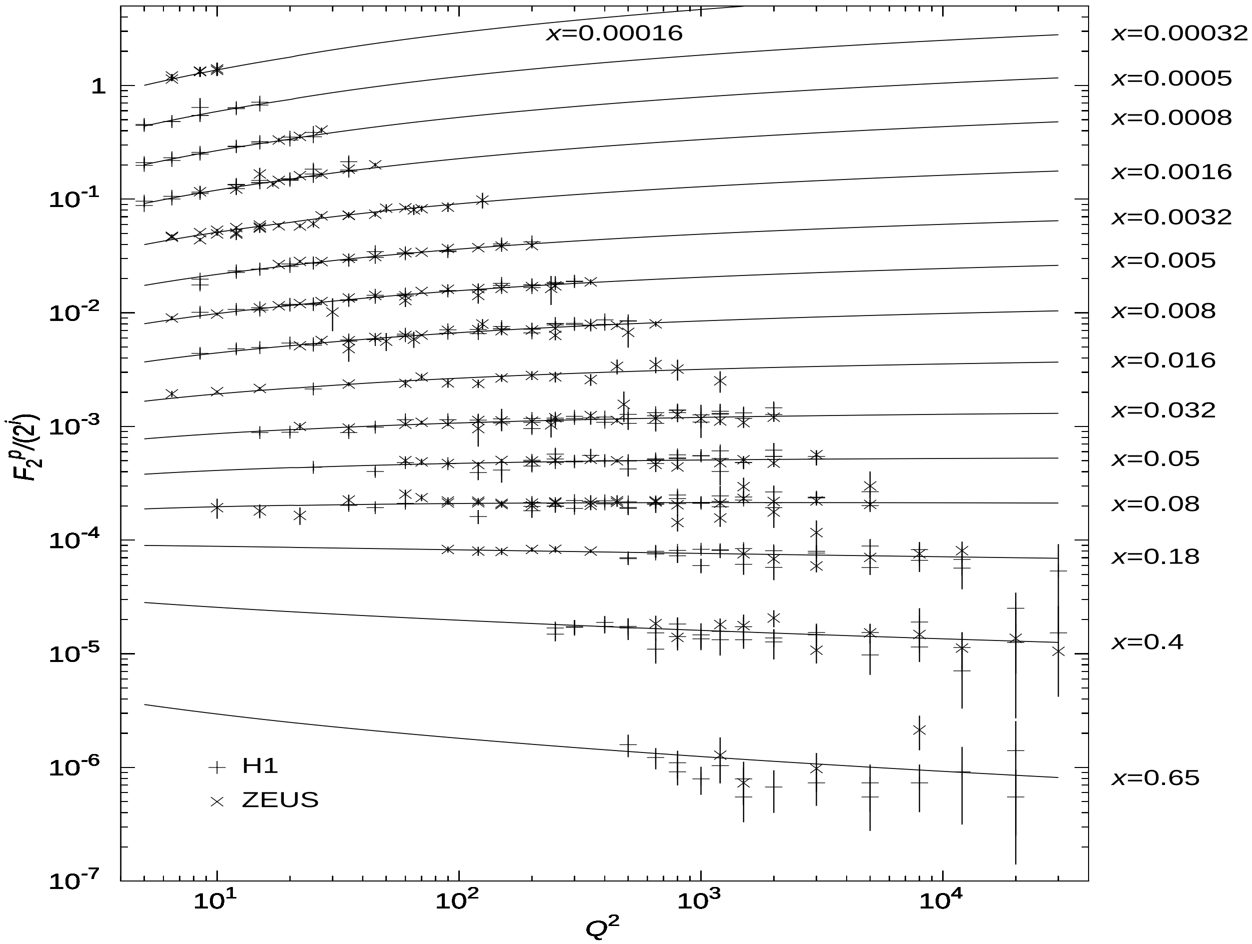
Problems of the conventional approach

- F_L at HERA – MRST (2001):



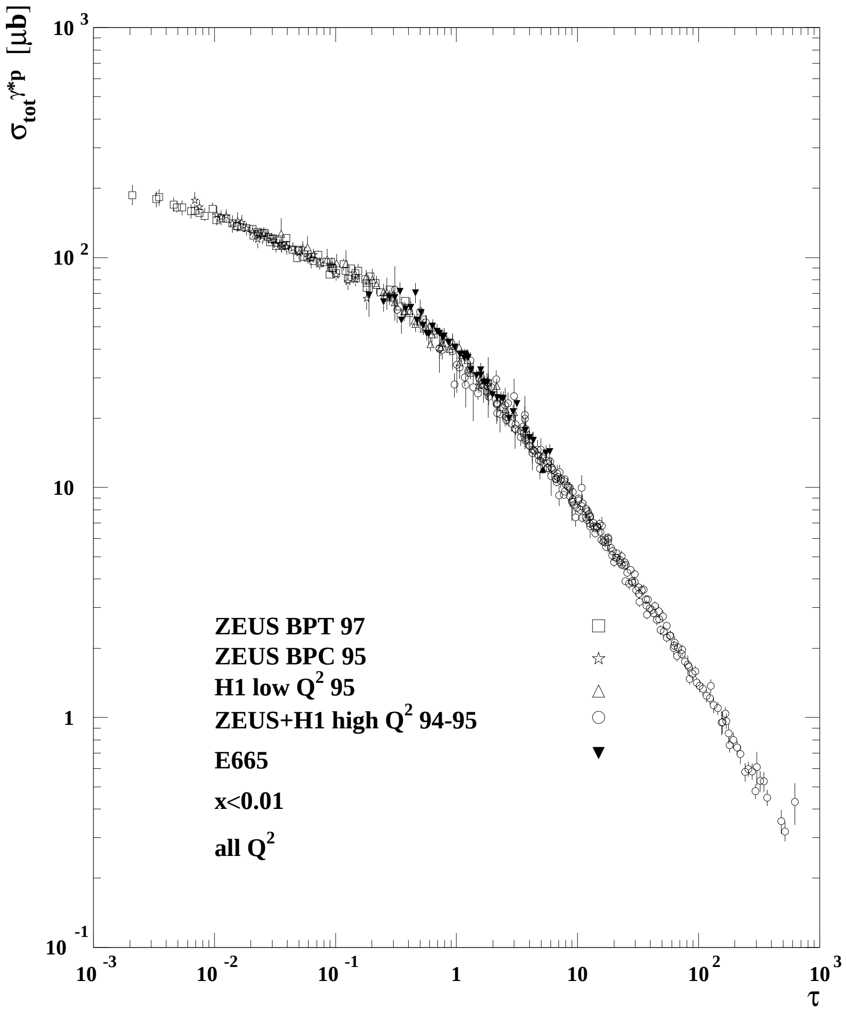
Geometrical scaling in DIS

- F_2 as a function of Q^2 and x, displayed in the conventional way :



Geometrical scaling in DIS

- Low x ($x < 10^{-2}$) part displayed as a function of $\tau = x^{\lambda} Q^2$
  Stasto, Golec-Biernat, Kwiecinski (2000)



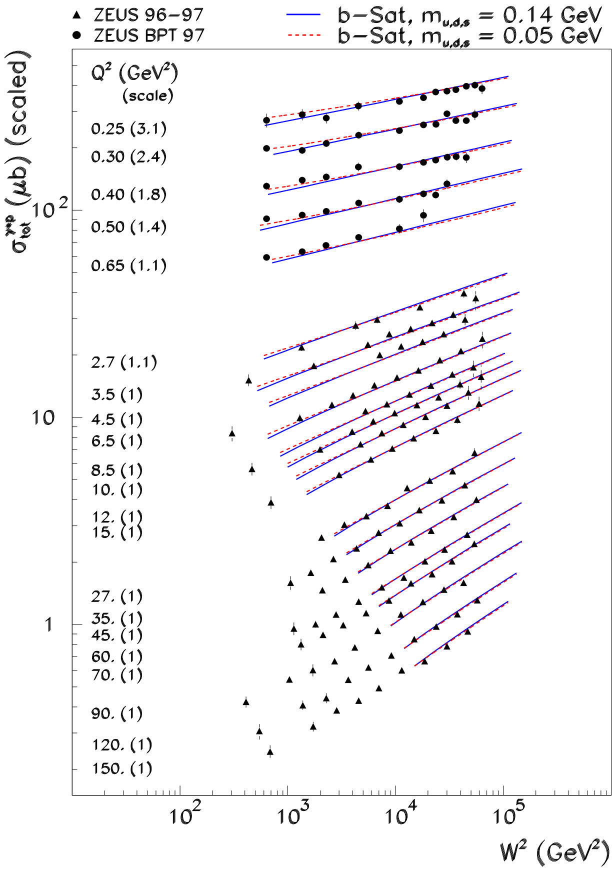
Geometrical scaling in DIS

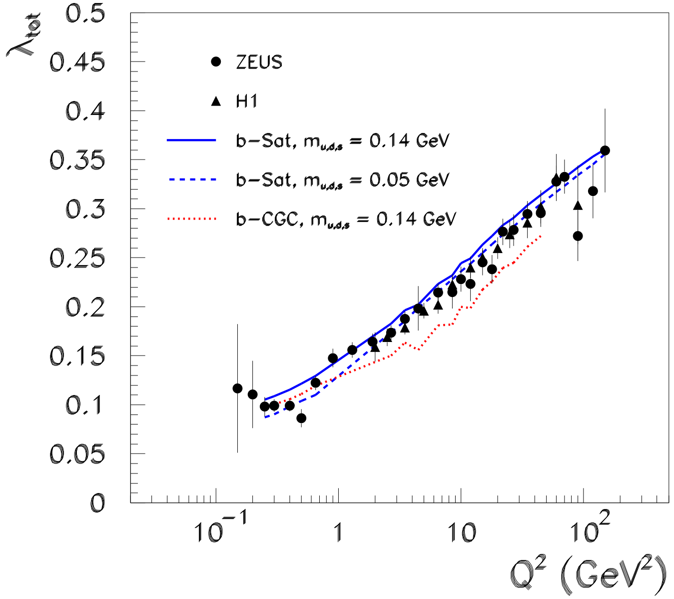
- Regardless of the underlying details, such a scaling suggests the existence of an x-dependent momentum scale that controls the dynamics which is relevant for F_2 at small x
- In particular, the scale Λ_{QCD} seems to have become largely irrelevant
- Moreover, this momentum scale grows when x gets smaller ($\lambda \approx 0.3$ is positive)

Exclusive reactions

- Simultaneous fit of inclusive F_2 and exclusive reactions at HERA within an impact parameter dependent dipole model :

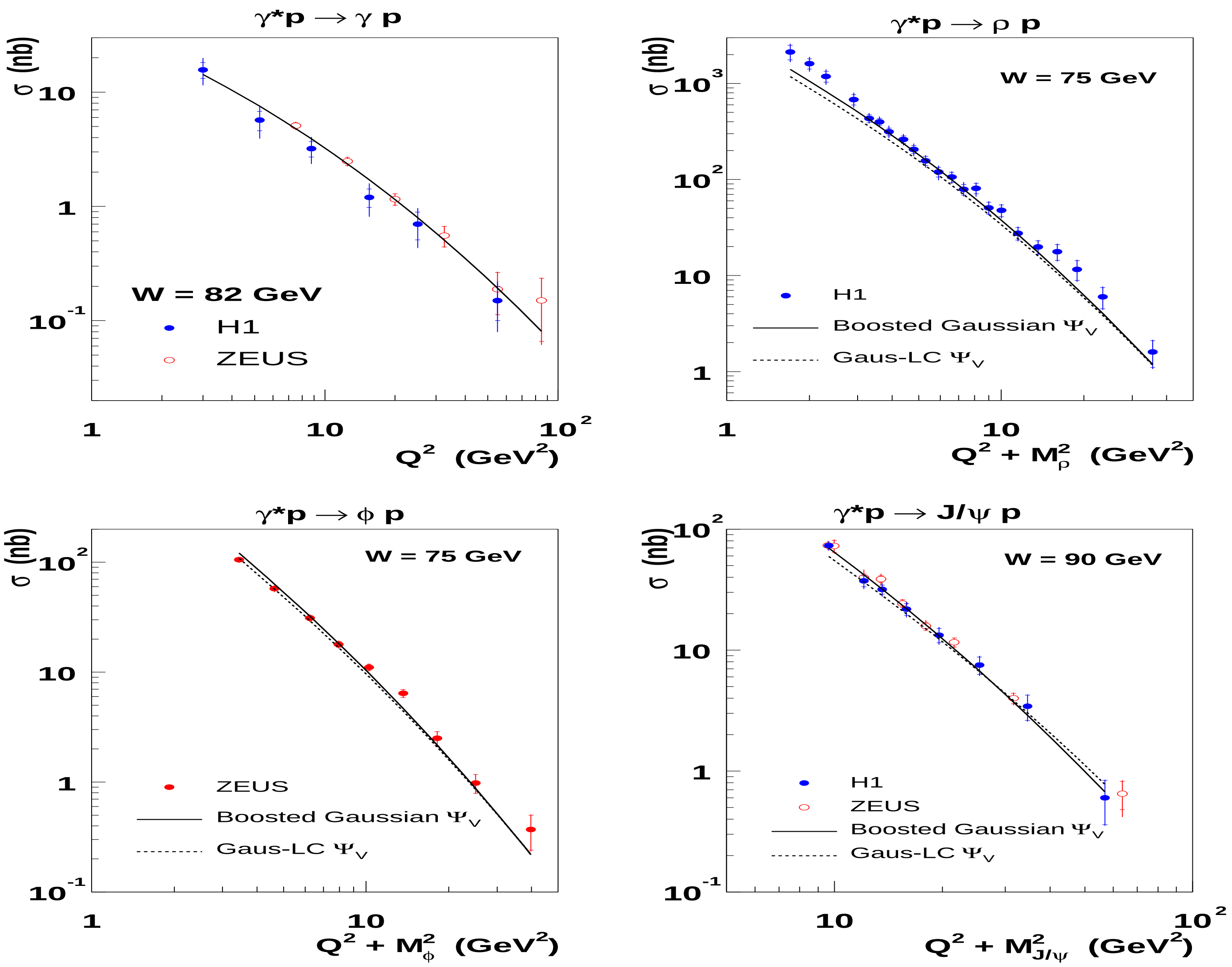
  Kowalski, Motyka, Watt (2006)





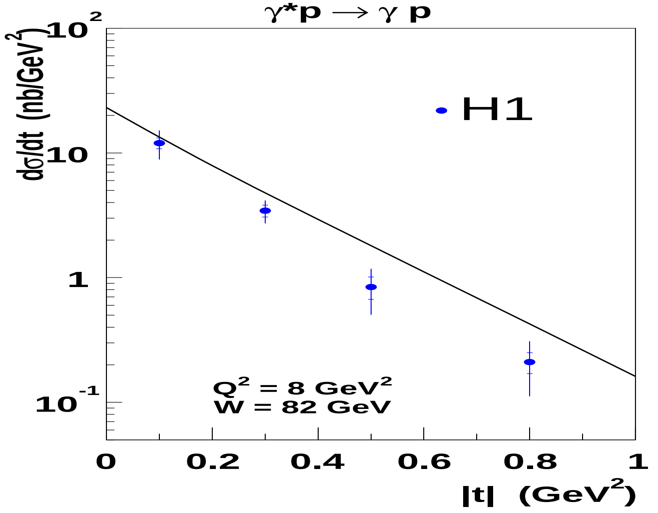
Exclusive reactions

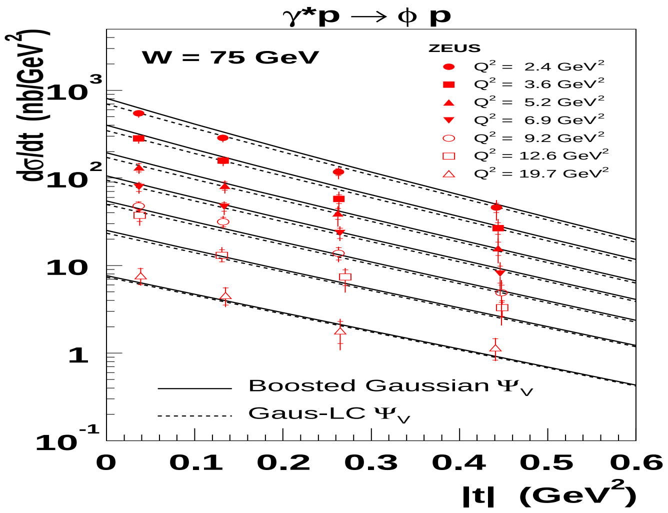
- Exclusive photon and vector meson production :

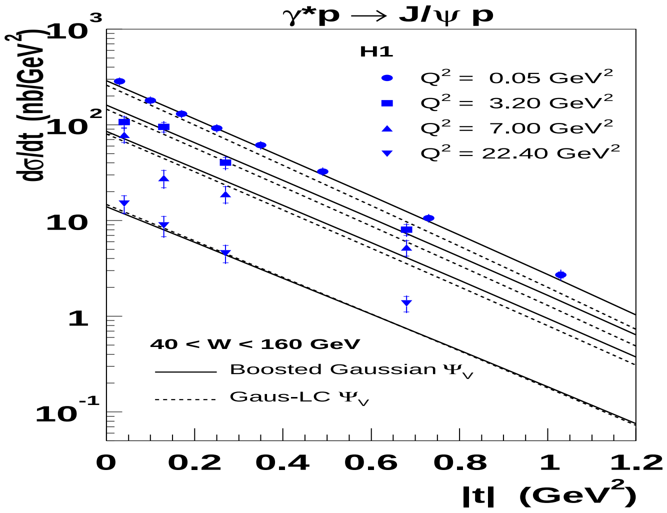


Exclusive reactions

- Exclusive photon and vector meson production :



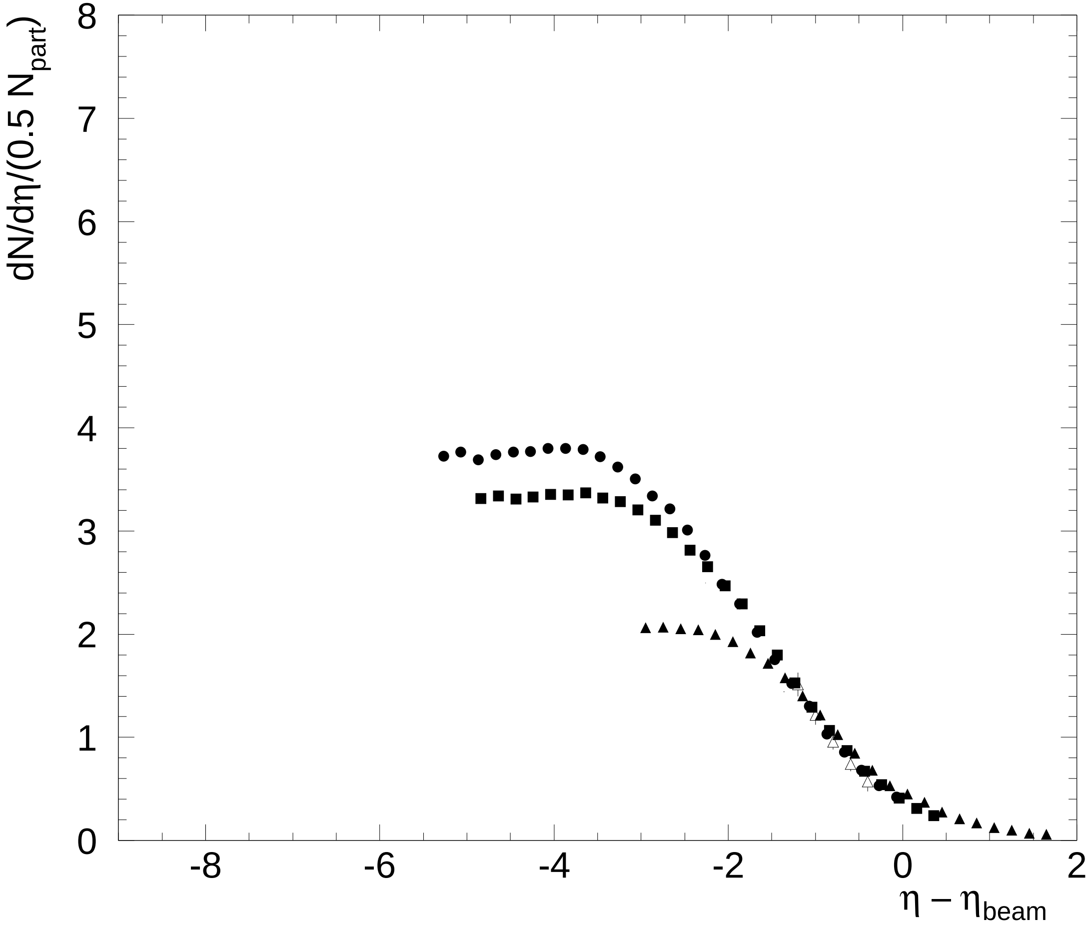




Limiting fragmentation (RHIC)

- Inclusive hadron spectrum at RHIC, shifted by the beam rapidity ($\sqrt{s} = 19.6, \;\; 64, \;\; 130, \;\; 200$ GeV) :

  data from PHOBOS, STAR and BRAHMS



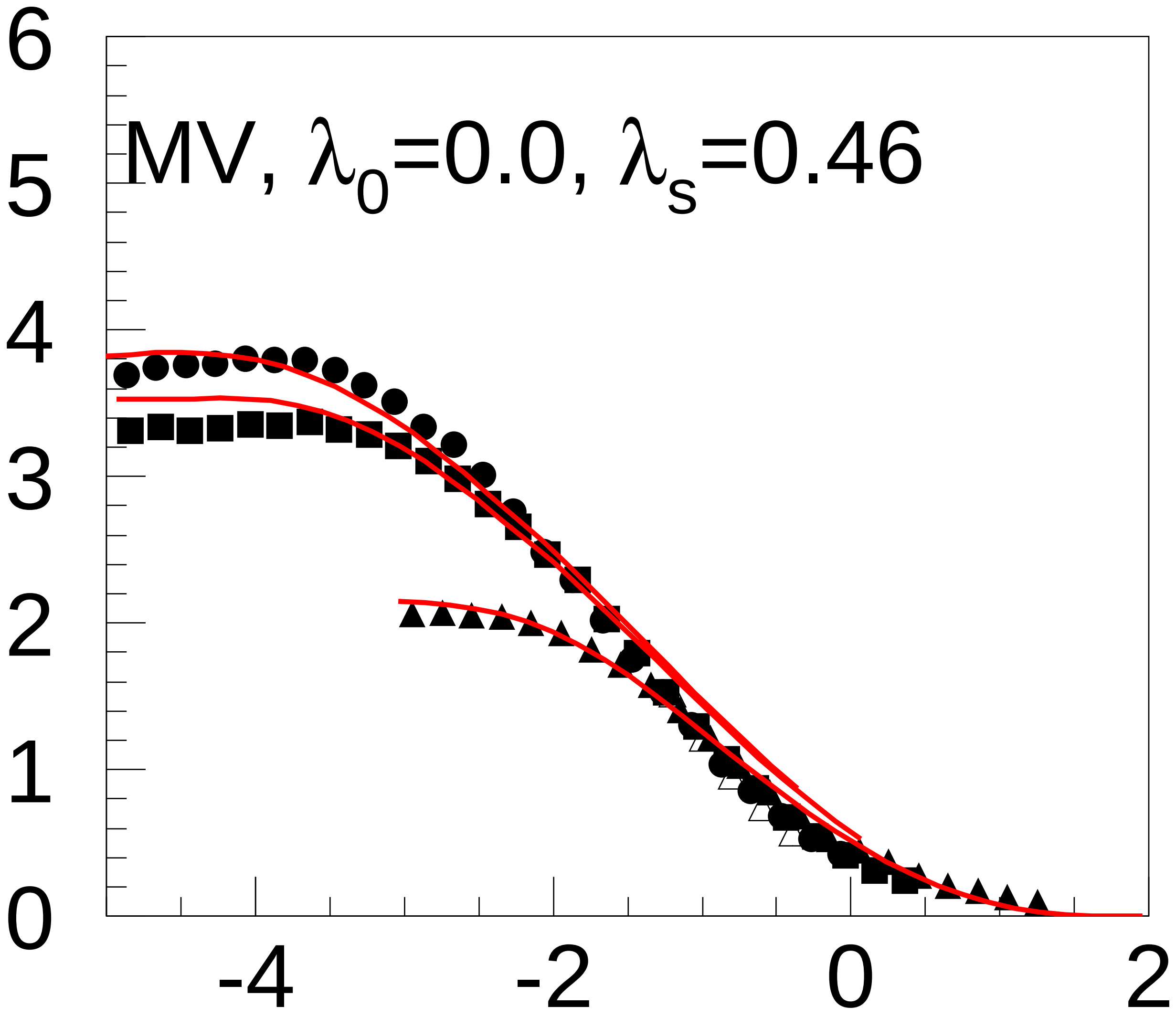
Limiting fragmentation (RHIC)

Jalilian-Marian (2002)
FG, Stasto, Venugopalan (2006)

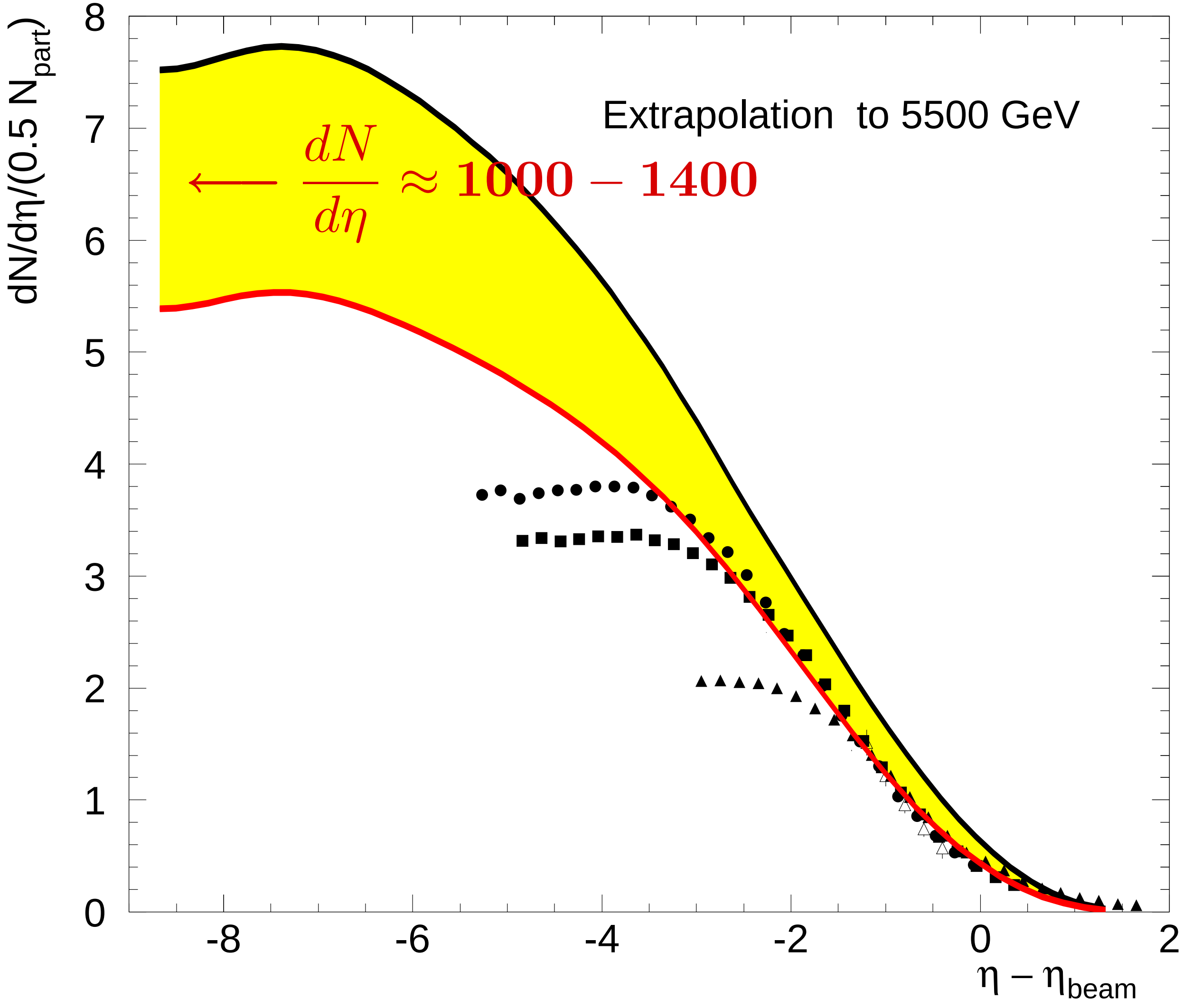
- Limiting fragmentation has a very natural – and robust – interpretation in the framework of gluon saturation. It only requires the following ingredients :
  - Approximate Bjorken scaling in the nucleus which is probed at large x
  - Unitarization of the dipole amplitudes in the other nucleus (probed at small x)
- Note : limiting fragmentation by itself does not tell anything about the dynamics in the saturated regime

  However, deviations from limiting fragmentation tell us something about the mechanisms by which one approaches the black disc limit

Fit of RHIC data

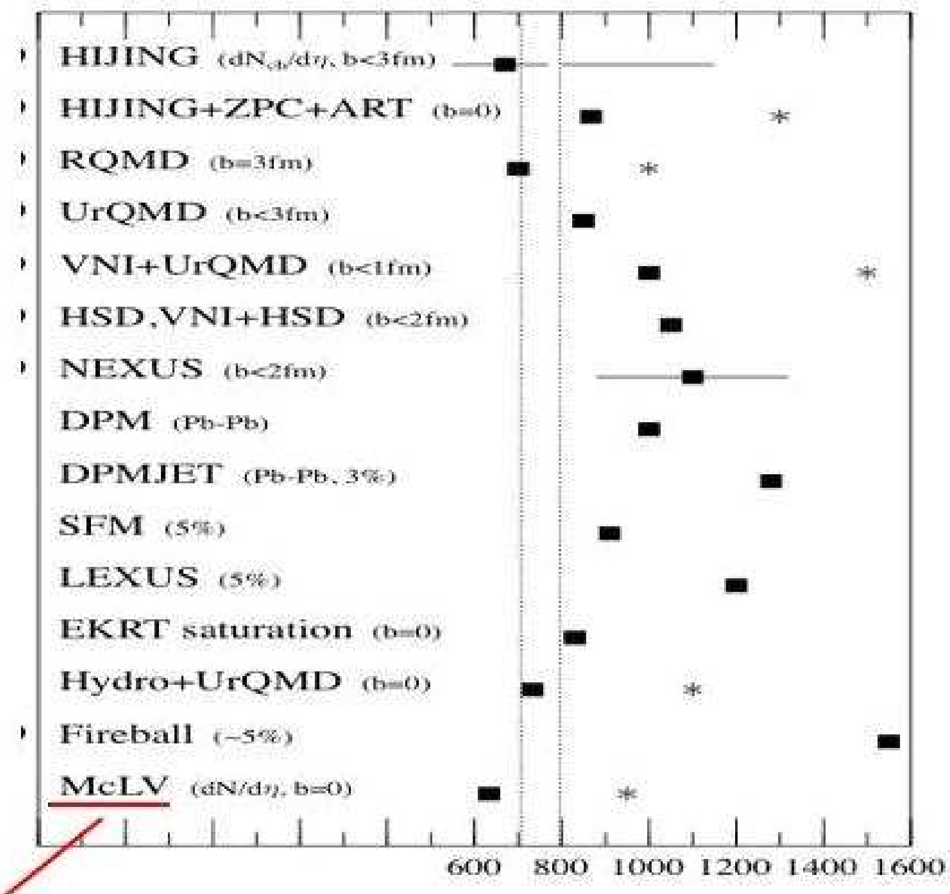


Extrapolation to LHC energy



Multiplicity at RHIC

- Predictions from different approaches vs. data :



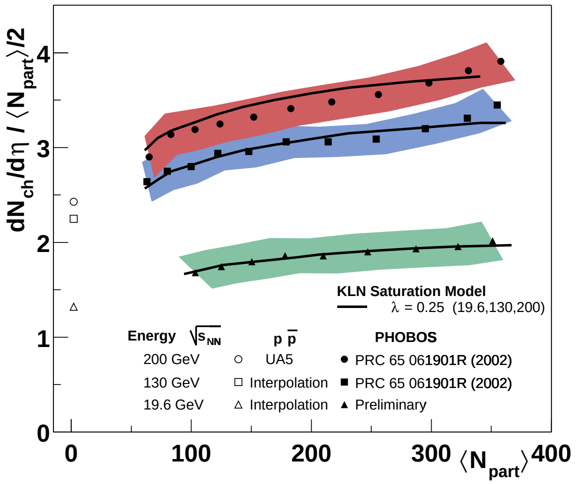
Eskola, QM 2001

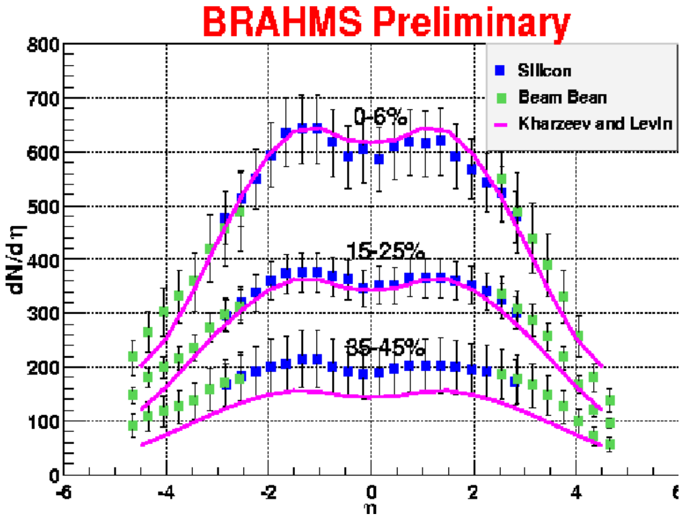
Krasnitz, RV

Multiplicity at RHIC

- N_{part} scaling and rapidity dependence :

  Kharzeev, Levin, Nardi (2001)

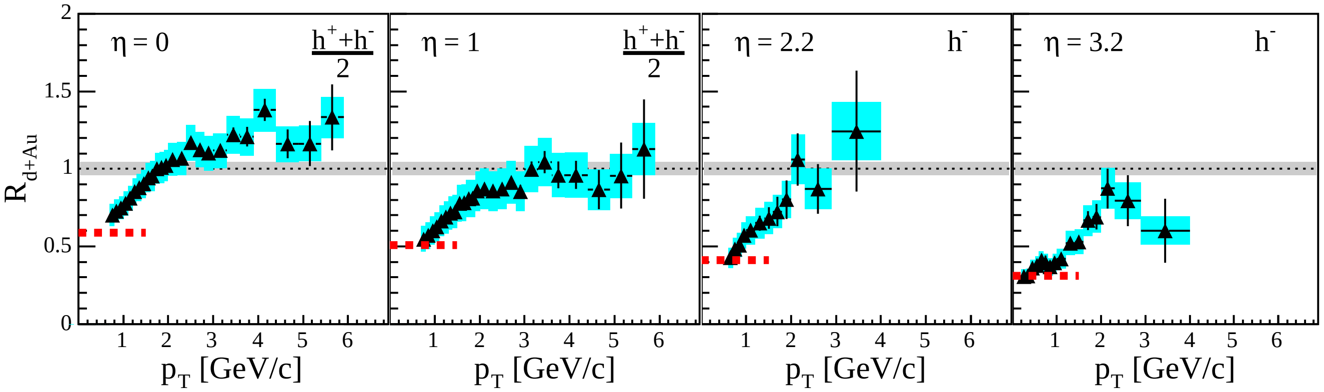




High pt suppression at large Y

- Results of the BRAHMS experiment at RHIC for deuteron-gold collisions :

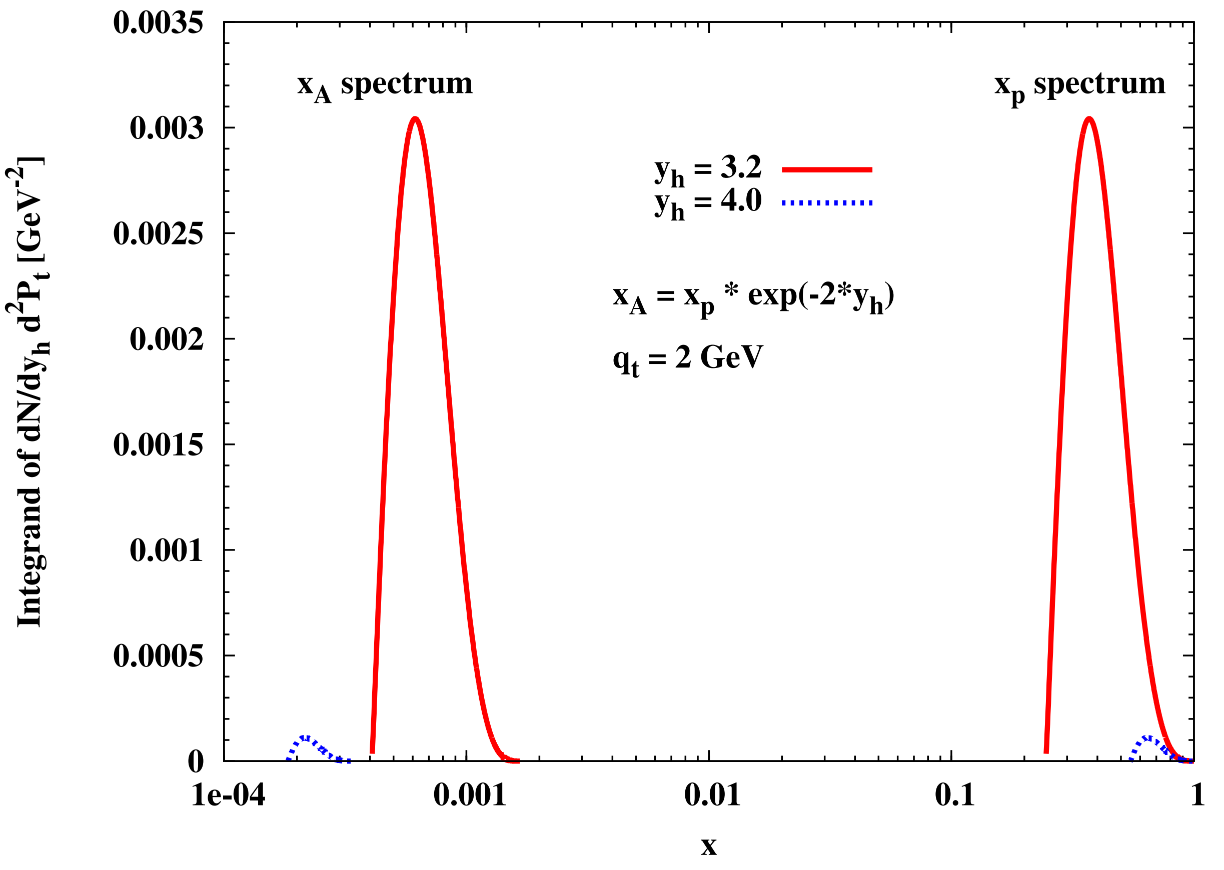
$$R_{\mathrm{dAu}} \equiv \frac{1}{N_{\mathrm{coll}}} \frac{\left.\frac{dN}{dp_\perp d\eta}\right|_{\mathrm{dAu}}}{\left.\frac{dN}{dp_\perp d\eta}\right|_{\mathrm{pp}}}$$



- At small rapidity, suppression at low $p_\perp$ and enhancement at high $p_\perp$ (multiple scatterings – Cronin effect)
- At large rapidity, suppression at all $p_\perp$'s (shadowing)

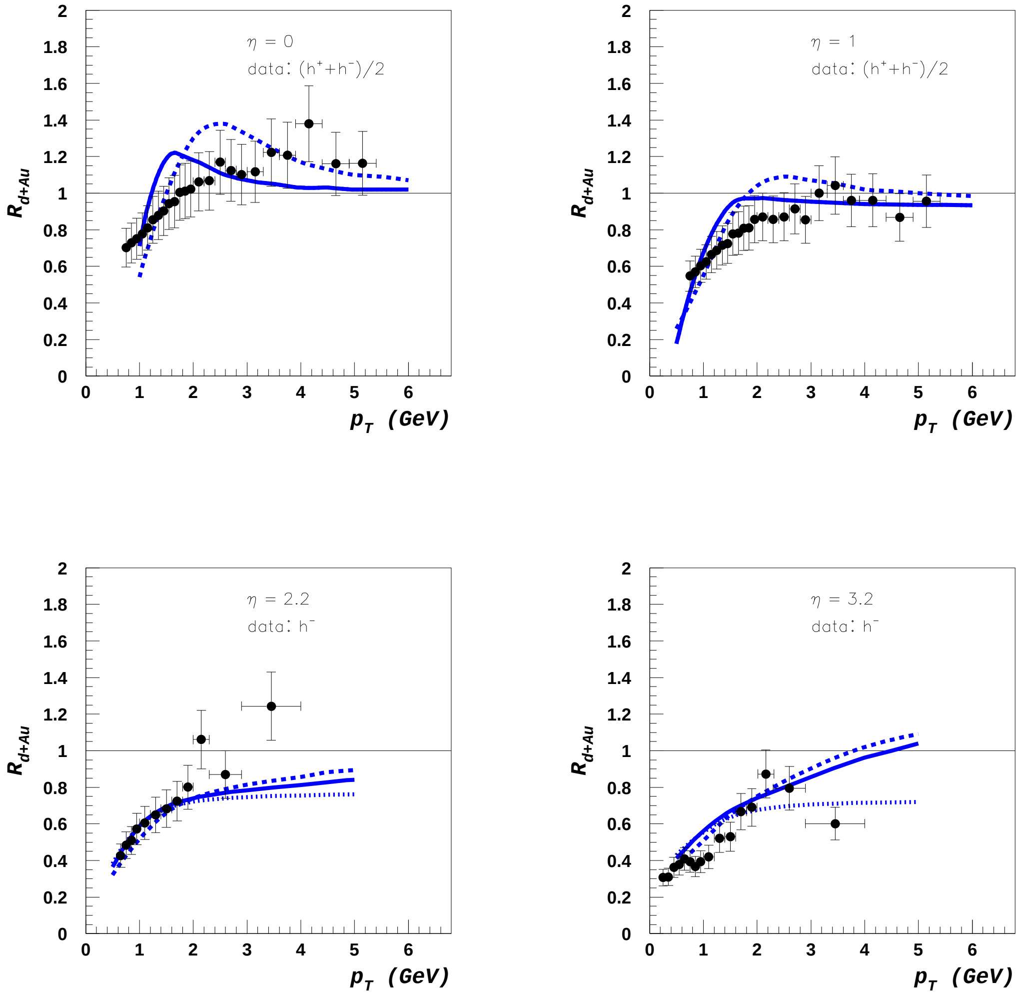
Kinematics

- Relevant values of $x_{1,2}$:



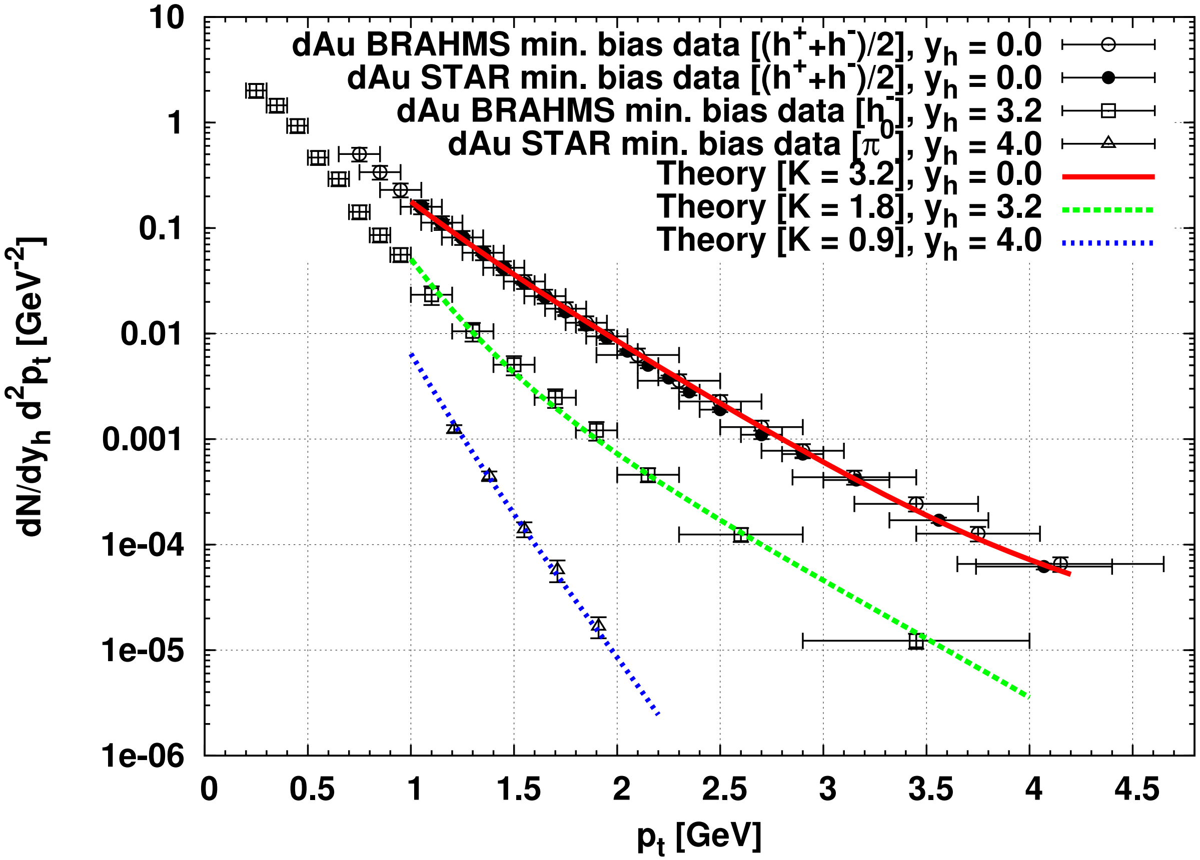
dA collisions at RHIC

- Kharzeev, Kovchegov, Tuchin (2005)

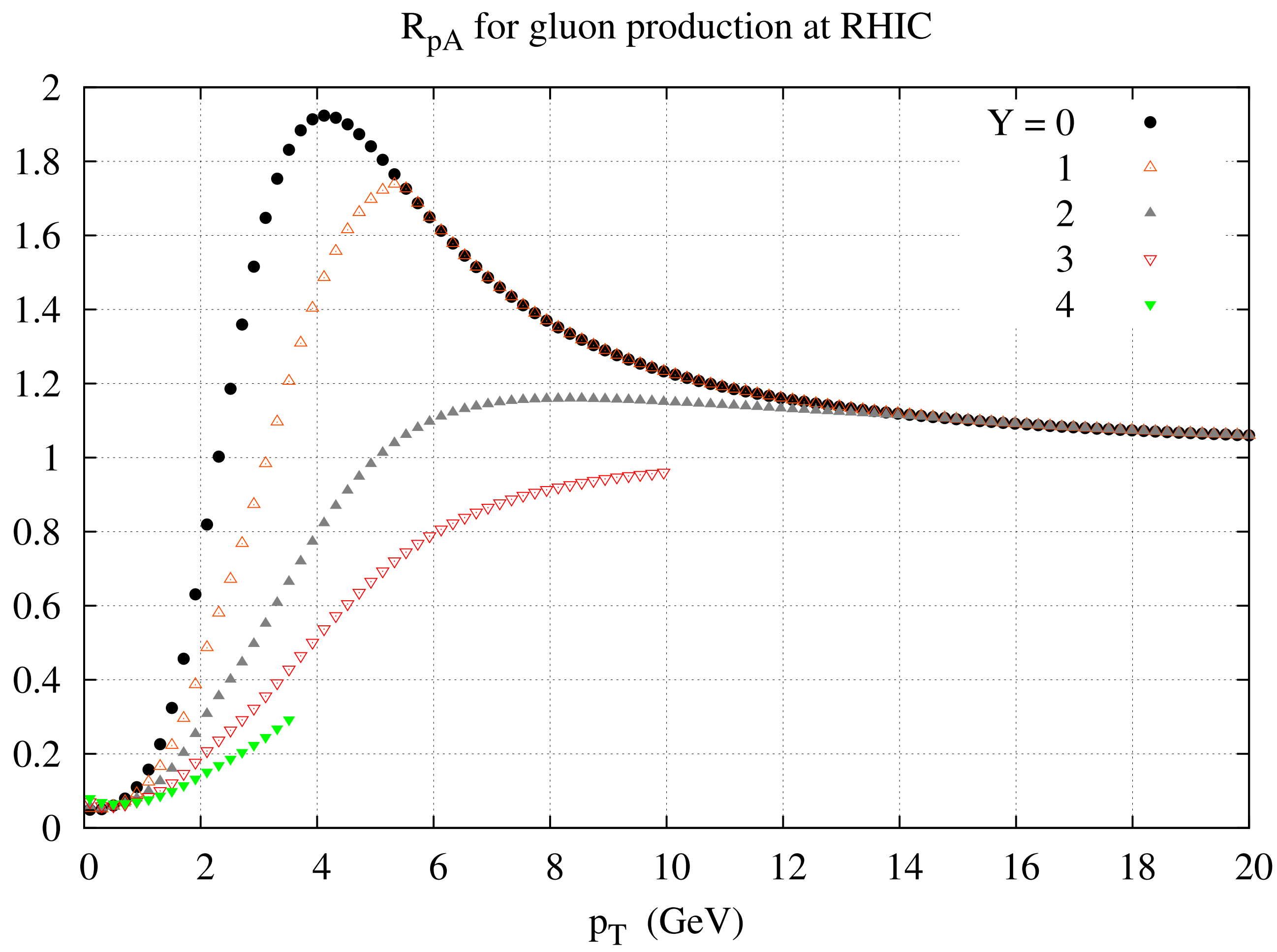


dA collisions at RHIC

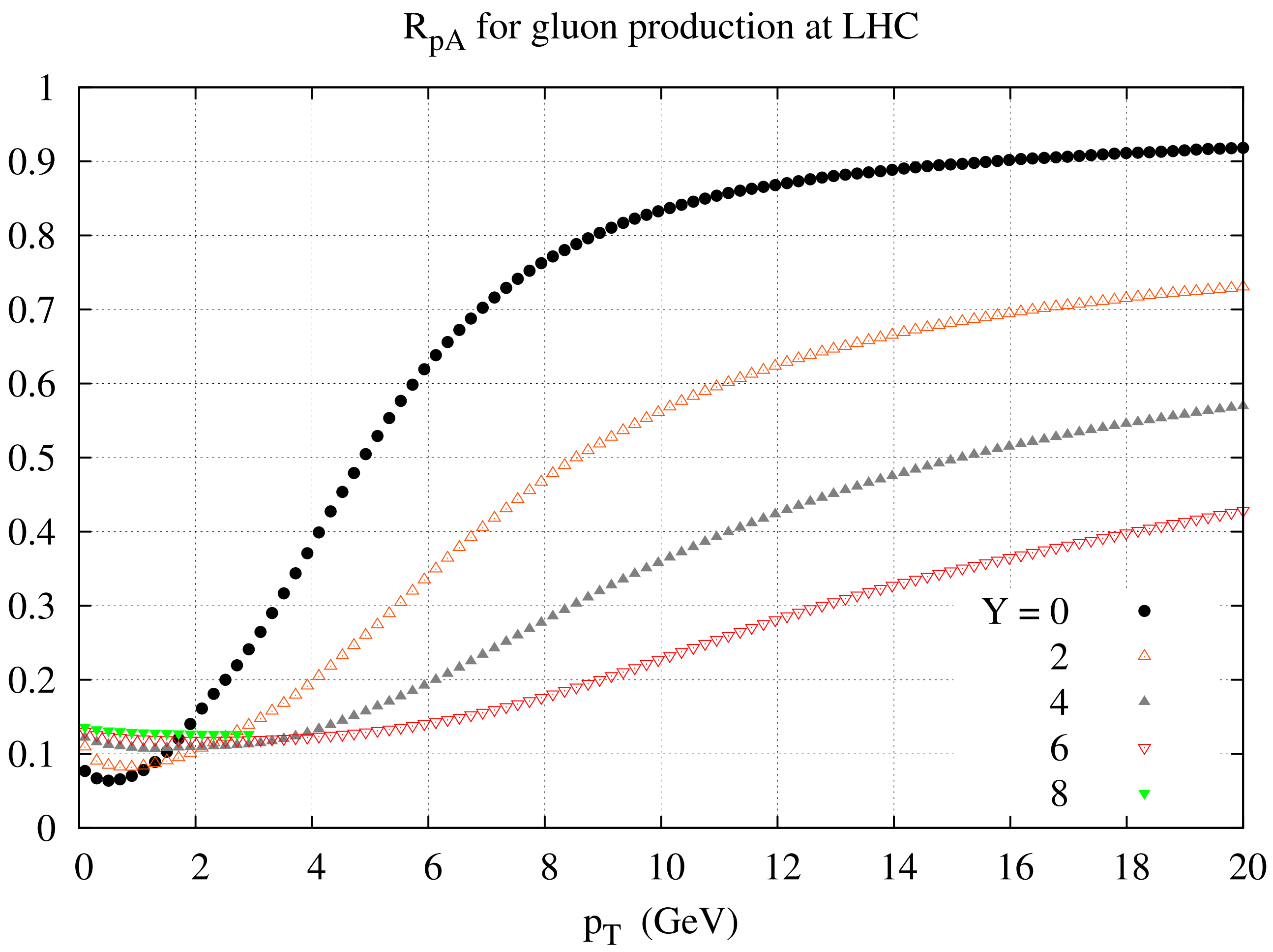
- Dumitru, Hayashigaki, Jalilian-Marian (2005 – 2006)



RdA at RHIC from the BK equation



RpA at LHC from the BK equation



pp, pA, AA colliders

vs eA colliders

AA collisions at the LHC

- Collision energy :
  - $\sqrt{s} = 14$ TeV for pp
  - $\sqrt{s} = 8.8$ TeV for pA
  - $\sqrt{s} = 5.5$ TeV for AA
- Very small values of x are achievable
- In AA collisions, there are lots of final state interactions, that may hide/blur the physics of the initial state

  ▷ in fact, if a thermalized plasma is formed, then by definition the only memory of the initial state lies in the temperature...

  ▷ only very inclusive observables, like the multiplicity, are robust enough to be probes of the initial state in such a situation

pA collisions at the LHC

- The general idea is to use the proton as a probe of the partonic content of the nucleus
- At forward rapidity (with respect to the proton) :
  - The proton, being probed at fairly large x, may be described by conventional DGLAP / Leading Twist approach
  - The nucleus is probed at very small x
- General remark : in hadron-hadron collisions, there is in principle a convolution in the momentum fractions x_1 and x_2. One has :

$$x_{1,2} = \sqrt{\frac{M^2 + K_\perp^2}{s}} \; e^{\pm Y} \quad ,$$

where $M, K_\perp, Y$ refer to the whole final state in this formula

▷ the more is measured about the final state, and the less convolution in x_1 and x_2 there is

eRHIC

- Main advantages :
  - The object used to probe the proton/nucleus is elementary and very accurately described by QED
  - By measuring the deflected electron, one knows the values of x and Q^2

    ▷ no convolution in x
  - High luminosity : HERA × 100 for unpolarized ep
  - Wider coverage in η than HERA
- What can be studied ?
  - Measure F_2 for light and heavy nuclei at small fixed x
  - Measure $F_{_L}$ directly (more sensitive than F_2 to higher twist corrections)
  - Semi-inclusive or exclusive quantities (e.g. DVCS, vector meson production)
  - Study of diffraction and rapidity gaps (10% – 20% of the events at HERA)

Conclusions

Conclusions

- Hadronic interactions at high energy – small x – are characterized by gluon staturation and the emergence of a new dimensionful scale Q_s

  The saturation scale grows at small x, indicating that this problem can be treated by weak coupling techniques

  The high energy scattering problem has some non-perturbative aspects, because of the large density of partons

- The LHC and eRHIC are complementary in order to study this regime of QCD :
  - High energies but somewhat “dirty” environment at the LHC
  - Cleaner environment and high luminosity at eRHIC