# Proton Structure

Object is to describe constituents of proton which are important for binding.

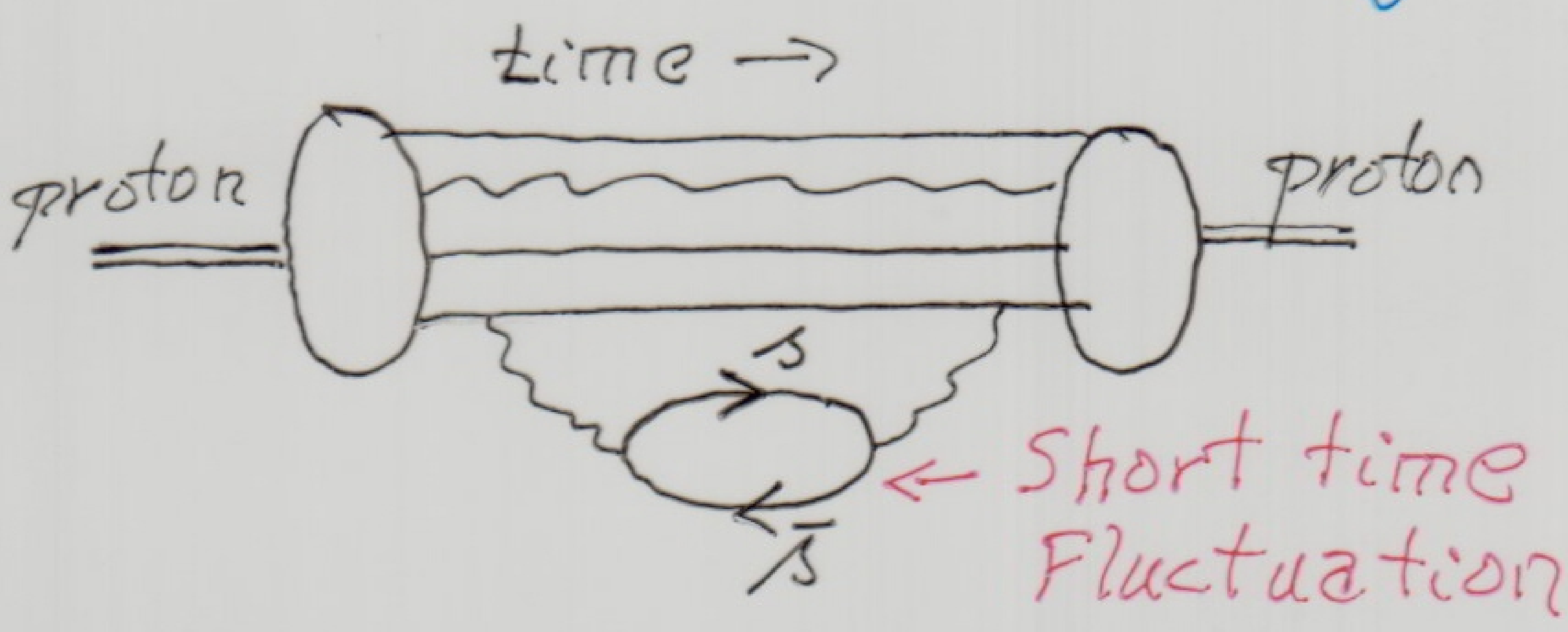
Large $Q^2$ and small $x$ not focus here.

Can hope to make strong contact with Lattice QCD

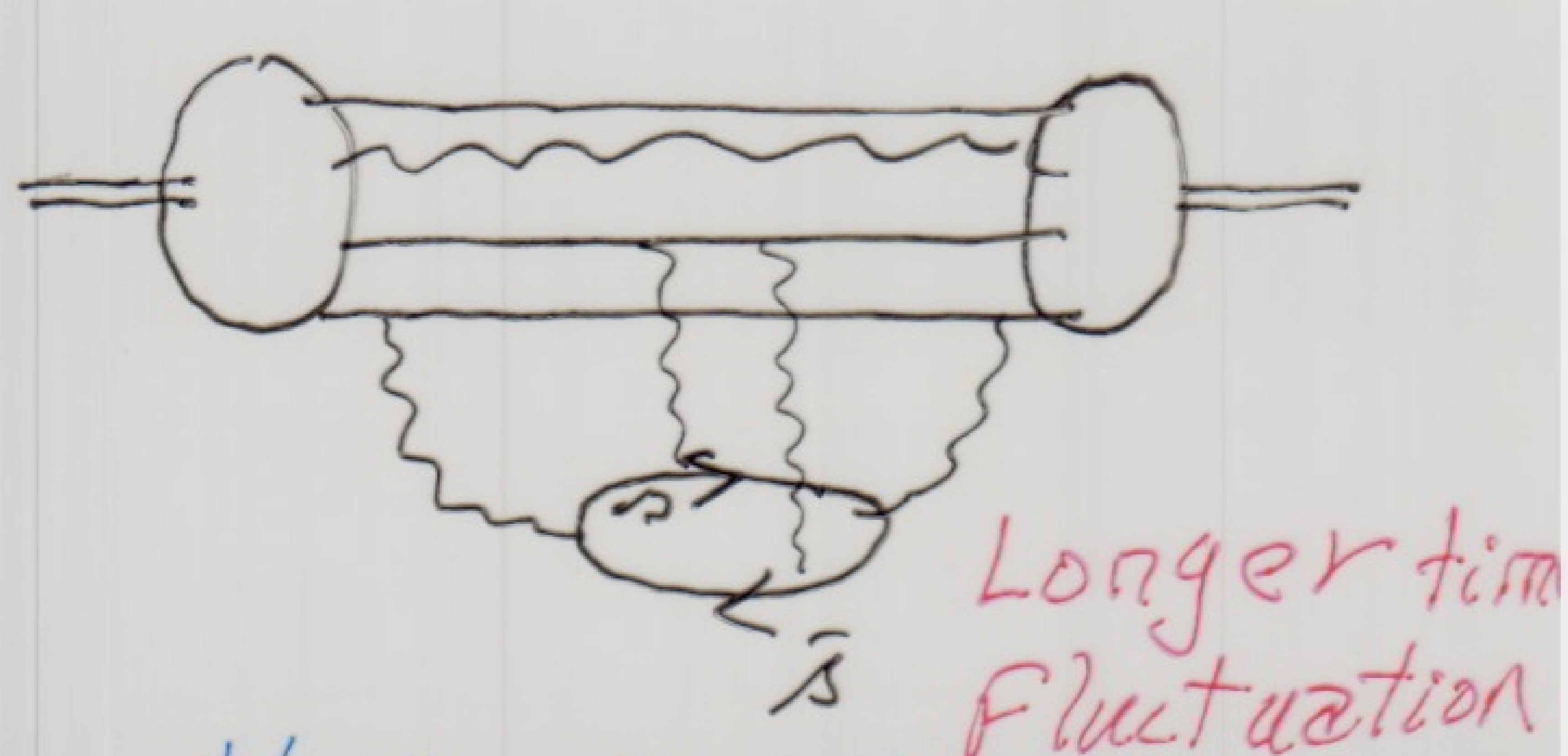
## 1. Strangeness as example

$$x[s(x,Q^2)+\bar{s}(x,Q^2)] \simeq \frac{1}{4}[x(u+\bar{u})+x(d+\bar{d})]$$

Is this interesting? It depends ...




$s, \bar{s}$ not interesting




this $s\bar{s}$ interesting

How do we tell which is the case?

If the $s$ and $\bar{s}$ have different spatial distributions, or if they are strongly polarized then they are important for the static properties of the proton.

## What do we know?

(a) SAMPLE, HAPPEX, A4

$G_M^s(Q^2 \simeq 0.1\,\text{GeV}^2) = 0.23 \pm 0.36 \pm 0.40$ SAMPLE

$G_E^s(Q^2 \simeq 0.1\,\text{GeV}^2) = -0.01 \pm 0.03$
$G_M^s(Q^2 \simeq 0.1\,\text{GeV}^2) = 0.55 \pm 0.28$ combined Fit HAPPEX MAMI

Recall that

$G_M^u(0) \simeq 2.47$ $G_M^d(0) = 0.32$

$G_E^u(0) = 4/3$ $G_E^d(0) = -1/3$

in quark model

If the s and $\bar{s}$ have identical $x_\perp$ and $x_{Bj}$ dependence then $G_E^s = G_M^s = 0$. A firm (precise) number for $G_M^s$ at various $Q^2$ would tell us a lot!

(b) $\Delta s$ and $\Delta\bar{s}$

Extracted from $g_1(x,Q^2)$ measurements and from semi-inclusive DIS

Still large uncertainties

RHIC $W^\pm$ data and $\Delta g$ from RHIC and COMPASS should help a lot.

$|\Delta s + \Delta\bar{s}| \gtrsim 0.1$ would indicate reasonably strongly interacting sea

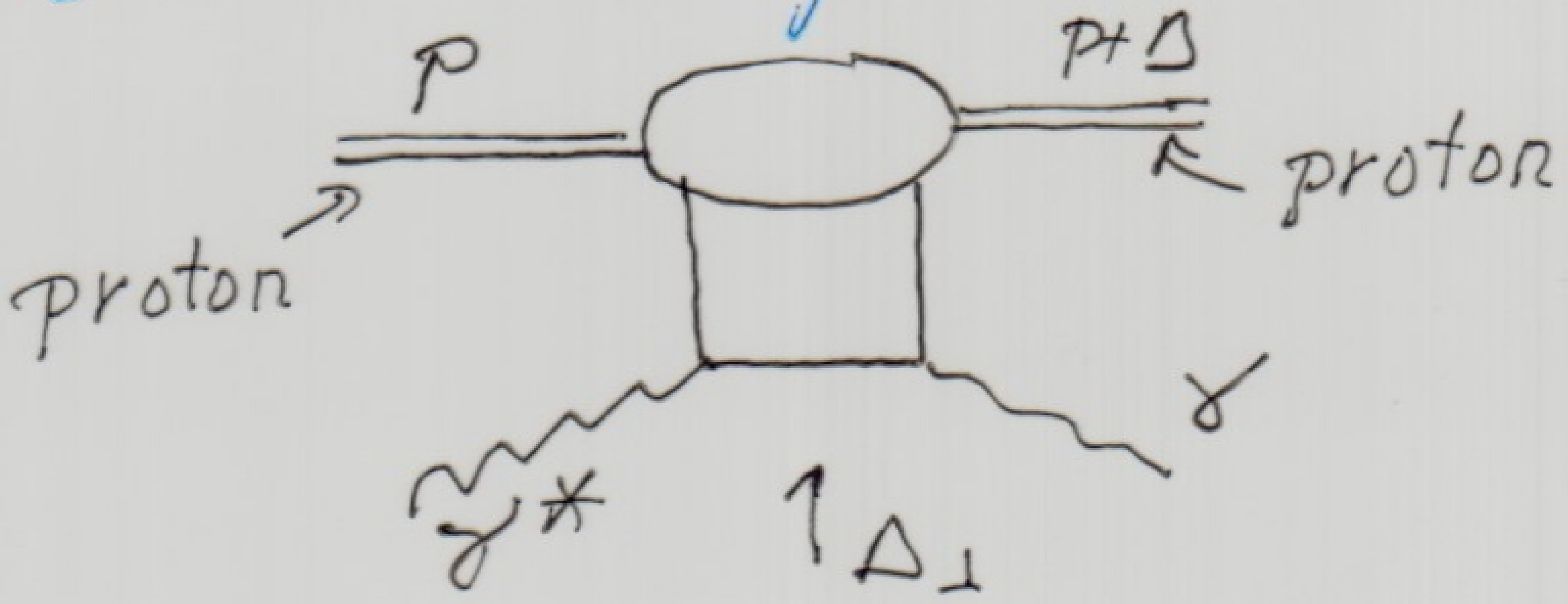
$|\Delta s + \Delta\bar{s}| \lesssim 0.05$ a weakly interacting sea

Should agree with Form Factor measurements and with lattice data eRHIC should lead to big improvement in $s(x,Q^2)$ and $\bar{s}(x,Q^2)$ both from $g_1$ and from <u>semi-inclusive</u> measurements.

## 2. Spatial distribution of partons

Form factors give some information on the spatial distribution of partons.

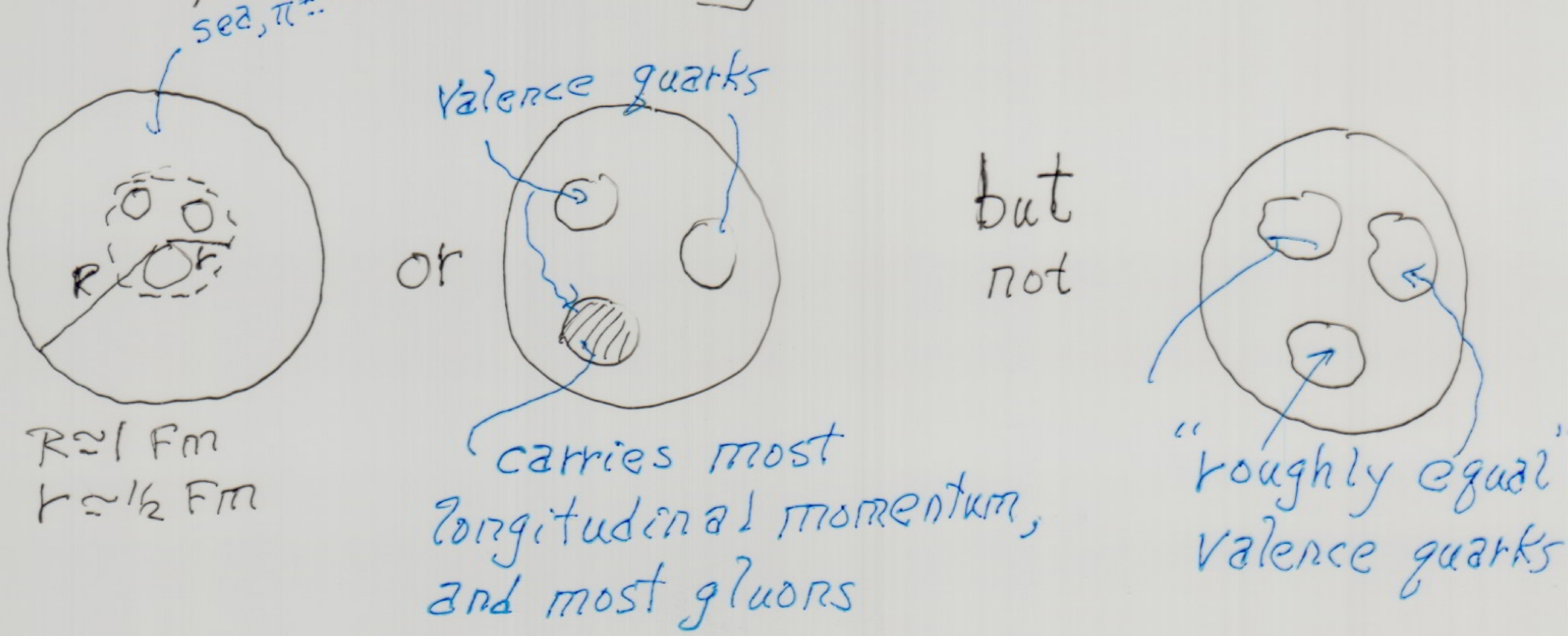
However, Generalized Parton Distributions, as determined, say, by deeply virtual Compton scattering are the best probe.




The $\Delta_\perp$ dependence can be Fourier transformed to give impact parameter dependence of quarks and antiquarks.

Regions of $x$ where sea is "unimportant" should have universal $b$-dependence

At small x one knows, from $J/\psi$ diffraction at HERA, that gluons are located within a transverse radius of 1/2 fm. This is a bit surprising since charge radius extends to $\simeq 1$ fm!




3. Ji Sum Rule

For DVCS measurements a good goal to strive for is the Ji sum rule

$$\frac{1}{2} = \frac{1}{2} \lim_{\Delta \to 0} \int x\,dx \left[ H(x,\xi,\Delta^2) + E(x,\xi,\Delta^2) \right] = \text{Spin + Orbital angular momentum of quarks and gluons}$$

Using lots of facts and lots of (reasonable) assumptions Diehl et al. conclude that valence quarks carry little orbital angular momentum!!

# 4. (Non)universality of the sea

At moderate $x$ $\bar{u}(x) \neq \bar{d}(x)$ CERN, Fermilab

Thus, sea is not completely universal (inert

Is $s(x) \stackrel{?}{=} \bar{s}(x)$ ?

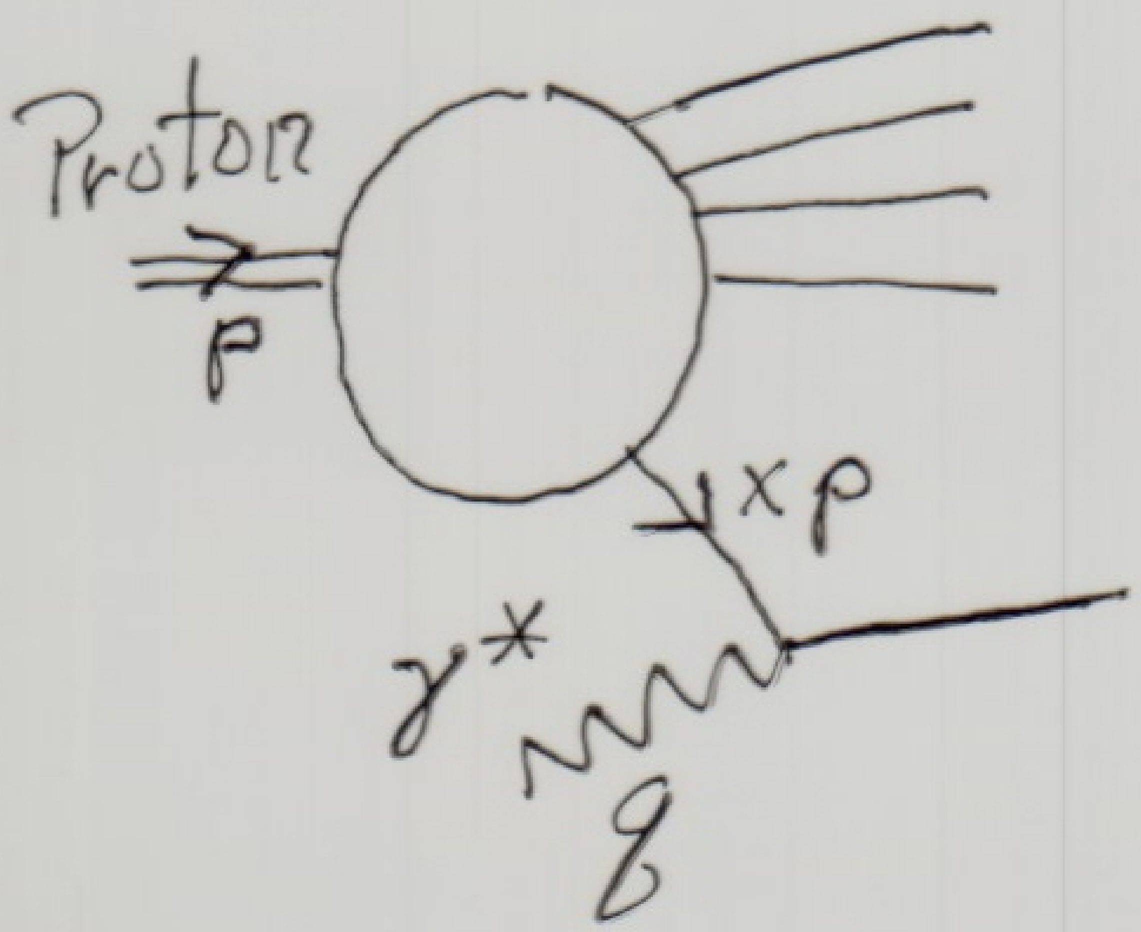
Is $u_{sea}(x) \stackrel{?}{=} \bar{u}(x)$ ? ...

Relation to quenched approximation in lattice gauge theory.

# Small-x Structure of Nuclei

## 1. Frame duality

### Bjorken Frame (Parton picture)




$$q = (q_0, q_\perp, 0)$$

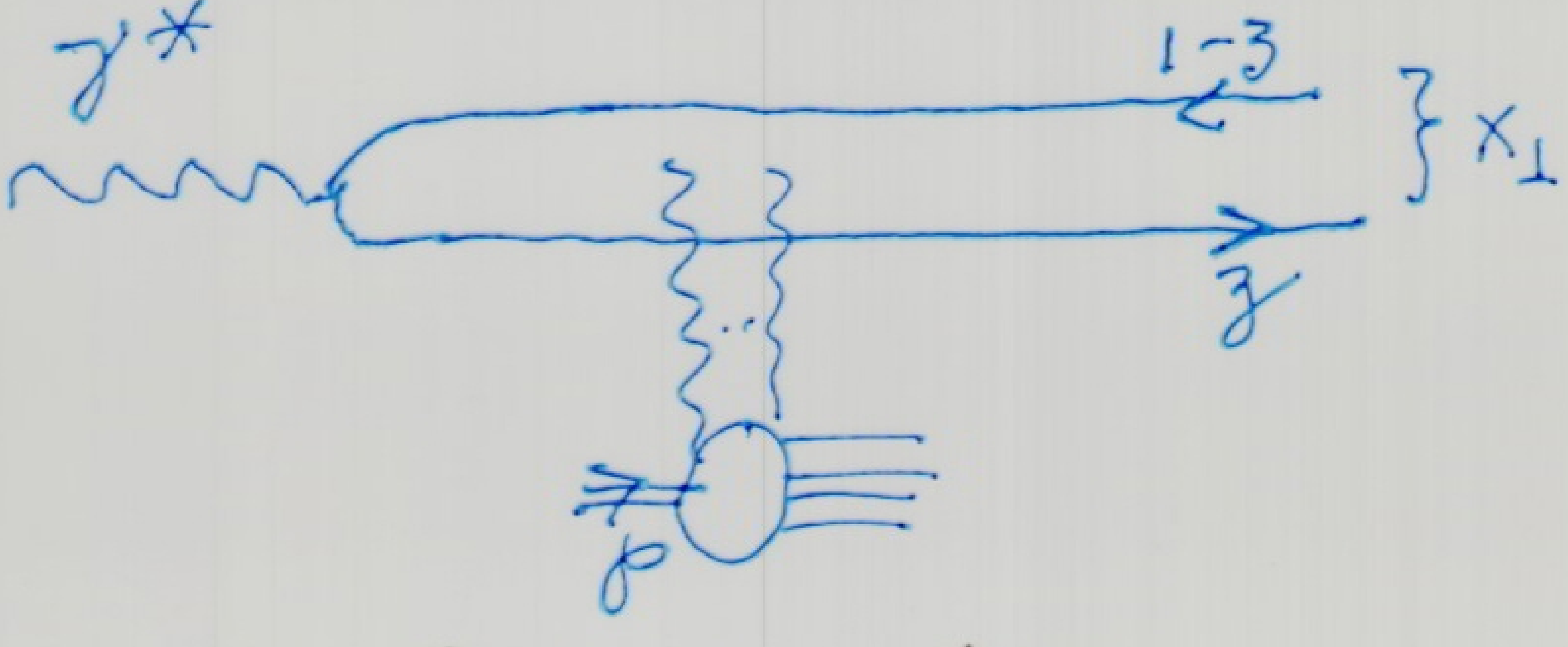
$$p \simeq \left(p + \frac{M^2}{2p}, 0, p\right)$$

$$x = \frac{Q^2}{2p\cdot q} \simeq \frac{Q^2}{2pq_0} \qquad q_0 \to 0 \text{ as } p \to \infty$$

$Q^2 \simeq q_\perp^2 \Rightarrow \Delta x_\perp \sim 2/Q$ is resolution scale of $\gamma^*$

$$F_2(x, Q^2) = \sum_f e_f^2\, x \left[ q_f(x, Q^2) + \bar{q}_f(x, Q^2) \right]$$

### Dipole Frame (useful at small x)




$$p = \left(p + \frac{M^2}{2p}, 0, 0, p\right)$$

$$q = \left(\sqrt{q^2 - Q^2}, 0, 0, q\right)$$

$Q/q$ small, but not too small

Evolution still in proton, but now looks like dipole-proton scattering

$$F_2(x, Q^2) = \frac{Q^2}{4\pi^2 \alpha_{em}} \int d^2x_\perp \int_0^1 dz \sum_f e_f^2 \left|\psi_T(x_\perp, Q, z)\right|^2 \sigma_{\text{dipole-proton}}(x_\perp, x)$$

Parton saturation $\leftrightarrow$ Unitarity limit of $\sigma_{\text{dip-pr}}$

## 2. The HERA Legacy

No definitive answer as to the physical picture of small-$x$ and moderate-$Q^2$ data.

Two broadly conflicting pictures.

(a) <u>DGLAP evolution</u>

Use DGLAP down to $Q^2 \gtrsim 1\ \text{GeV}^2$.

Good fits; many parameters.

More sea quarks than gluons at lowest $Q^2$.

Gluon distribution dangerously close to zero at lowest $Q^2$. ($F_L$ measurement would be very interesting.)

(b) Color Glass Condensate - Saturation picture

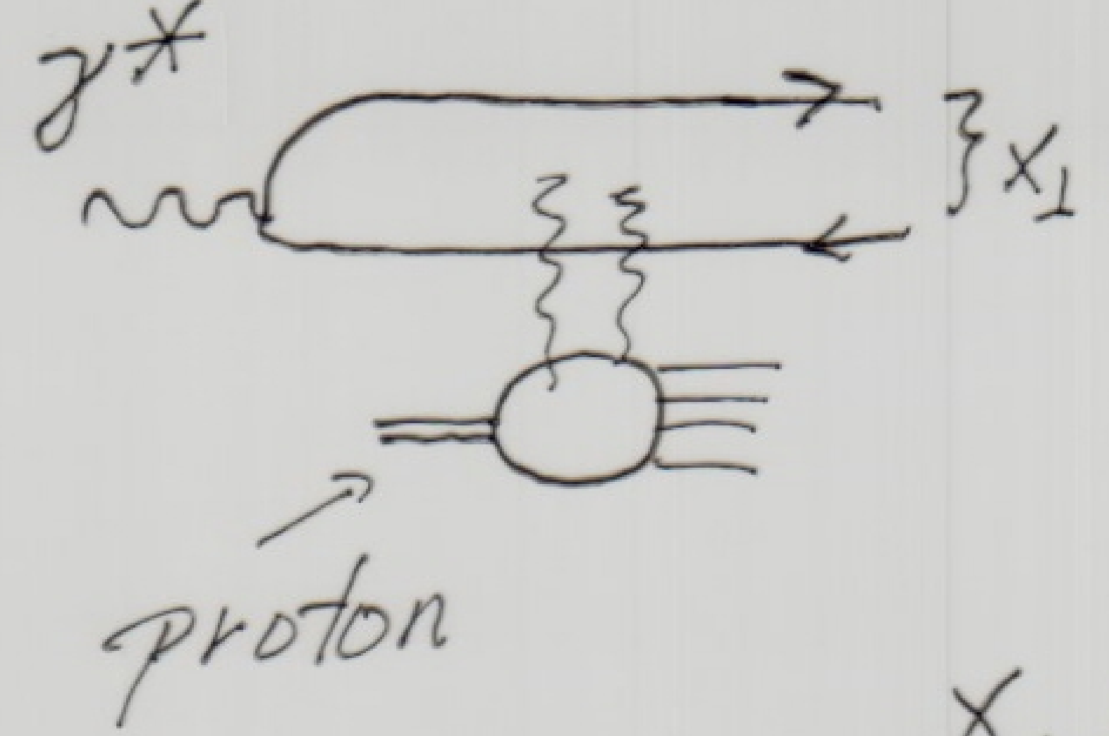
All higher twists important, and coherent, at $Q^2 = 1-2\ \text{GeV}^2$.

Gluon distribution is large (saturation)

Golec-Biernat and Wüsthoff model serves as illustrative picture

$$F_2(x,Q^2) = \frac{Q^2}{4\pi^2\alpha_{em}} \int d^2x_\perp \int_0^1 dz \sum_F e_F^2 |\psi_T(x_\perp, z, Q)|^2 \sigma_0 \left(1 - e^{-x_\perp^2 Q_s^2/4}\right)$$

$$F_2^D(x,Q^2) = \frac{Q^2}{4\pi^2\alpha_{em}} \int d^2x_\perp \int_0^1 dz \sum_F e_F^2 |\psi_T(x_\perp, z, Q)|^2 \frac{1}{2} \sigma_0 \left(1 - e^{-x_\perp^2 Q_s^2/4}\right)^2$$




$$S(x_\perp, x) = e^{-x_\perp^2 \bar{Q}_s^2/4}$$

$x_\perp < \frac{2}{\bar{Q}_s}$ weakly interacting dipole

$x_\perp > \frac{2}{\bar{Q}_s}$ strongly interacting dipole

G-B,W leads to two important phenomenological regularities of the data

(i) Geometric scaling $F_2 = F_2(Q^2/\bar{Q}^2(x))$

(ii) $F_2^D/F_2$ roughly $x$ and $Q^2$ independent

but, more elaborate models in the same spirit, like Kowalski-Teaney model, get these properties only "accidentally".

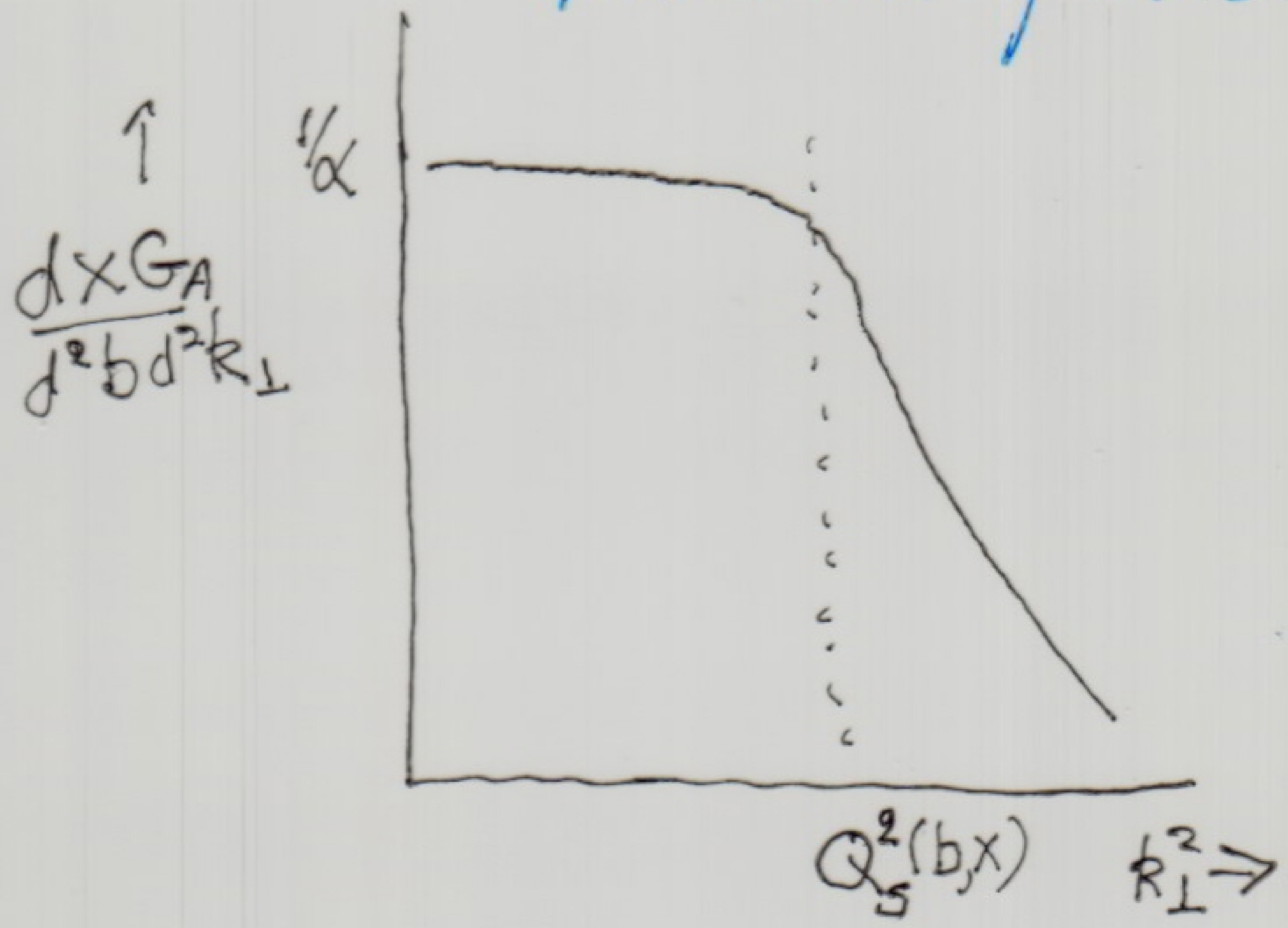
## 3. The major issues

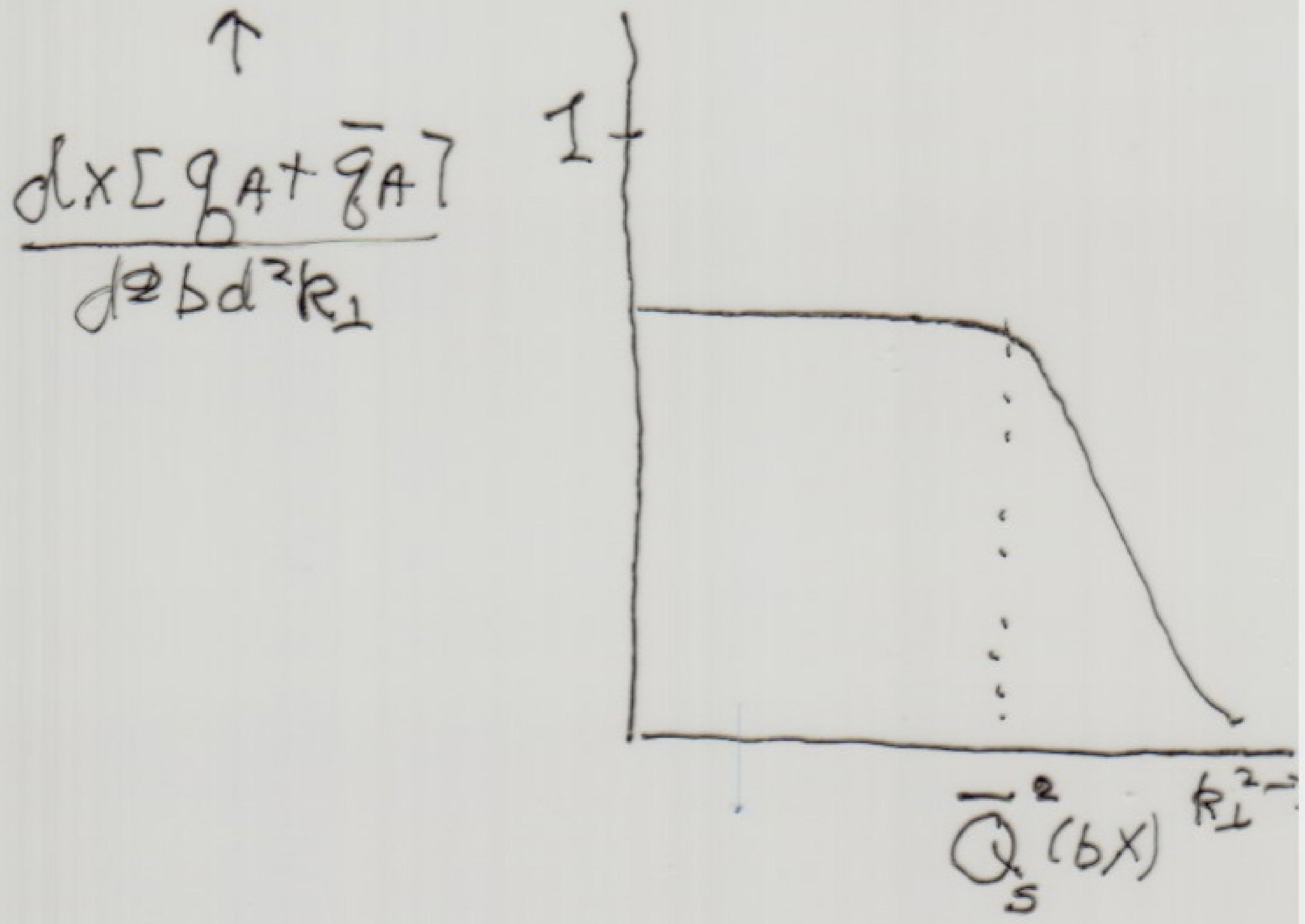
(a) What are the nuclear parton densities $x[q_{Af}(x,Q^2), \bar{q}_{Af}(x,Q^2), G_A(x,Q^2)]$?

Important for first-principles calculation of heavy ion physics.

Important to understand small-$x$ dynamics

(b) To what extent, and at what scale, are nuclear parton densities saturated?




Saturation, the CGC, is naturally expressed in terms of unintegrated distributions

(c) What is the relationship between nuclear shadowing and the Color Glass Condensate?

(d) The LHC, especially the heavy ion program, could be an extraordinary laboratory for small-x physics. However, hadronic reactions are always <u>much</u> more difficult to interpret than DIS. The ability to do LHC small-x physics will be enhanced by a good eRHIC program.

LHC: Central region, $p_\perp \simeq 3\,\mathrm{GeV}$ $\quad x \simeq 10^{-3}$

eRHIC: $\quad Q^2 \simeq 3$ $\quad x \simeq 10^{-3}$

reasonable overlap between LHC and eRHIC

# 4. Shadowing and Saturation in the M-V model

## (a) Gluons redistributed but not shadowed

In MV model

$$\frac{d^2 xG_A(x,Q^2)}{d^2b\, d^2 l_\perp} = \frac{N_c^2-1}{4\pi^4 \alpha N_c}\int d^2x_\perp\, \frac{1-e^{-x_\perp^2 Q_s^2/4}}{x_\perp^2}\, e^{-i l_\perp \cdot x_\perp}$$

nucleon gluon distribution

$$Q_s^2 = \frac{8\pi^2 \alpha N_c}{N_c^2-1}\sqrt{R^2-b^2}\,\rho\, xG_N = \frac{9}{4}\bar{Q}_s^2$$

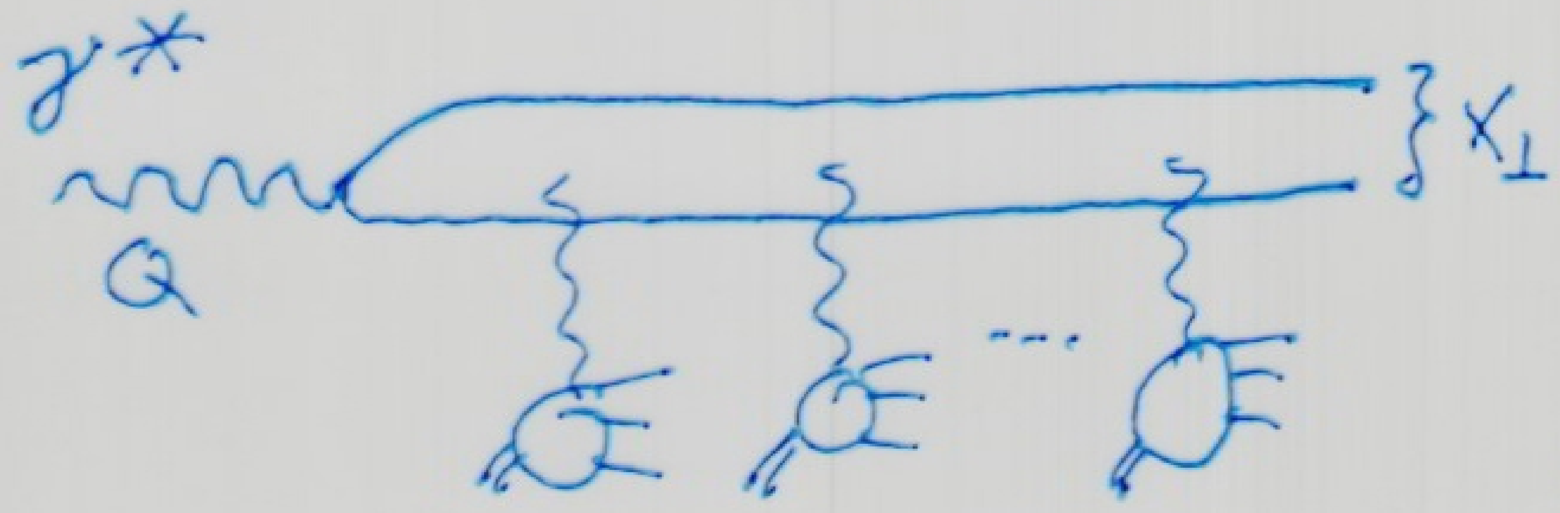
nuclear density

nuclear radius

$$x G_A(x,Q^2) = \int \Theta(Q^2-l_\perp^2)\, d^2l_\perp\, d^2b\, \frac{dxG_A}{d^2b\, d^2l_\perp} = A\, G_N(x,Q^2)$$

so long as $Q^2/Q_s^2 \gg 1$. Thus there is no gluon shadowing here, but gluons are rearranged in phase space.

## (b) Quarks are shadowed



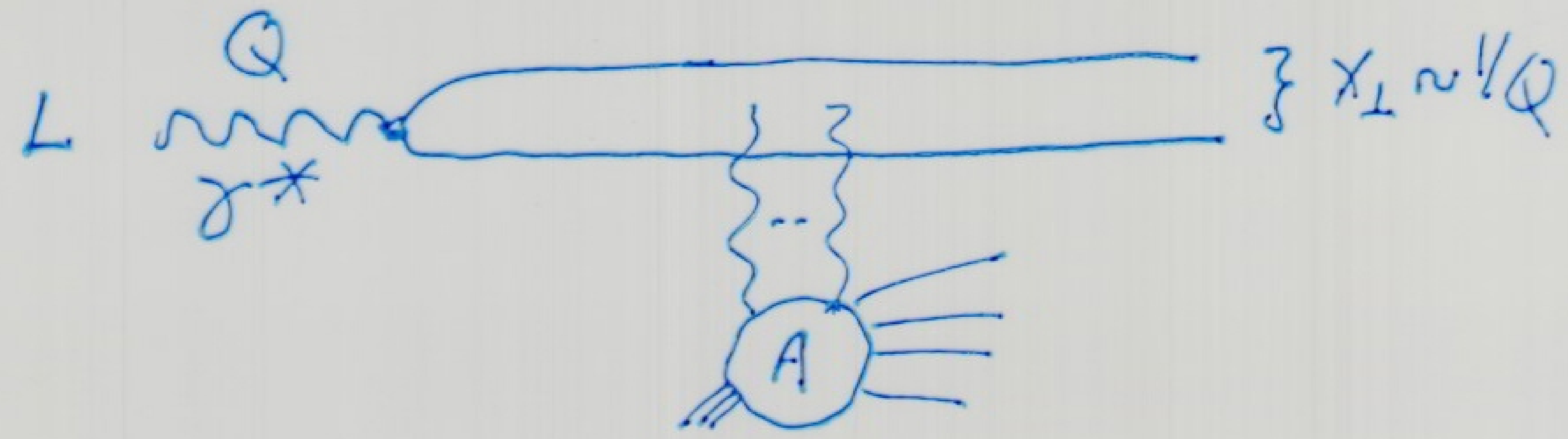

When $x_\perp > 2/\bar{Q}_s$ dipole gets strongly shadowed. Unintegrated quark distribution is strongly shadowed for $k_\perp < \bar{Q}_s$ but not for $k_\perp > \bar{Q}_s$. This gives leading twist shadowing when $Q^2/\bar{Q}_s^2 \gg 1$. Shadowing goes away logarithmically

$$\frac{x[q_A(x,Q^2)+\bar{q}_A(x,Q^2)]}{A\ x[q_N(x,Q^2)+\bar{q}_N(x,Q^2)]} \underset{Q^2/\bar{Q}_s^2 \gg 1}{\simeq} \frac{\ln Q^2/\bar{Q}_s^2}{\ln Q^2/\mu^2} \simeq 1 - \frac{\ln \bar{Q}_s^2/\mu^2}{\ln Q^2/\mu^2}$$

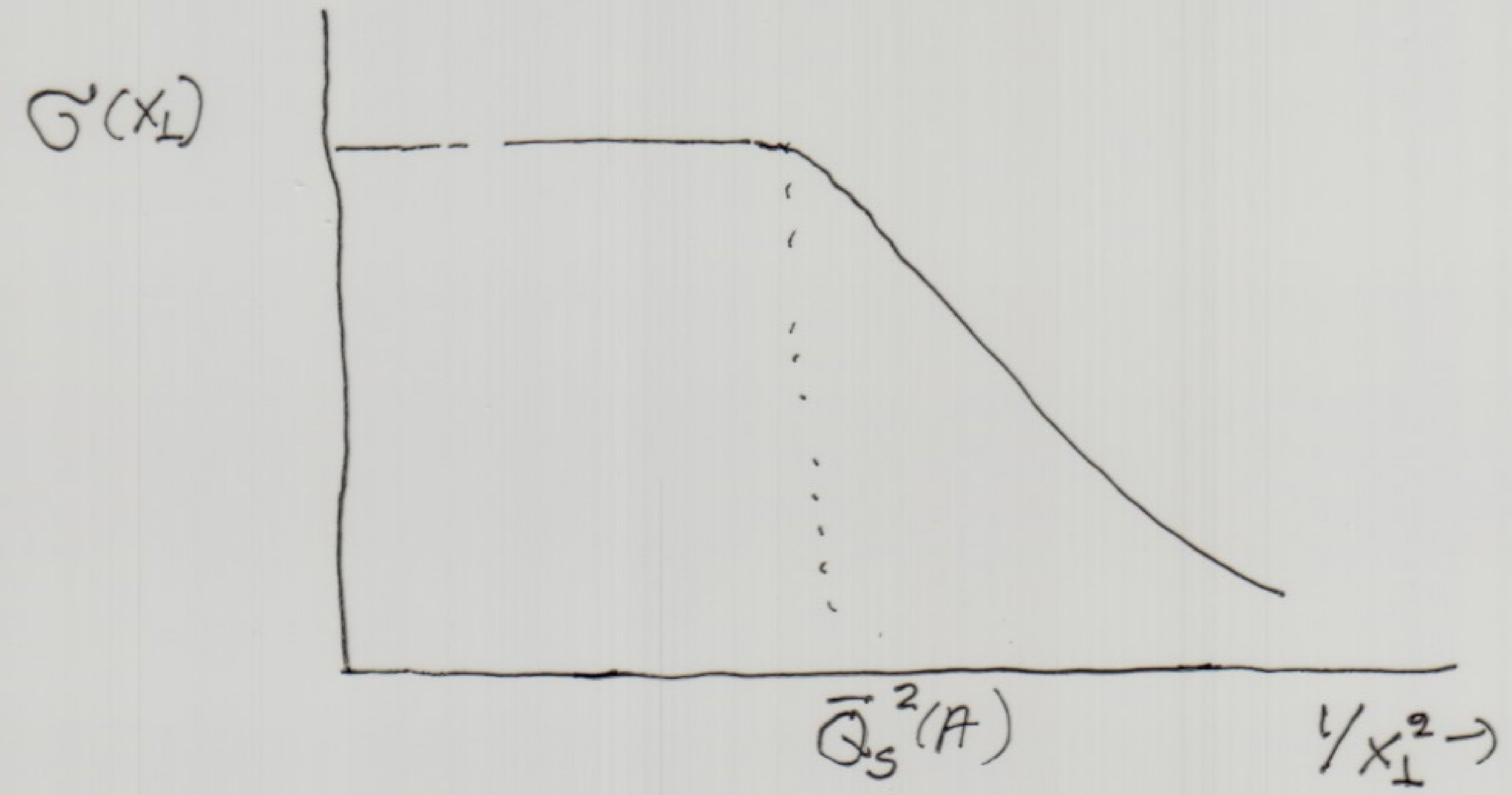
## 5. Shadowing and the CGC more generally

### (a) $F_L$

$F_L$ is especially interesting because it gives the cross section for scattering of a dipole of size $x_\perp \sim 1/Q$ on a nucleus.




When $Q$ is large this cross section can be identified with $xG_A(x,Q^2)$. One expects



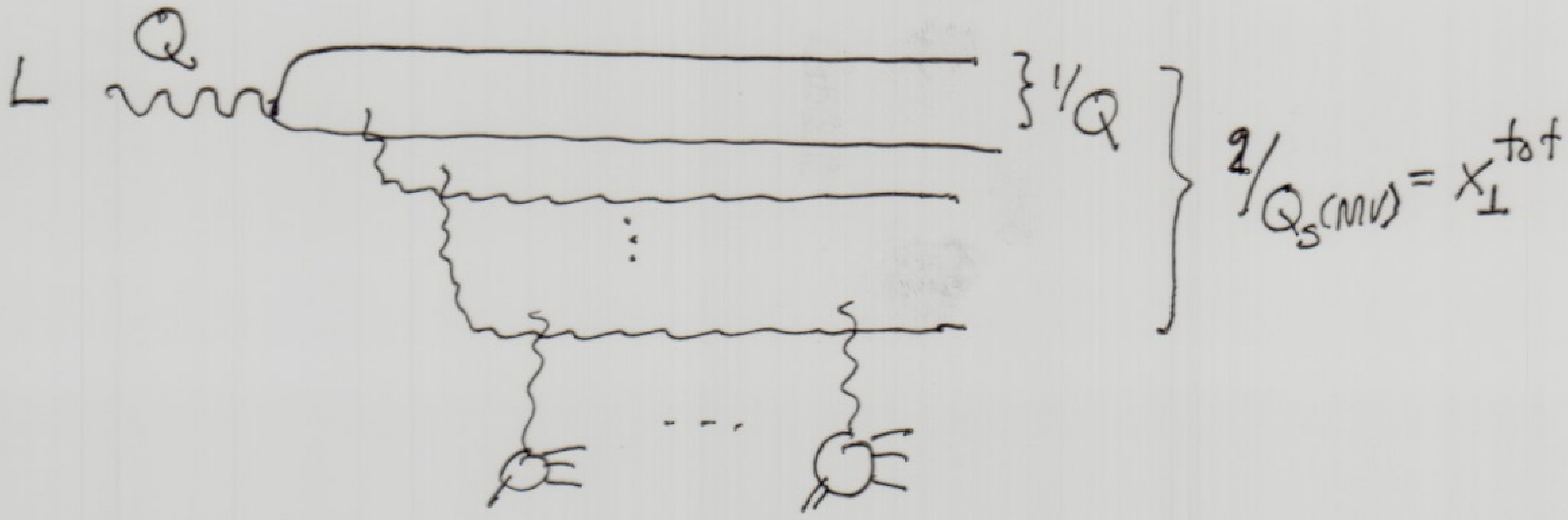

This gives a measurement of an (impact parameter averaged) $\bar{Q}_s(A)$ and its $x$ and $A$-dependence.

## (b) Shadowing and the CGC

(i) IF $Q^2/Q_s^2(A)$ not too large BFKL evolution dominates. Shadowing is strong and

$$R = \frac{\sigma_A}{A\sigma_N} = \text{const}\ A^{-\frac{1}{3}\lambda_0} = \frac{xG_A}{A\,xG_N} \qquad \lambda_0 \simeq 0.37$$

Leading twist



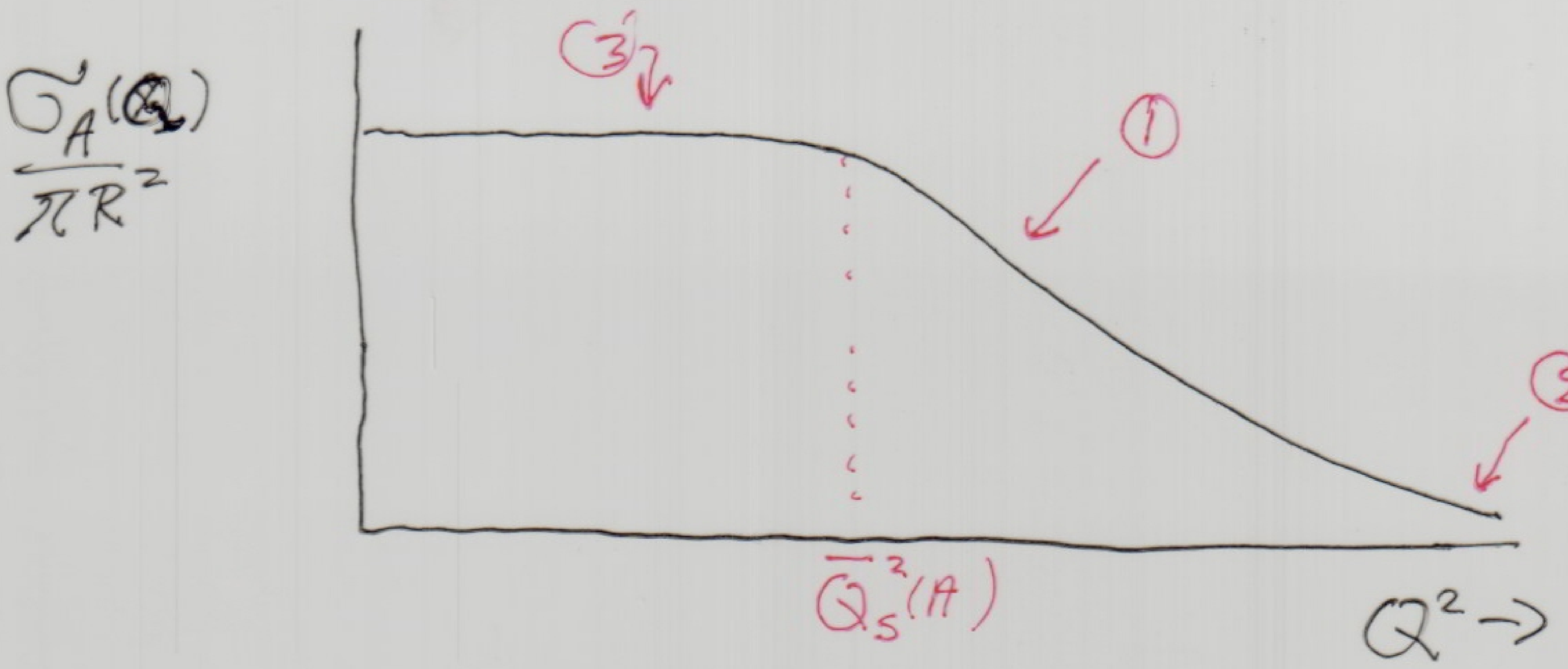

(ii) IF $Q^2/Q_s^2(A) \gg 1$ DGLAP evolution dominates and shadowing is parametrically small. ($\lambda_0 \to 0$ in above)

Leading twist

$R = \text{const}$ (slowly varying in $A$ and $Q^2$)

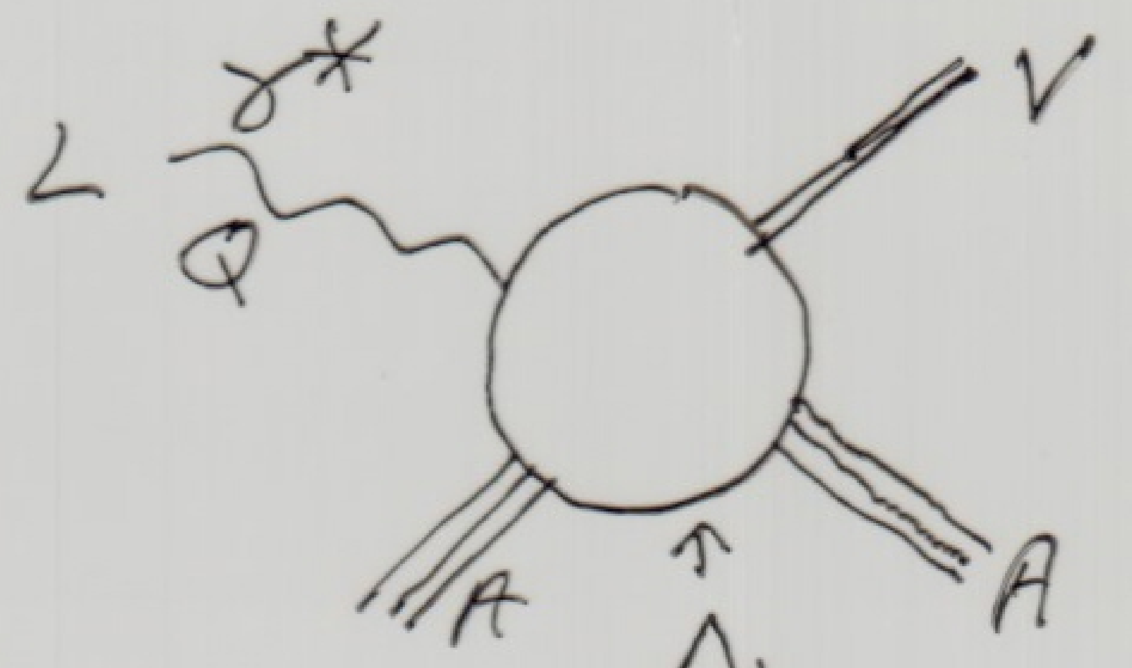
$R \simeq 1 - \frac{\ln Q_s^2/\mu^2}{\ln Q^2/\mu^2}$ in one gluon emission approx.

(iii) IF $Q^2/Q_s^2(A) \lesssim 1$ R is very small



## 6. Vector meson production

As in DVCS the use of vector mesons allows a transformation to impact parameter spac




$$S(x, b, x_\perp) = 1 - \text{Const} \int d^2\Delta \, e^{-i\Delta_\perp \cdot b} \sqrt{\frac{d\sigma}{d\Delta_\perp^2}}$$

"known", depends on vector meson exclusive wavefunction

In looking at nuclear effects the dependence on vector meson wavefunction should largely cancel. Should be good way to get

$$\frac{d\, x G_A(x, b, Q^2)}{d^2 b}$$

when $Q^2$ large, and to study

$$1 - S = \frac{d\sigma_{dipole-A}}{d^2 b}$$

when $Q^2 \simeq \bar{Q}_s^2(A)$.