L02

- Physics is a study of the Universe: every thing you see and not see around you.
  - To find the underlying regularity, symmetry, predictability
  - Use models and current understanding of phenomenon and make hypotheses
  - Perform experiments and test them to gain further understanding, get insights and find new phenomenon
- Tools we use:
  - Logical Reasoning, Mathematics, Experimental Equipment & instruments and lots of common sense

L02 Units

- **Units:** Numbers without proper units often mean nothing!
- Table of Standard International (SI) Units in the book
  - Length, time, mass, current, force,…..
  - Conversions: 1 mile = 1.61 km
  - Speed limits: 55miles/hr = 55 miles/hr x 1.61 km/mile
  - 88.5 km/hr = 88.5 km/hr x 1000 m/km x 1 hr/3600 s
  - 24.6 m/s
- Use UNITS effectively to check results

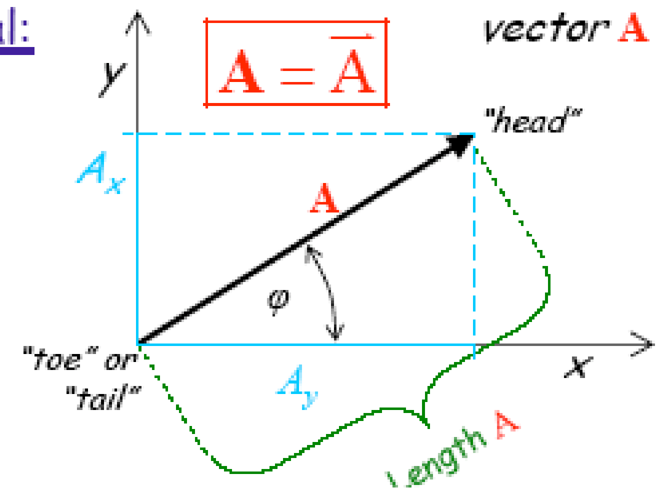
$$\text{Period } T = \sqrt{\frac{L(\text{Length of the pendulum})\ m}{g(\text{acceleration due to gravity})\ m/s^2}}$$

- **Practical HINT for use in future:** Keep symbols as long as reasonably possible in your calculations. Put in numbers in the last possible stage!

Vectors

- Vectors are quantities those have both **<u>magnitude and direction</u>** **such as displacement, velocity, force**
- Compare with scalar (non-directional quantities) such as: distance, speed, work done, volume
- Need an agreed upon reference system
- Using Coordinates:
  - Polar $\vec{A} = (A, \phi)$
  - Orthogonal $$\vec{A} = A_x \vec{i} + A_y \vec{j}$$
  - Relation:

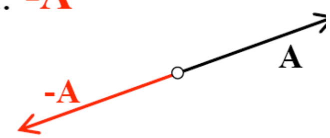
- graphical:



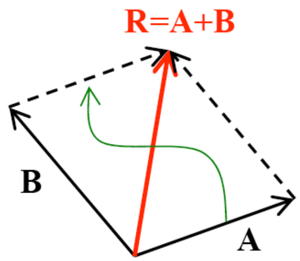
$$A_x = A\cos(\phi) \qquad A_y = A\sin(\phi)$$

Vector manipulation

- Inverse of a vector **A**: **-A**
  - components: $-A_x$, $-A_y$

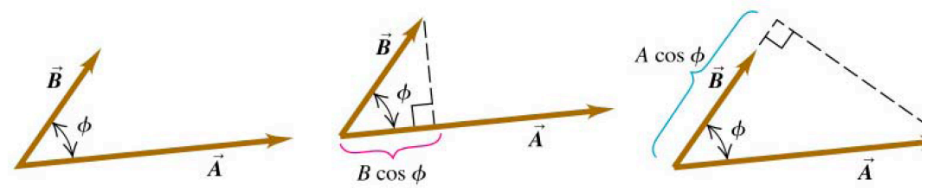


- Vector Addition: **R**=**A**+**B**
  - Graphically: add vectors "head-to-toe"



  - using (orthogonal) coordinates:
    - $R_x = (A+B)_x = A_x + B_x$
    - $R_y = (A+B)_y = A_y + B_y$
  - easily expandable:
    - **R** = **A**+**B**+**C**+ …
    - Subtraction = addition of inverse of vector:
      R = **A**-**B** = **A**+(-**B**)
    - From 2 to 3 dimensions: *z*-coordinates…

- Dot-Product (Scalar Product) of two Vectors:
  - $\mathbf{A}\cdot\mathbf{B} = AB\cos\phi = A(B\cos\phi) = B(A\cos\phi)$
    $= A_xB_x + A_yB_y + \ldots$



- Cross-Product (Vector Product) will be discussed later, when we will need it…

Lecture 2

- Motion:
  - the displacement or position of an object as function of time: $\mathbf{x}(t)$ (vector!)
  - it was the study of motion that led Newton to the breakthrough in mechanics:

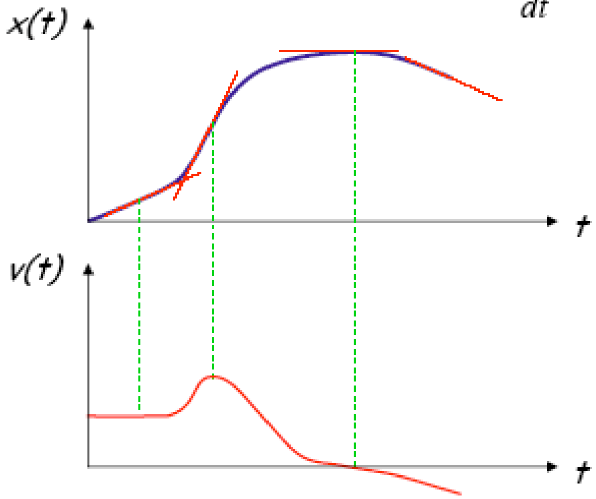
$$\mathbf{F} = m\,\mathbf{a}$$

- Definitions:

$$\text{Motion} \equiv \mathbf{x}(t)$$

$$\text{Velocity (vector):} \quad \mathbf{v}(t) \equiv \frac{d\mathbf{x}(t)}{dt}$$

$$\text{Acceleration (vector):} \quad \mathbf{a}(t) \equiv \frac{d\mathbf{v}(t)}{dt}$$



- Thus, given the motion $\mathbf{x}(t)$ we easily find the $\mathbf{v}(t)$ and $\mathbf{a}(t)$ by successive differentiation...

- On the other hand, if the (vector) Force $\mathbf{F}$ is known as function of time , then with Newton's Law $\underline{\mathbf{F} = m\,\mathbf{a}}$, we obtain $\mathbf{a}(t)$;
  (i.e. we then know acceleration $\mathbf{a}$ at any point t in time...)

  With $\mathbf{a}(t)$, $\mathbf{v}_0$ and $\mathbf{x}_0$, we can obtain the full motion $\mathbf{x}(t)$...

L02

- Often, we want to obtain the motion from the acceleration $\mathbf{a}(t)$; (the acceleration can be had from the knowledge of the force $\mathbf{F}$ and Newton's Law: $\mathbf{F} = m\,\mathbf{a}$):

$$\frac{d\mathbf{v}}{dt} \equiv \mathbf{a} \quad \Rightarrow \quad d\mathbf{v} = \mathbf{a}dt \quad \Rightarrow \quad \int_{\mathbf{v}_0}^{\mathbf{v}(t)} d\mathbf{v} = \int_0^t \mathbf{a}dt$$

$$\Rightarrow \quad \mathbf{v}(t) - \mathbf{v}_0 = \int_0^t \mathbf{a}dt \quad \Rightarrow \quad \boxed{\mathbf{v}(t) = \mathbf{v}_0 + \int_0^t \mathbf{a}dt}$$

For constant $\mathbf{a}$: $\displaystyle\int_0^t \mathbf{a}dt = \mathbf{a}t \quad \Rightarrow \quad \boxed{\mathbf{v}(t) = \mathbf{v}_0 + \mathbf{a}t}_{\text{constant acceleration}}$

- Integrating once more, we find the motion:

$$\frac{d\mathbf{x}}{dt} \equiv \mathbf{v} \quad \Rightarrow \quad d\mathbf{x} = \mathbf{v}dt \quad \Rightarrow \quad \int_{\mathbf{x}_0}^{\mathbf{x}(t)} d\mathbf{x} = \int_0^t \mathbf{v}dt$$

$$\Rightarrow \quad \mathbf{x}(t) - \mathbf{x}_0 = \int_0^t \mathbf{v}dt \quad \Rightarrow \quad \boxed{\mathbf{x}(t) = \mathbf{x}_0 + \int_0^t \mathbf{v}dt}$$

For constant $\mathbf{a}$: $\displaystyle\int_0^t \mathbf{v}dt = \int_0^t (\mathbf{v}_0 + \mathbf{a}t)dt = \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2$

$$\Rightarrow \quad \boxed{\mathbf{x}(t) = \mathbf{x}_0 + \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2}_{\text{constant acceleration}}$$

- Summary:
  - Definitions:

Motion $\equiv \mathbf{x}(t)$

Velocity (vector): $\displaystyle\mathbf{v}(t) \equiv \frac{d\mathbf{x}(t)}{dt} \quad \Leftrightarrow \quad \mathbf{x}(t) - \mathbf{x}_0 = \int_0^t \mathbf{v}dt$

Acceleration (vector): $\displaystyle\mathbf{a}(t) \equiv \frac{d\mathbf{v}(t)}{dt} \quad \Leftrightarrow \quad \mathbf{v}(t) - \mathbf{v}_0 = \int_0^t \mathbf{a}dt$

  - For constant acceleration $\mathbf{a}$:

$$\mathbf{v}(t) - \mathbf{v}_0 = \mathbf{a}t$$

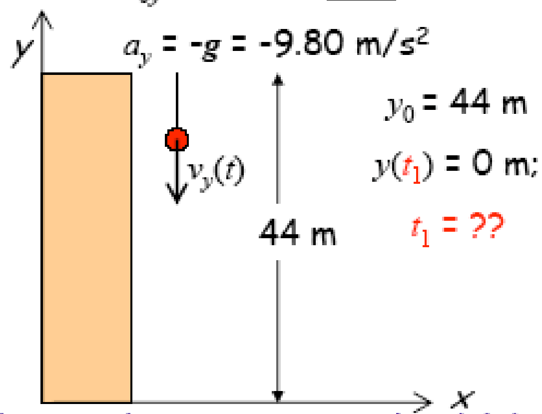
$$\mathbf{x}(t) - \mathbf{x}_0 = \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2 \quad \text{(parabola)}$$

$$\mathbf{v}(t)^2 - \mathbf{v}_0^{\ 2} = 2\mathbf{a}\left(\mathbf{x}(t) - \mathbf{x}_0\right)$$

L02

- Exercises:
  - Ball is dropped from rest from the Physics Building (44 m). When does it hit the ground?
  - Note: the gravitational acceleration is constant (near Earth's surface):
    g = 9.80 m/s^2, downwards
  - Strategy:
    - 1: make sketch with axes, and other details
    - 2: consider the problem: what is requested; what is going on; what is the physics; can the problem be split in successive parts?
    - 3: pick a solution strategy and start the solution. Try to keep the solution in symbols as much as possible...
    - 4: Check your solution for units, reasonableness, ...

- Sketch: v_{0y} = 0 (from rest)



  - Only one direction is involved (y)
  - Acceleration is constant
  - v_y varies with t
- Solution- need to find t_1 given y_0, v_{0y}, a_y

$$\mathbf{x}(t) = \mathbf{x}_0 + \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2$$

rewrite: $y(t_1) = y_0 + \mathrm{v}_{0y} t_1 + \frac{1}{2}\mathrm{a}_y t_1^{\,2}$

$$\Rightarrow \quad 0 = y_0 + 0t_1 - \frac{1}{2}gt_1^{\,2} \quad \Rightarrow \quad 0 = y_0 - \frac{1}{2}gt_1^{\,2}$$

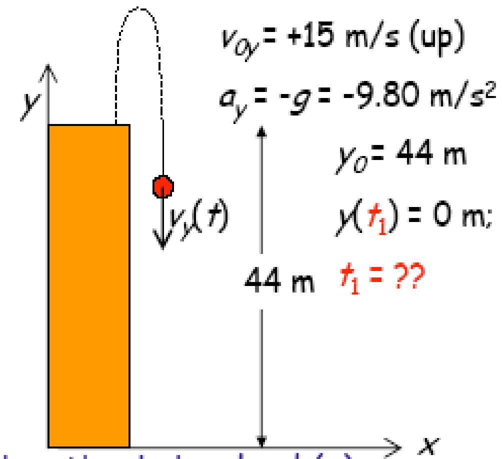
$$\Rightarrow \quad \frac{1}{2}gt_1^{\,2} = y_0$$

$$\Rightarrow \quad t_1 = \sqrt{\frac{2y_0}{g}} = \sqrt{\frac{2\times 44\ \mathrm{m}}{9.80\ \mathrm{m/s^2}}} = 2.9965967\cdots\ \mathrm{s} = 3.00\ \mathrm{s}$$

L02

- Q2: What is the ball's speed just before it hits the ground (at t=3.00 s)?
  - simple: $v(t=3.00\text{ s}) = v_{0y} - gt$ = ?
    = 0 - 9.80x3.00 m/s
    = - 29.4 m/s (i.e. down)
- Q3: A person on the ground throws a ball straight up at 20 m/s (~50 mi/hr); how high will it go?
  - simple: $y - y_0 = (v_y^2 - v_{0y}^2)/2a$
    $y - 0 = (0 - v_{0y}^2)/(-2g)$
    $y = v_{0y}^2/2g$
    $= 400\ \text{m}^2\text{s}^{-2}/19.60\ \text{ms}^{-2} = 20.4\text{m}$
- Q4: How long before the ball returns?
  - simple: v(at max. height) = $v_{0y} - gt_{\text{max}}$
    thus: $t_{\text{max}} = v_{0y}/g = 20\ \text{ms}^{-1}/9.80\ \text{ms}^{-2}$
    = 2.04 s $\Rightarrow t_{\text{total}}$ = 4.08 s
- Q5: A ball thrown straight up stays 4 s up in the air before it hits ground again; how high did it go?
  - e.g.: $v_y - v_{0y} = -gt$; at t_{tot} =4 s: $v_y = -v_{0y}$
    thus: $-2v_{0y} = -gt_{\text{tot}}$, and v_{0y} = 19.8 m/s -> Q3

- A ball is thrown upwards from the roof with v_{0y}= 15 ms^{-1}. When does it hit ground?
  - Sketch:



  - Only one direction is involved (y)
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  - v_y varies with t
- Solution- need to find t_1 given y_0, v_{0y}, a_y

$$\mathbf{x}(t) = \mathbf{x}_0 + \mathbf{v}_0 t + \tfrac{1}{2}\mathbf{a}t^2$$

rewrite: $y(t_1) = y_0 + \mathbf{v}_{0y}t_1 + \tfrac{1}{2}\mathbf{a}_y t_1^2$

$$\Rightarrow \quad 0 = y_0 + v_{0y}t_1 - \tfrac{1}{2}gt_1^2 \quad \Rightarrow \quad \text{Quadratic Eqn in } t_1!$$

$$\Rightarrow \quad t_1 = \frac{-v_{0y} \pm \sqrt{v_{0y}^2 + 2gy_0}}{-g} = \frac{-15 \pm \sqrt{225 + 862}}{-9.80} = \cancel{-1.83\text{ s}},\ \underline{4.89\text{ s}}$$

Check: