RIKEN PHENIX SPIN FEST @ WAKO

Lectures on Perturbative QCD

or

from basic principles to current applications

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Outline of the lectures

Lecture 1: basic ideas; exploring the QCD final state

Lecture 2: origin of singularities; infrared safety

Lecture 3: QCD initial-state; factorization; renormalization

Lecture 4: more on factorization & renormalization; pdfs

Lecture 5: applications in hadron-hadron collisions; spin

Lecture 5

hadron-hadron collisions

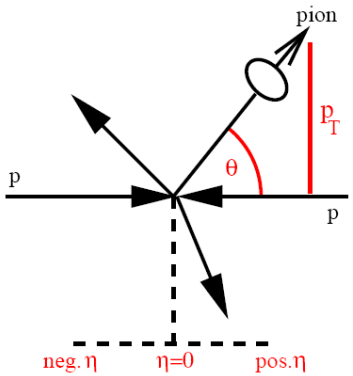
RHIC-spin

pQCD approach for hadron-hadron collisions

starting point:
- exploit universality of pdf's
- invoke *factorization* Libby, Sterman; Ellis et al.; Amati et al.; Collins et al.; . . .

  $\longrightarrow$ way to separate long-distance (= non-perturbative) from short-distance (= perturbative) phenomena

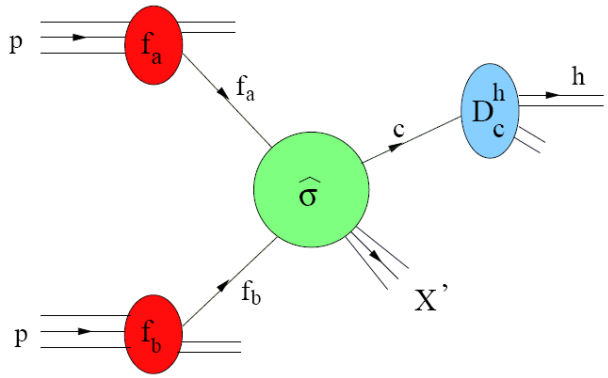
example: *unpolarized* high-p_T single-inclusive hadron production



factorization

$\longrightarrow$

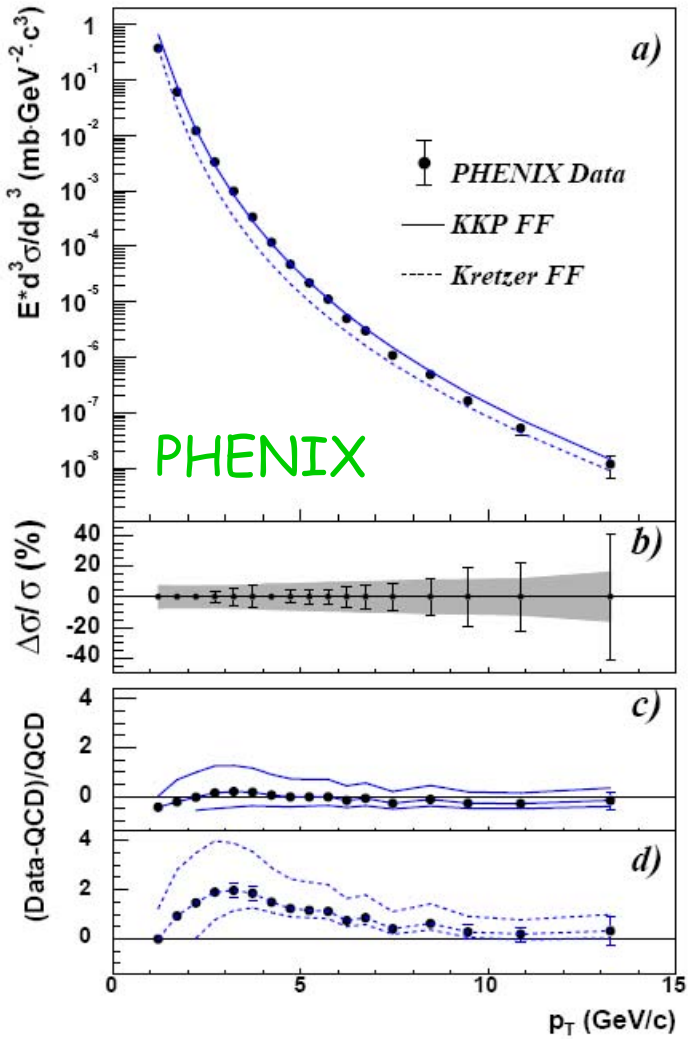
theorem



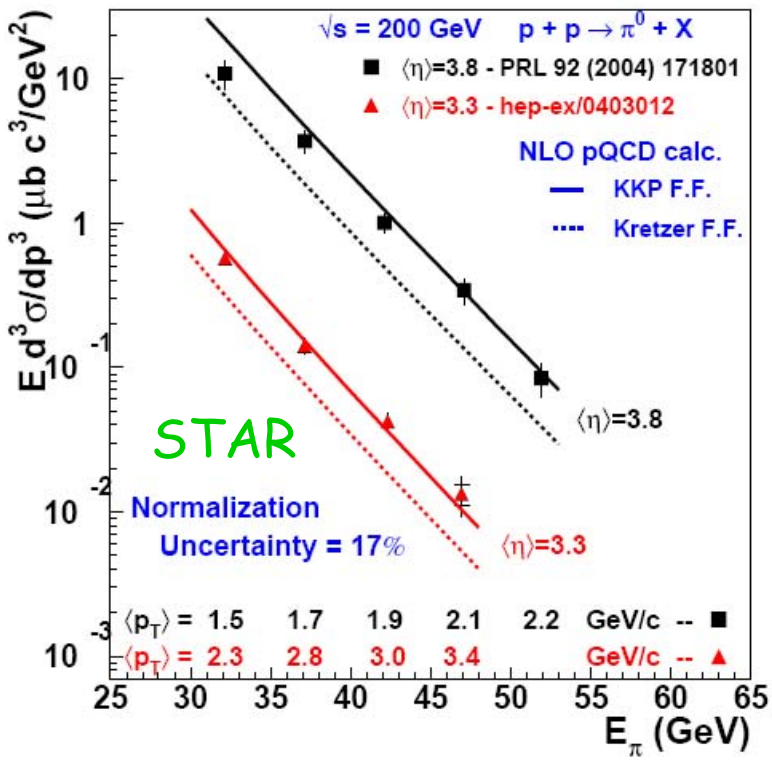
ingredients: parton densities $f_a(x_a)$, $f_b(x_b)$, fragmentation fcts. $D_c^h(z)$,

hard partonic cross section $\hat{\sigma}$

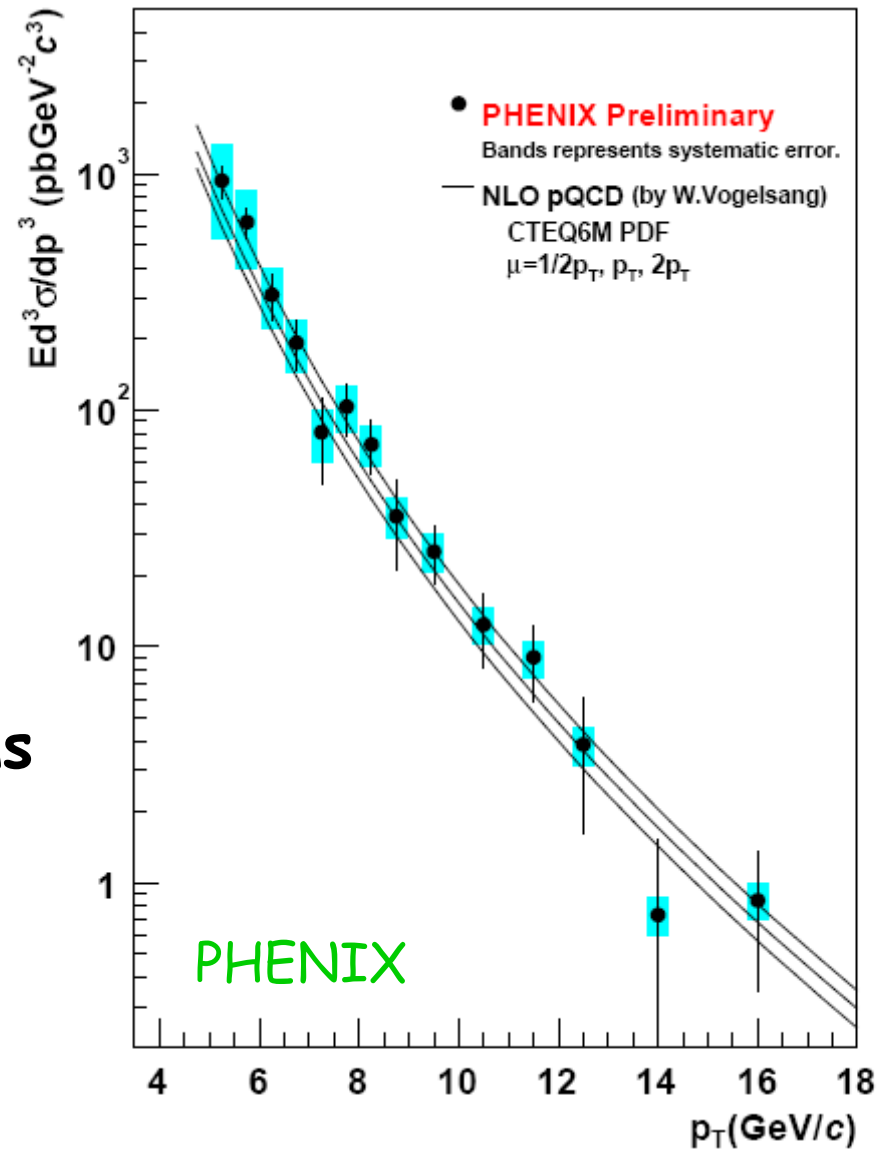
so far pQCD seems to work very well ...



inclusive high-p_T pions



and also for prompt photons



Recall some important things from Lectures 1-4

measured cross section *must not* depend on theoretical conventions

$\rightarrow$ e.g. $\mu_f \frac{d\sigma}{d\mu_f} = 0$ (example of a *"renormalization group eq."*)

$d\hat{\sigma}(x_a, x_b, z_c, \mu_f \rightarrow \mu_f + d\mu_f, \mu'_f)$ dictates scale behavior of $f_{a,b}(x_{a,b}, \mu_f)$

roughly

$$\frac{d\hat{\sigma}(\mu_f)}{d\ln\mu_f} \otimes f_a(x_a, \mu_f) \otimes f_b(x_b, \mu_f) \otimes D_c^h(z_c, \mu'_f) \quad +$$

$$\hat{\sigma}(\mu_f) \otimes \frac{df_a(x_a, \mu_f)}{d\ln\mu_f} \otimes f_b(x_b, \mu_f) \otimes D_c^h(z_c, \mu'_f) \quad +$$

$$\hat{\sigma}(\mu_f) \otimes f_a(x_a, \mu_f) \otimes \frac{df_b(x_b, \mu_f)}{d\ln\mu_f} \otimes D_c^h(z_c, \mu'_f) \quad = \quad 0$$

result: $\frac{df_a(x_a,\mu_f)}{d\ln\mu_f}$ have to obey DGLAP evolution!

Recall some important things from Lectures 1-4

There is **catch** . . . we work with a perturbative expansion in α_s:

$$d\hat{\sigma} = \underbrace{d\hat{\sigma}^{(0)}}_{\text{LO}} + \underbrace{\alpha_s\, d\hat{\sigma}^{(1)}}_{\text{NLO}} + \underbrace{\alpha_s^2\, d\hat{\sigma}^{(2)}}_{\text{NNLO}} + \ldots$$

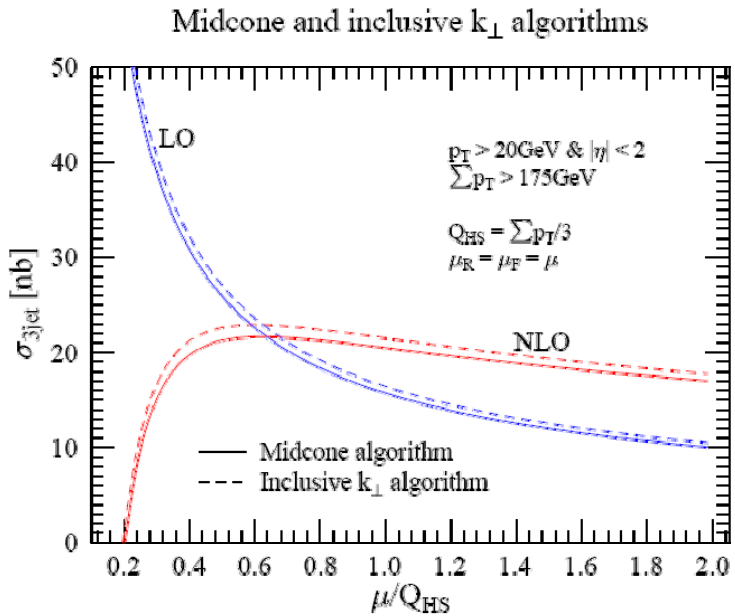
number of partons

complexity of calculation

μ_f cancellation only happens to all orders

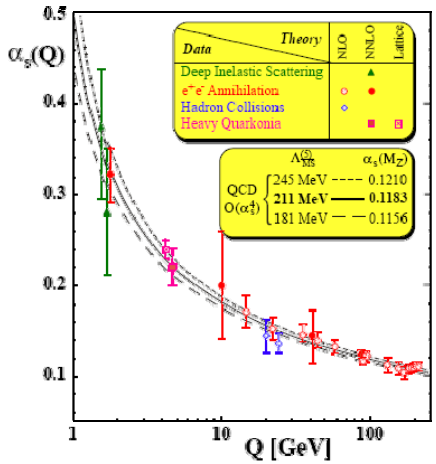
LO	:	no cancellation whatsoever
NLO	:	cancellation starts to work
NNLO	:	better and better
. . .		

→ motivation for higher order calculations

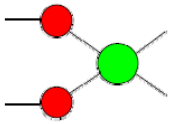


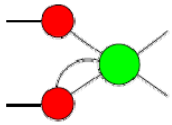
example: 3-jets @ TeVatron (Z. Nagy)

Recall some important things from Lectures 1-4

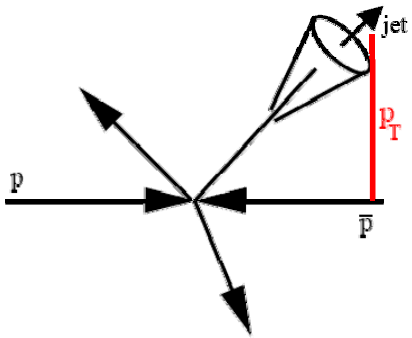


Question: How to choose all these scales μ_f, μ'_f, μ_r ?

for our factorized framework  to work . . .

- we have to be in the perturbative domain: $\alpha_s(\mu) \ll 1$
- multi-parton correlations  have to be negligible

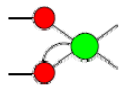
→ **minimum requirement**: at least one hard (= large) scale present, usually p_T



$$\mu_f \simeq \mu'_f \simeq \mu_r = \mathcal{O}(p_T)$$

if $p_T \gtrsim 2\,\mathrm{GeV}$ (no clear borderline!!)

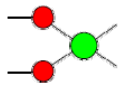
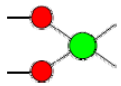
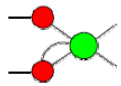
· $\alpha_s(\mu_r) \ll 1$ → pert. expansion should work

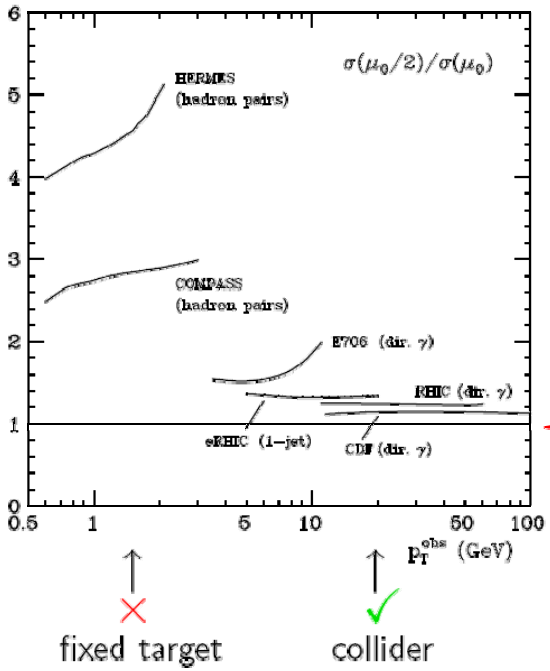
·  $\propto \left(\frac{\Lambda}{p_T}\right)^n$ → hopefully small

pQCD @ colliders

in terms of *reliability* colliders have a big advantage over fixed target experiments:

large c.m.s. energy $\sqrt{S}$ $\rightarrow$ very large p_T-range up to $p_T^{\max} = \sqrt{S}/2$ accessible

- pQCD should be applicable plus  *not*  +  + . . .
- bonus: for same p_T, scale cancellation works better the higher $\sqrt{S}$



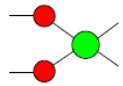
typical ratios of cross sections for two choices of μ_f

fixed target:
HERMES, COMPASS, E706

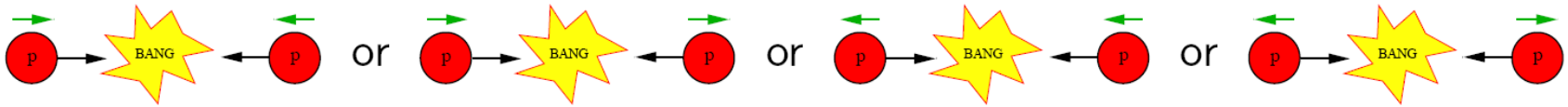
collider:
eRHIC, RHIC, CDF

ratio of cross sections should be close to 1

Polarized cross sections

next question: How does this "factorized picture"  work out w/ polarization?

with longitudinally polarized protons you can perform *four different experiments*:



label proton spin states by their helicity

$d\sigma(+,+)$ $\quad$ $d\sigma(+,-)$ $\quad$ $d\sigma(-,+)$ $\quad$ $d\sigma(-,-)$

so far we have discussed the unpolarized (*helicity-averaged*) cross section:

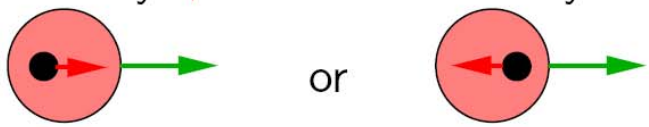
$$d\sigma \equiv \frac{1}{2} \cdot \frac{1}{2} \left[d\sigma(+,+) + d\sigma(+,-) + d\sigma(-,+) + d\sigma(-,-) \right]$$

let's have a closer look at what happens to $d\sigma(+,+)$ at the *partonic level* . . .

Polarized cross sections (cont.)

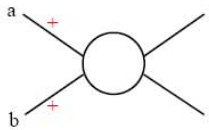
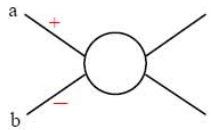
recall: in a proton with helicity $+$ we can find a parton with

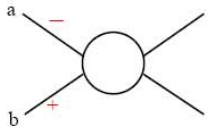
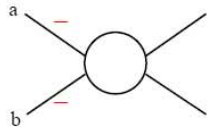
helicity $+$ or helicity $-$



$$f^{\rightarrow}_{\rightarrow} \equiv f^{+}_{+} \qquad\qquad f^{\rightarrow}_{\leftarrow} \equiv f^{+}_{-}$$

$\rightarrow$ hadronic $d\sigma(+,+)$ decomposes into *four partonic reactions*:

$$= f^{+}_{a,+} \otimes f^{+}_{b,+} \otimes$$  $$+ f^{+}_{a,+} \otimes f^{+}_{b,-} \otimes$$ 

$$+ f^{+}_{a,-} \otimes f^{+}_{b,+} \otimes$$  $$+ f^{+}_{a,-} \otimes f^{+}_{b,-} \otimes$$ 

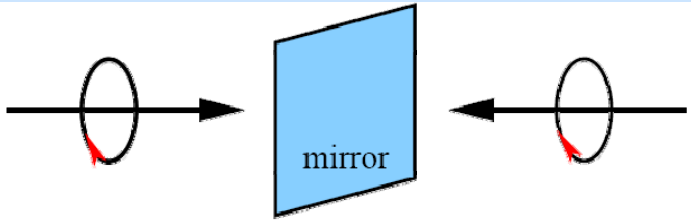
$$= f^{+}_{a,+} \otimes f^{+}_{b,+} \otimes d\hat{\sigma}(+,+) \quad + f^{+}_{a,+} \otimes f^{+}_{b,-} \otimes d\hat{\sigma}(+,-)$$

$$+ f^{+}_{a,-} \otimes f^{+}_{b,+} \otimes d\hat{\sigma}(-,+) \quad + f^{+}_{a,-} \otimes f^{+}_{b,-} \otimes d\hat{\sigma}(-,-)$$

and similarly for $d\sigma(+,-)$, $d\sigma(-,+)$, and $d\sigma(-,-)$

Polarized cross sections (cont.)

strong interactions invariant under *parity*:



parity flips helicity $\rightarrow f_+^+ = f_-^-$ and $f_-^+ = f_+^-$

This simplifies everything considerably and one obtains

$$
\begin{aligned}
d\sigma &\equiv \frac{1}{4}\left[d\sigma(+,+) + d\sigma(+,-) + d\sigma(-,+) + d\sigma(-,-)\right] \\
&= \frac{1}{4}\sum_{a,b}(f_{a,+}^+ + f_{a,-}^+) \otimes (f_{b,+}^+ + f_{b,-}^+) \\
&\qquad \otimes \left[d\hat{\sigma}(+,+) + d\hat{\sigma}(+,-) + d\hat{\sigma}(-,+) + d\hat{\sigma}(-,-)\right] \\
&\equiv \sum_{a,b} f_a \otimes f_b \otimes d\hat{\sigma}_{ab}
\end{aligned}
$$

. . . our well-known factorized result for the *unpolarized cross section*

Polarized cross sections (cont.)

But which combination of polarized cross sections contains the desired $\Delta f_{a,b}$?

This one . . .

$$
\begin{aligned}
d\Delta\sigma &\equiv \frac{1}{4}\left[d\sigma(+,+) - d\sigma(+,-) - d\sigma(-,+) + d\sigma(-,-)\right] \\
&= \frac{1}{4}\sum_{a,b}(f^{+}_{a,+} - f^{+}_{a,-}) \otimes (f^{+}_{b,+} - f^{+}_{b,-}) \\
&\qquad \otimes \left[d\hat{\sigma}(+,+) - d\hat{\sigma}(+,-) - d\hat{\sigma}(-,+) + d\hat{\sigma}(-,-)\right] \\
&\equiv \sum_{a,b} \Delta f_a \otimes \Delta f_b \otimes d\Delta\hat{\sigma}_{ab}
\end{aligned}
$$

- defines the spin-dependent $\vec{p}\vec{p}$ cross section measurable at RHIC
- for parity-conserved QCD processes this simplifies further:

$$
\begin{aligned}
d\Delta\sigma &= \frac{1}{2}\left[d\sigma(+,+) - d\sigma(+,-)\right] \quad (\mathrm{hadron-level}) \\
d\Delta\hat{\sigma} &= \frac{1}{2}\left[d\hat{\sigma}(+,+) - d\hat{\sigma}(+,-)\right] \quad (\mathrm{parton-level})
\end{aligned}
$$

Polarized cross sections (cont.)

upshot: to compute spin-dependent cross sections we have a similar factorized "picture" as discussed earlier

one only has to sprinkle some $\Delta's$ where appropriate

example: *polarized* high-p_T single-inclusive pion production $\vec{p}(P_a)\vec{p}(P_b) \to \pi(P^\pi)X$

long-distance

from exp.; μ-dep.: $d\sigma/d\mu = 0$ (pQCD)

$$\frac{d\Delta\sigma^{\vec{p}\vec{p}\to\pi X}}{dp_T d\eta} = \sum_{abc}\int dx_a\, dx_b\, dz_c\, \Delta f_a(x_a,\mu_f)\, \Delta f_b(x_b,\mu_f)\, D_c^\pi(z_c,\mu'_f) \times \frac{d\Delta\hat{\sigma}^{ab\to cX'}}{dp_T d\eta}(x_aP_a, x_bP_b, P^\pi/z_c, \mu_f, \mu'_f, \mu_r) + \mathcal{O}(\frac{\lambda}{p_T})^n$$

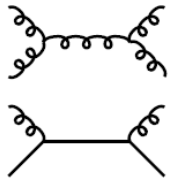
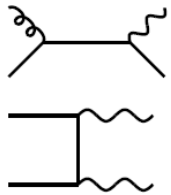
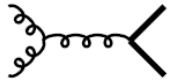
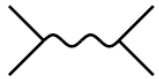
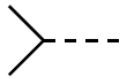
short-distance

calculable in pQCD: power series in α_s

power corrections

neglected

Cross sections relevant for RHIC spin

Reaction	Dom. partonic process	probes	LO Feynman diagram
$\vec{p}\vec{p} \to \pi + X$ [61, 62]	$\vec{g}\vec{g} \to gg$ $\vec{q}\vec{g} \to qg$	Δg	
$\vec{p}\vec{p} \to \text{jet(s)} + X$ [71, 72]	$\vec{g}\vec{g} \to gg$ $\vec{q}\vec{g} \to qg$	Δg	(as above)
$\vec{p}\vec{p} \to \gamma + X$ $\vec{p}\vec{p} \to \gamma + \text{jet} + X$ $\vec{p}\vec{p} \to \gamma\gamma + X$ [67, 73, 74, 75, 76]	$\vec{q}\vec{g} \to \gamma q$ $\vec{q}\vec{g} \to \gamma q$ $\vec{q}\vec{\bar{q}} \to \gamma\gamma$	Δg Δg $\Delta q, \Delta\bar{q}$	
$\vec{p}\vec{p} \to DX, BX$ [77]	$\vec{g}\vec{g} \to c\bar{c}, b\bar{b}$	Δg	
$\vec{p}\vec{p} \to \mu^+\mu^- X$ (Drell-Yan) [78, 79, 80]	$\vec{q}\vec{\bar{q}} \to \gamma^* \to \mu^+\mu^-$	$\Delta q, \Delta\bar{q}$	
$\vec{p}\vec{p} \to (Z^0, W^\pm)X$ $p\vec{p} \to (Z^0, W^\pm)X$ [78]	$\vec{q}\vec{\bar{q}} \to Z^0,\ \vec{q}'\vec{\bar{q}} \to W^\pm$ $\vec{q}'\bar{q} \to W^\pm, q'\vec{\bar{q}} \to W^\pm$	$\Delta q, \Delta\bar{q}$	

Δg

$\Delta\bar{q}$

Cross sections: Born approximation

at $\mathcal{O}(\alpha_s^2)$ (LO = "Born" = "tree-level") one has:

eight relevant QCD $2 \to 2$ parton-parton scattering processes $ab \to cd$

$$
\begin{aligned}
qq' &\to qq' \\
q\bar{q} &\to q'\bar{q}' \\
qq &\to qq \\
q\bar{q} &\to q\bar{q} \\
q\bar{q} &\to gg \\
gg &\to q\bar{q} \\
qg &\to qg \\
gg &\to gg
\end{aligned}
$$

unpol.: boils down to *four* different processes $qq' \to qq'$, $qq \to qq$, $q\bar{q} \to gg$, $gg \to gg$

all other processes related by *crossing symmetry*: $ab \to cd \leftrightarrow a\bar{d} \to c\bar{b}$

polarized: cannot use crossing, e.g., $\vec{q}\vec{\bar{q}} \to gg \neq \vec{q}\vec{g} \to qg$

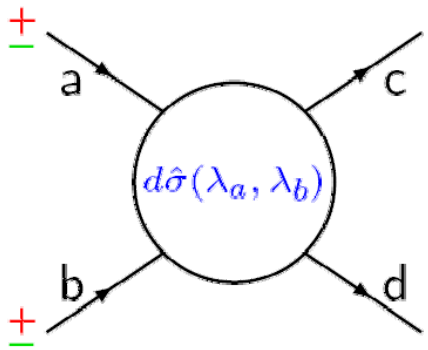
Cross sections: kinematics

computation of LO cross sections is pretty straightforward:

$$a(p_a, \lambda_a) + b(p_b, \lambda_b) \to c(p_c) + d(p_d)$$

$\uparrow$ $\uparrow$

helicities $\lambda_{a,b} = \pm 1$



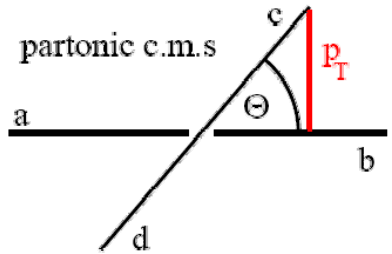
- the helicity-dependent partonic QCD cross sections can be written as

$$d\hat{\sigma}(\lambda_a, \lambda_b) = d\hat{\sigma} + \lambda_a \lambda_b \, d\Delta\hat{\sigma}$$

$\to$ no single-spin asymmetries for QCD processes at Born-level

- dependence of $d\hat{\sigma}(\lambda_a, \lambda_b)$ on kinematics conveniently expressed in terms of

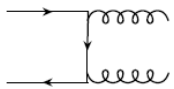
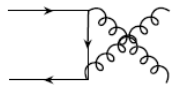
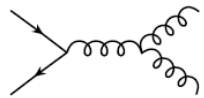
$$\begin{aligned} s &\equiv (p_a + p_b)^2 \\ t &\equiv (p_a - p_c)^2 = -\sqrt{s}\, p_T\, e^{-\eta} \\ u &\equiv (p_a - p_d)^2 = -\sqrt{s}\, p_T\, e^{\eta} \\ \text{where} \quad & \eta = -\ln \tan(\Theta/2) \end{aligned}$$



Cross sections: computation (basic steps)

example: $q(p_a, \lambda_a)\, \bar{q}(p_b, \lambda_b) \to g(p_c)\, g(p_d)$

1. step: write down all relevant *Feynman diagrams*

$M_1 =$  $M_2 =$  $M_3 =$ 

2. step: get amplitudes M_i by applying *Feynman rules* for vertices, propagators, and external lines

3. step: compute total amplitude squared $MM^* = |M|^2 = |M_1 + M_2 + M_3|^2$

4. step: attach flux factor $1/2s$ for incoming partons, a *phase space factor* $\frac{d^3 p_i}{(2\pi)^3 (2E_i)}$ for each produced parton, and four-momentum conservation $(2\pi)^4 \delta^{(4)}(p_a + p_b - p_c - p_d)$ to obtain

$$d\hat{\sigma}(s, t, u, \lambda_a, \lambda_b) = \frac{1}{2s} |M|^2(s, t, u, \lambda_a, \lambda_b) \frac{d^3 p_c}{(2\pi)^3 (2E_c)} \frac{d^3 p_d}{(2\pi)^3 (2E_d)} (2\pi)^4 \delta^{(4)}(p_a + p_b - p_c - p_d)$$

Diracology

after some algebra and "*Dirac*ology" one obtains the full list of LO QCD processes:

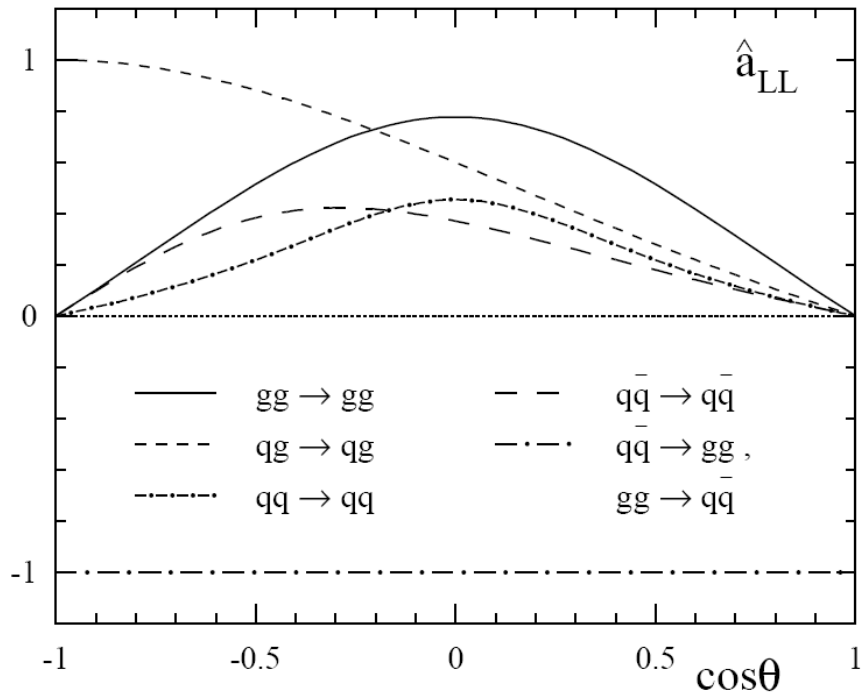
process	$\lvert M\rvert^2(+,+)/g_s^4$	$\lvert M\rvert^2(+,-)/g_s^4$
$qq' \to qq'$	$8s^2/(9t^2)$	$8u^2/(9t^2)$
$q\bar{q} \to q'\bar{q}'$	0	$8(t^2+u^2)/(9s^2)$
$qq \to qq$	$8(s^2/t^2+s^2/u^2-2s^2/(3tu))/9$	$8(u^2/t^2+t^2/u^2)/9$
$q\bar{q} \to q\bar{q}$	$8s^2/(9t^2)$	$8(u^2/t^2+t^2/s^2+u^2/s^2-2u^2/(3st))/9$
$q\bar{q} \to gg$	0	$64(t^2+u^2)/(27ut)-16(t^2+u^2)/(3s^2)$
$gg \to q\bar{q}$	0	$(u^2+t^2)/(3ut)-3(t^2+u^2)/(4s^2)$
$qg \to qg$	$2s^2/t^2-8s^2/(9us)$	$2u^2/t^2-8u^2/(9us)$
$gg \to gg$	$9(2s^2/(ut)-su/t^2-st/u^2)/2$	$9(6-2s^2/(ut)-2u/t^2-st/u^2-2ut/s^2)/2$

and similar for $qg \to q\gamma$ (prompt photons), $q\bar{q} \to l\bar{l}$ (Drell-Yan), $gg, q\bar{q} \to Q\bar{Q}$ (heavy flavors), . . .

Partonic spin asymmetries at LO

some observations:

- $gg \to gg$ dominates at $\Theta = 90°$ ($\eta = 0$; $t = u = -s/2$) followed by $qg \to qg$
- most processes have $\hat{a}_{\rm LL} \equiv d\Delta\hat{\sigma}/d\hat{\sigma} > 0$, $q\bar{q}$ annihilation has -1



"analyzing power"

Limitations of LO estimates

LO pQCD results/predictions

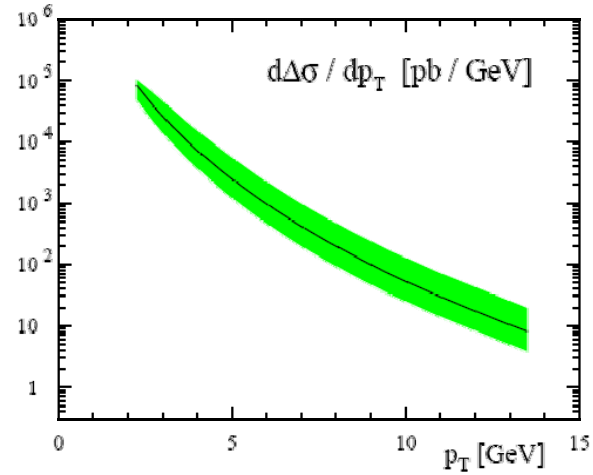
- are useful for quick & dirty estimates of *qualitative* features of a process

- suffer from large theoretical uncertainties

  example:

  $\vec{p}(P_a)\vec{p}(P_b) \to \pi(P^\pi)X$ @ high-p_T

  band: variation $p_T/2 \leq \mu_f \leq 2p_T$



upshot: NLO corrections are in general a must if it comes to numbers

- expect much reduced dependence on unphysical scales $\mu_{f,r}$

- NLO corrections are often sizable

- may effect the sensitivity to Δg and its extraction from data

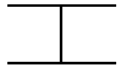
going beyond the LO is in

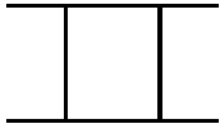
every aspect a major enterprise

...here are the basic steps

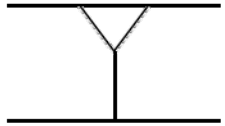
NLO corrections in a nutshell

at $\mathcal{O}(\alpha_s^3)$ (NLO) one has to consider:

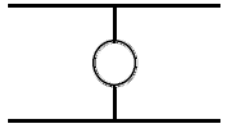
- one-loop (virtual) corrections to all LO $2 \to 2$  processes



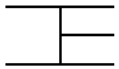
"box"



"vertex"



"selfenergy"

- all conceivable $2 \to 3$  parton-parton scattering processes

  this includes additional gluon emission to existing LO processes

  $$qq' \to qq'g,\ q\bar{q} \to ggg,\ gg \to ggg, \text{ etc.}$$

  as well as genuine NLO processes not possible at $\mathcal{O}(\alpha_s^2)$

  $$qg \to qq'\bar{q}',\ qg \to qq\bar{q}, \text{ etc.}$$

NLO corrections in a nutshell (cont.)

complication: all diagrams have infinities in four dimensions

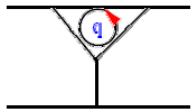
three types of singularities will appear:

recall: Lectures 1 & 2

- *ultraviolet (UV) singularities*:

  due to the integration of the unobserved loop-momentum up to infinity

$$\int_0^\infty d^4q$$



- *infrared (UV) singularities*:

  emission of soft gluons ($E_g \to 0$)

$$\frac{1}{p_q \cdot p_g} = [E_q E_g (1 - \cos\Theta_{qg})]^{-1}$$

- *collinear or mass singularities*:

  collinear emission ($\Theta_{qg} \to 0$)

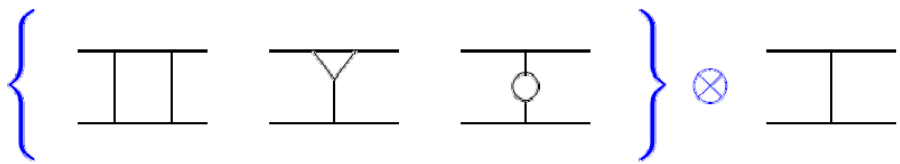
$$\frac{1}{p_q \cdot p_g} = [E_q E_g (1 - \cos\Theta_{qg})]^{-1}$$

  (not possible for massive quarks)

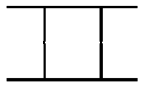
NLO corrections in a nutshell (cont.)

a closer look at virtual corrections:

at $\mathcal{O}(\alpha_s^3 = g_s^6)$ only the interference of 1-loop and Born amplitudes contributes:



- kinematics described by s, t, u like for a $2 \to 2$ Born process
- IR+UV divergencies show up as $1/\varepsilon$ and $1/\varepsilon^2$
- main technical challenge: computation of loop-integrals

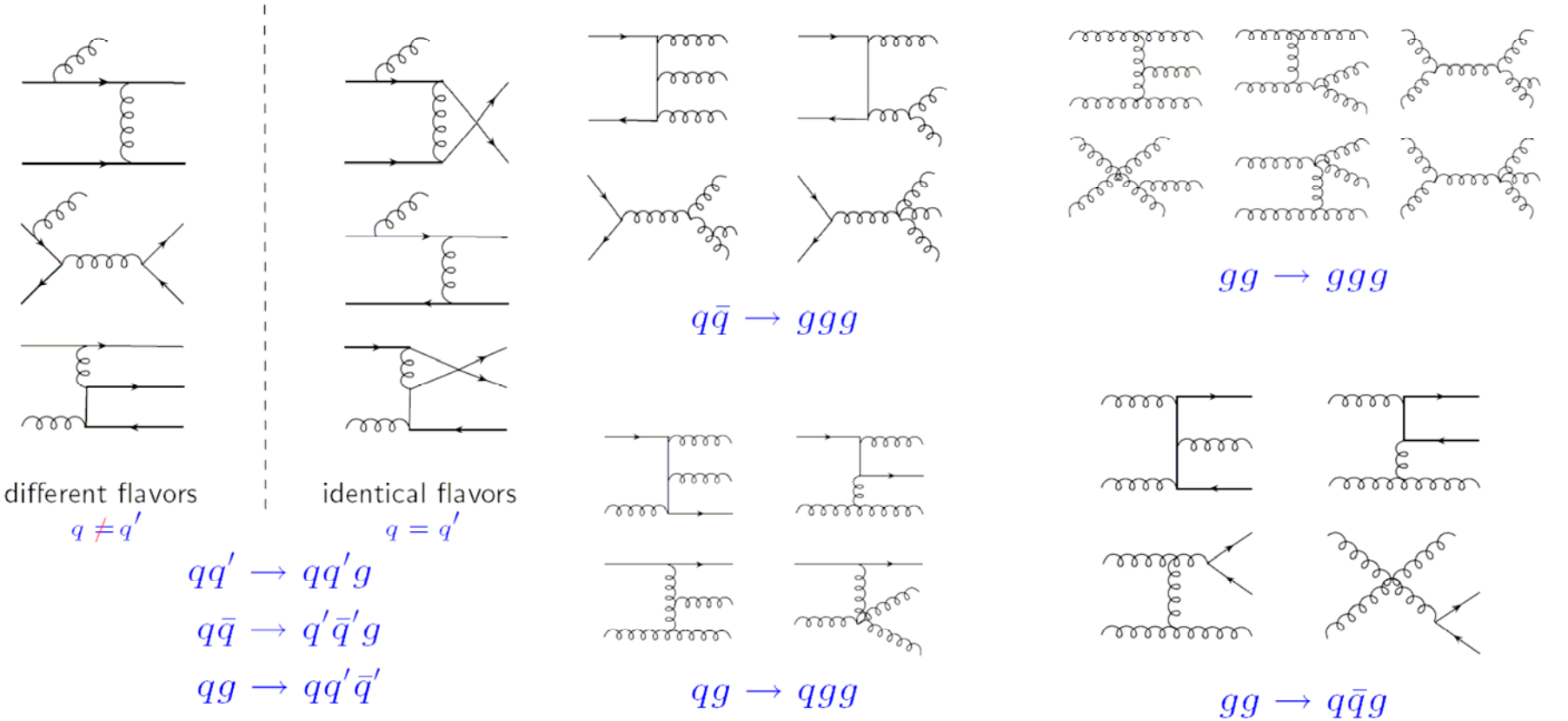
e.g.:  $\sim \int d^n q \, \frac{q^\mu [q^\nu [q^\rho]]}{q^2 (q+p_a)^2 (q+p_a-p_c)^2 (q+p_a-p_c-p_d)^2}$

can be reduced to large set of n-dim. scalar integrals Passarino, Veltman

renormalization: UV poles "used" to define physical coupling $g_s \to$ UV finite

NLO corrections in a nutshell (cont.)

a closer look at $2 \to 3$ corrections: here is a list of some typical diagrams



NLO corrections in a nutshell (cont.)

main technical challenge here:

- integration over phase space $\frac{d^{(n-1)}p_i}{(2\pi)^{(n-1)}(2E_i)}$ of all unobserved partons

example: calculation of *single-inclusive* pion cross section:

typical $2 \to 3$ process: $gg \;\to\; q \;\; (\bar{q}g)$

fragments: $q \to \pi X$ integrated out

"Monte-Carlo approach":

idea: separate off divergent regions of p.s.

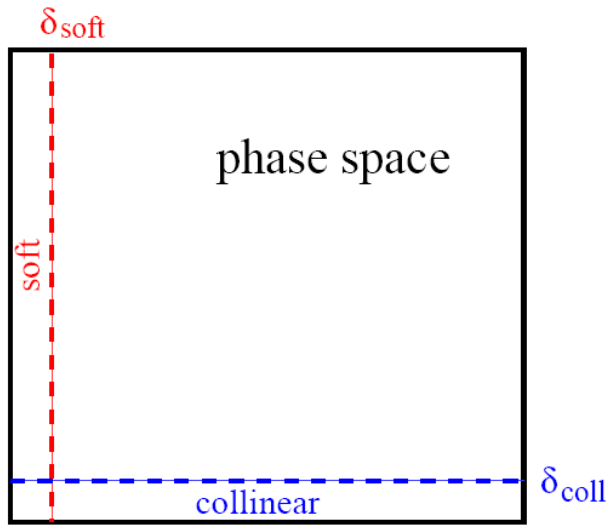
integrate "rest" in 4 dimensions

✓ suitable for complicated observables

✓ exp. cuts easy to build in

× delicate numerical cancellations

× slow → not very suitable for fitting pdfs



NLO corrections in a nutshell (cont.)

"largely analytical approach":

idea: do entire phase space integration analytically in n dimensions

[p.s. best parameterized by two angles $\theta_{1,2}$ in rest frame of the two unobserved partons]

$$d\Delta\hat{\sigma}_{2\to 3} \sim \ldots \int d\theta_1 d\theta_2 \sin^{1-2\varepsilon}\theta_1 \sin^{-2\varepsilon}\theta_2 \, \Delta \, |M_{2\to 3}|^2$$

extensive partial fractioning boils everything down to

$$I^{(k,l)} = \int \frac{d\theta_1 \, \sin^{1-2\varepsilon}\theta_1 \, d\theta_2 \, \sin^{-2\varepsilon}\theta_2}{(1+\cos\theta_1)^k (1 + A\cos\theta_1 + B\sin\theta_1\cos\theta_2)^l}$$

which can be done *analytically* $\rightarrow$ IR+collinear poles

strength/weakness of method:

- ✓ straightforward, fast numerics
- ✓ well suited for pdf analyses
- × usually limited to single-incl. x-secs
- × exp. cuts not easy to build in

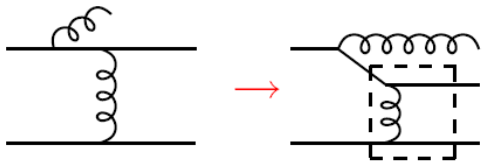
NLO corrections in a nutshell (cont.)

final step: cancellation of remaining singularities *then* $\varepsilon \to 0$

- UV singularities have already been taken care of by renormalizing α_s
- IR singularities cancel in sum of one-loop and $2 \to 3$ contributions

  [essence of Kinoshita, Lee, Nauenberg & Bloch, Nordsieck theorems for "IR-safe" observables]
- remaining collinear singularities have to be removed by factorization

  [factorization = renormalization of bare parton densities and fragmentation fcts.]

e.g.:  $\sim \frac{1}{\varepsilon} \int dx \Delta P_{qq}(x) \Delta\hat{\sigma}_{qq \to qq}$

introduce two arbitrary factorization scales μ_f and μ'_f

initial-/final-state singularities

$\swarrow$ $\searrow$

$\Delta f(x, \mu_f)$ $\quad$ $D_f^\pi(z, \mu'_f)$

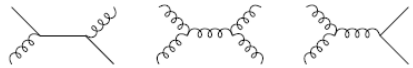
good news: most of the NLO corrections have been computed recently

evol. kernels	ΔP_{ij}	NLO		Mertig, van Neerven; Vogelsang
hadrons	$\vec{p}\vec{p} \to H + X$	NLO	•	De Florian; Jäger, Schäfer, MS, Vogelsang
	$p\vec{p} \to \vec{H} + X$	NLO		Jäger et al. (soon)
jets	$\vec{p}\vec{p} \to \text{jet(s)} + X$	NLO	•	De Florian et al.; Jäger,MS, Vogelsang
prompt γ	$\vec{p}\vec{p} \to \gamma + X$	NLO	•	Gordon, Vogelsang; Contogouris et al.
	$\vec{p}\vec{p} \to \gamma\gamma + X$	NLO		Coriano, Gordon
	$p\vec{p} \to \vec{\gamma} + X$	NLO		Vogelsang
γ + jet	$\vec{p}\vec{p} \to \gamma + \text{jet} + X$	NLO		Gordon
γ + charm	$\vec{p}\vec{p} \to \gamma + c + X$	NLO		Berger et al. ($m_c = 0$)
heavy quarks	$\vec{p}\vec{p} \to Q\bar{Q}X$	NLO	•	Bojak, MS
Drell-Yan	$\vec{p}\vec{p} \to (\gamma^*)X$	NLO		Weber; Gehrmann;
		NNLO		Smith et al.
vector bosons	$\vec{p}\vec{p} \to (Z^0, W^\pm)X$	NLO		Weber; Gehrmann
	$p\vec{p} \to (Z^0, W^\pm)X$	NLO	•	Weber; Gehrmann

some results coming up

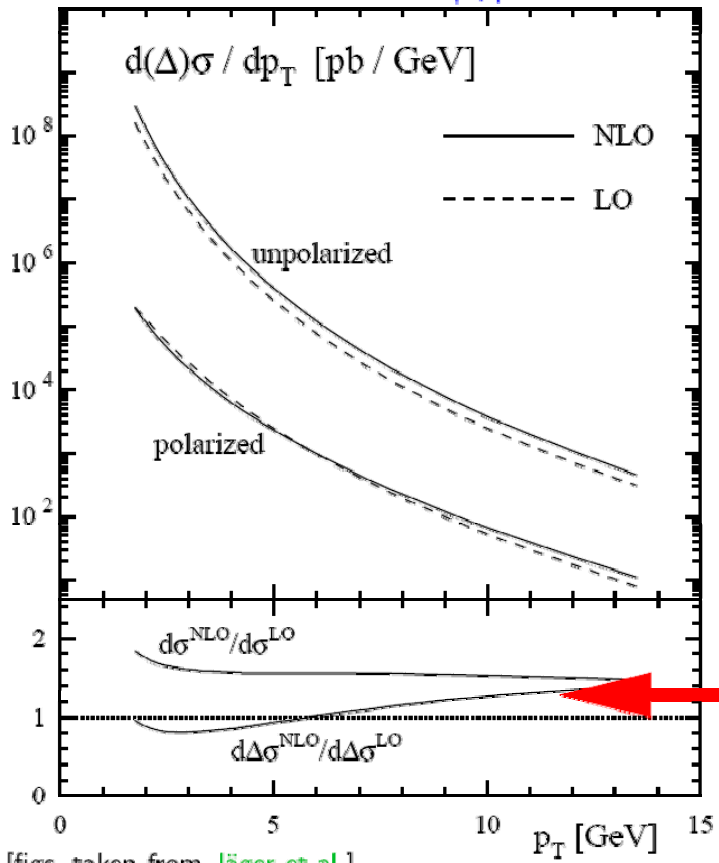
Processes one-by-one

- **high-p_T hadrons:**

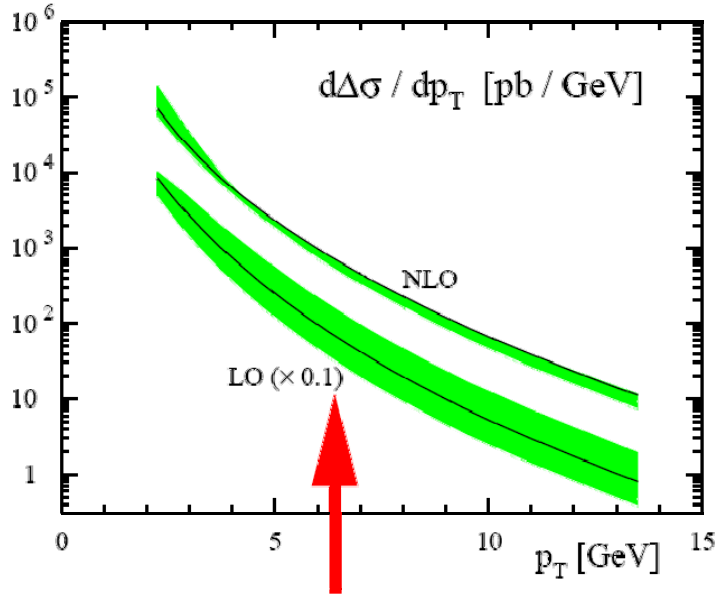


NLO: Jäger, MS, Schäfer, Vogelsang; de Florian

$\sqrt{S} = 200\,\mathrm{GeV}$, $|\eta| \leq 0.38$



[figs. taken from Jäger et al.]



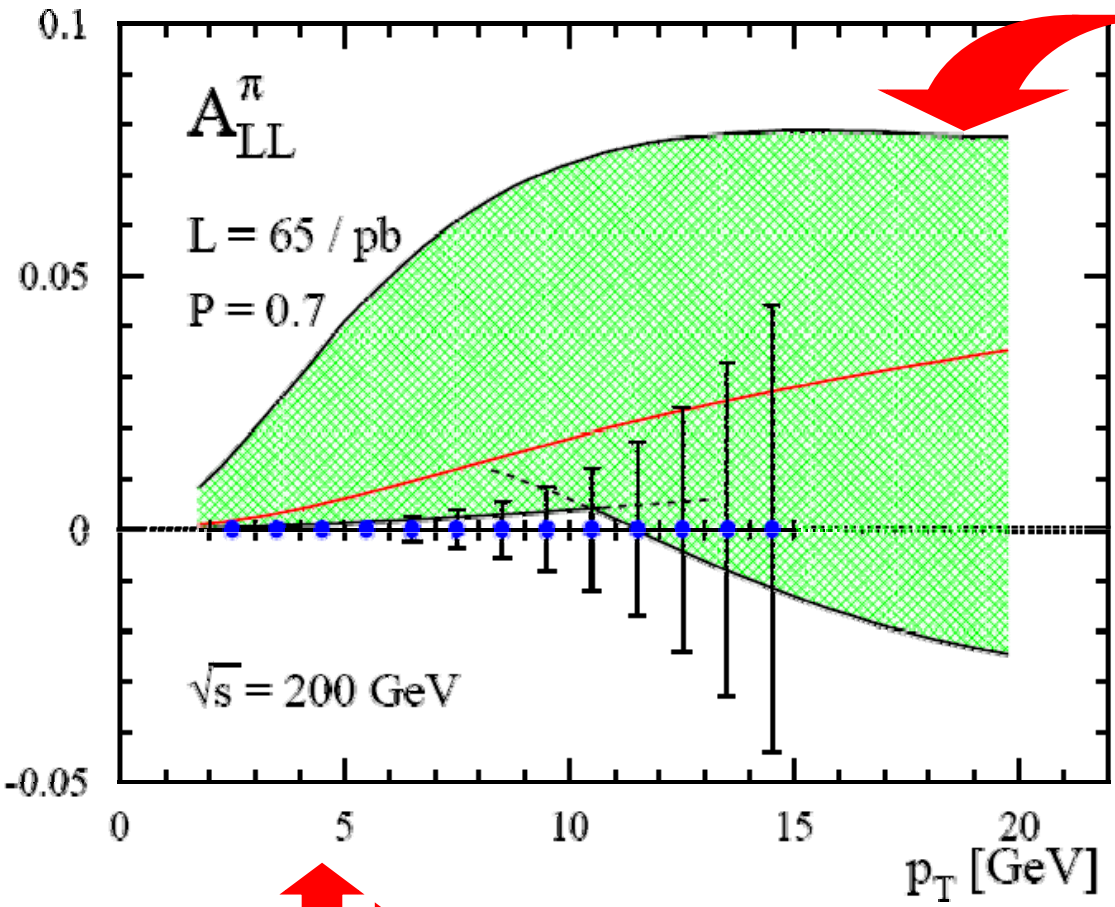
μ_f dependence much reduced in NLO

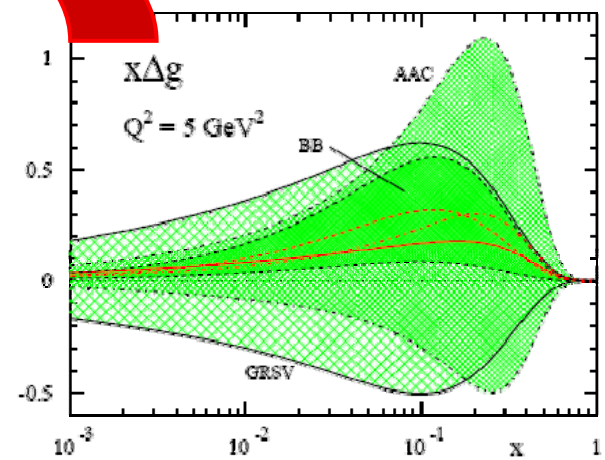
NLO corrections different for $d\Delta\sigma$ and $d\sigma$

$\rightarrow$ do not cancel in $A_{LL} = d\Delta\sigma/d\sigma$

Processes one-by-on (cont.)

expectations for spin asymmetry:

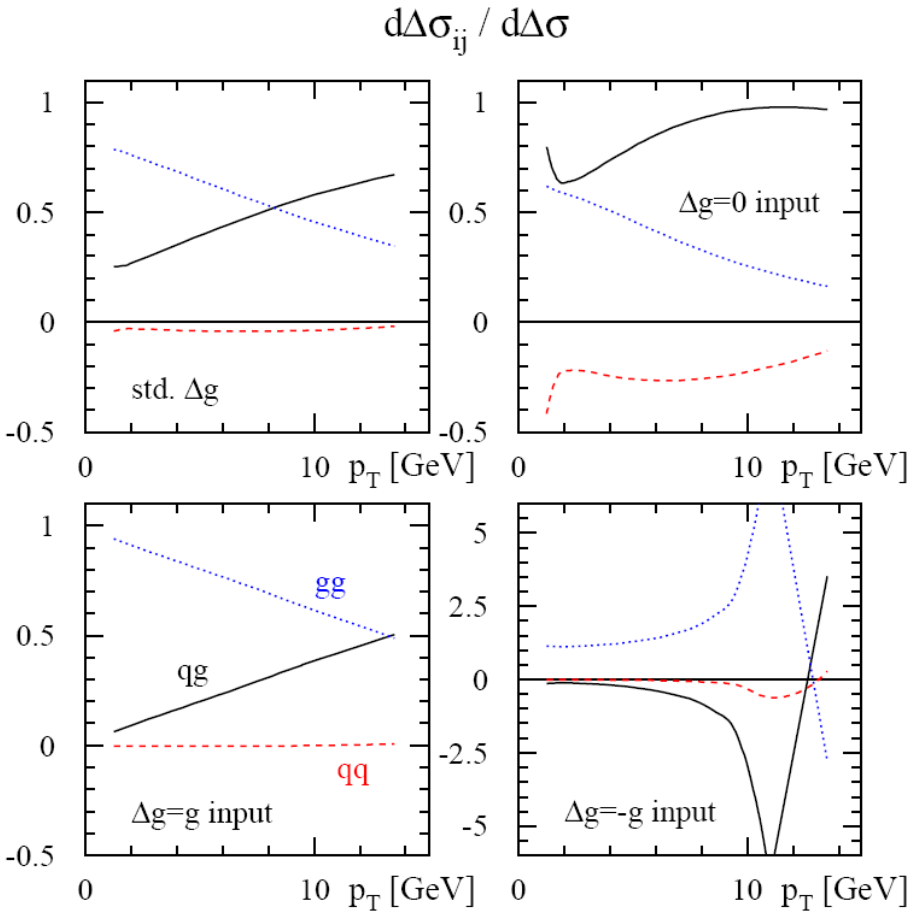




based on very different gluon polarizations Δg

note: A_{LL} positive at small p_T (dominance of gg scattering)

Processes one-by-one (cont.)



subprocess contributions:

fairly independently on what we assume about Δg we find:

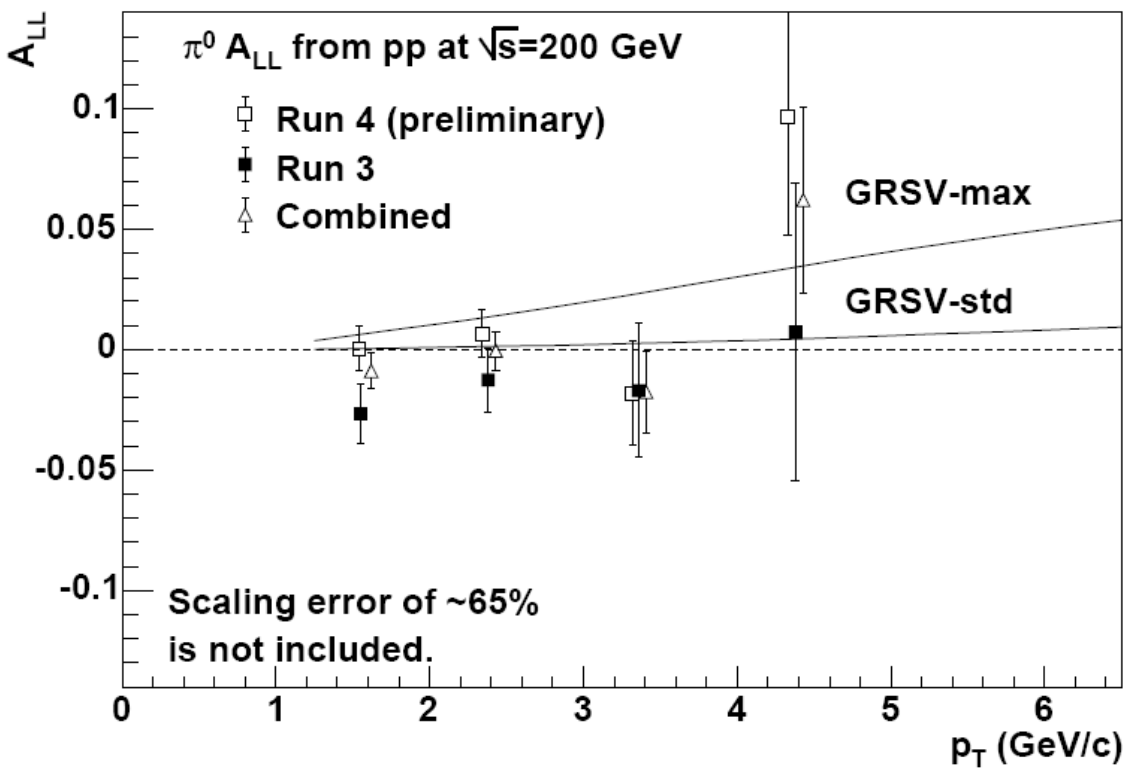
gg processes:
dominate for $p_T \lesssim 10\,\mathrm{GeV}$

qg processes:
take over for $p_T \gtrsim 10\,\mathrm{GeV}$

qq processes:
always small unless p_T very large

Processes one-by-one (cont.)

first results on A_{LL} from PHENIX:

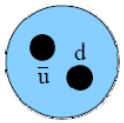


conclusions: ???

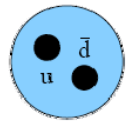
Processes one-by-one (cont.)

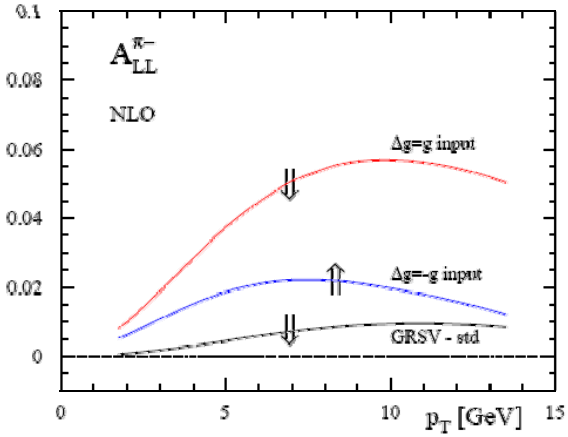
Jäger, MS, Vogelsang

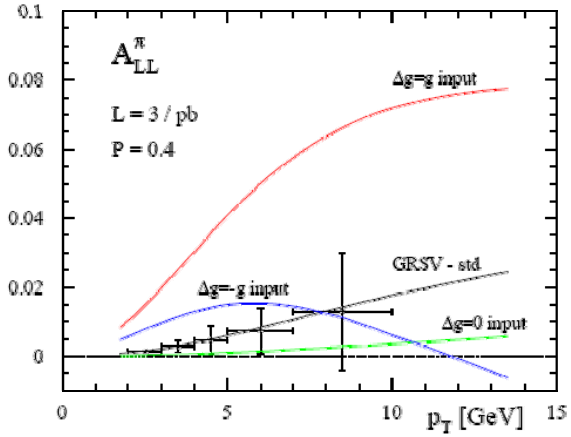
with more luminosity PHENIX can go to higher p_T at $\langle\eta\rangle \simeq 0$ *plus* $A_{LL}^{\pi^\pm}$ vs. $A_{LL}^{\pi^0}$

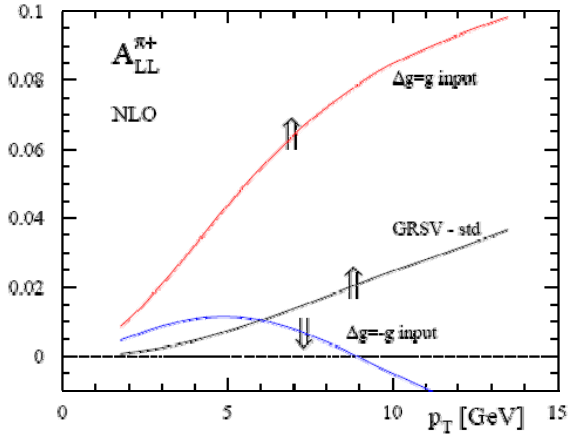
π^- 

π^0

π^+ 





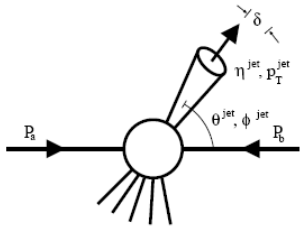


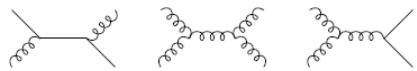
idea: qg starts to dominate for $p_T \gtrsim 5\,\mathrm{GeV}$ and $D_u^{\pi^+} > D_u^{\pi^0} > D_u^{\pi^-}$, $D_g^{\pi^+} = D_g^{\pi^-}$

expect: sensitivity to sign of Δg, e.g., positive Δg: $A_{LL}^{\pi^+} > A_{LL}^{\pi^0} > A_{LL}^{\pi^-}$

Processes one-by-one (cont.)

- **high-p_T jets:**

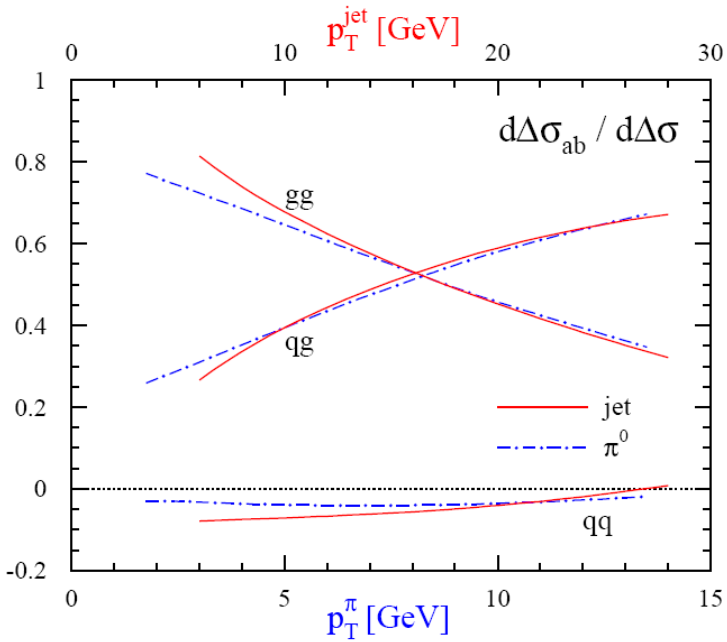




NLO: Jäger, MS, Vogelsang; de Florian, Frixione, Signer, Vogelsang

jet = bunch of particles in a small pencil-like cone; all final-state sing. cancel

jet production proceeds through the same partonic subprocesses as π-production:



π's have roughly $\langle z \rangle \simeq 0.5$:

$\rightarrow$ π with $p_T \simeq$ jet with $2p_T$

comparison to hadrons:

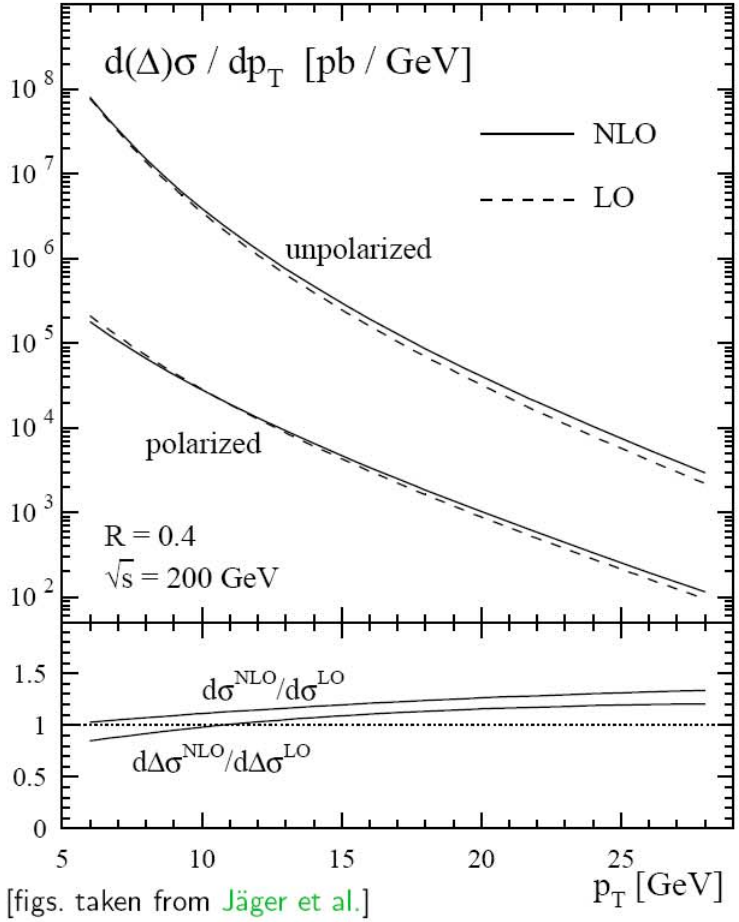
✓ much higher rates

✓ no uncertainties from $D(z)$

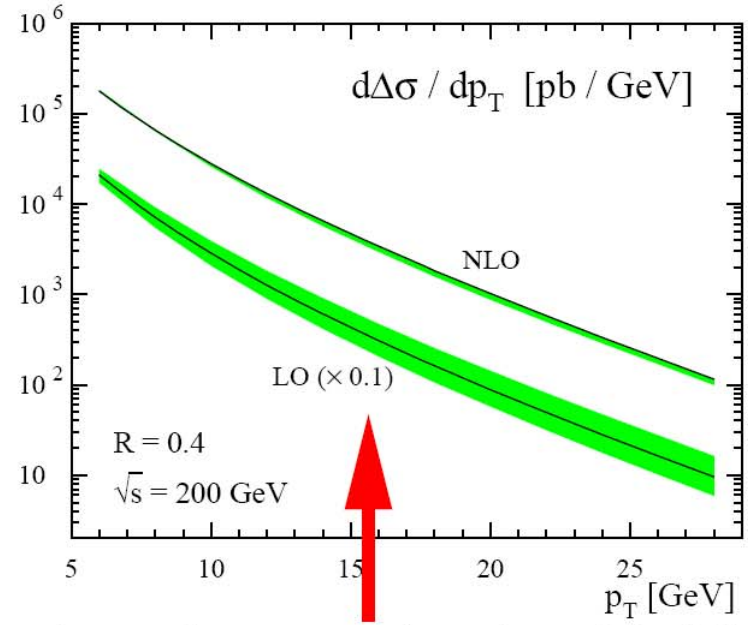
× dependence on precise definition of jet

Processes one-by-one (cont.)

pQCD results for jet-production at $\sqrt{S} = 200\,\mathrm{GeV}$, $R_{\mathrm{cone}} = 0.4$ (SCA), $|\eta| \leq 1$:



[figs. taken from Jäger et al.]

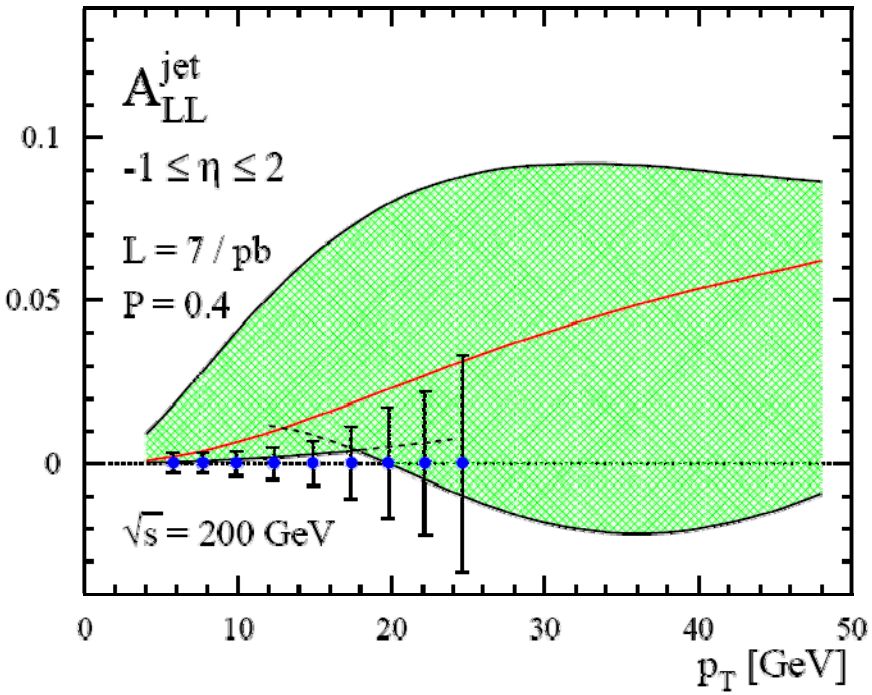


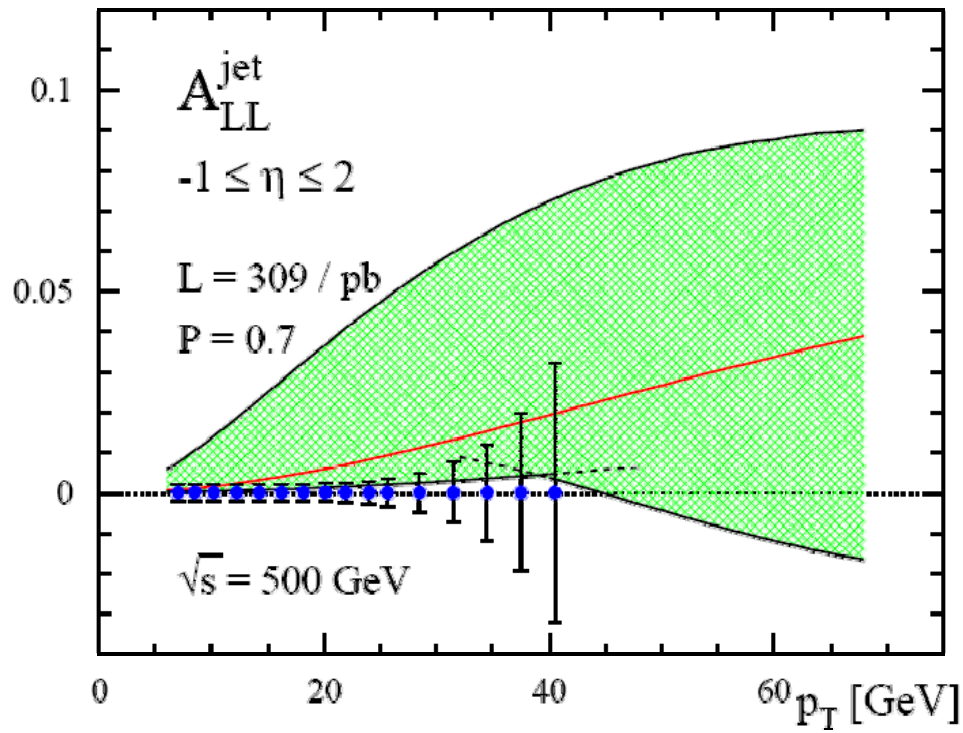
μ_f dependence much reduced in NLO

theoretical uncertainties even smaller than for hadrons

Processes one-by-one (cont.)

expectations for spin asymmetries:





again: no sensitivity to sign of Δg at small p_T

summary on inclusive jets and hadrons:

- offer excellent short-term prospects for a first determination of Δg

  caveat: for sign of Δg one needs measurements up to $p_T \simeq 10 \div 15\,\mathrm{GeV}$

- very important: measurement of *absolute* cross sections not "only" of A_{LL}

  [even unpolarized pp cross sections never before measured at $\sqrt{S} = 200 \div 500\,\mathrm{GeV}$]

- unpolarized "benchmark x-secs" agree very well with pQCD so far

  [even down to unexpectedly small p_T]

long-term goals:

need much higher luminosities, beam polarizations, and/or $\sqrt{S} = 500\,\mathrm{GeV}$

- production of prompt photons $pp \to \gamma X$:

  a supposedly clean signature for Δg

- production of heavy flavors $pp \to c\bar{c}X$, $pp \to b\bar{b}X$:

  extend Δg measurements to smaller x

- study of $W^{\pm}$ boson production:

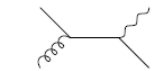
  a very clean tool to gather further information on Δq and $\Delta\bar{q}$ separately

- look for weird things beyond the Standard Model:

  this is always an issue at colliders . . .

Processes one-by-one (cont.)

- **high-p_T prompt photons (plus jets):**



NLO: Gordon, Vogelsang; Contogouris et al.; Frixione, Vogelsang

idea: a "classical", supposedly clean signature for Δg

· only two rather than eight LO QCD processes: $qg \to q\gamma$ (dominant) and $q\bar{q} \to g\gamma$

· Δg does not enter squared $\to$ sign of Δg not an issue here

on the downside:

· photons are much less abundant $\to$ need a lot of luminosity

· prompt photon data abandoned in all unpolarized pdf analyses CTEQ, MRST

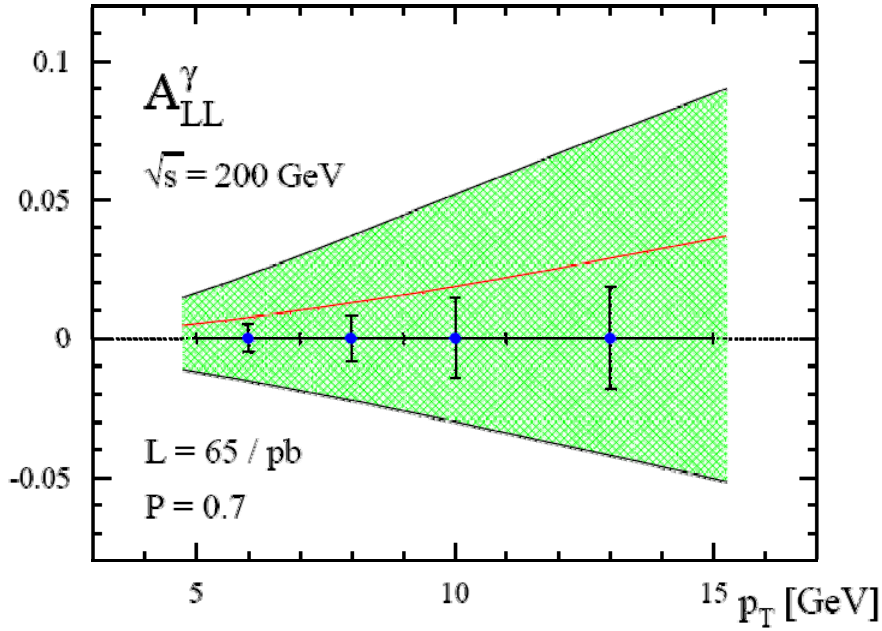
reason: some trouble with fixed target data; high p_T jets give better constraints

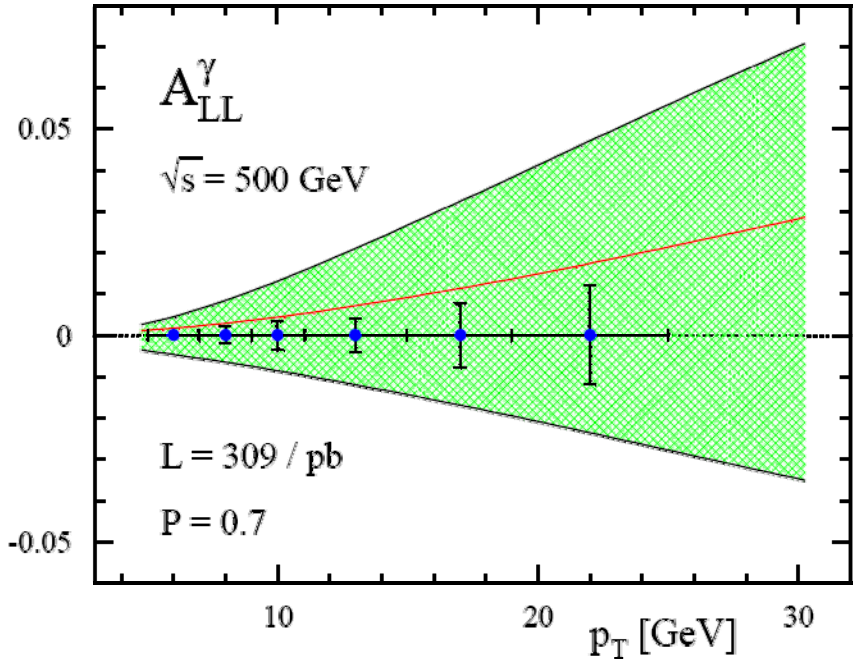
positive aspect: problems have pushed theoretical efforts to go beyond NLO

[soft gluon/threshold (and k_T) resummations]Laenen et al.; Catani et al.; Li; Kidonakis Owens; . . .

Processes one-by-one (cont.)

expectations for spin asymmetries (single-inclusive photons):

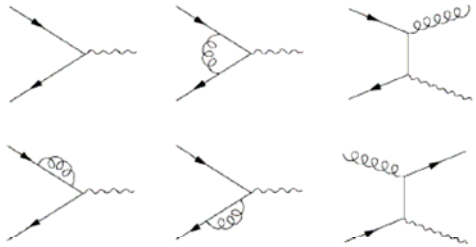




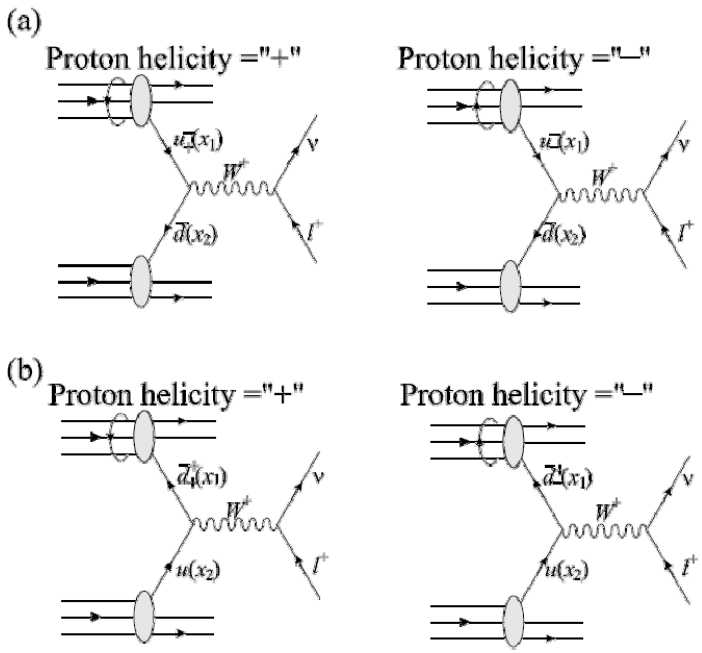
Processes one-by-one (cont.)

- **$W^{\pm}$ boson production:**

  NLO: Weber; Kamal; Gehrmann; Smith et al.



idea: weak interactions maximally parity violating



$W^{\pm}$ selects parton helicity $\rightarrow$ A_L sufficient

in LO one has e.g. Leader, Sridhar; Bourrely, Soffer

$$A_L^{W^+}(y) = \frac{-\Delta u(x_a)\bar{d}(x_b) + \Delta\bar{d}(x_a)u(x_b)}{u(x_a)\bar{d}(x_b) + \bar{d}(x_a)u(x_b)}$$

where $x_{a,b} = \frac{M_W}{\sqrt{S}} e^{\pm y}$

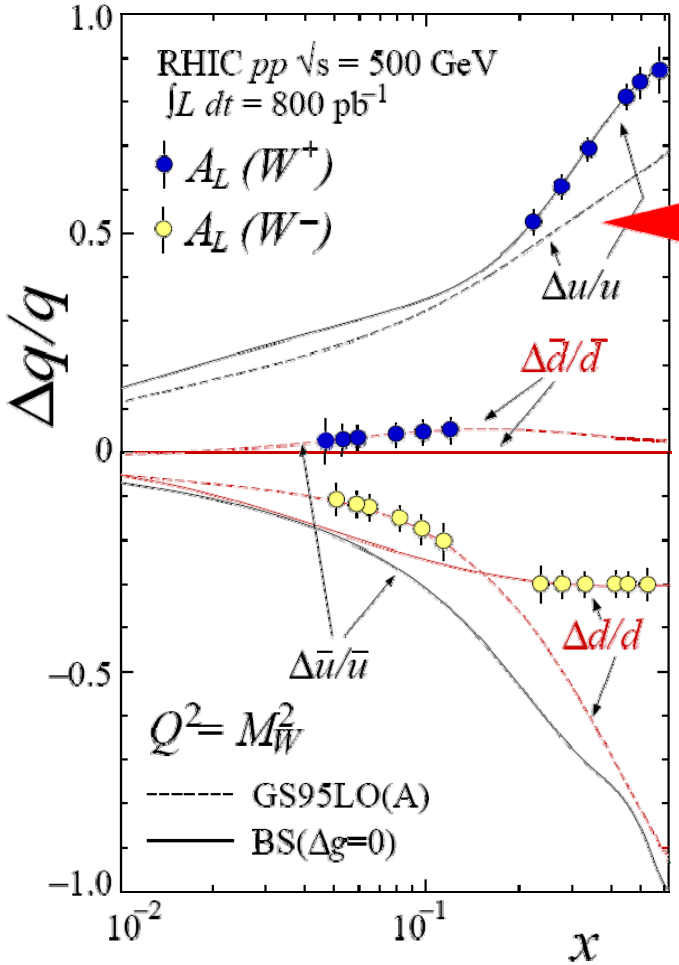
y large & $\left\{ \begin{array}{l} \text{pos. } (x_a > x_b) \\ \text{neg. } (x_a < x_b) \end{array} \right.$: $A_L^{W^+} \simeq \left\{ \begin{array}{l} -\Delta u(x_a)/u(x_a) \\ \Delta\bar{d}(x_a)/\bar{d}(x_a) \end{array} \right.$

$A_L^{W^-}(y)$: $u \leftrightarrow d$ $\rightarrow$ probes $\Delta d/d$ and $\Delta\bar{u}/\bar{u}$

ideal tool for a separation of Δu, Δd, $\Delta\bar{u}$, and $\Delta\bar{d}$

Processes one-by-one (cont.)

expected sensitivity on $\Delta q/q$ from $A_L^{W^\pm}(y)$: [fig. taken from Bunce et al.]



for RHIC kinematics:

Δu, Δd probed at $x \gtrsim 0.2$

$\Delta\bar{u}$, $\Delta\bar{d}$ probed at $0.045 \lesssim x \lesssim 0.12$

complication: detector issues

have to observe $W^\pm$ via decay but

neither PHENIX nor STAR are hermetic

→ missing E_T cannot be reconstructed

→ get $y(W)$ only from charged lepton

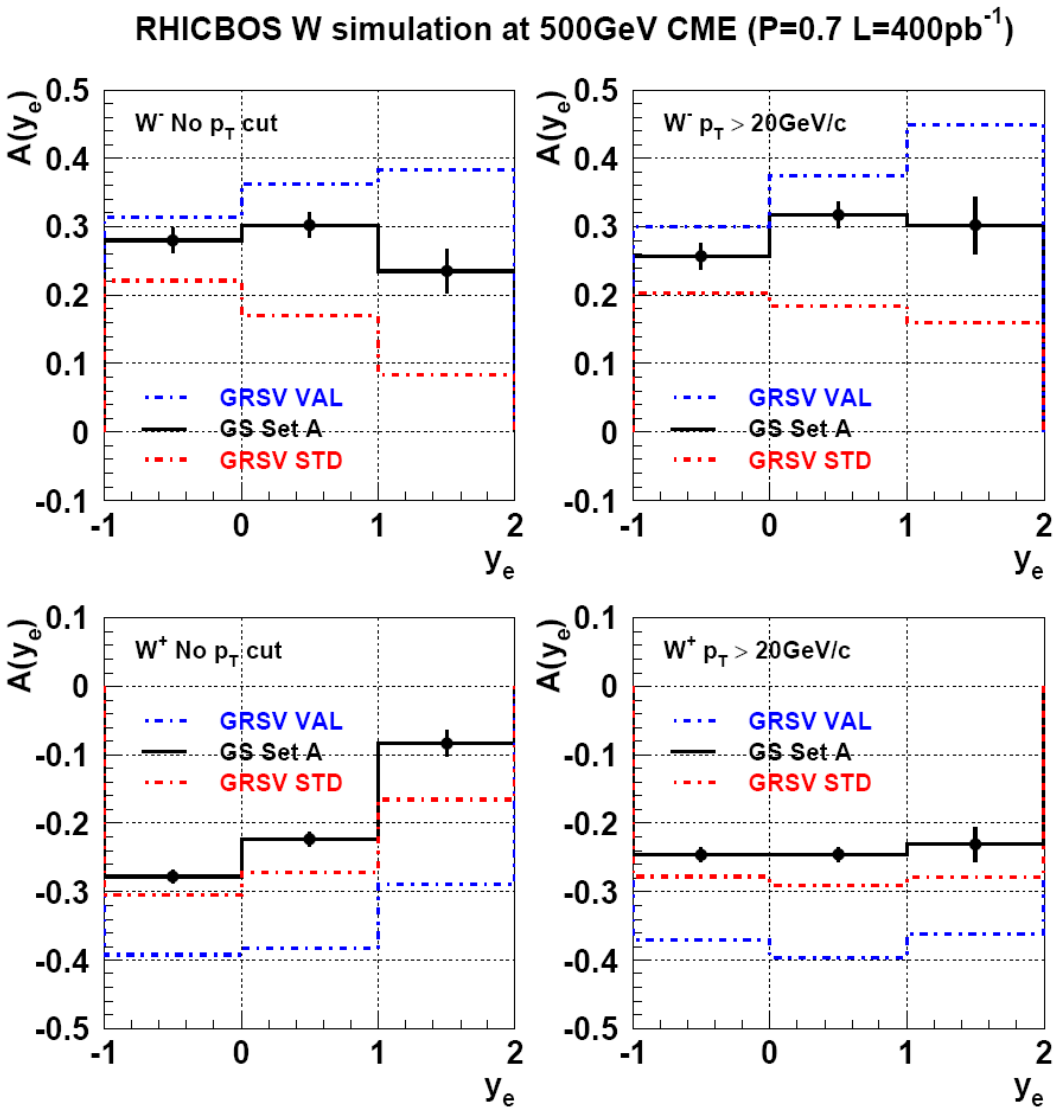
good news:

full NLO *lepton-level* MC available

Nadolsky, Yuan

Processes one-by-one (cont.)

Nadolsky, Yuan



I better stop here .....

.... thanks for your interest in pQCD

... and your patience