RIKEN PHENIX SPIN FEST @ WAKO

Lectures on Perturbative QCD
or
from basic principles to current applications

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Outline of the lectures

Lecture 1: basic ideas; exploring the QCD final state

Lecture 2: origin of singularities; infrared safety

Lecture 3: QCD initial-state; factorization; renormalization

Lecture 4: more on factorization & renormalization, pdfs

Lecture 5: applications in hadron-hadron collisions, spin

Literature & useful links

Lecture notes & write-ups:

Wu-Ki Tung: *Perturbative QCD and the Parton Structure of the Nucleon* (from www.cteq.org)

!!! Dave Soper: *Basics of QCD Perturbation Theory* (hep-ph/9702203)

J. Collins, D. Soper, G. Sterman: *Factorization of Hard Processes in QCD* (hep-ph/0409313)

CTEQ Collaboration: *Handbook of Perturbative QCD* (Rev. Mod. Phys. 67 (1995) 157 or from www.cteq.org)

Talks & lectures on the web:

!!! annual CTEQ summer schools (tons of material !): *www.cteq.org*

1st summer school on QCD Spin Physics @ BNL: *www.bnl.gov/qcdsp*

Lecture 4

more on factorization & renormalization
pdf's and their evolution

today we try to answer these questions:

- What does renormalization?
- Pdfs are universal, so what is their formal definition?
- What should I do with all these arbitrary scales?
- What is a factorization scheme?

afterwards (Lecture 5) comes the "fun part":

we apply all the concepts we have learned
to study hadron-hadron collisions

Reminder Lecture 3: Factorization

the physical structure fct. is independent of μ_f
(this will lead to the concept of renormalization group eqs.)

$$F_2(x,Q^2) = x \sum_{a=q,\bar{q}} e_q^2 \int_x^1 \frac{d\xi}{\xi} f_a(\xi,\mu_f^2) \left[\delta(1-\frac{x}{\xi}) + \frac{\alpha_s(\mu_r)}{2\pi} \underbrace{\left[P_{qq}\left(\frac{x}{\xi}\right) \ln\frac{Q^2}{\mu_f^2} + (C_2^q - z_{qq})\left(\frac{x}{\xi}\right)\right]}_{\text{short-distance "Wilson coefficient"}}\right]$$

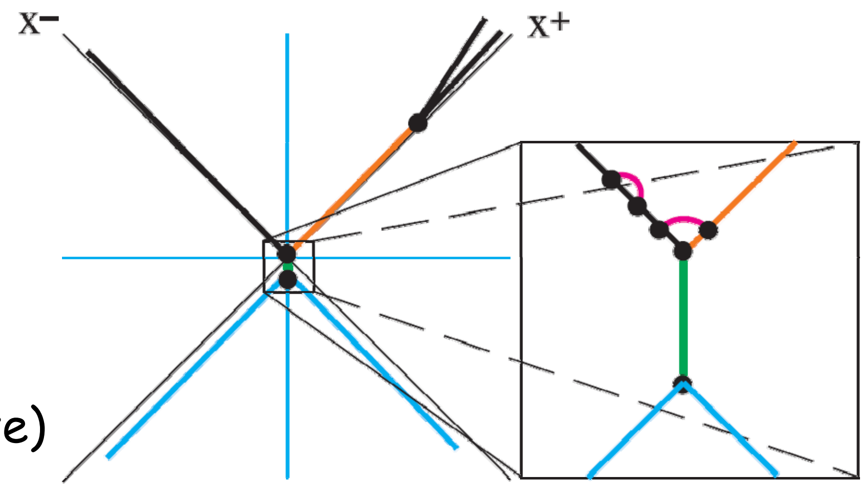
yet another scale: μ_r
due to the renormalization
of ultraviolet divergencies

What renormalization does

so far we have discussed infinities related to
long-time/distance physics (soft/collinear emissions)

these singularities cancel for infrared safe observables
or can be systematically removed (factorization) by hiding them
in some non-perturbative parton or fragmentation functions

there is also a class of ultraviolet infinities related to the smallest time scales/distances:



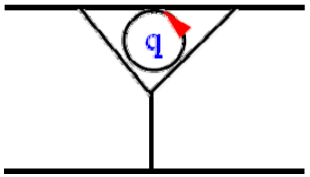
we can insert perturbative corrections
to vertices and propagators ("loops")

loop momenta can be very large (infinite)
leading to virtual fluctuations on very
short time scales/distances

What renormalization does (cont.)

we cannot compute virtual corrections without introducing a suitable regulator for divergent loop integrations

$$\int_0^\infty d^4q$$



again, you have the choice:

- **ultraviolet cut-off scale M**
  intuitive and transparent
  but works only in NLO
- 4-2ε **dimensional regularization**
  calculations more involved
  works in general

again, infinities in disguise as large logarithms or as 1/ε

again, the regulator should drop out in the end after renormalization

What renormalization does (cont.)

factorization and renormalization play similar roles
- at opposite ends of the energy range of pQCD

in IR factorization bare parton densities absorb all long-distance physics and acquire a factorization scale dependence

similarly

in UV renormalization the bare strong coupling absorbs all very-short time physics and acquires a renormalization scale dep.

both scale parameters μ_f and μ_r are not intrinsic to QCD
they tell us how we did the factorization/renormalization

Renormalization: executive summary

ALRIGHT RUTH, I ABOUT GOT THIS ONE RENORMALIZED.

Running coupling revisited

we use α_s (and pdfs) to absorb UV (IR) divergencies

→ we cannot predict the value of α_s (or pdfs) in pQCD

however, a key prediction of pQCD is their **scale variation**

the physical idea behind this is beautiful & simple:

a *measurable* cross section dσ has to be *independent* of μ_r and μ_f

$$\mu_{r,f}\frac{d\sigma}{d\mu_{r,f}} = \frac{d\sigma}{d\ln\mu_{r,f}} = 0$$

→ renormalization group equations

→ all we need is a reference measurement at some scale μ_0

Running coupling revisited (cont.)

recipe: compute a QCD cross section dσ to a certain order in pQCD

use $\frac{d\sigma}{d\ln\mu_r} = 0$ to derive the RGE for α_s

$$\boxed{\frac{d\alpha_s(\mu)}{d\ln\mu} = -\underbrace{\frac{\beta_0}{2\pi}\alpha_s^2}_{\text{LO}} - \underbrace{\frac{\beta_1}{4\pi^2}\alpha_s^3}_{\text{NLO}} - \underbrace{\frac{\beta_2}{64\pi^3}\alpha_s^4}_{\text{NNLO}} + \ldots}$$

depending on how many orders you have considered you find

$$\beta_0 = 11 - \frac{2}{3}N_f,\ \ \beta_1 = 51 - \frac{19}{3}N_f,\ \ldots$$

NNNLO available!
Larin, van Ritbergen, Vermaseren

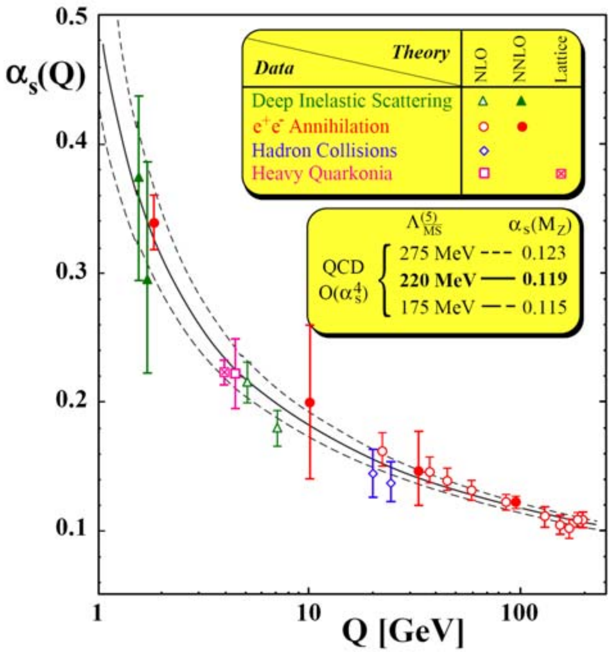
perhaps the best known quantity in pQCD

let's solve the RGE for α_s at lowest order ...

Running coupling revisited (cont.)

integrating the RGE from some boundary condition $\alpha_s(\mu_0)$ yields:

$$\boxed{\alpha_s(\mu) = \frac{\alpha_s(\mu_0)}{1 + \frac{\beta_0}{4\pi}\alpha_s(\mu_0)\ln\left(\frac{\mu^2}{\mu_0^2}\right)} \equiv \frac{4\pi}{\beta_0 \ln\left(\mu^2/\Lambda_{QCD}^2\right)}}$$



often convenient to introduce the QCD scale parameter Λ_{QCD}

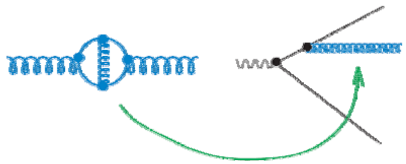
β_0 positive

we confirm asymptotic freedom!

α_s decreases as μ increases

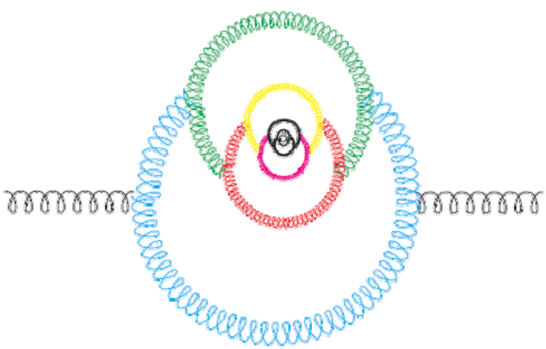
Running coupling revisited (cont.)

physical interpretation of the RGE solution: $\alpha_s(\mu) = \dfrac{\alpha_s(\mu_0)}{1 + \frac{\beta_0}{4\pi}\alpha_s(\mu_0)\ln\left(\frac{\mu^2}{\mu_0^2}\right)}$



the effect of small time physics $\Delta t \ll 1/\mu_r$ is removed from the perturbative calculation and accounted for by adjusting α_s

the RGE sums all leading effects of short time fluctuations:



expand $\alpha_s(\mu)$ for small $\alpha_s(\mu_0)$:

$$\begin{aligned}\alpha_s(\mu) \;&\simeq\; \alpha_s(\mu_0) - \left(\frac{\beta_0}{\pi}\right)\ln\frac{\mu^2}{\mu_0^2}\,\alpha_s^2(\mu_0) \\ &+\; \left(\frac{\beta_0}{\pi}\right)^2 \ln^2\frac{\mu^2}{\mu_0^2}\,\alpha_s^3(\mu_0) + \ldots\end{aligned}$$

The science and art of choosing scales

let us look at the most transparent **example: $e^+e^- \rightarrow$ hadrons**

$$\sigma_{\rm tot} = \frac{4\pi\alpha^2}{s} N_c \left(\sum_q e_q^2\right) [1 + \Delta_{\rm QCD}]$$

QCD corrections can be expressed as an power series:

$$\begin{aligned}\Delta_{\rm QCD}(\mu_r) &= \frac{\alpha_s(\mu_r)}{\pi} + [1.4092 + 1.9167 \ln \frac{\mu_r^2}{s}] \left(\frac{\alpha_s(\mu_r)}{\pi}\right)^2 \\ &+ [-12.805 + 7.8179 \ln \frac{\mu^2}{s} + 3.674 \ln^2 \frac{\mu^2}{s}] \left(\frac{\alpha_s(\mu_r)}{\pi}\right)^3 + \ldots\end{aligned}$$

in lowest order we have no clue what the scale is

in higher orders both α_s and the coeff. fct. depend logarith. on the scale: cancellations

&

μ_r should be of the order of s

Choosing scales (cont.)

lessons to learn: (applies in general, also for μ_f)

- arbitrary scales should be of the order of the hard scale characterizing the process (here s) to avoid large logs in the coeff. fcts. which might spoil the pert. series
- if we truncate the series $\Delta_{\text{QCD}} = \sum_{n=1}^{\infty} c_n(\mu_r)\alpha_s^n(\mu_r)$ after the first N terms, there will be a residual scale dependence

$$\frac{d}{d\ln\mu_r}\sum_{n=1}^{N} c_n(\mu_r)\alpha_s^n(\mu_r) \sim \mathcal{O}\left(\alpha_s^{N+1}(\mu_r)\right)$$

➡ the harder we work, the less the final result depends on the artificial scales

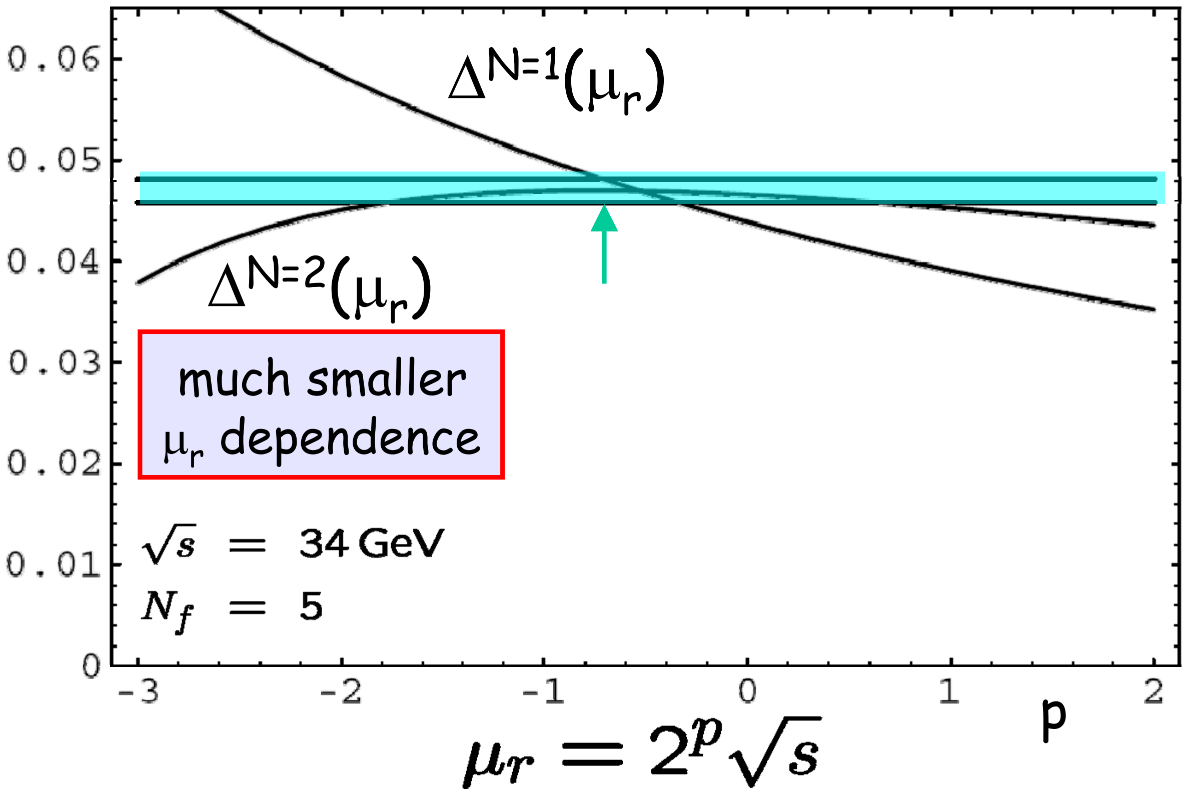
prime motivation for NLO pQCD calculations!

Choosing scales (cont.)

numerical example (again $e^+e^- \to$ hadrons):

from Dave Soper's CTEQ lectures

have to think about the theor. error caused by truncation of the series



error *estimate*:

central μ_r value:

$$\frac{d\Delta(\mu_r)}{d\ln\mu_r} = 0$$

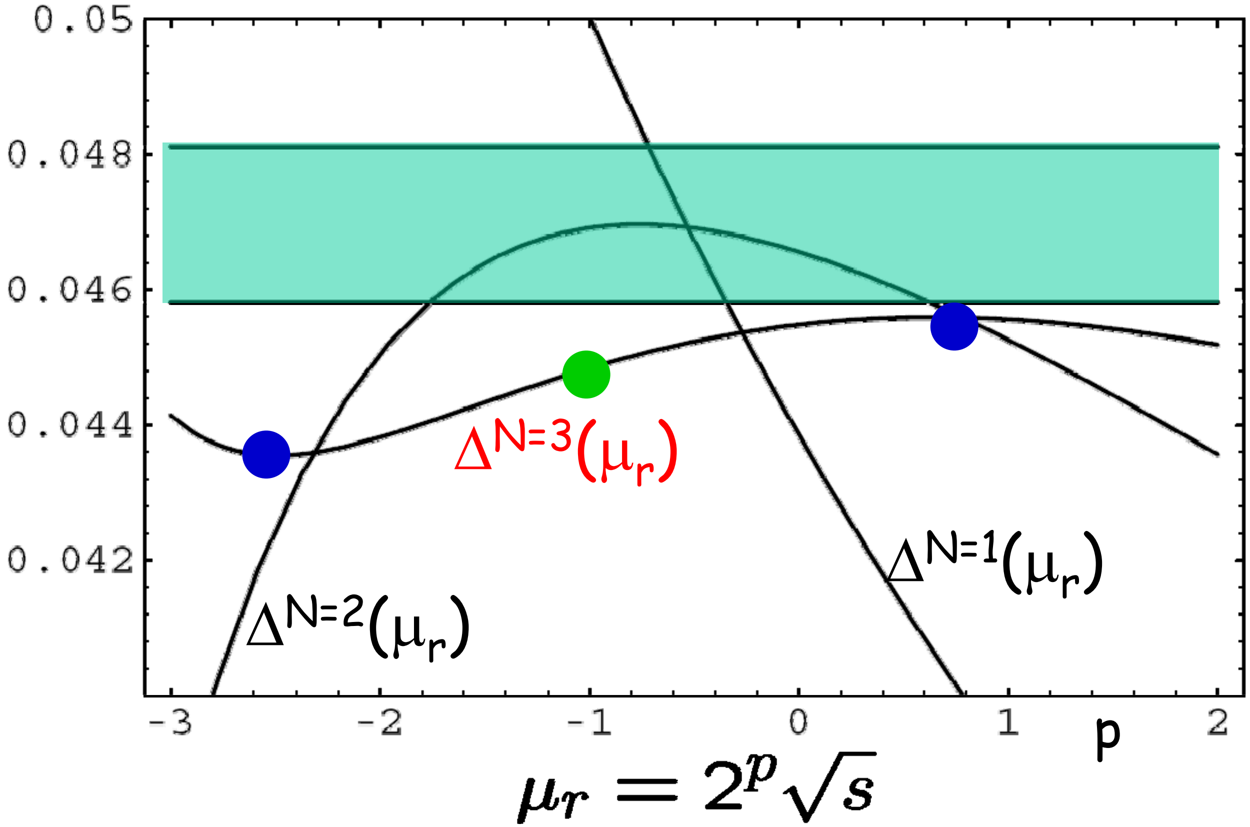
error band:

vary μ_r by factor 2 around centr. value

Choosing scales (cont.)

How good is such an error estimate?

add another order (N=3) and zoom in ...



$\Delta^{N=3}$ less scale dep. as expected

two places where μ_r-dep. stationary

take average as new central value

error estimate was slightly off

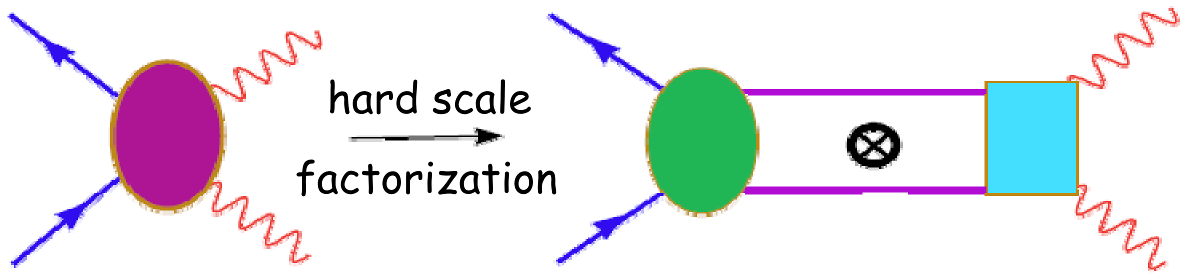
Reminder Lecture 3: Factorization

$$
\begin{aligned}
F_2(x,Q^2) \; &= \; x \sum_{a=q,\bar{q}} e_q^2 \int_x^1 \frac{d\xi}{\xi} f_a(\xi,\mu_f^2) \\
&\left[\delta\left(1-\frac{x}{\xi}\right) + \frac{\alpha_s(\mu_r)}{2\pi} \underbrace{\left[P_{qq}\left(\frac{x}{\xi}\right) \ln\frac{Q^2}{\mu_f^2} + \left(C_2^q - z_{qq}\right)\left(\frac{x}{\xi}\right)\right]}_{\text{short-distance "Wilson coefficient"}}\right]
\end{aligned}
$$

choice of the factorization scheme

Factorization schemes

pictorial representation of factorization:



e.g. $F_2(x,Q^2) \sim \sum_a f_a(x,\mu_f) \otimes \hat{F}_2^a(x,\frac{Q}{\mu_f})$

the separation between long- and short-distance physics is not unique

1. **choice of** μ_f: defines borderline between long-/short-distance
2. **choice of scheme**: re-shuffling finite pieces

Factorization schemes (cont.)

whatever we do, we *demand* that the measurable F_2 does *not* change

1. **choice of** μ_f: renormalization group/evolution eqs. for pdfs (similar to $\alpha_s(\mu_r)$ discussed above) $\longrightarrow$ next topic!

2. **what exactly is a factorization scheme?**

recall: we absorb all long-distance singularities into the bare pdfs

but we can exploit the freedom to absorb more finite stuff

this is controlled by coefficients z_{ij} above

and simply *defines* a factorization scheme!

important: the scheme *in*dependence of physical quantities is guaranteed as long as pdfs and hard cross sections are combined in the same factorization scheme

Definition of parton densities

parton densities are **universal**

there must be a process-independent **precise definition**

we have to specify the factorization scheme:

most common choice: modified *minimal* (!) subtraction ($\overline{\mathrm{MS}}$) scheme
(very closely linked to dim. regularization)

less often used: DIS scheme

the other "extreme" – a maximal subtraction scheme for DIS
(DIS stc. fct. F_2 retains its LO form)

classic definition of pdfs through their Mellin moments through Wilson's operator product expansion

Bardeen, Buras, Duke, Muta

Definition of parton densities (cont.)

we will use a **more physical formulation** in Bjorken-x space:
matrix elements of *bi-local operators* on the light-cone

Curci, Furmanski, Petronzio; Collins, Soper

for quarks:

$$f_a(\xi,\mu_f) = \frac{1}{2}\int \frac{dy^-}{2\pi} e^{i\xi p^+ y^-} \langle p|\overline{\Psi}_a(0,y^-,\vec{0})\gamma^+ \mathcal{F}\Psi_a(0)|p\rangle_{\overline{\mathrm{MS}}}$$

Fourier transform such that $k^+ = \xi\, p^+$

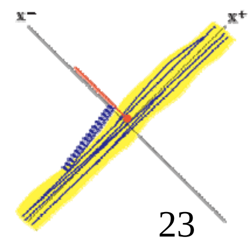
recreates quark at $x^+=0$ and $x^-=y^-$

annihilates quark at $x^\mu=0$

interpretation as number operator in "$A^+ = 0$ gauge"

in general we need a "**gauge link**" for a gauge invariant definition:

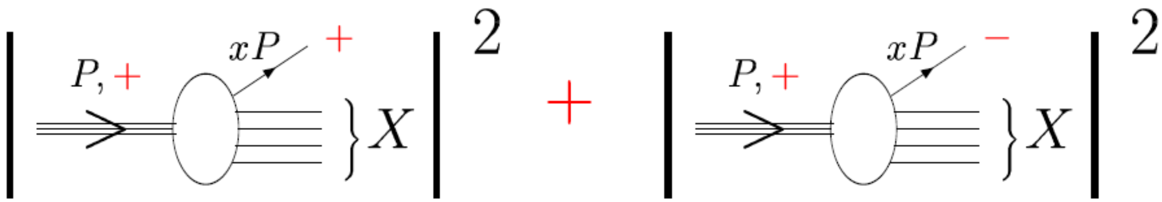
$$\mathcal{F} = \mathcal{P}\exp\left(-ig\int_0^{y^-} dz^- A_c^+(0,z^-,\vec{0})T_c\right)$$



Definition of parton densities (cont.)

pictorial representation:

$q(x) =$

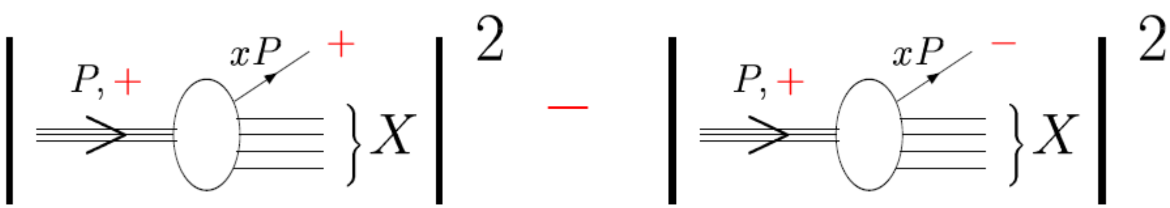


remarks:

- similar definition for the gluon distribution
- easy to include spin ($\gamma^{+} \to \gamma^{+}\gamma_5$)

see, e.g., D. Soper
hep-lat/9609018

$\Delta q(x) =$



- only collinear factorization discussed (gauge links become very important for def. of transverse momentum dep. pdfs)

Reminder Lecture 3: Factorization

the physical structure fct. is independent of μ_f
(this will lead to the concept of renormalization group eqs.)

both, pdf's and the short-dist. coefficient depend on μ_f
(choice of μ_f: shifting terms between long- and short-distance parts)

$$F_2(x,Q^2) = x \sum_{a=q,\bar{q}} e_q^2 \int_x^1 \frac{d\xi}{\xi} f_a(\xi,\mu_f^2) \underbrace{\left[\delta(1-\frac{x}{\xi}) + \frac{\alpha_s(\mu_r)}{2\pi}\left[P_{qq}\left(\frac{x}{\xi}\right)\ln\frac{Q^2}{\mu_f^2} + (C_2^q - z_{qq})\left(\frac{x}{\xi}\right)\right]\right]}_{\text{short-distance "Wilson coefficient"}}$$

Scale evolution of parton densities

the last thing we have to discuss is the μ_f **dependence**:

main idea as for RGE for $\alpha_s(\mu_r)$

a physical quantity, e.g., F_2, should not depend on μ_f

look at simplified version (one quark flavor)

$$\underbrace{F_2(x,Q^2)}_{\text{physical}} = \underbrace{q(x,\mu_f)}_{\text{pdf}} \otimes \underbrace{\hat{F}_2(x,\frac{Q}{\mu_f})}_{\text{hard cross section}}$$

convenient to first turn nasty convolution $\otimes$ into simple product

tool: Mellin transform

$$\boxed{f(n) \equiv \int_0^1 dx\, x^{n-1} f(x)}$$

Scale evolution of parton densities (cont.)

this is often an important tool – let's see how it works:

$$\int_0^1 dx x^{n-1} \left[\int_x^1 \frac{dy}{y} f(y) g\left(\frac{x}{y}\right) \right]$$

$$= \int_0^1 dx x^{n-1} \int_0^1 dy \int_0^1 dz\, \delta(x - zy) f(y) g(z)$$

$$= \int_0^1 dy \int_0^1 dz (zy)^{n-1} f(y) g(z)$$

$$= \underline{f(n)\, g(n)} \quad ✓$$

so we find

$$F_2(x, Q^2) = \underbrace{q(x, \mu_f) \otimes \hat{F}_2(x, \frac{Q}{\mu_f})}_{\text{convolution}} \xrightarrow{\text{Mellin}} \underbrace{q(n, \mu_f) \hat{F}_2(n, \frac{Q}{\mu_f})}_{\text{simple product}}$$

Scale evolution of parton densities (cont.)

now we can compute $\dfrac{dF_2(x,Q^2)}{d\ln\mu_f} = 0$:

$$\Leftrightarrow \quad \frac{dq(n,\mu_f)}{d\ln\mu_f}\widehat{F}_2(n,\frac{\mu_f}{Q}) + q(n,\mu_f)\frac{d\widehat{F}_2(n,\frac{\mu_f}{Q})}{d\ln\mu_f} = 0$$

$$\Leftrightarrow \quad \frac{d\ln\widehat{F}_2(n,\frac{Q}{\mu_f})}{d\ln\frac{Q}{\mu_f}} = \underbrace{\frac{d\ln q(n,\mu_f)}{d\ln\mu_f} \equiv -\gamma_{qq}(n)}_{\text{DGLAP evolution equation}}$$

anomalous dimension

solve it

$$\boxed{q(n,\mu_f) = q(n,\mu_0)\exp\left[-\gamma_{qq}(n)\ln\left(\frac{\mu_f}{\mu_0}\right)\right]}$$

Scale evolution of parton densities (cont.)

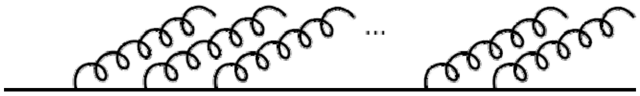
the anomalous dimensions $\gamma_{ij}(n)$ are nothing but Mellin transforms of the splitting functions $P_{ij}(x)$ multiplying the IR singularities:

$$\gamma_{qq}(n) = -\frac{\alpha_s}{2\pi} P_{qq}(n)$$

physical interpretation of the evolution eqs.:

$$\boxed{\text{RGE resums collinear emissions to all orders}}$$

to see this expand the solution in α_s:



$$\exp[\ldots] = 1 + \frac{\alpha_s}{2\pi} P_{qq}(n) \ln \frac{\mu_f}{\mu_0} + \frac{1}{2} \left[\frac{\alpha_s}{2\pi} P_{qq}(n) \ln \frac{\mu_f}{\mu_0} \right]^2 + \ldots$$

Scale evolution of parton densities (cont.)

in full glory (including gluons) the DGLAP eqs. read

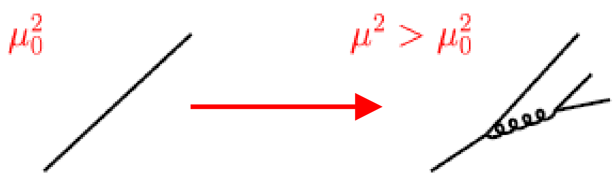
Dokshitzer, Gribov, Lipatov, Altarelli, Parisi

calculable in pQCD

$$\frac{d}{d\ln\mu}\begin{pmatrix} q(x,\mu) \\ g(x,\mu) \end{pmatrix} = \int_x^1 \frac{dz}{z} \begin{pmatrix} \mathcal{P}_{qq} & \mathcal{P}_{qg} \\ \mathcal{P}_{gq} & \mathcal{P}_{gg} \end{pmatrix}_{(z,\alpha_s)} \cdot \begin{pmatrix} q(x/z,\mu) \\ g(x/z),\mu) \end{pmatrix}$$

integro-differential eqs. are readily solved in Mellin moment space

think of evolution as the effect of increasing the resolution scale



recent achievement: NNLO splitting functions Moch, Vermaseren, Vogt

(about 10000 diagrams, 100000 integrals)

Proofs of factorization theorems

to prove the validity of factorization to all orders of pQCD is a highly theoretical and technical matter

serious proofs exist only for a limited number of processes such as DIS and Drell-Yan

Libby, Sterman; Ellis et al.; Amati et al.; Collins et al.; ...

faith in factorization rests on existing calculations and the success of pQCD in explaning data

we will *assume* factorization in the following

the renormalizibility of a non-abelian gauge theory like QCD was demonstrated by 't Hooft and Veltman

1999

now we have studied all relevant concepts of perturbative QCD !!

recap: salient features of pQCD

- strong interactions, yet perturbative method is applicable
- confined quarks, yet calculations based on free partons can describe large classes of processes

keys to resolve the apparent dilemma:

- asymptotic freedom
- infrared safety
- factorization theorems & renormalizibility

tomorrow we will apply the techniques to study

polarized proton-proton collisions at RHIC