RIKEN PHENIX SPIN FEST @ WAKO

Lectures on Perturbative QCD
or
from basic principles to current applications

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Outline of the lectures

Lecture 1: basic ideas; exploring the QCD final state

Lecture 2: origin of singularities; infrared safety

Lecture 3: QCD initial-state; factorization; renormalization

Lecture 4: more on factorization & renormalization; pdfs

Lecture 5: applications in hadron-hadron collisions; spin

Literature & useful links

Lecture notes & write-ups:

Wu-Ki Tung: *Perturbative QCD and the Parton Structure of the Nucleon* (from www.cteq.org)

!!! Dave Soper: *Basics of QCD Perturbation Theory* (hep-ph/9702203)

J. Collins, D. Soper, G. Sterman: *Factorization of Hard Processes in QCD* (hep-ph/0409313)

CTEQ Collaboration: *Handbook of Perturbative QCD* (Rev. Mod. Phys. 67 (1995) 157 or from www.cteq.org)

Talks & lectures on the web:

!!! annual CTEQ summer schools (tons of material !): *www.cteq.org*

1st summer school on QCD Spin Physics @ BNL: *www.bnl.gov/qcdsp*

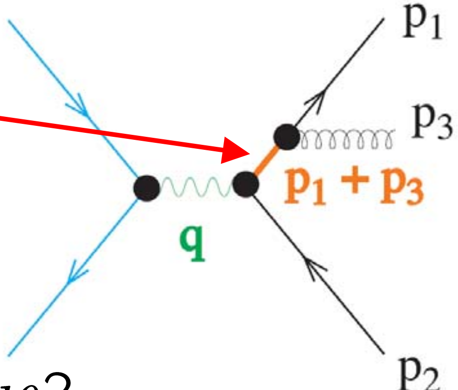
Lecture 2

origin & cancellation of singularities
infrared safety

Origin of the singularities
space-time picture

soft/collinear limit: internal propagator goes on-shell

$$\frac{1}{(p_1+p_3)^2} = \frac{1}{2E_1E_3(1-\cos\theta_{13})}$$



calculation gives

$$d\sigma \propto \int \underbrace{E_3 dE_3 d\theta_{13}^2}_{\text{phase space factor}} \underbrace{\left[\frac{\theta_{13}}{E_3\theta_{13}^2}\right]^2}_{\text{matrix element}} = \int \frac{dE_3}{E_3}\frac{d\theta_{13}^2}{\theta_{13}^2}$$

logarithmically divergent

note: "soft quarks" (here $E_1 \to 0$) never lead to singularities

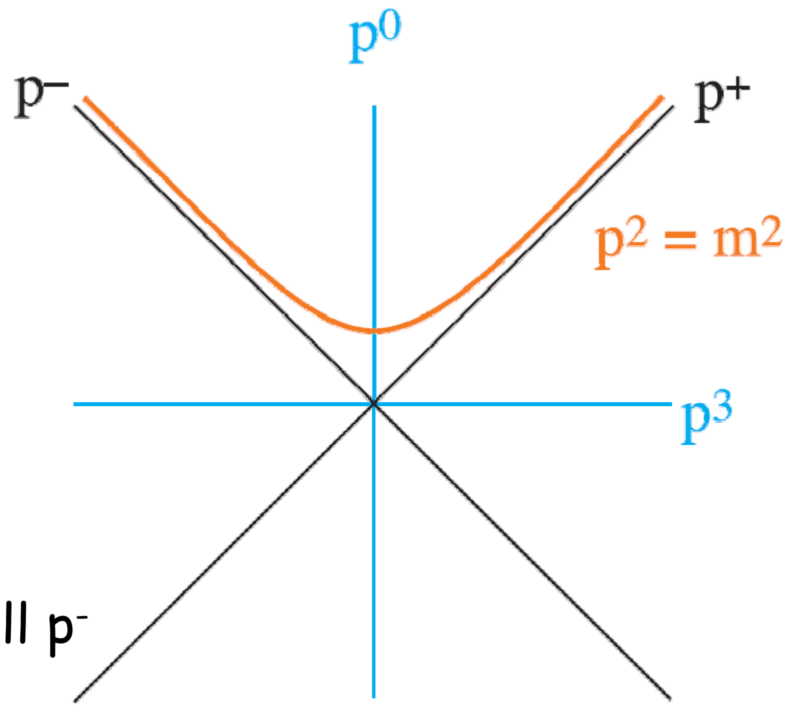
How far does the internal parton propagate in space-time?

Origin of the singularities space-time picture (cont.)

detour: light-cone coordinates

$$p^\mu = (p^0, p^1, p^2, p^3)$$
$$\rightarrow (p^+, p^-, p^1, p^2)$$

$$\boxed{p^\pm \equiv (p^0 \pm p^3)/\sqrt{2}}$$



➡ particle with large momentum in +p^3 direction has large p^+ and small p^-

for the invariant p^2 we find:

$$\xrightarrow[p^+ > 0,\ p^- > 0]{\text{on-shell: } p^2=m^2} \quad \mu \quad - \quad \frac{}{2p^+}$$

Origin of the singularities space-time picture (cont.)

... and similarly for the coordinate space: $\boxed{x^{\pm} \equiv (x^0 \pm x^3)/\sqrt{2}}$

$$\text{momentum space} \xleftrightarrow{\text{Fourier transform}} \text{coordinate space}$$

➡ x^- is conjugate to p^+ and x^+ is conjugate to p^- because

-

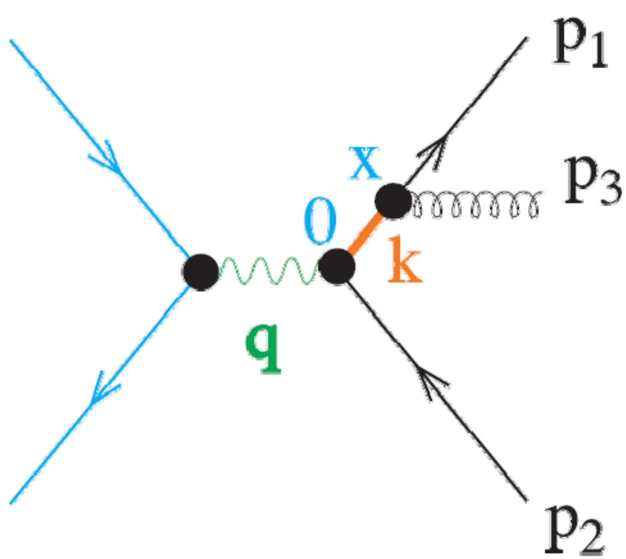
⬌ large $p^+ \longrightarrow$ small x^-

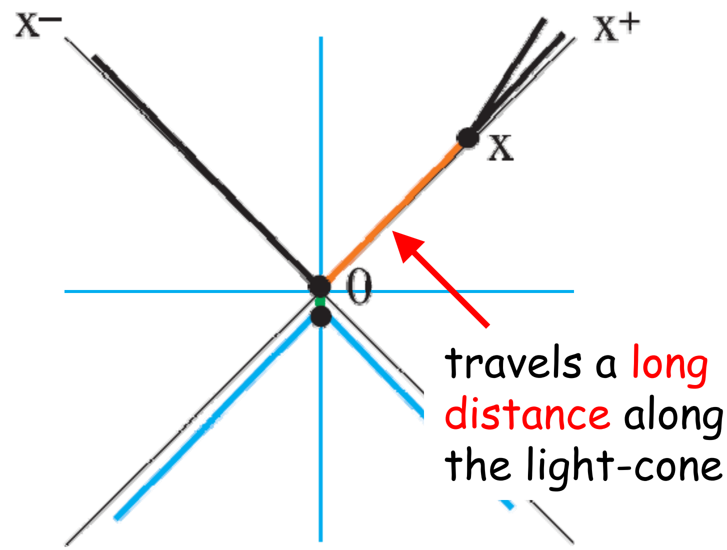
small $p^- \longrightarrow$ large x^+

What does this imply for our propagator going on-shell?

Origin of the singularities space-time picture (cont.)

define $k \equiv p_1+p_3$; let k^+ define the p^+ direction





soft/collinear limit: $k^2 \approx 0$

$$
\begin{aligned}
k^+ &\simeq \sqrt{s}/2 \quad \text{large} & \xrightarrow{\text{Fourier}} \quad x^+ &\simeq 1/k^- \quad \text{large} \\
k^- &\simeq (\vec{k}_T^2 + k^2)/\sqrt{s} \quad \text{small} & x^- &\simeq 1/k^+ \quad \text{small}
\end{aligned}
$$

upshot: soft/collinear singularities arise from interactions that happen a long time after the creation of the quark/antiquark pair

pQCD is not applicable at long-distance

so ...... What to do with the long-distance physics associated with these soft/collinear singularities? Is there any hope that we can predict some reliable numbers to compare with experiment?

to answer this, we have to formulate the

concept of infrared safety

Cancellation of singularities
infrared safety

infrared safety: proper connection between the "right" physics questions and the partonic pQCD calculations

recall from 1st lecture: soft/collinear singularities appear when 2->3 kinematics reduces to 2->2 kinematics at the boundaries of phase space

meaningful infrared-safe observables must be insensitive to the indistinguishable 2->2, 2->3 origin of the long-distance interactions

➡ cancellation of singularities

Cancellation of the singularities infrared safety (cont.)

Kunszt, Soper

formal definition of infrared safety:

study inclusive observables which do not distinguish between (n+1) partons and n partons in the soft/collinear limit

$$
\begin{aligned}
\mathcal{I} &= \frac{1}{2!}\int d\Omega_2 \frac{d\sigma[2]}{d\Omega_2} \mathcal{S}_2(p_1,p_2) \\
&+ \frac{1}{3!}\int d\omega_2 dE_3 d\Omega_3 \frac{d\sigma[3]}{d\Omega_2 dE_3 d\Omega_3} \mathcal{S}_3(p_1,p_2,p_3) \\
&+ \ldots
\end{aligned}
$$

measurement fcts.
(define observable)

infrared safe *if* [for $\lambda = 0$ (soft) and $0 < \lambda < 1$ (collinear)]

$$\mathcal{S}_{n+1}(p_1,\ldots,(1-\lambda)p_n,\lambda p_n) = \mathcal{S}_n(p_1,\ldots,p_n)$$

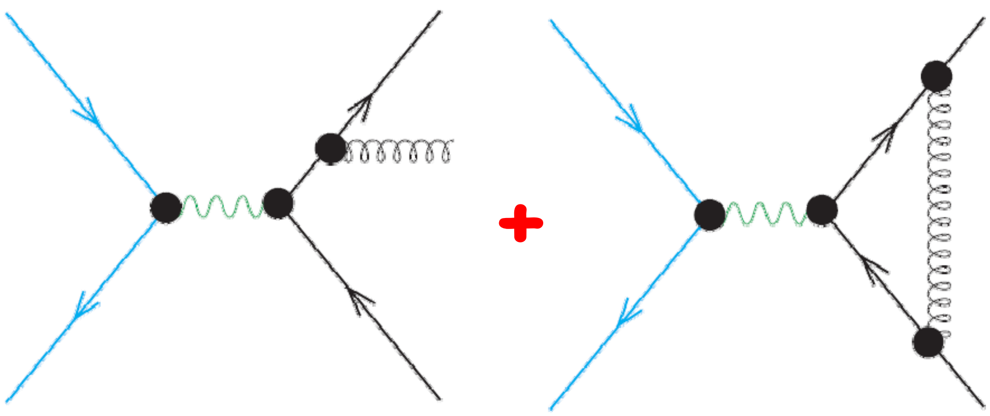
Cancellation of the singularities infrared safety (cont.)

What is the physics behind this formal requirement?

at a level of a pQCD calculation (at $\mathcal{O}(\alpha_s)$, i.e., n=2)

$$S_{n+1}(p_1, \ldots, (1-\lambda)p_n, \lambda p_n) = S_n(p_1, \ldots, p_n)$$

➡ singularities of real gluon emission and virtual corrections cancel in the sum

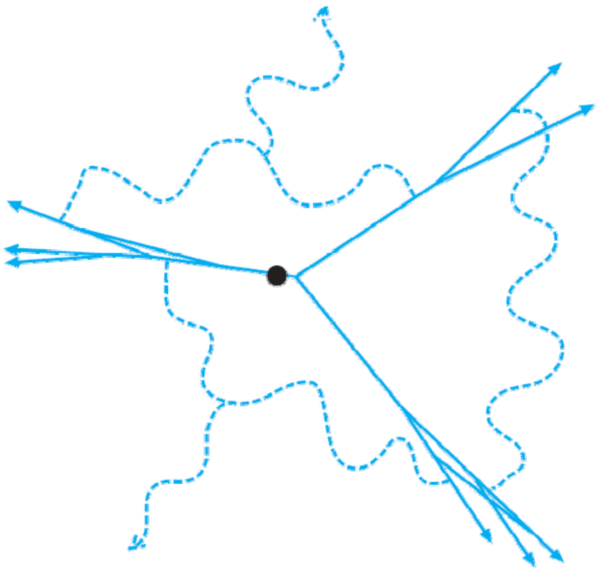


extension of famous theorems by Kinoshita-Lee-Nauenberg and Bloch-Nordsieck

Cancellation of the singularities infrared safety (cont.)

or in more physical terms:

QCD suggests a jet-like structure of final-state hadrons with additional soft partons communicating over long-distance



cannot resolve soft and collinear partons experimentally

➡ intuitively reasonable that a theoretical calculation *can be infrared safe* as long as it is insensitive to long-distance physics (not a priori guaranteed)

let's discuss some examples ...

Examples of infrared safe observables
total cross section

simplest case: $e^+e^- \to$ hadrons

$$\boxed{\mathcal{S}_n(p_1,\ldots,p_n) = 1}$$

fully inclusive quantity ↔ we don't care what happens at long-distance

- the produced partons will *all* hadronize with *probability one*
- we do not observe a specific type of hadron (i.e. sum over a *complete set of states*)
- we sum over all degenerate kinematic regions

formal argument: **unitarity**

infrared safe by definition

most important application:

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = N_c \sum e_q^2 \,(1+\Delta_{\rm QCD})$$

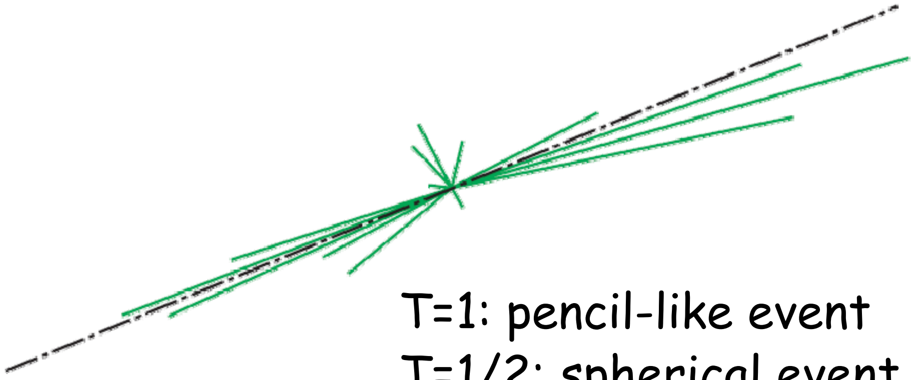
measures sum of quark charges and the number of quark colors

Examples of infrared safe observables
thrust

somewhat less trivial: thrust distribution $d\sigma/dT$

$$\begin{aligned} S_n(p_1,\ldots,p_n) &= \delta(T - T_n(p_1,\ldots,p_n)) \\ T_n(p_1,\ldots,p_n) &\equiv \max|_{\vec{n}} \frac{\sum_{i=1}^{n} |\vec{p}_i \cdot \vec{n}|}{\sum_{i=1}^{n} |\vec{p}_i|} \end{aligned}$$

procedure:
vary unit vector $\vec{n}$ to maximize the sum of the projections of $\vec{p}_i$ on $\vec{n}$



T=1: pencil-like event
T=1/2: spherical event

Examples of infrared safe observables thrust (cont.)

Why is thrust infrared safe?

$$T_n(p_1, \ldots, p_n) \equiv \max|_{\vec{n}} \frac{\sum_{i=1}^{n} |\vec{p}_i \cdot \vec{n}|}{\sum_{i=1}^{n} |\vec{p}_i|}$$

- contributions from soft particles with $\vec{p}_i \rightarrow 0$ drop out
- a collinear splitting does not change the thrust:

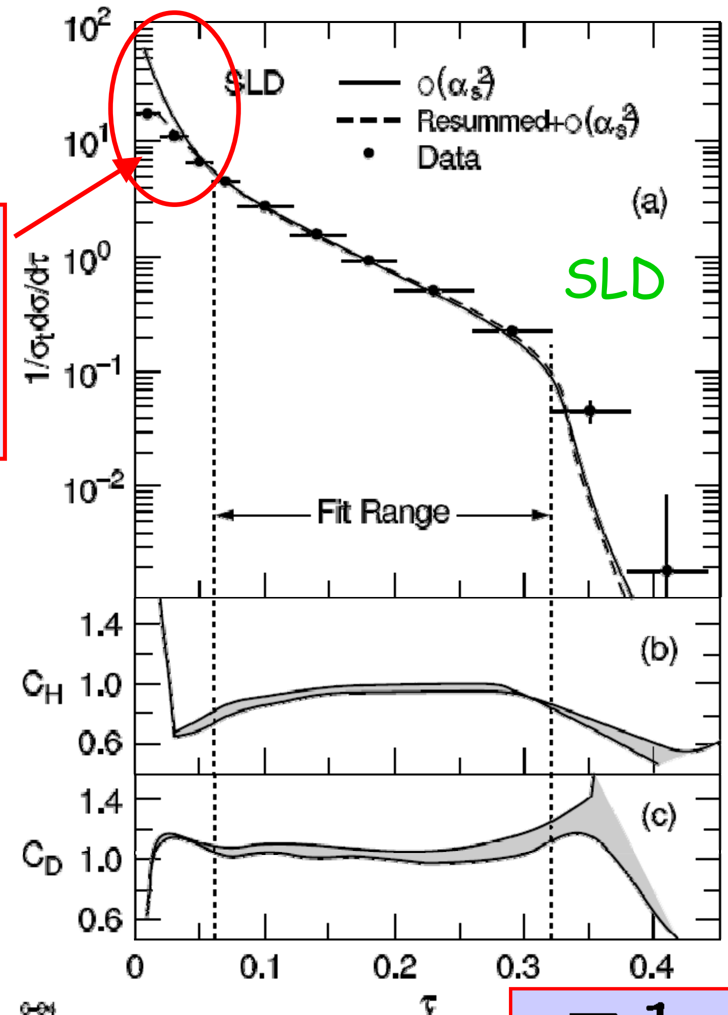
$$|(1-\lambda)\vec{p}_i \cdot \vec{n}| \;+\; |\lambda \vec{p}_i \cdot \vec{n}| = |\vec{p}_i \cdot \vec{n}|$$
$$|(1-\lambda)\vec{p}_i| \;+\; |\lambda \vec{p}_i| = |\vec{p}_i|$$

infrared safe

Examples of infrared safe observables thrust (cont.)

data on the thrust distribution tell us more:

perfect agreement with theory but some trouble for $T \to 1$ in fixed-order pQCD



$\boxed{\tau \equiv 1 - T}$

problem: remnants of soft/collinear singularities lead to large logarithms in each order of α_s: $(\alpha_s \ln^2[1-T])^n$

➡ spoil convergence of series

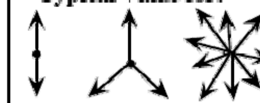
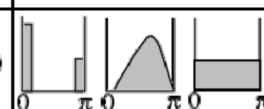
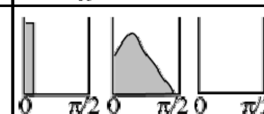
solution: re-organization of series to resum large logs to all orders (exponentiation) Catani, Trentadue; Greco; ...

Examples of infrared safe observables other event-shape variables

there is a long list of infrared safe observables:

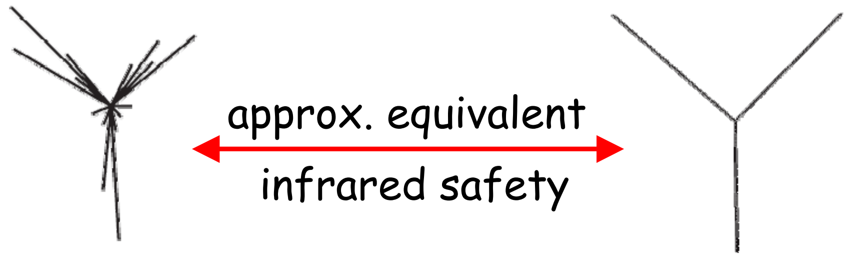
event-shapes: fertile ground for comparison between theory and experiment

- validity of pQCD calculations
- many ways to test SU(3) (color factors)
- spin of quarks and gluons
- measurements of α_s

Name of Observable	Definition	Typical Value for: 			QCD calculation
Thrust	$T = \max_{\vec{n}} \left(\frac{\Sigma_i \lvert \vec{p}_i \vec{n} \rvert}{\Sigma_i \lvert \vec{p}_i \rvert} \right)$	1	≥2/3	≥1/2	(resummed) $O(\alpha_s^2)$
Thrust major	Like T, however T_{maj} and $\vec{n}_{maj}$ in plane $\perp \vec{n}_T$	0	≤1/3	≤1/√2	$O(\alpha_s^2)$
Thrust minor	Like T, however T_{min} and $\vec{n}_{min}$ in direction $\perp$ to $\vec{n}_T$ and $\vec{n}_{maj}$	0	0	≤1/2	$O(\alpha_s^2)$
Oblateness	$O = T_{maj} - T_{min}$	0	≤1/3	0	$O(\alpha_s^2)$
Sphericity	$S = 1.5\,(Q_1 + Q_2)$; $Q_1 \le \ldots \le Q_3$ are Eigenvalues of $S^{\alpha\beta} = \frac{\Sigma_i p_i^\alpha p_i^\beta}{\Sigma_i p_i^2}$	0	≤3/4	≤1	none (not infrared safe)
Aplanarity	$A = 1.5\,Q_1$	0	0	≤1/2	none (not infrared safe)
Jet (Hemisphere) masses	$M_\pm^2 = (\Sigma_i E_i^2 - \Sigma_i \vec{p}_i^{\,2})_{i \in S_\pm}$ ($S_\pm$: Hemispheres $\perp$ to $\vec{n}_T$) $M_H^2 = \max(M_+^2, M_-^2)$ $M_D^2 = \lvert M_+^2 - M_-^2 \rvert$	0 0	≤1/3 ≤1/3	≤1/2 0	(resummed) $O(\alpha_s^2)$
Jet broadening	$B_\pm = \frac{\Sigma_{i \in S_\pm} \lvert \vec{p}_i \times \vec{n}_T \rvert}{2\,\Sigma_i \lvert \vec{p}_i \rvert}$; $B_T = B_+ + B_-$ $B_W = \max(B_+, B_-)$	0 0	≤1/(2√3) ≤1/(2√3)	≤1/(2√2) ≤1/(2√3)	(resummed) $O(\alpha_s^2)$
Energy-Energy Correlations	$EEC(\chi) = \sum_{events} \sum_{i,j} \frac{E_i E_j}{E_{vis}^2} \int_{\chi - \frac{\Delta\chi}{2}}^{\chi + \frac{\Delta\chi}{2}} \delta(\chi - \chi_{ij})$				(resummed) $O(\alpha_s^2)$
Asymmetry of EEC	$AEEC(\chi) = EEC(\pi - \chi) - EEC(\chi)$				$O(\alpha_s^2)$
Differential 2-jet rate	$D_2(y) = \frac{R_2(y - \Delta y) - R_2(y)}{\Delta y}$				(resummed) $O(\alpha_s^2)$

taken from S. Bethke, hep-ex/0001023

Examples of infrared safe observables
n-jet cross section

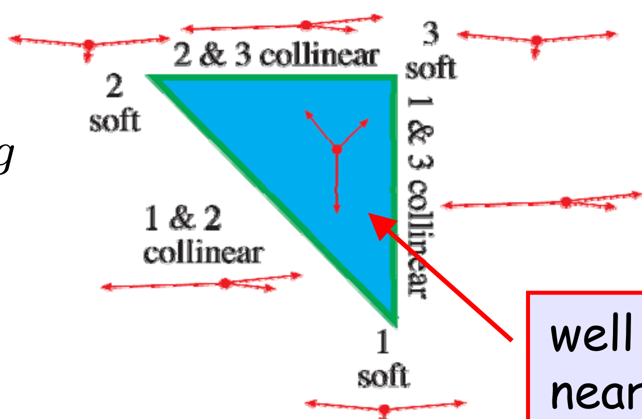


real physical event
with 3 hadron-jets

theor. jet event
with 3 parton-jets

But what is a jet *exactly* ?

recall:
$e^+e^- \to q\bar{q}g$



well inside: 3-jets
near edges: 2-jets

jet "measure"/"algorithm":

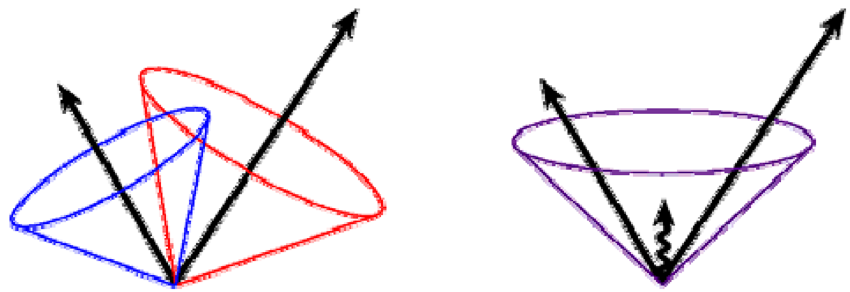
classify the final-state of hadrons (exp.) or partons (th.) according to the number of jets

"2 or 3" depends on algorithm

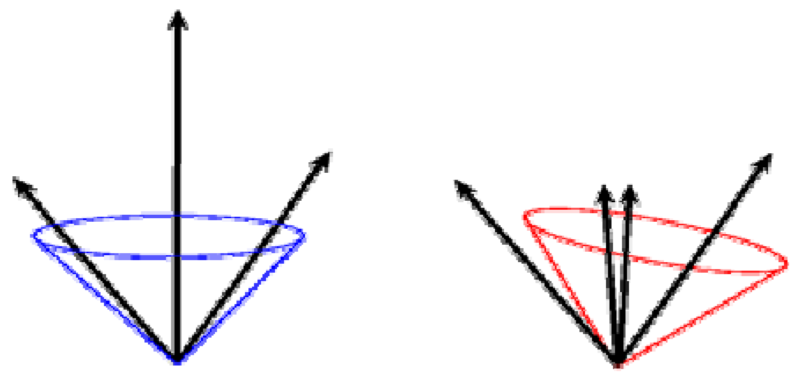
Examples of infrared safe observables n-jet cross section (cont.)

basic requirements for a jet definition:

- adding an infinit. soft parton should not change the number of jets



- replacing a parton by a collinear pair of partons should not change the number of jets



- in addition: insensitive to long. boosts, insensitive to hadronization

Examples of infrared safe observables n-jet cross section (cont.)

there are many algorithms to choose from!

two classes: "recombination/clustering" or "cone"

n-jet vs. (n+1)-jet rate depends on algorithm

➡ have to choose the *same* jet definition in exp. and theory

one has to be very careful when comparing jet results

between different experiments (usually diff. algorithms!)

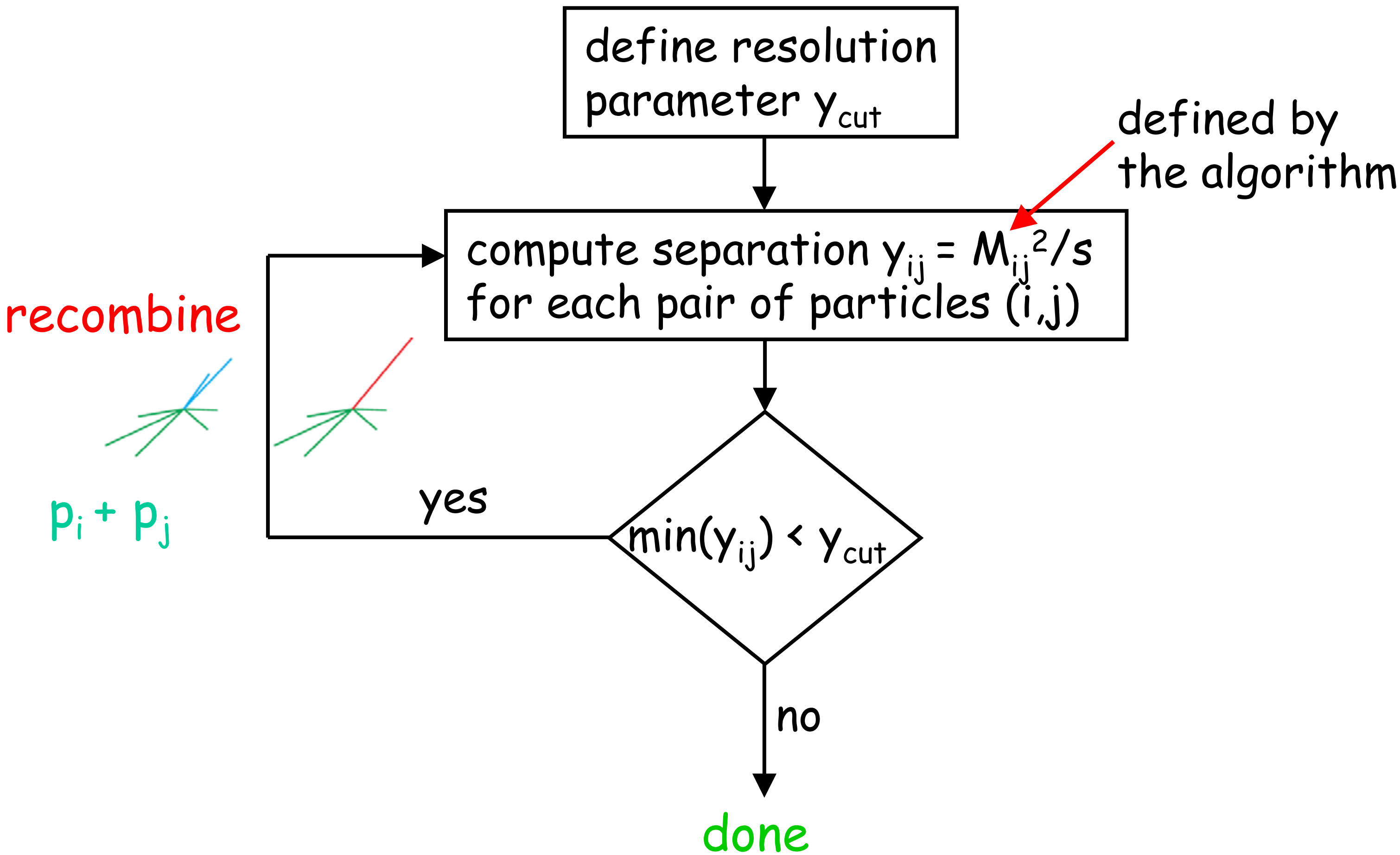
between experiment and theory

between different theoretical calculations

we shall outline some jet algorithms next ...

Examples of infrared safe observables n-jet cross section (cont.)

generic recombination algorithm:

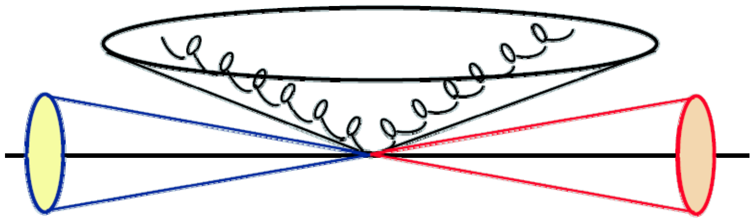


Examples of infrared safe observables n-jet cross section (cont.)

classic example: JADE algorithm

$$\boxed{M_{ij}^2 = 2E_iE_j(1-\cos\theta_{ij})}$$

big problem!



two soft gluons i,j with large relative angle will be merged (if $y_{ij} < y_{cut}$) into 3rd spurious jet

way out: DURHAM/k_T algorithm

Catani, Dokshitzer, Olsson, Turnock, Webber

$$\boxed{M_{ij}^2 = 2\min(E_i^2, E_j^2)(1-\cos\theta_{ij})}$$

reduces to (for small θ_{ij}):

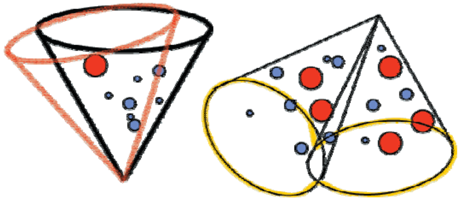
$$M_{ij}^2 \approx \min(k_{Ti}^2, k_{Tj}^2)$$

safe!

Examples of infrared safe observables
n-jet cross section (cont.)

jet algorithms for hadron colliders:

cone type



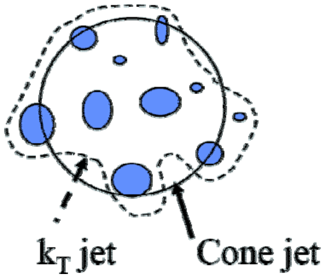
- long. boost invariant cone size

$$R = \sqrt{\Delta\eta^2 + \Delta\phi^2}$$

- intuitive
- for every "seed" sum all cells within cone radius R
- recompute jet direction and iterate
- merge/split overlapping cones

not always infrared safe
(e.g. the Snowmass algorithm)

k_T type



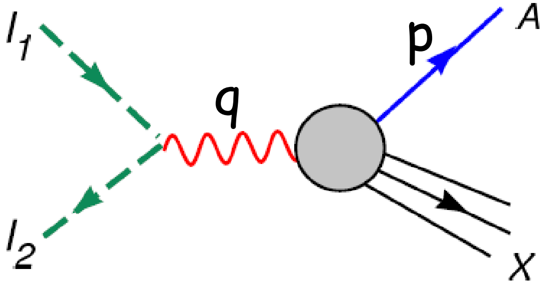
- merge pairs in order of increasing rel. transverse momentum until terminates
- infrared safe
- no overlapping jets
- no bias from seeds
- every parton assigned to jet
- flexible "edges"

pQCD cannot give all the answers
but it does cover a lot of ground
despite the long-distance "problem"

the **concept of factorization** allows us to
compute cross sections for a much wider
class of processes than considered so far

Electron-positron annihilation factorization

consider the inclusive cross section for $e^+e^- \to A(p) + X$:



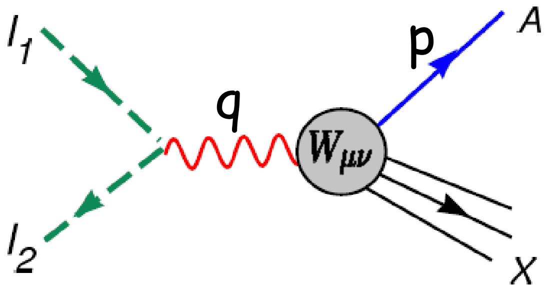
e.g. ($A = \pi$): $\boxed{\dfrac{d\sigma(e^+e^- \to \pi + X)}{dE_\pi}}$

not infrared safe by themselves!

before giving up here let's meet another powerful concept:
the remarkable property of factorization

Electron-positron annihilation factorization (cont.)

What does it mean?

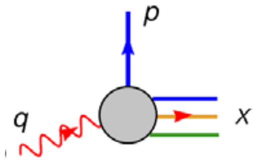


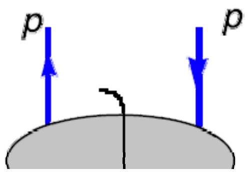
leptonic tensor

hadronic tensor

strategy: try to factorize the physical observable into a calculable infrared safe and a non-calculable but universal piece

hadronic tensor $W_{\mu\nu}$:



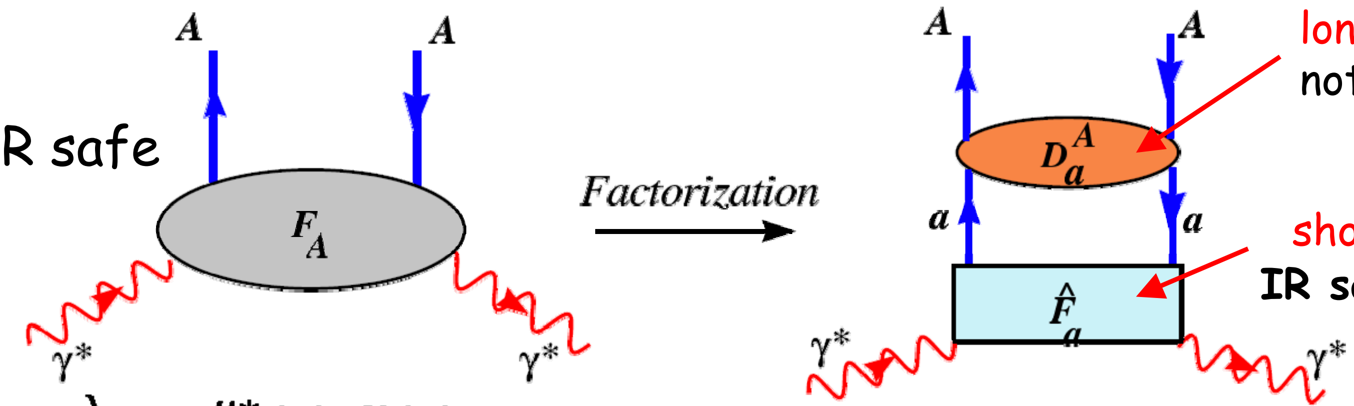


square of the hadronic scattering ampl.
summed over all final-states X except $A(p)$

Electron-positron annihilation factorization (cont.)

factorization = isolating and absorbing infrared singularities

not IR safe



long-distance
not calculable

short distance
IR safe, calculable

$$F_a^\lambda = \epsilon_\lambda^{\mu *}(q)\epsilon_\lambda^\nu(q)W_{\mu\nu}$$

λ=L,T (polarization of virtual photon)

we have to apply this "trick" for each *observed* hadron in the initial and final state to obtain a finite cross section

we will continue to discuss
concept of factorization
and its implications
in the next lectures