RIKEN PHENIX SPIN FEST @ WAKO

Lectures on Perturbative QCD

or

from basic principles to current applications

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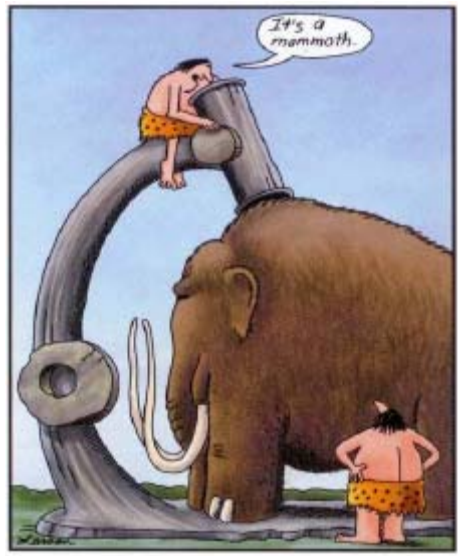
Appetizer: Collider Physics & QCD

Collider experiments are QCD laboratories ...

(and World's best microscopes!)

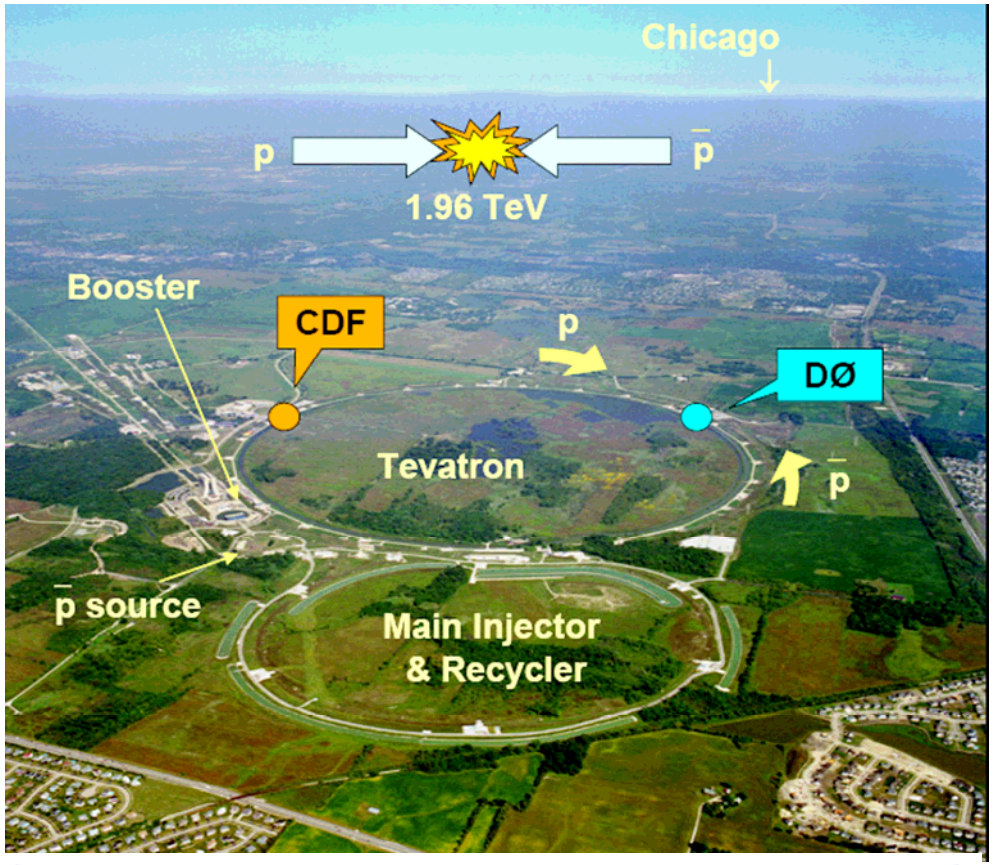


Early microscopes

- vast source of information about hadronic structure
- gluons are often "key players"
- challenge our understanding of strong interactions
- push theorists to perform more and more refined calculations

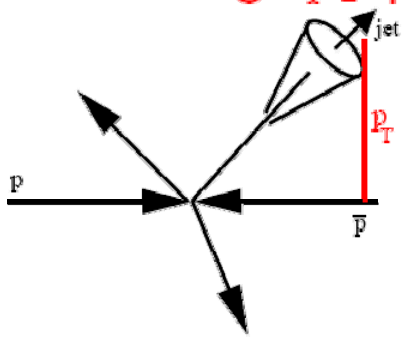
... a success story ever since

One (of many) examples:

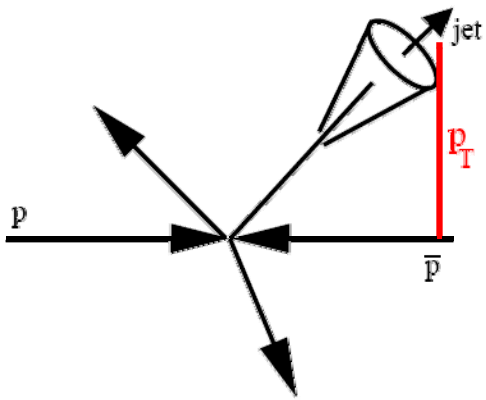


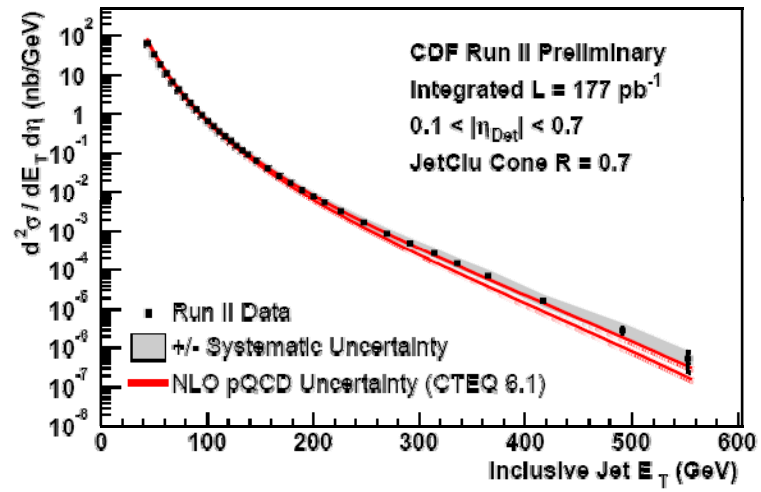
Fermilab

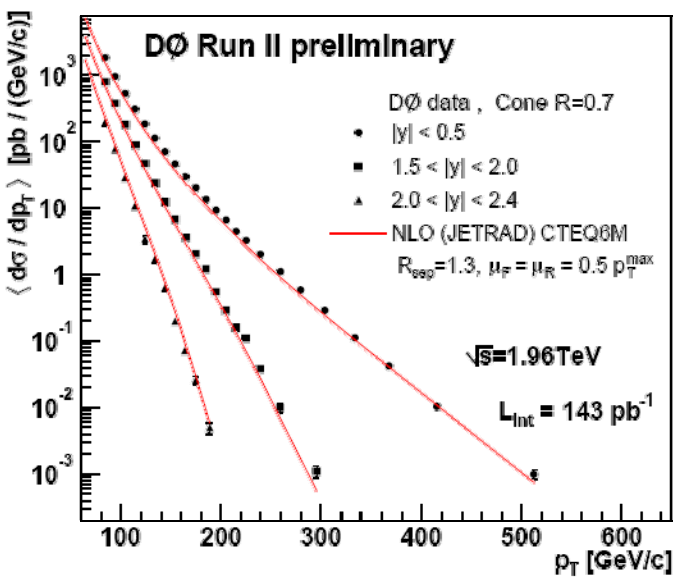
- largest hadron-hadron collider
- *unpolarized* p and $\bar{p}$ beams
- c.m. energy up to $\sqrt{S} = 1.96\,\mathrm{TeV}$
- two major experiments: DØ and CDF
- major discovery: top quark
- probes QCD hard interactions in many different high-p_T processes



Fermilab's TeVatron cont.







- jet cross section in agreement with expectations from pert. QCD calculations
- similar for other processes: heavy flavors, prompt photons, electroweak bosons, . . .

Questions to be addressed in the lectures

- What are the general features of QCD?

  keywords: asymptotic freedom; infrared safety; origin of "singularities"

- How to relate QCD to experiment?

  keywords: partons; pQCD; factorization; cross sections; some kinematics

- How reliable is a theoretical calculation?

  keywords: scale dependence; NLO; resummations

- How to include spin?

  keywords: polarized cross section; spin asymmetries

Disclaimer: 5 hours are not enough to learn how to do an actual calculation! You should learn about the basic principles and the meaning of some of the buzzwords theorists like to use!

Outline of the lectures

Lecture 1: basic ideas; exploring the QCD final state

Lecture 2: origin of singularities; infrared safety

Lecture 3: QCD initial-state; factorization; renormalization

Lecture 4: more on factorization & renormalization; pdfs

Lecture 5: applications in hadron-hadron collisions; pdfs

Literature & useful links

Lecture notes & write-ups:

Wu-Ki Tung: *Perturbative QCD and the Parton Structure of the Nucleon* (from www.cteq.org)

!!! Dave Soper: *Basics of QCD Perturbation Theory* (hep-ph/9702203)

J. Collins, D. Soper, G. Sterman: *Factorization of Hard Processes in QCD* (hep-ph/0409313)

CTEQ Collaboration: *Handbook of Perturbative QCD* (Rev. Mod. Phys. 67 (1995) 157 or from www.cteq.org)

Talks & lectures on the web:

!!! annual CTEQ summer schools (tons of material !): *www.cteq.org*

1st summer school on QCD Spin Physics @ BNL: *www.bnl.gov/qcdsp*

Lecture 1

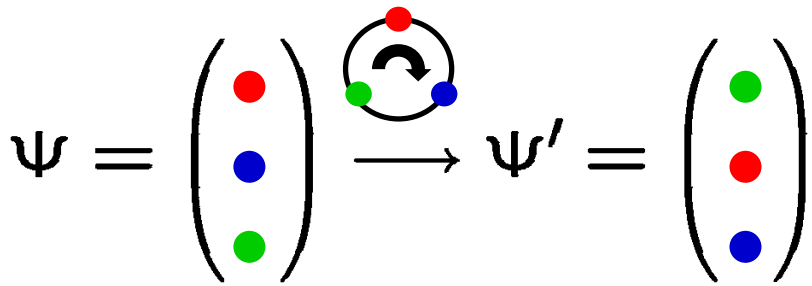
basic ideas
exploring the QCD final state

What is QCD?

guiding principle for all field theories: local gauge invariance of the underlying Lagrangian

here: local SU(3) rotations in color space

spin-$\frac{1}{2}$ quark fields
come as colors triplets



local SU(3) invariance dictates:

- 8 massless spin-1 gluons
- all interactions between quarks and gluons

non-abelian group structure:

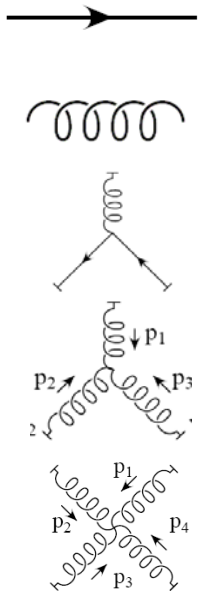
- algebra: $[T_a, T_b] = i f_{abc} T_c$
- color factors: $C_A = 3,\ C_F = \frac{4}{3}, T_F = \frac{1}{2}$

What is QCD? (cont.)

all physics related to strong interactions can be encoded in the Lagrangian of QCD!

for perturbative calculations we simply read off the Feynman rules

$$
\begin{aligned}
\mathcal{L}_{\mathrm{QCD}} &= \bar{\Psi}(i\partial_\mu\gamma^\mu - m)\Psi \\
&- (\partial_\mu A_\nu - \partial_\nu A_\mu)^2 \\
&- g\bar{\Psi}A^a_\mu T_a\gamma^\mu\Psi \\
&- \frac{1}{2}g(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)f_{abc}A^{\mu b}A^{\nu c} \\
&- \frac{1}{4}g^2 f_{abc}A^b_\mu A^c_\nu f_{ade}A^{\mu d}A^{\nu e}
\end{aligned}
$$



like in QED

non abelian

(we ignore here complications due to the gauge-fixing term)

the gauge principle is elegant and powerful but any theory must ultimately stand (or fall) by its success

QCD is the theory of strong interactions
– how can we make use of perturbative methods?

confinement

non-perturbative
hadronic structure

e.g. Lattice QCD

asymptotic freedom

hard scattering
cross sections

perturbative methods

interplay

our goal: probe hadronic structure with
weakly interacting quanta of asymptotic freedom

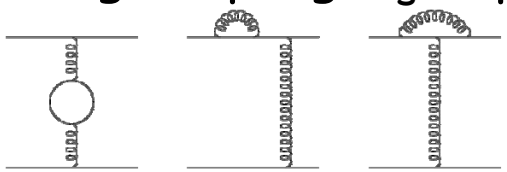
What is asymptotic freedom?

rough qualitative picture (for the moment):

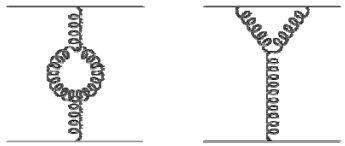
we get back to this when we discuss renormalization!

Gross, Wilczek; Politzer ('73/'74) (Nobel prize 2004)

value of strong coupling α_s depends on distance (i.e. energy)

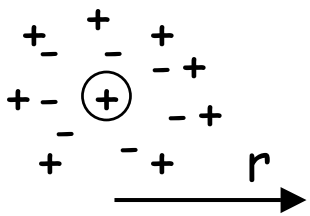


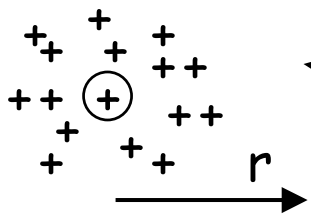
"screening" of the charge



"anti-screening"

like QED





non abelian

$\alpha_s(r) \uparrow$ if $r \downarrow$

$\alpha_s(r) \downarrow$ if $r \downarrow$

Who wins?

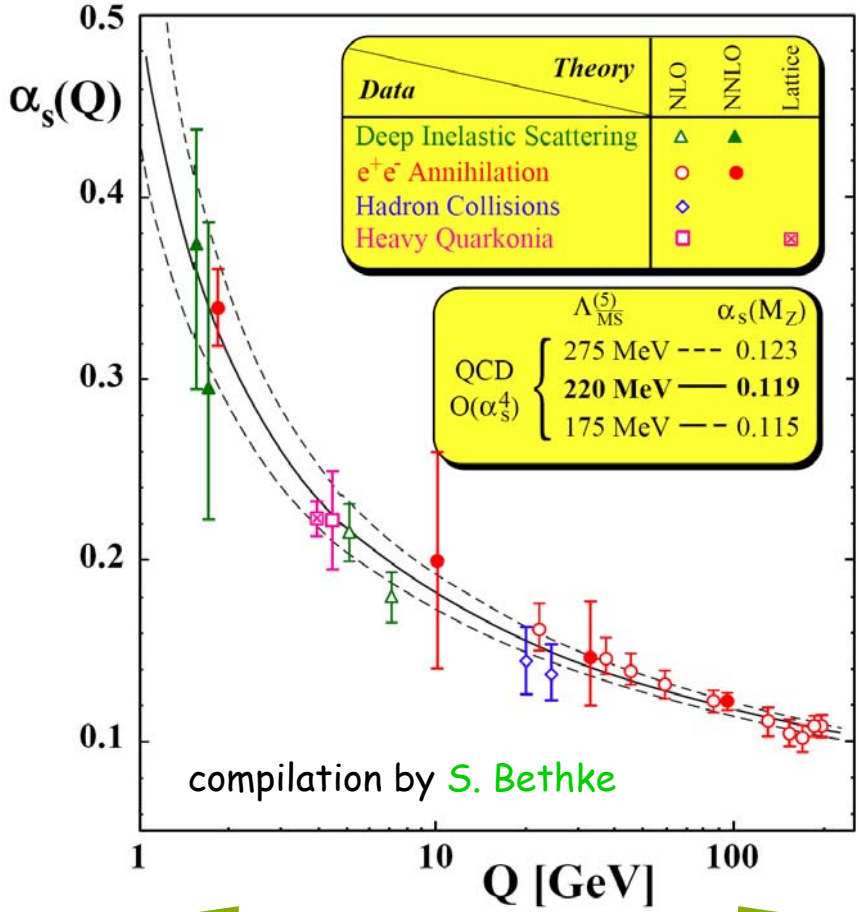
$$\alpha_s(Q^2) = \frac{g^2}{4\pi} \approx \frac{4\pi}{(11 - \frac{2}{3}N_f)\ln(Q^2/\Lambda^2)}$$

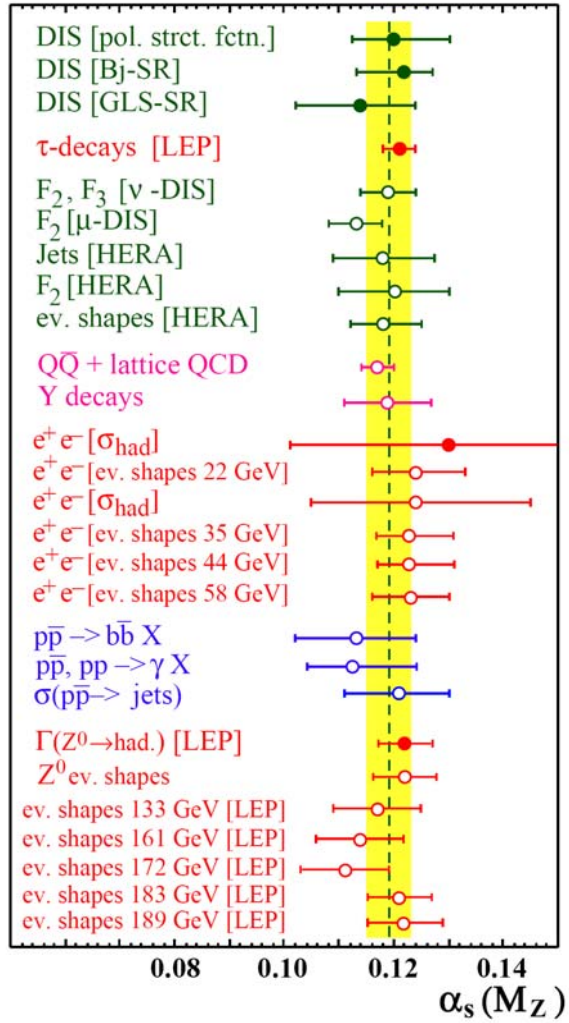
$Q \sim 1/r$

coupling at some reference scale Q_0

What is asymptotic freedom? (cont.)

test of QCD: experimental confirmation of asymptotic freedom





confinement (?) asymp. freedom

upshot: a strongly interacting theory at long-distance can become weakly interacting at short-distance

Is this enough to explain the success of the parton model and pQCD?

NO!

asymptotic freedom "only" enables us to compute interactions of quarks and gluons at short-distance

- detectors are a long-distance away
- experiments only see hadrons not free partons

to establish the crucial connection between theory and experiment we need two more things:

- infrared safety
- factorization

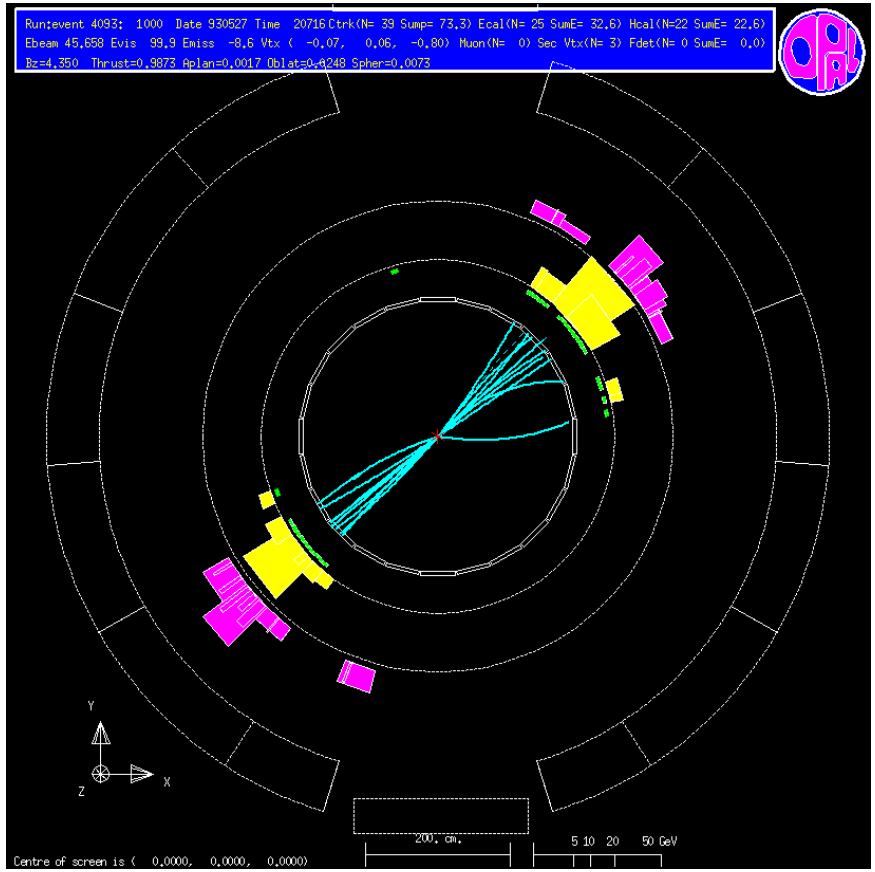
infrared safety: there are classes of quantities which are independent of long-distance physics, hence calculable in pQCD

factorization: there is an even wider class of processes which can be factored into *universal* long-distance and short-distance pieces (*process-dep. but calculable*)

let's study electron-positron annihilation to see what this is all about ...

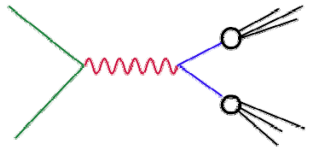
Electron-positron annihilation: exploring the QCD final-state

most of the hadronic events at LEP have two back-to-back jets

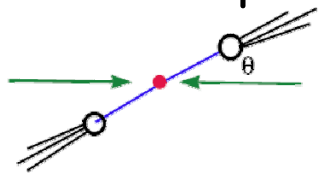


jet: pencil-like collection of hadrons

- jets resemble features of underlying 2->2 hard process $e^+e^- \rightarrow q\bar{q}$



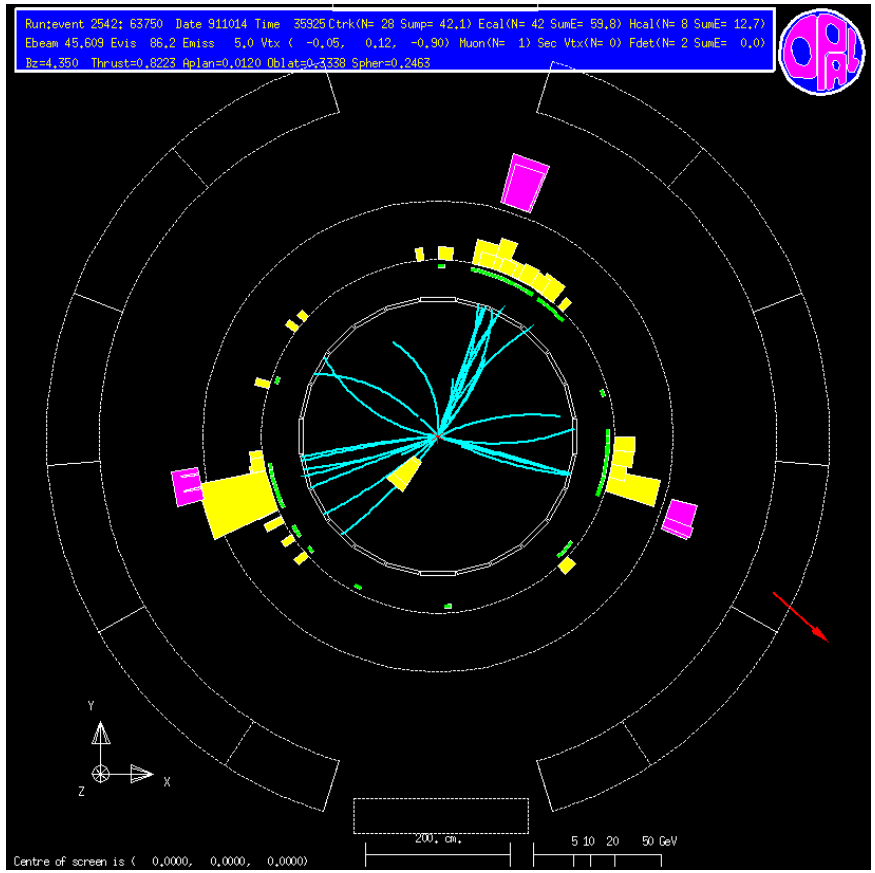
- angular distribution of jet axis w.r.t. beam axis as predicted for spin-$\frac{1}{2}$ quarks



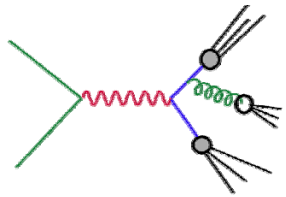
Electron-positron annihilation: exploring the QCD final-state (cont.)

about 10% of the events have a third jet

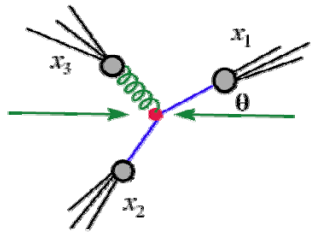
first discovered at DESY-PETRA in 1979



- jets resemble features of underlying 2->3 hard process $e^+e^- \longrightarrow q\bar{q}g$



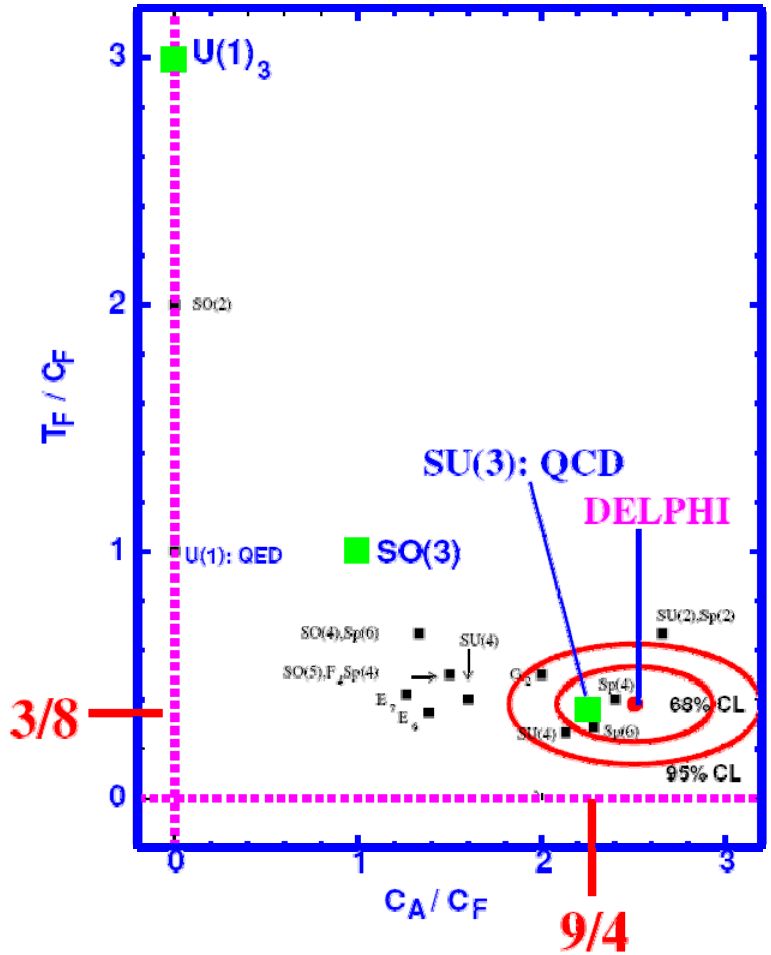
- 10% rate consistent with $\alpha_s \simeq 0.1$ (measurement of α_s)
- angular distribution of jets w.r.t. beam axis as expected for spin-1 gluons



Electron-positron annihilation: exploring the QCD final-state (cont.)

some events even have a fourth jet

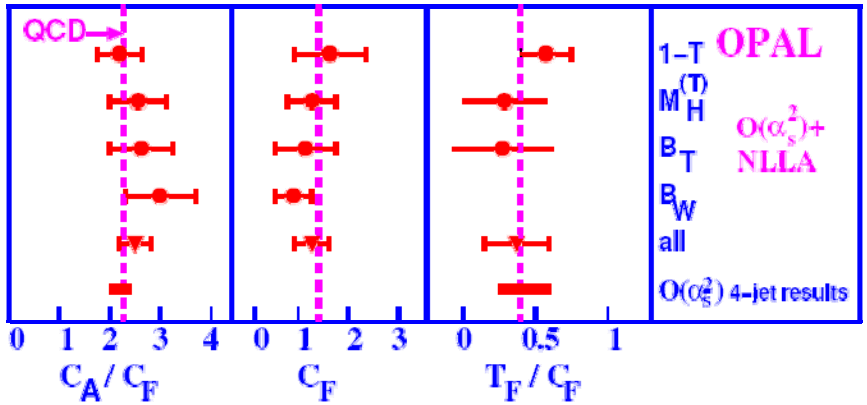
extensively studied at LEP



- angular correlations between four jets depend on

$$\frac{C_A}{C_F} = \frac{9}{4}, \quad \frac{T_F}{C_F} = \frac{3}{8}$$

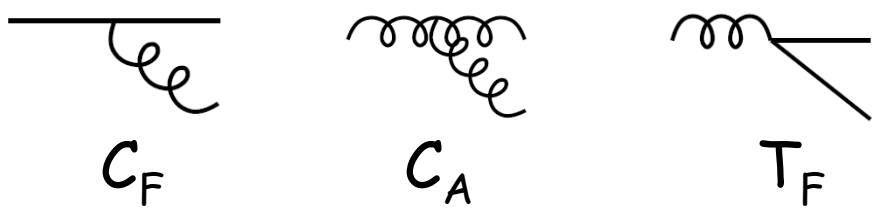
- even better constraints from combined fits including event shapes (thrust, etc.)



upshot: e^+e^- experiments play a vital role in establishing QCD as the correct theory of strong interactions and SU(3) as the underlying gauge group

<u>unique & classic precision tests</u>:

- confirm spin $\frac{1}{2}$ of the quarks and spin 1 of the gluons
- measurements of the strong coupling constant
- measurements of the color factors
  (most important numbers in QCD besides α_s)
  specify relative probabilities of parton radiation in QCD

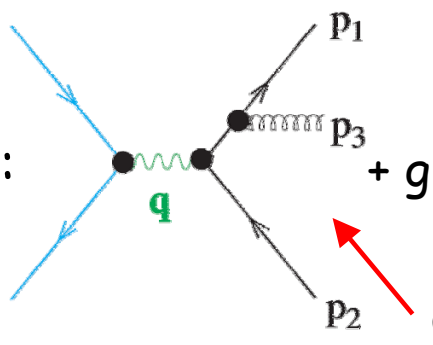


Electron-positron annihilation: exploring the QCD final-state (cont.)

seems to work very well –

let us try to actually compute the cross section for $e^+e^- \to q\bar{q}g$

simple, only two amplitudes:



+ gluon radiation from antiquark

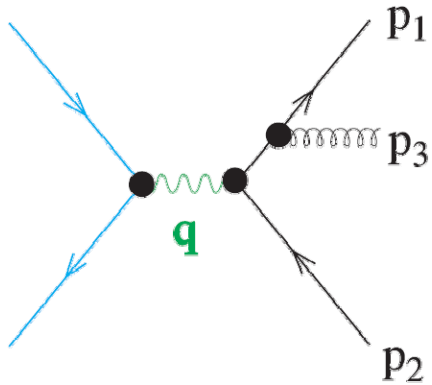
all partons massless

some kinematics first:

- available center-of-mass energy: $\sqrt{s}$, $q^\mu q_\mu = q^2 = s$
- energy fractions: $\boxed{x_i \equiv \frac{2E_i}{\sqrt{s}}}$, $i = 1, 2, 3$
- energy conservation implies: $\sum x_i = \frac{2(\sum p_i) \cdot q}{s} = 2$

Electron-positron annihilation: exploring the QCD final-state (cont.)

the angles between two outgoing partons can be expressed by the energy fractions:



$$2p_1 \cdot p_3 = (p_1 + p_3)^2 = (q - p_2)^2 = s - 2q \cdot p_3$$

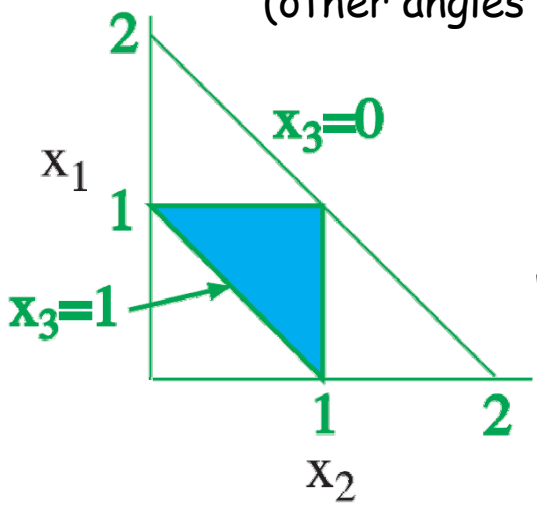
$$\Leftrightarrow \quad 2E_1E_3(1 - \cos\theta_{13}) = s(1 - x_2)$$

$$\Leftrightarrow \quad \underline{x_1x_3(1 - \cos\theta_{13}) = 2(1 - x_2)}$$

(other angles by cycl. permutation)

$$\boxed{0 \leq x_i \leq 1}$$

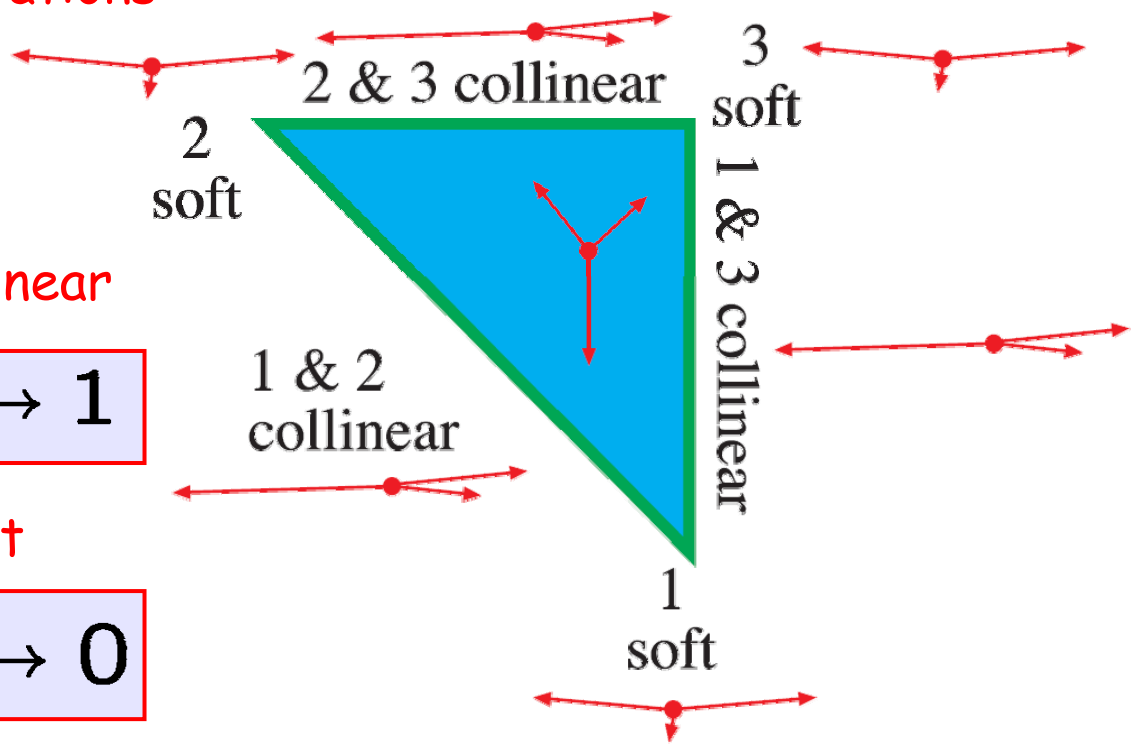
allowed values for x_i lie within a triangle



massless "Dalitz plot"

Electron-positron annihilation: exploring the QCD final-state (cont.)

at the boundaries of phase space we encounter special kinematic configurations:



- edges: two partons collinear

$$\theta_{13} \to 0 \Leftrightarrow x_2 \to 1$$

- corners: one parton soft

$$p_i^\mu \to 0 \Leftrightarrow x_i \to 0$$

What does this imply for the cross section?

Electron-positron annihilation: exploring the QCD final-state (cont.)

the cross section for $e^+e^- \to q\bar{q}g$ reads:

$$\frac{d\sigma}{dx_1 dx_2} = \sigma_0 \frac{\alpha_s}{2\pi} C_F \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

$$\frac{4\pi\alpha_{\rm em}}{s}\sum e_q^2$$

collinear singularities:

$$(1-x_1) \to 0$$

(gluon collinear to antiquark)

$$(1-x_2) \to 0$$

(gluon collinear to quark)

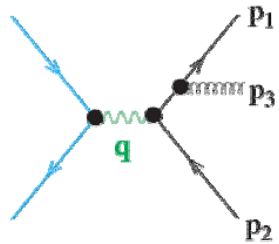
soft singularity:

gluon soft: $x_3 \to 0$

$$\Updownarrow$$

$$x_1 \to 1, \quad x_2 \to 1$$

Electron-positron annihilation: exploring the QCD final-state (cont.)



the singularities can be made more explicit by rewriting the cross section:

$$\frac{d\sigma}{dE_3\, d\cos\theta_{13}} = \sigma_0 \frac{\alpha_s}{2\pi} C_F \frac{f(E_3, \theta_{13})}{E_3(1-\cos\theta_{13})}$$

if we now attempt to integrate over phase space

either $\int_0^1 d\cos\theta_{13}$ or $\int_0^{E_3^{\max}} dE_3$

we end up with infinity as the answer!

Do we observe a breakdown of pQCD already here?

NO! Perturbative QCD only tries to tell us that we are not doing the right thing!
Our cross section is not defined properly, it is not infrared safe!

the lesson is:

whenever the 2->3 kinematics collapses to an effective 2->2 kinematics due to

- the emission of a soft gluon
- a collinear splitting of a parton into two partons

we have to be more careful and work harder!

general feature of QCD not limited to $e^+e^- \to q\bar{q}g$

next task:

we have to understand the origin of the singularities

let's have some coffee first ...