Polarized PDFs with EIC: Global Analysis of World Data

Swadhin Taneja
(Stony Brook University)
D. de Florian, R. Sassot, M. Stratmann, W. Vogelsang, K. Boyle and A. Deshpande

Introduction

- Parton distribution functions (Pdfs).
  - Essential input in high energy calculation
  - Assessing uncertainties is a challenge
    - Non-gaussian sources of uncertainties from pQCD(e.g. higher order corrections, power law corrections etc.)
    - Parametrization choice of pdf at an input energy scale
    - Including experimental statistical and systematic errors
  - All these sources of errors are studied individually and are eventually combined systematically .

Fit complications

- Problem is more complicated than it appears
- For example,
  - Large number of data points ($\sim$ 467 in DSSV)
  - Many experiments ($\sim$ 9)
  - A variety of physical processes ($\sim$ 5− 6 and growing) with diverse characteristics, precision, and error determination.
  - Many independent fitting parameters ($\sim$ 20)

Specifics of DSSV

DSSV, **Phys.Rev.D80:034030,2009**

“data selection”

“classic” inclusive DIS data

routinely used in PDF fits

! $\Delta q + \Delta \bar{q}$

semi-inclusive DIS data

so far only used in DNS fit

! flavor separation

first RHIC pp data (never used before)

! Δg

467 data pts in total (10% from RHIC)

experiment	data type	data points fitted
EMC, SMC	DIS	34
COMPASS	DIS	15
E142, E143, E154, E155	DIS	123
HERMES	DIS	39
HALL-A	DIS	3
CLAS	DIS	20
SMC	SIDIS, $h^{\pm}$	48
HERMES	SIDIS, $h^{\pm}$	54
	SIDIS, $\pi^{\pm}$	36
	SIDIS, $K^{\pm}$	27
COMPASS	SIDIS, $h^{\pm}$	24
PHENIX (in part prel.)	200 GeV pp, π^0	20
PHENIX (prel.)	62 GeV pp, π^0	5
STAR (in part prel.)	200 GeV pp, jet	19
TOTAL:		467

Marco Stratmann, Spin’08

Setup of DSSV

- Parametrization, defined at $Q_0^2 = 1$ GeV^2

$$x\Delta f_j(x, \underbrace{1\,\text{GeV}}_{\text{input scale}}) = N_j\, x^{\alpha_j}\,(1-x)^{\beta_j}\left[1 + \overbrace{\kappa_j\sqrt{x}}^{\text{possible nodes}} + \gamma_j x\right]$$

for sea quarks and delta g , simple forms $\kappa_j = 0$

- Strong coupling constant, α_s , from MRST, also use MRST for positivity bounds

$$|\Delta\sigma| \leq \sigma$$

- Positivity constraint for large x imposed via

$$|\Delta f| \leq f$$

Setup of DSSV

- avoid assumptions on parameters unless data cannot discriminate:

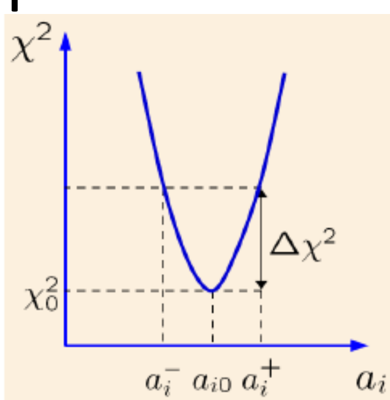
impose: $\alpha_{\bar{u}} = \alpha_{u+\bar{u}}$ $\alpha_{\bar{d}} = \alpha_{\bar{s}} = \alpha_{d+\bar{d}}$

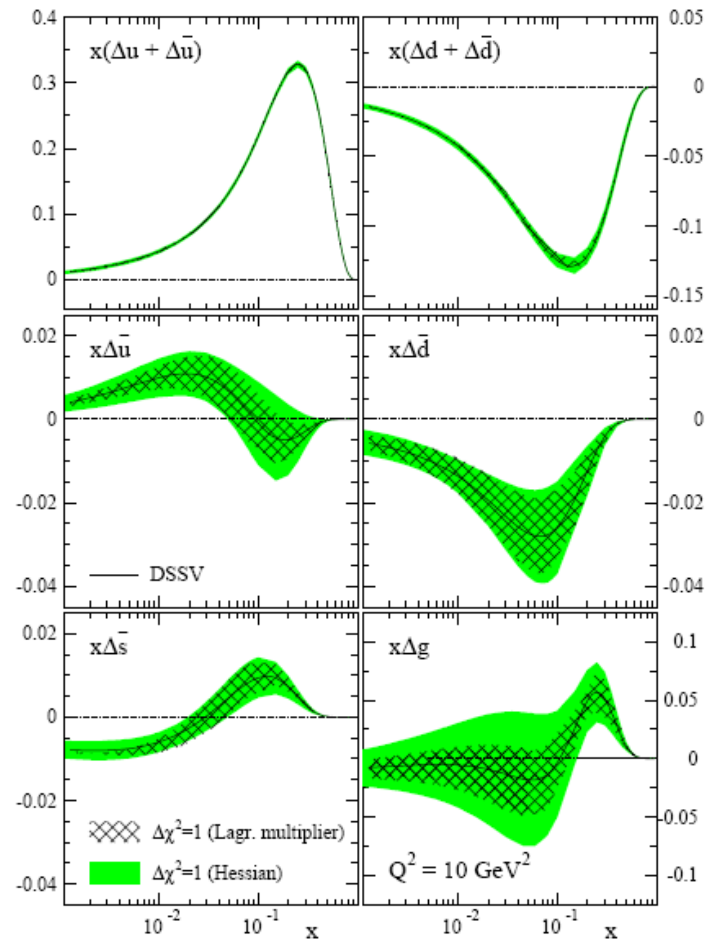
$$\Delta s = \Delta \bar{s}$$

- Large x , x--> 1, behavior is unconstrained, as there are no data sensitive to > ~0.6
- Allows for SU(3) symmetry breaking with a χ^2 penalty.

Estimating pdfs uncertainties from experiment

- Two methods:
  - Hessian : assumes gaussian errors, explores vicinity of χ^2_{min} in quadratic approx.
  - Lagrange multiplier: explores non parabolic χ^2 dependence on observable





Marco Stratmann, Spin'08

Lagrange multiplier in global analysis

- Minimize a new function,

$$\chi'^2 = \chi^2 + \lambda \, \Delta f^{[a,b]} \qquad \Delta f = \Delta\Sigma, \Delta G$$

- With "λ" as a Lagrange multiplier and "$\Delta f^{[a,b]}$" the moment of "f" in x range [a,b] ,

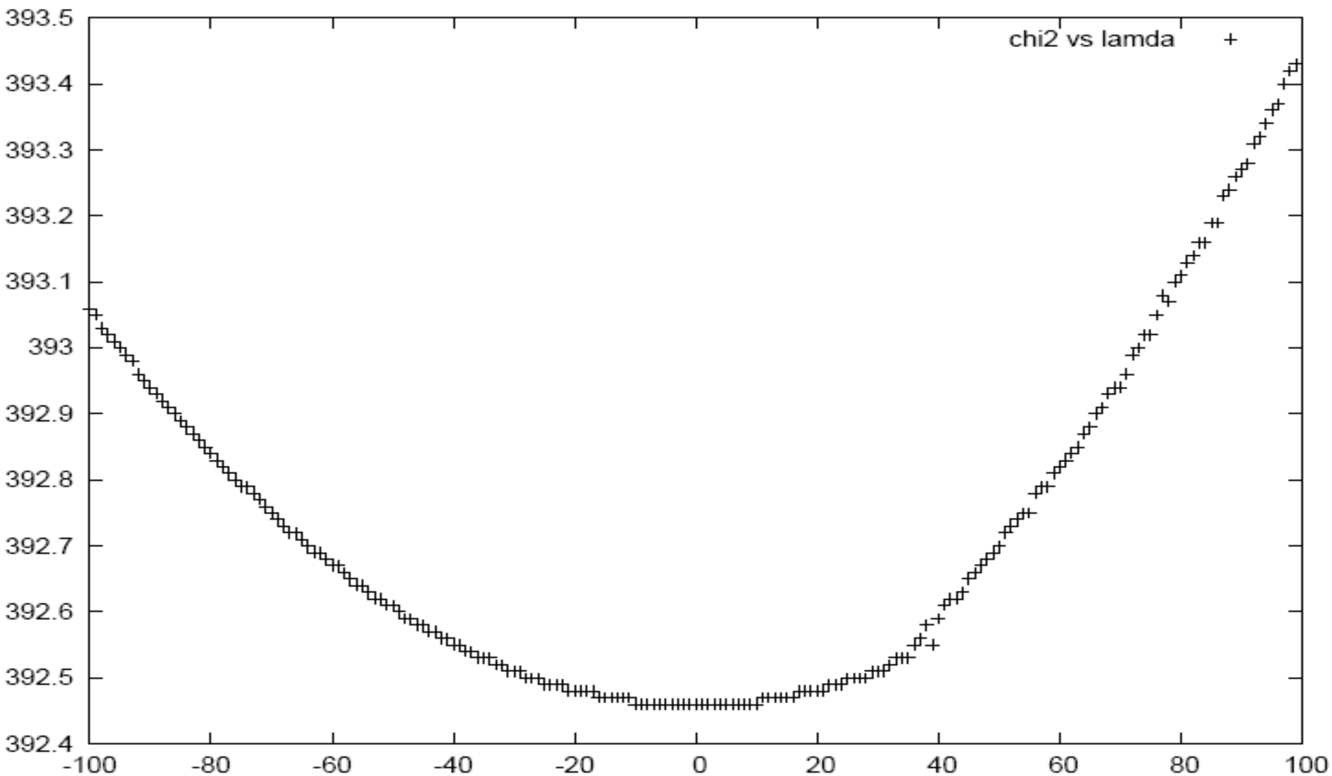
$$\Delta f^{[a,b]} \equiv \int_a^b dx \Delta f(x, \mu^2)$$

CTEQ, **JHEP 0207:012,2002**.

Studies & Results

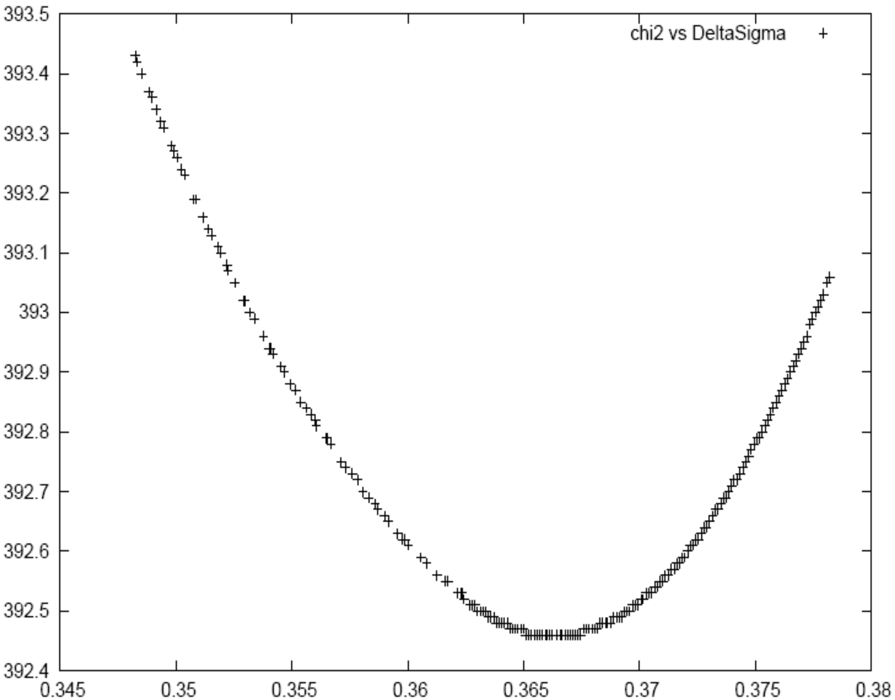
χ^2 distribution vs. ΔΣ (x range 0.001, 1)

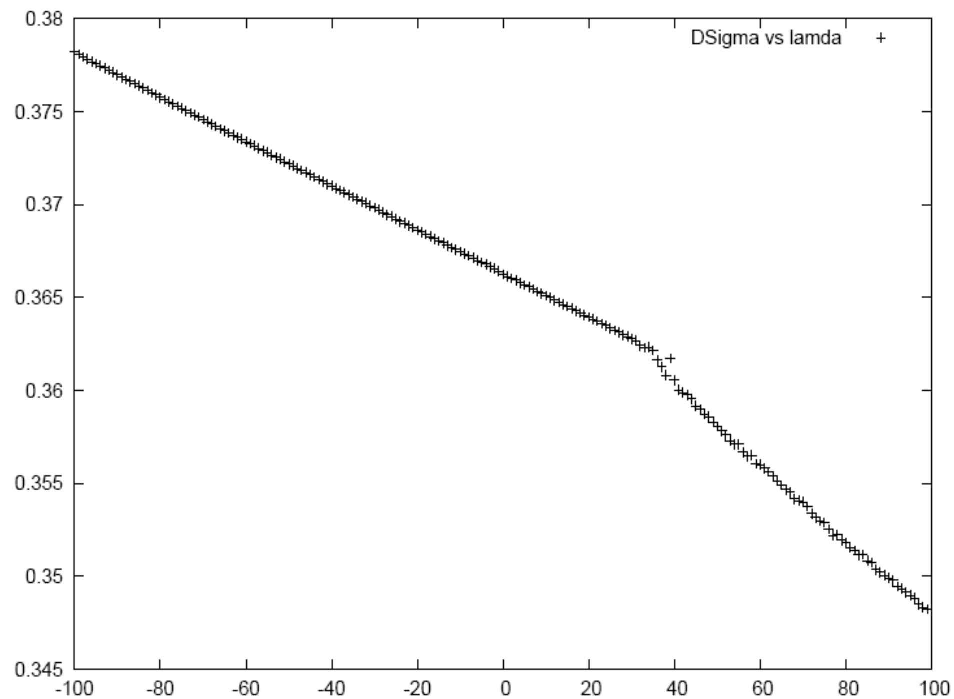
- $$\chi^2 = \sum_i \frac{(T_i - E_i)^2}{\delta E_i^2}$$ vs $$\Delta\Sigma^{[a,b]} \equiv \int_a^b dx \Sigma(x, \mu^2 = 10GeV^2)$$



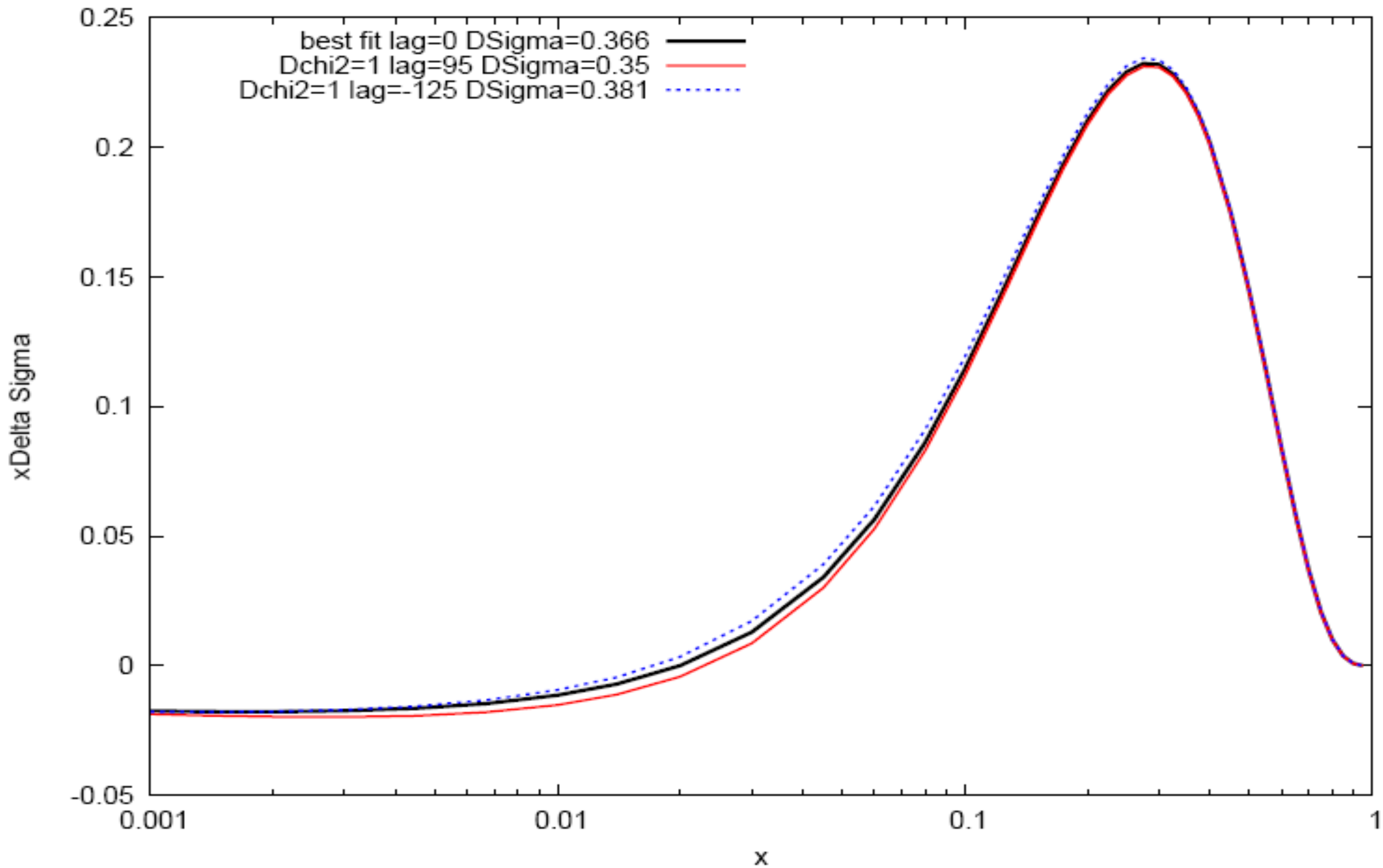
χ^2 distribution vs. ΔΣ (x range 0.001, 1)

- $\chi^2 = \sum_i \frac{(T_i - E_i)^2}{\delta E_i^2}$ vs $\Delta\Sigma^{[a,b]} \equiv \int_a^b dx \Sigma(x, \mu^2 = 10GeV^2)$



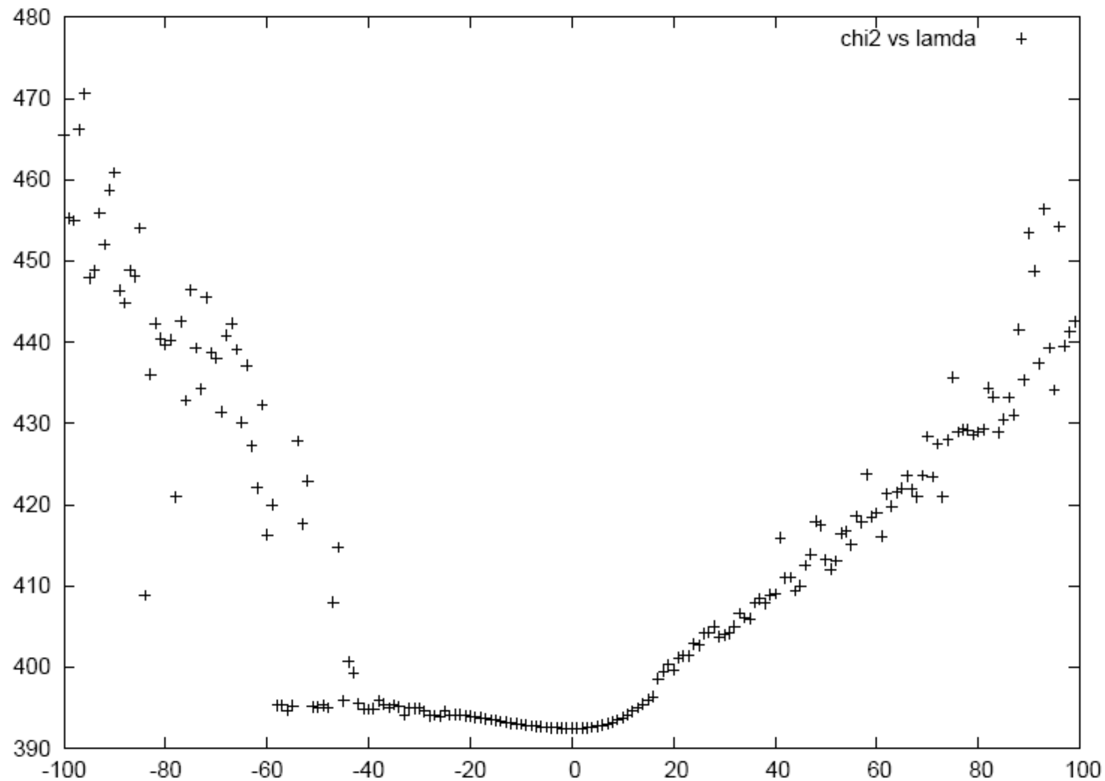


Polarized quark x distribution and uncertainties at $\Delta\chi^2 = 1$ from constraints on $\Delta\Sigma$ (x range 0.001, 1)



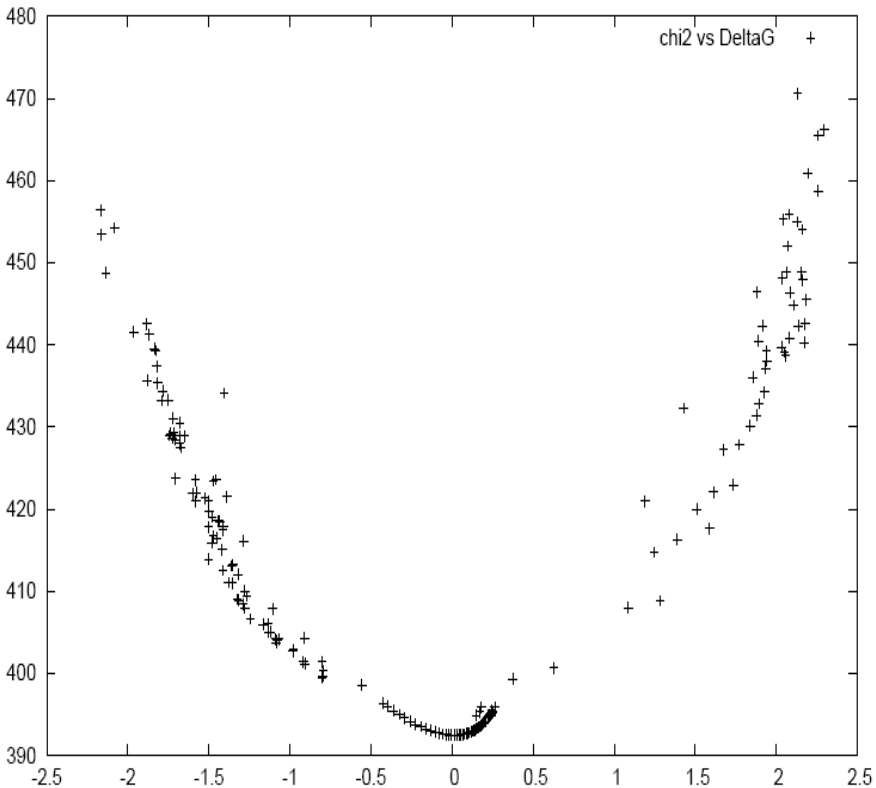
χ^2 distribution vs. ΔG (x range 0.001, 1)

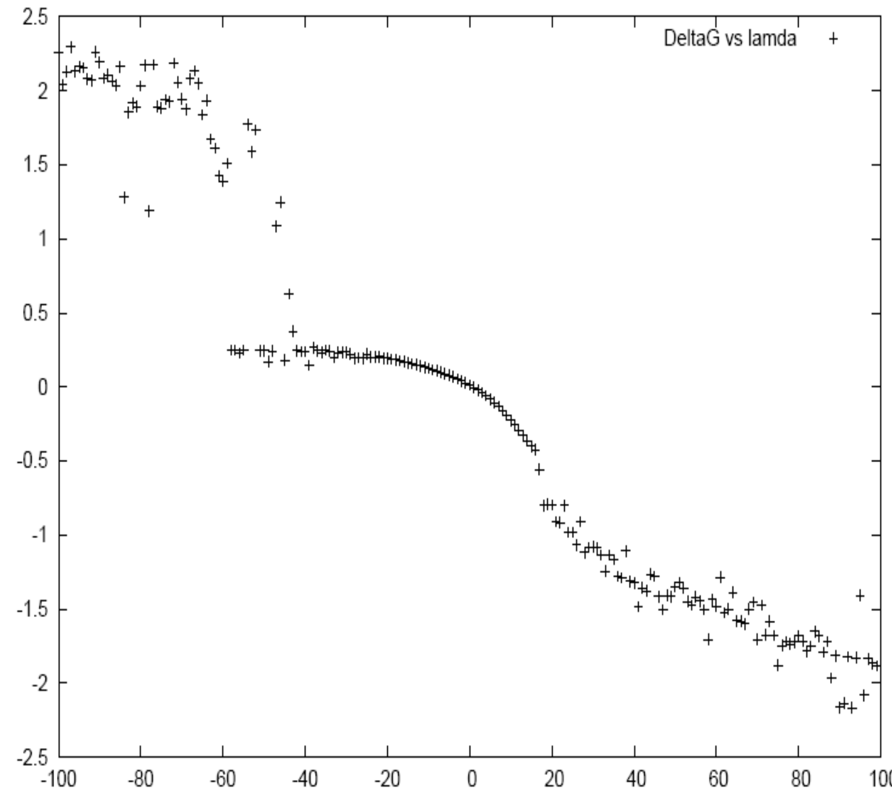
- $\chi^2 = \sum_i \frac{(T_i - E_i)^2}{\delta E_i^2}$ vs $\Delta G^{[a,b]} \equiv \int_a^b dx g(x, \mu^2 = 10 GeV^2)$



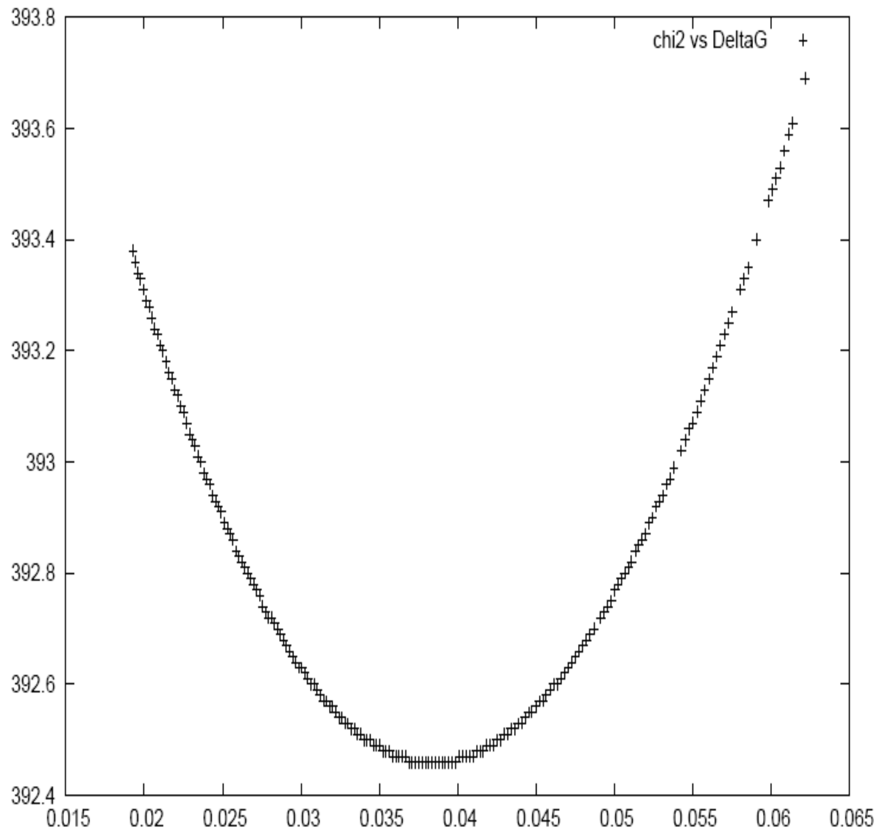
χ^2 distribution vs. ΔG (x range 0.001, 1)

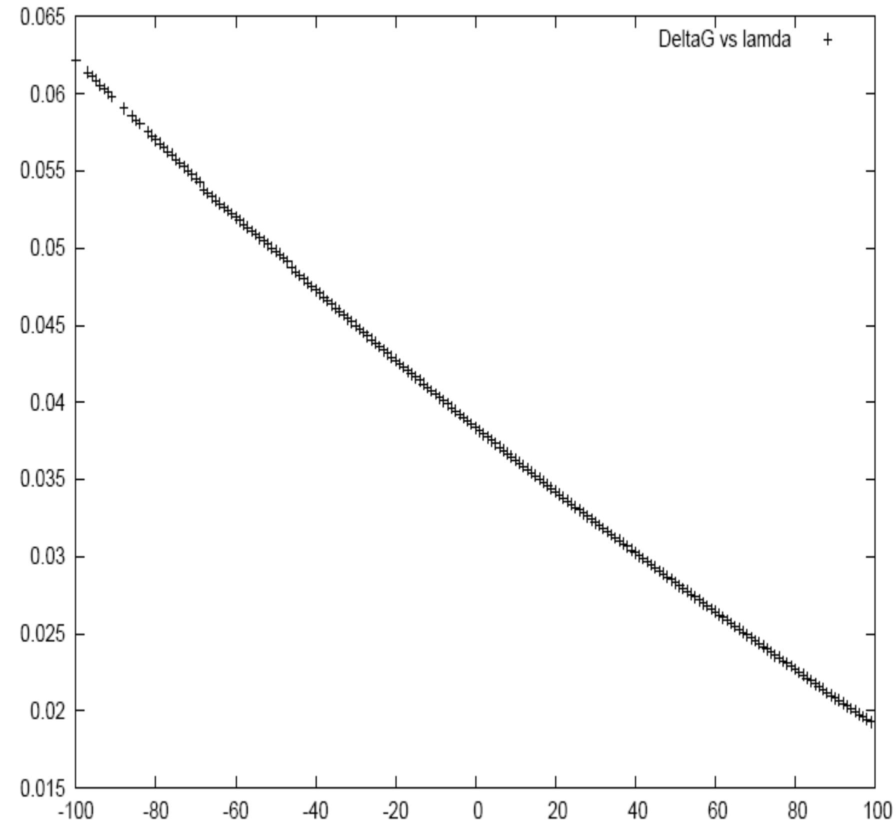
- $\chi^2 = \sum_i \frac{(T_i - E_i)^2}{\delta E_i^2}$ vs $\Delta G^{[a,b]} \equiv \int_a^b dx g(x, \mu^2 = 10GeV^2)$



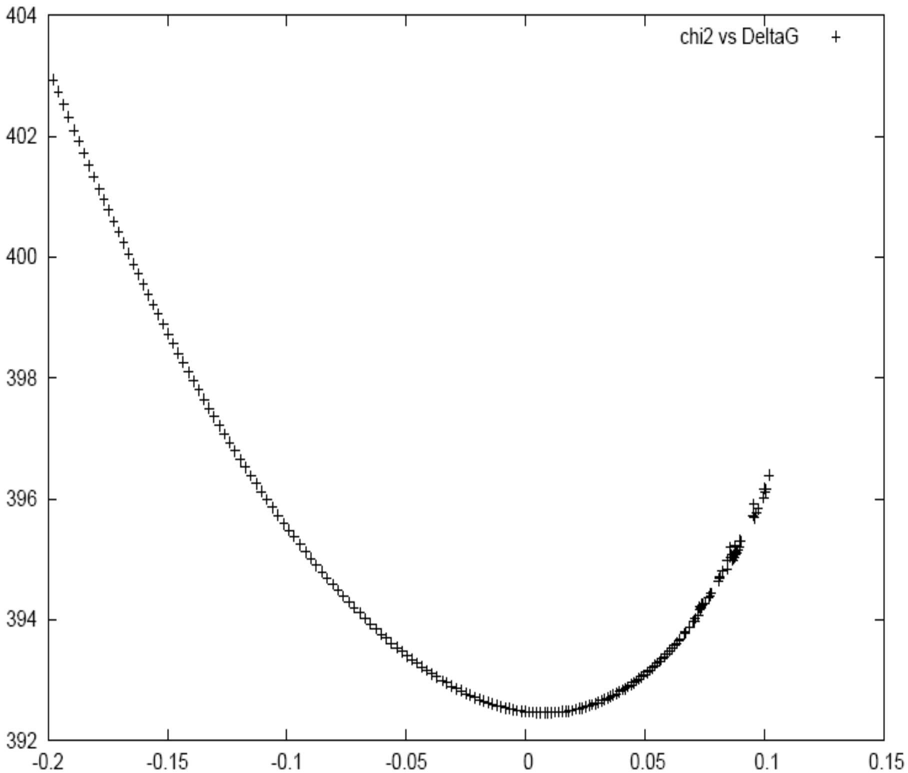


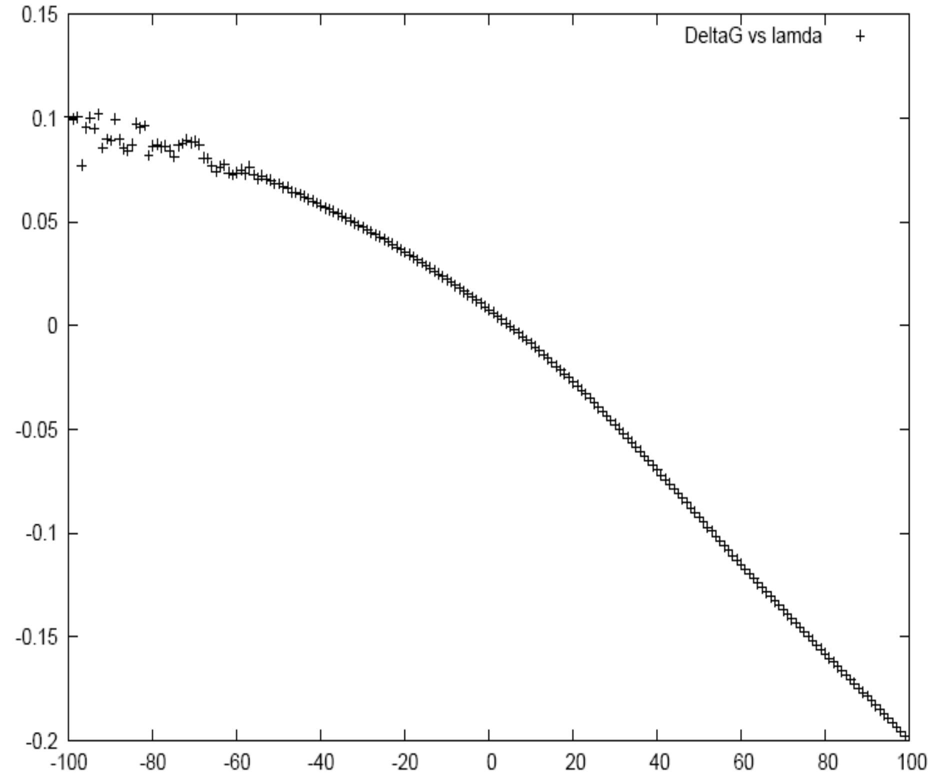
χ^2 distribution vs. ΔG (x range 0.2, 1)



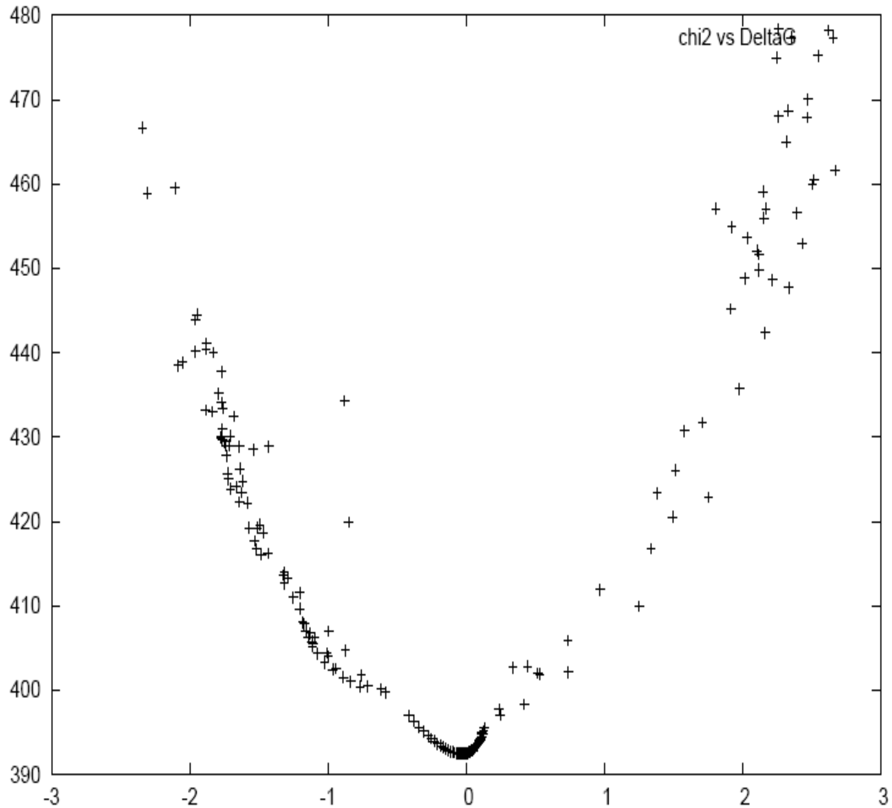


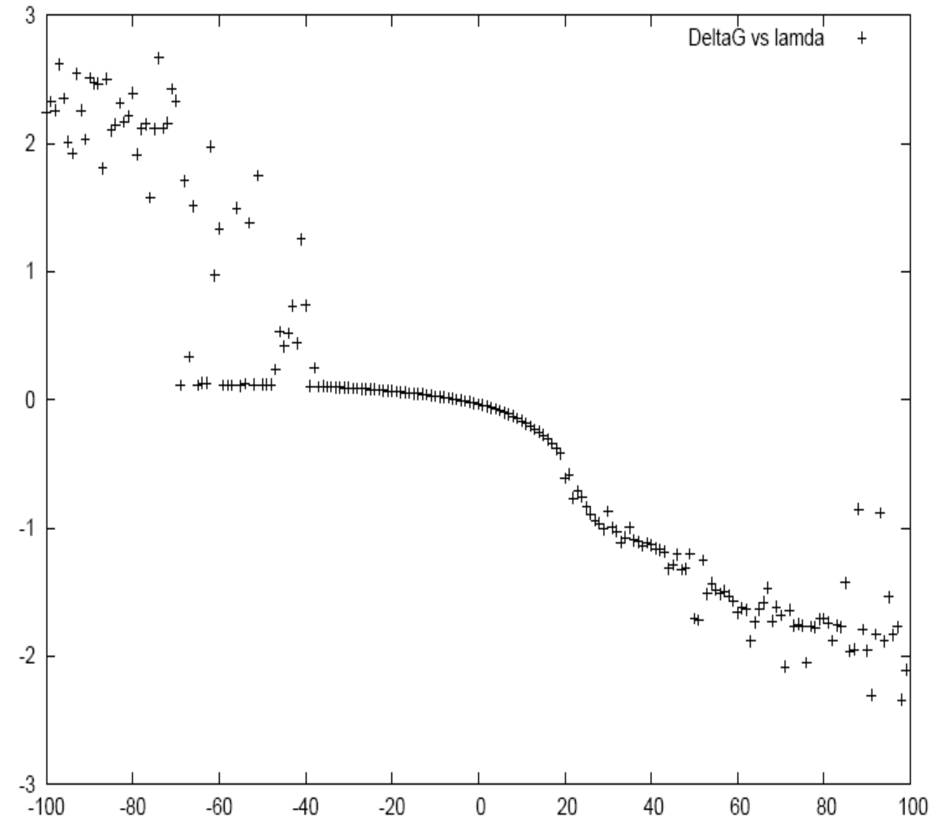
χ^2 distribution vs. ΔG (x range 0.05, 0.2)



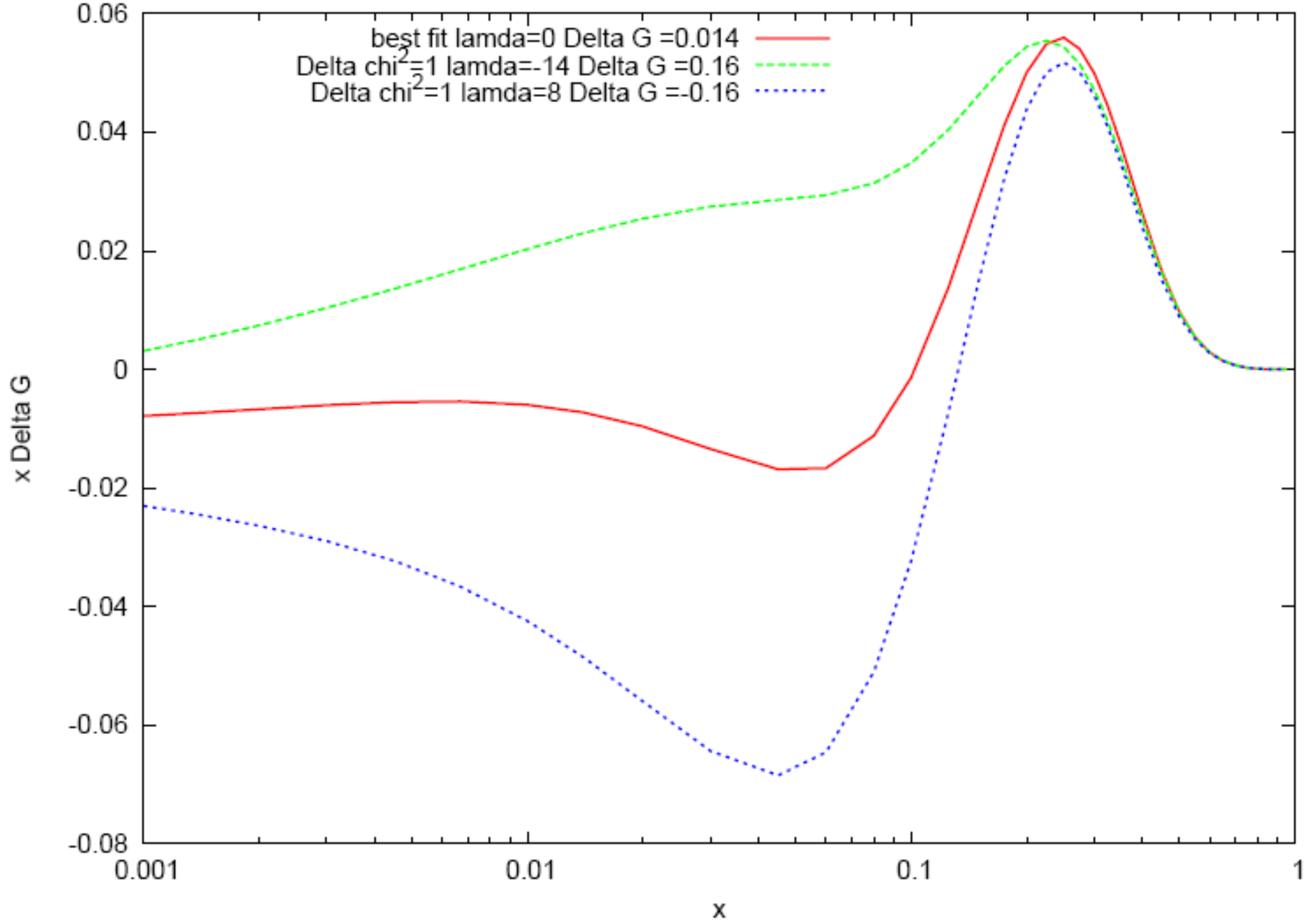


χ^2 distribution vs. ΔG (x range 0.001, 0.05)

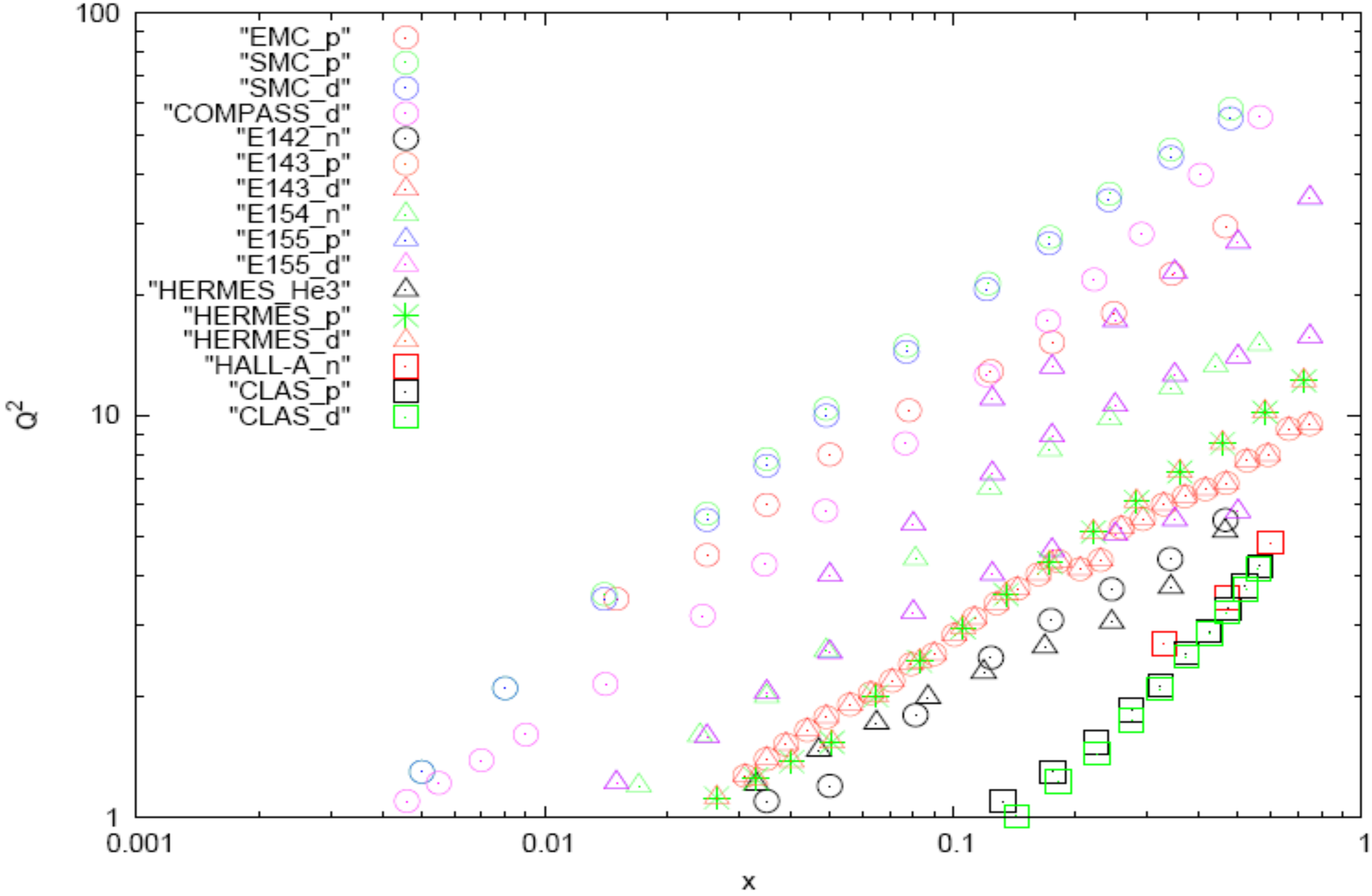




Polarized gluon x distribution and uncertainties at $\Delta\chi^2 = 1$ from constraints on ΔG (x range 0.001, 1)



Kinematic x, Q^2 range of used data sets

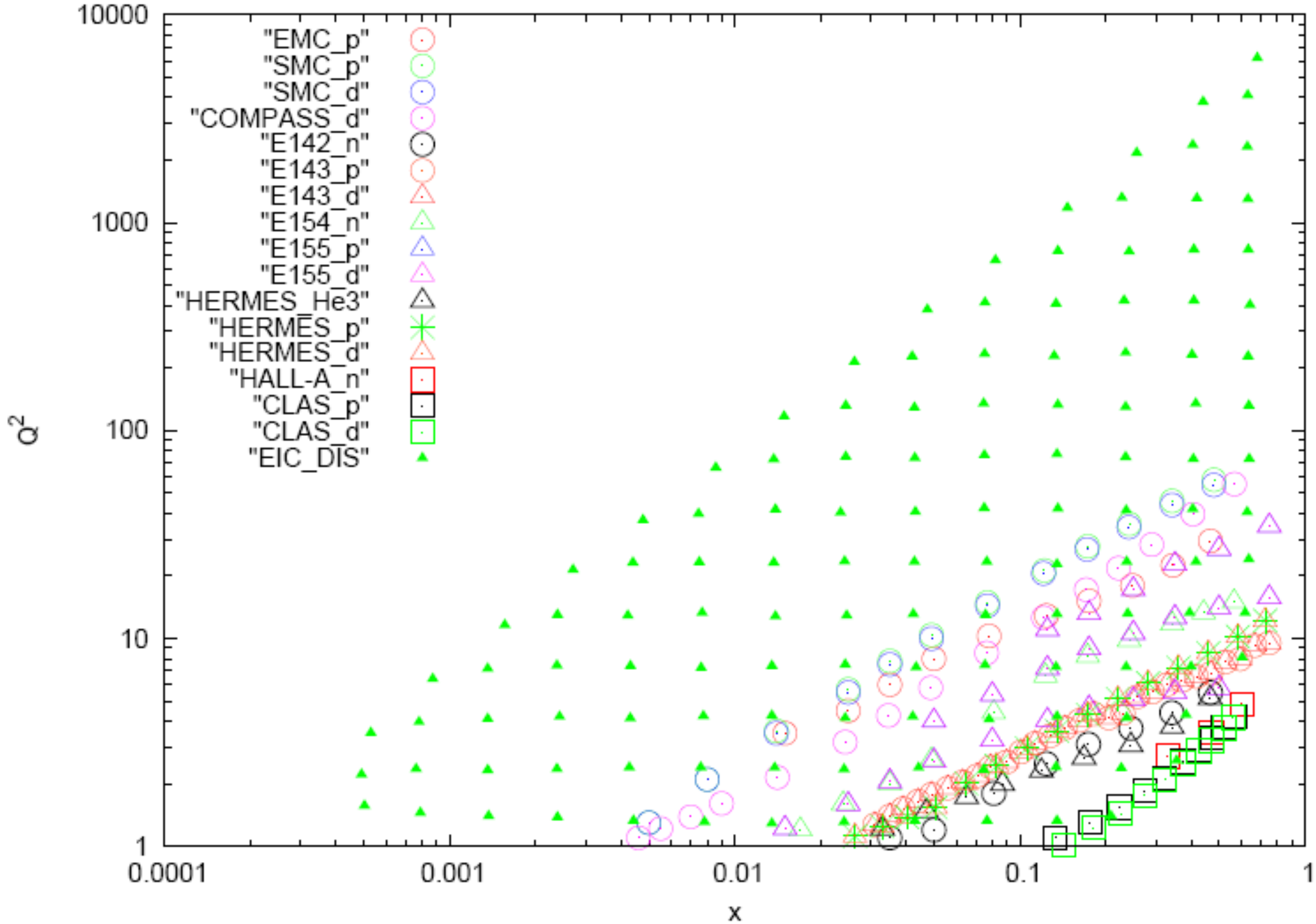


EIC Simulation

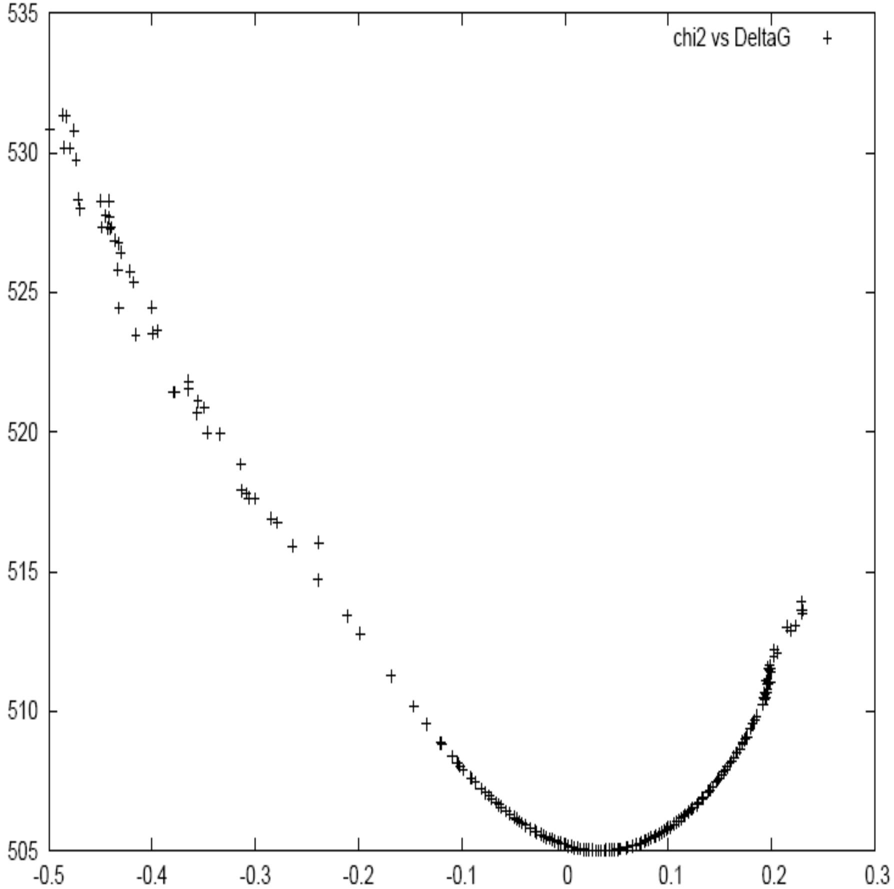
Michael Savastio

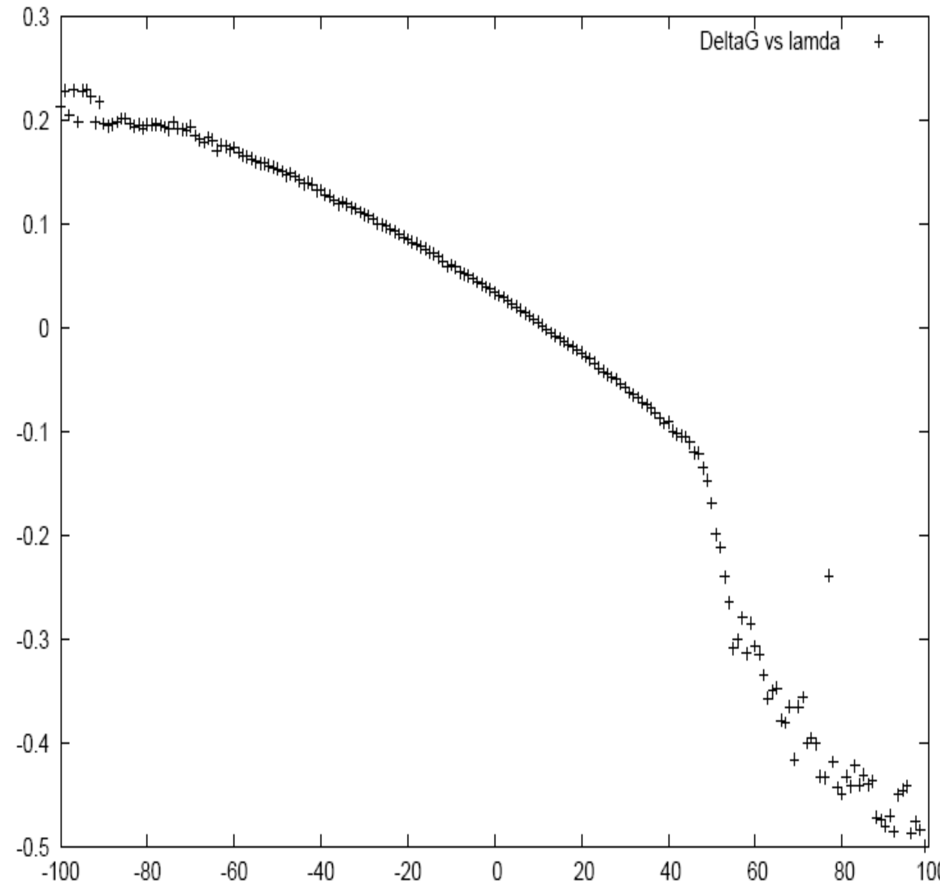
- Use PYTHIA to produce cross section weighted x and Q^2 bins. Energies used 10x250 GeV.
- Generate number of deep inelastic events, N, for a integrated luminosity of 6 fb^{-1} in those bins.
- Statistical uncertainty in each bin is then given by 1/ $\sqrt{N}$.
- Convert this to uncertainty in A_1^p .
- Generate new A_1^p by randomizing around the best fit from DSSV.

Kinematic x, Q^2 range of used data sets + EIC simulated data

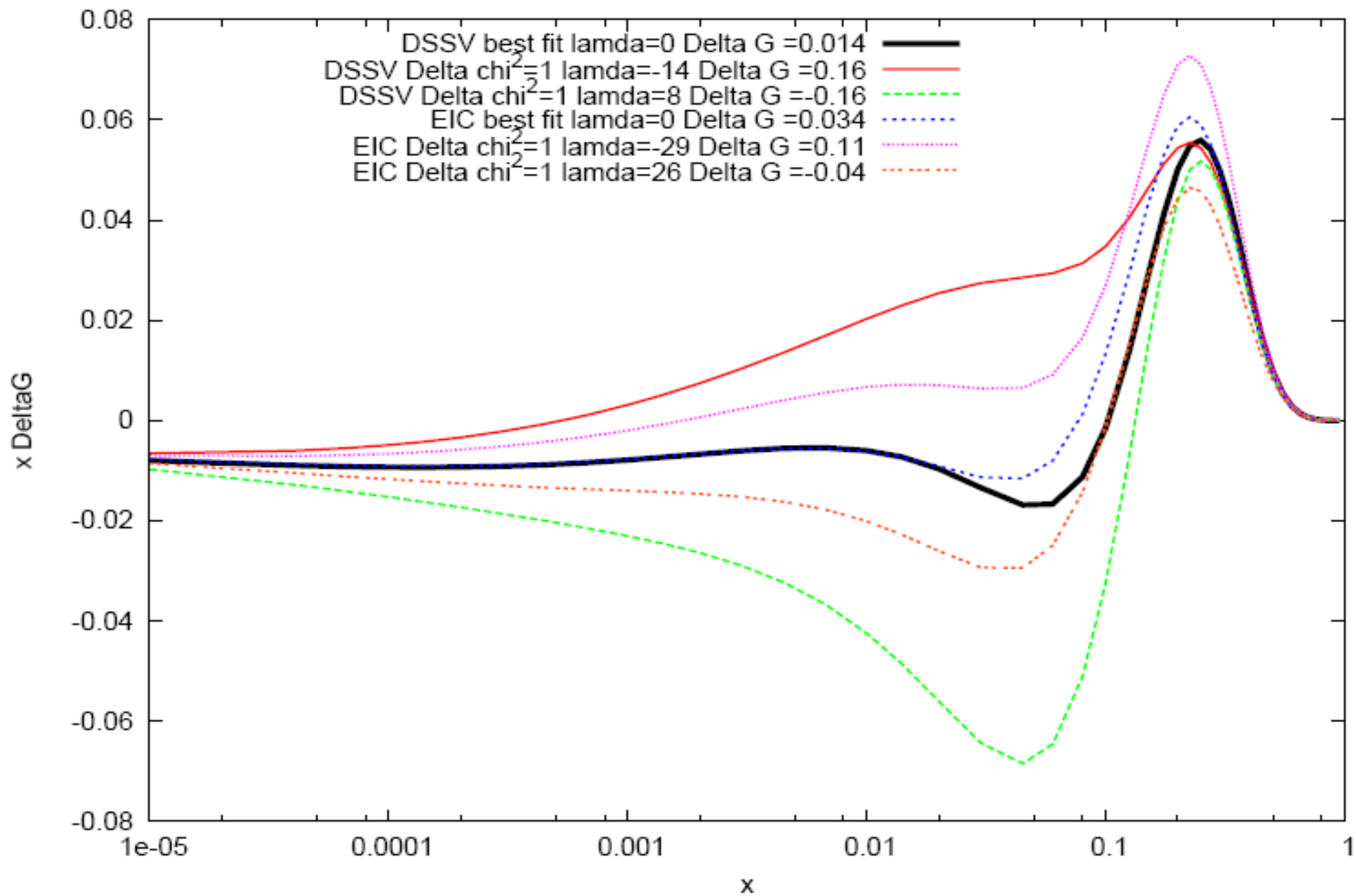


χ^2 distribution vs. ΔG (x range 0.001, 1) using EIC simulated data





Rough EIC implications for gluon polarization uncertainties



Effort to constrain x distribution of polarized gluon

- Ignores correlation between x regions.

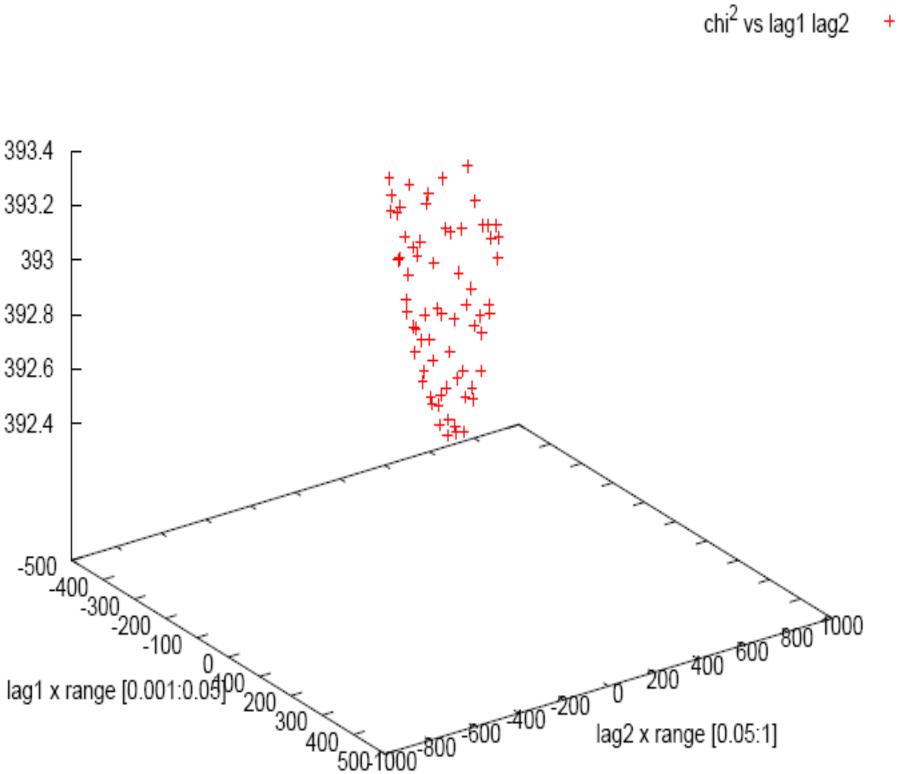
$$\chi'^2 = \chi^2 + \lambda \, \Delta f^{[a,b]}$$

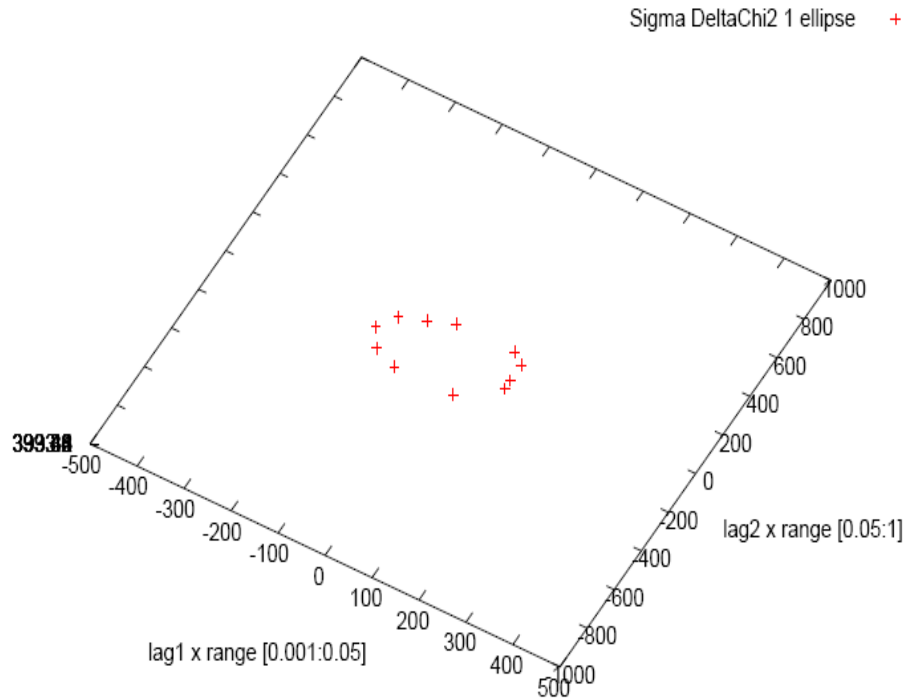
- Splitting the x region, meaningfully, and constraining these regions simultaneously

$$\chi'^2 = \chi^2 + \lambda_1 \Delta f^{[a,b_1]} + \lambda_2 \Delta f^{[b_1,b]}$$

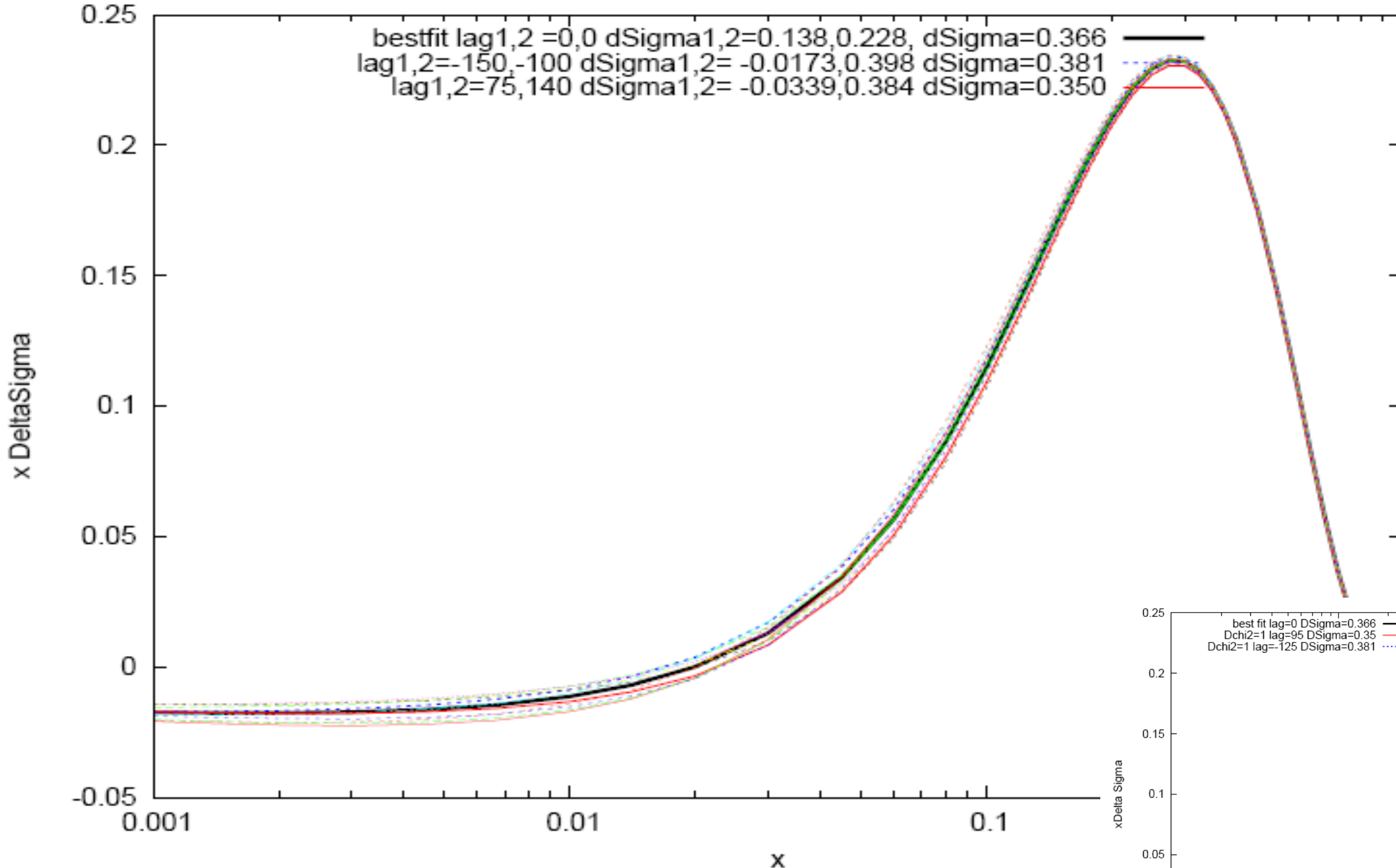
Note: Here forth only data included by DSSV is used.

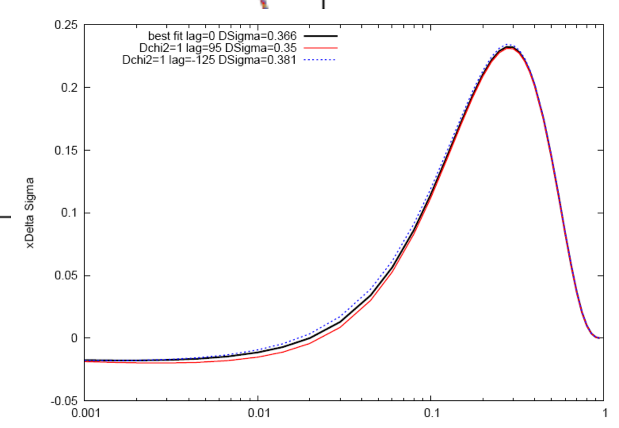
χ^2 distribution vs. λ_1, λ_2
($\Delta\Sigma$ constrained in x range [0.001, 0.05] and [0.05, 1])



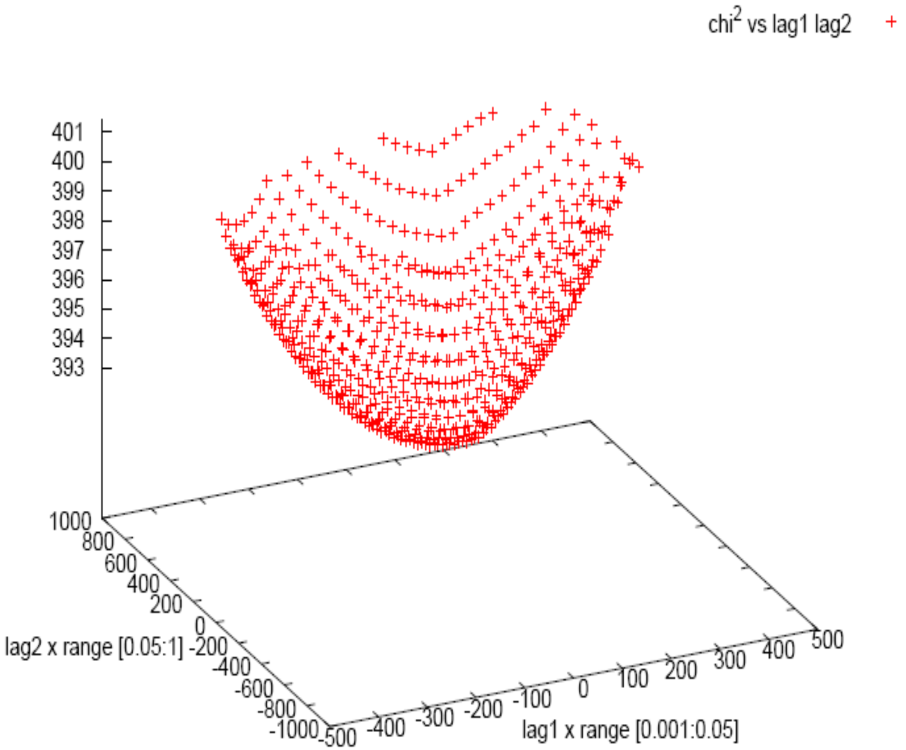


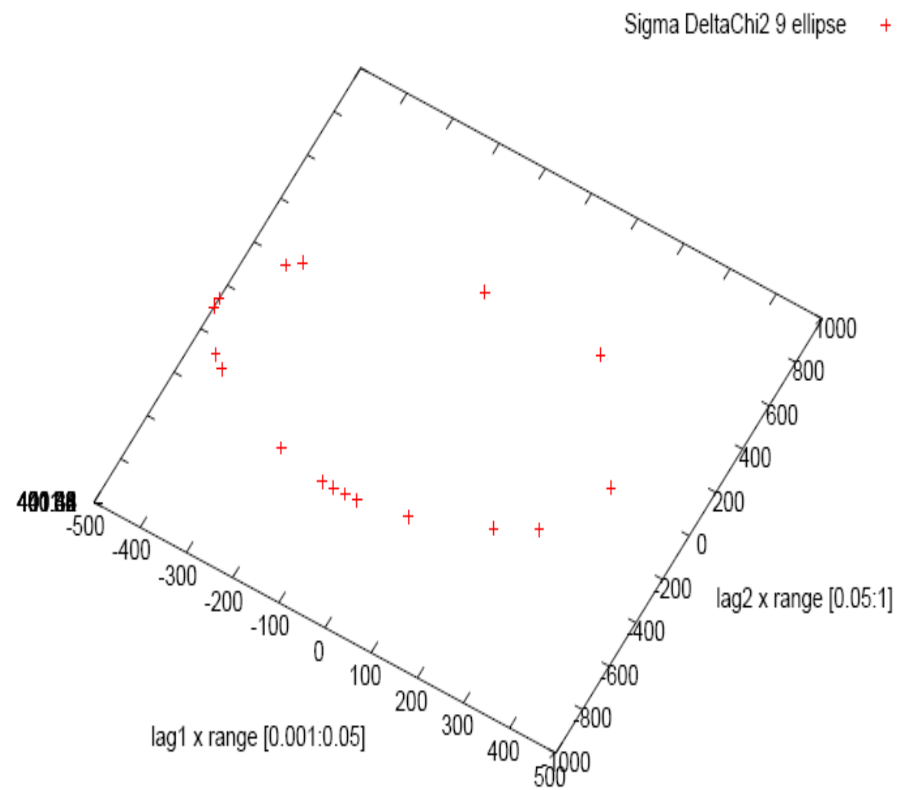
Polarized quarks x distribution and uncertainties at $\Delta\chi^2 = 1$ from constraints on ΔΣ's (x range [0.001, 0.05] and [0.05, 1])



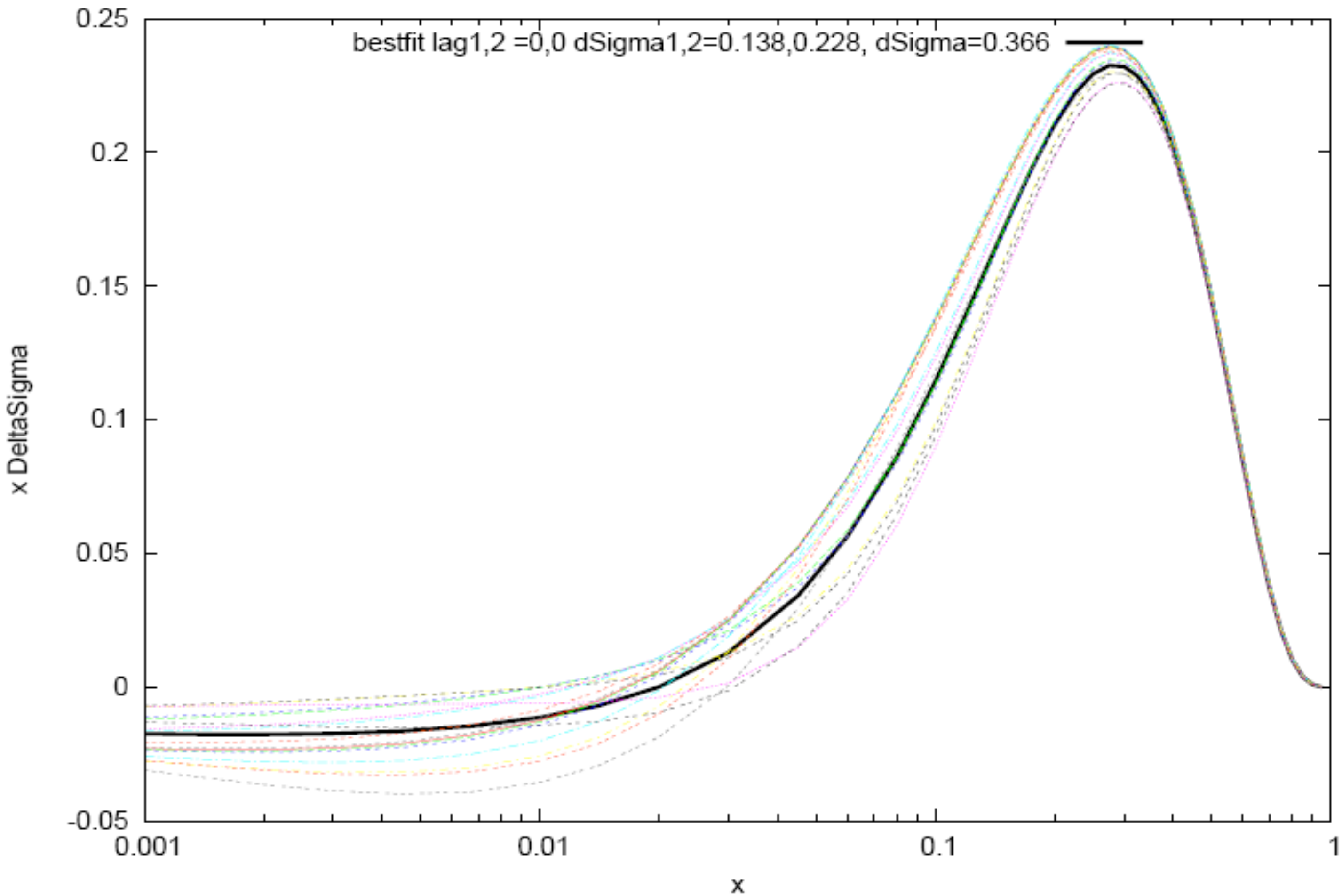


χ^2 distribution vs. λ_1 , λ_2 at $\Delta \chi^2 = 9$ level
($\Delta\Sigma$ constrained in x range [0.001, 0.05] and [0.05, 1])

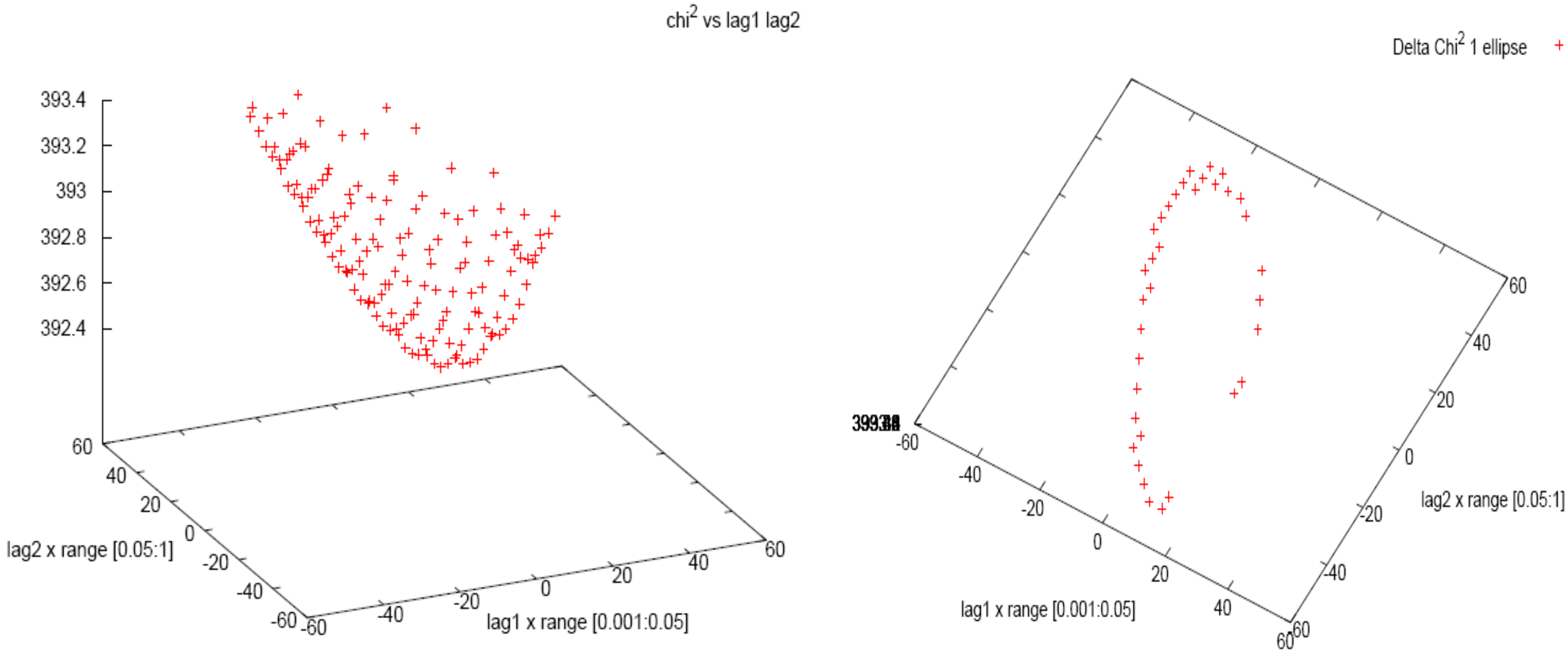




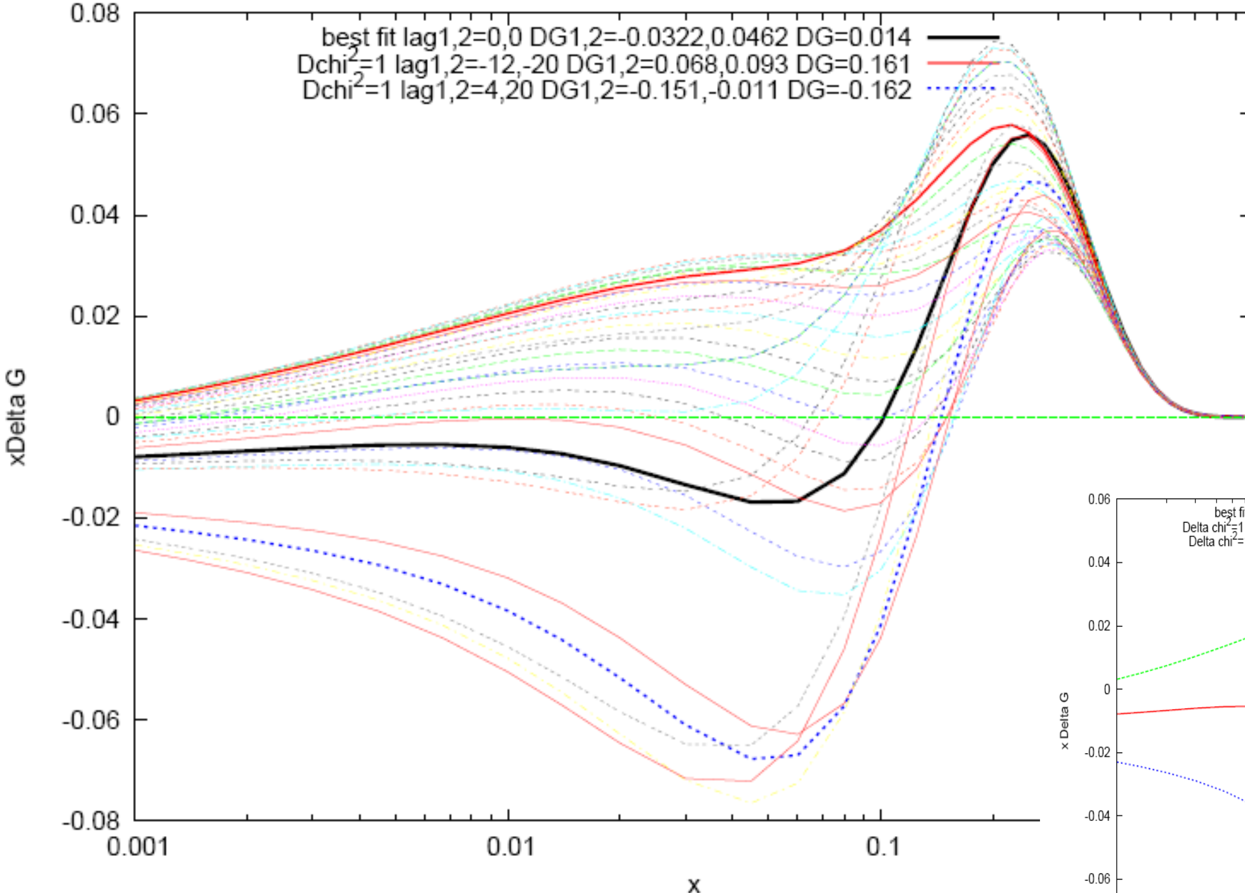
Polarized quarks x distribution and uncertainties at $\Delta\chi^2 = 9$ from constraints on ΔΣ's (x range [0.001, 0.05] and [0.05, 1])

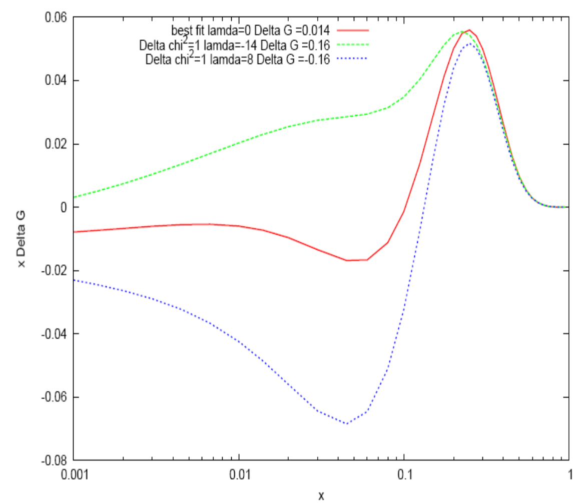


χ^2 distribution vs. λ_1 , λ_2 (ΔG constrained in x range [0.001, 0.05] and [0.05, 1])

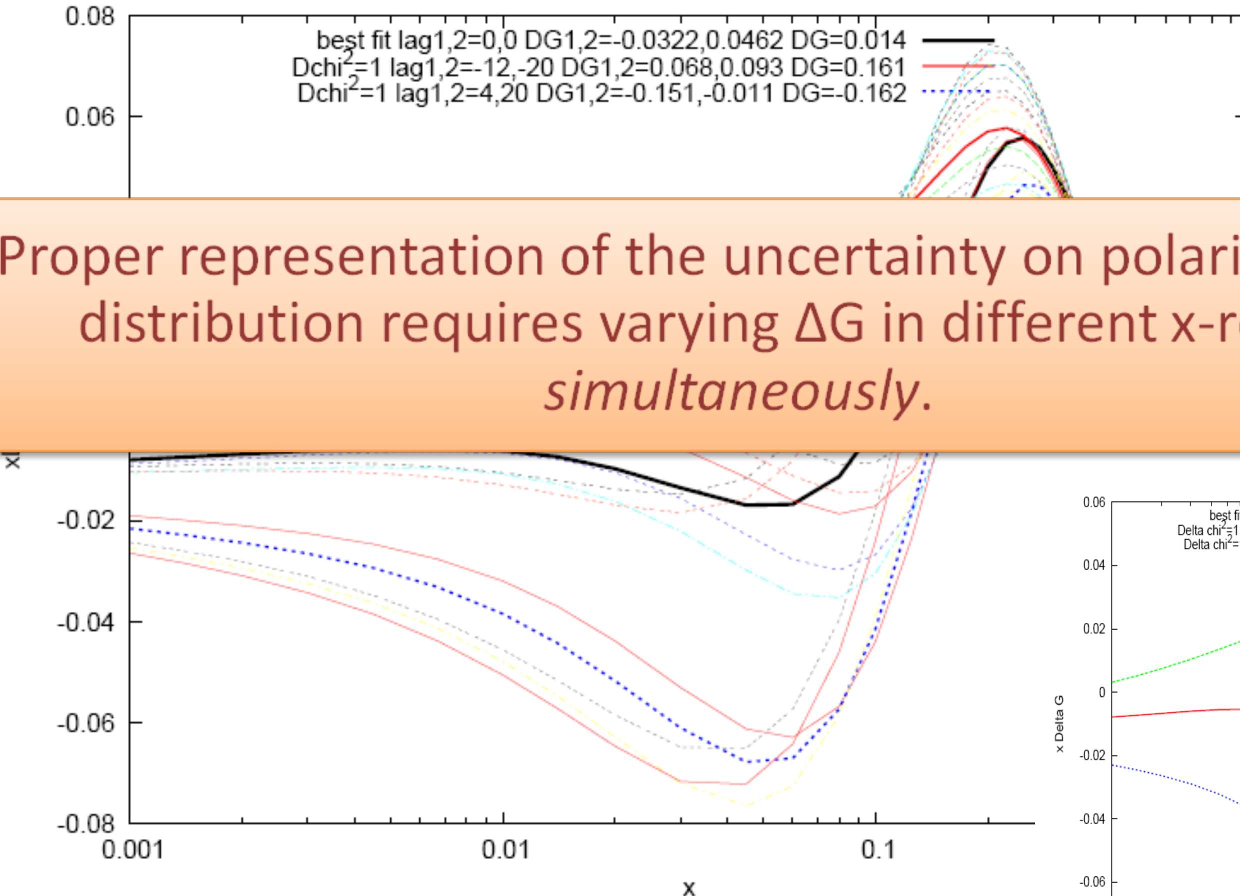


Polarized gluon x distribution and uncertainties at $\Delta\chi^2 = 1$ from constraints on ΔG's (x range [0.001, 0.05] and [0.05, 1])

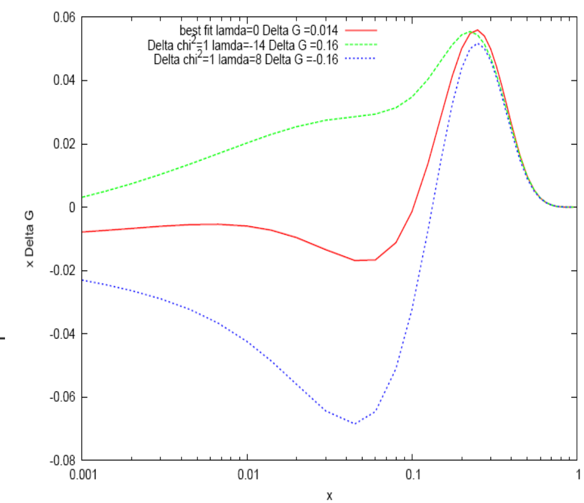




Polarized gluon x distribution and uncertainties at $\Delta\chi^2 = 1$ from constraints on ΔG's (x range [0.001, 0.05] and [0.05, 1])



Proper representation of the uncertainty on polarized gluon distribution requires varying ΔG in different x-regions *simultaneously*.



Outlook

- Include experimental systematic uncertainty properly (K. Boyle, A. Deshpande, E. Aschenauer).
- Other uncertainties (DSSV, KB and AD)
  - alpha strong, parameterization, energy scales.
  - role of higher twists.
- Include new sets of data e.g. charged pion, direct photon, di-jet from RHIC, new COMPASS SIDIS data.

Back up

$$\gamma = \frac{2Mx}{\sqrt{Q^2}} = \frac{\sqrt{Q^2}}{\nu}.$$

$$\frac{A_{\parallel}}{D} = (1+\gamma^2)\,\frac{g_1}{F_1}$$

$$\Delta u_{\text{tot}} - \Delta d_{\text{tot}} = (F+D)[1+\varepsilon_{\text{SU(2)}}]$$

1.269±0.003

fitted (end up close to zero)

$$\Delta u_{\text{tot}} + \Delta d_{\text{tot}} - 2\Delta s_{\text{tot}} = (3F-D)[1+\varepsilon_{\text{SU(3)}}]$$

0.586±0.031

$$\Delta\Sigma \equiv \Sigma_u + \Sigma_d + \Sigma_s = (3F-D) + 3\Delta\Sigma_s$$

TABLE II: Parameters $\{a_i^0\}$ describing our optimum NLO ($\overline{\mathrm{MS}}$) $x\Delta f_i$ in Eq. (28) at the input scale $\mu_0 = 1\,\mathrm{GeV}$.

flavor i	N_i	α_i	β_i	γ_i	η_i
$u+\bar{u}$	0.677	0.692	3.34	-2.18	15.87
$d+\bar{d}$	-0.015	0.164	3.89	22.40	98.94
$\bar{u}$	0.295	0.692	10.0	0	-8.42
$\bar{d}$	-0.012	0.164	10.0	0	98.94
$\bar{s}$	-0.025	0.164	10.0	0	-29.52
g	-131.7	2.412	10.0	0	-4.07

Lagrange Multiplier

- Minimize a function *while* maintaining a constraint.
- For example , maximize/minimize this:

$$f(x,y) = x + y$$ (Function)

$$x^2 + y^2 = 1$$ (Constraint)

$$f(\sqrt{2}/2, \sqrt{2}/2) = \sqrt{2} \text{ and } f(-\sqrt{2}/2, -\sqrt{2}/2) = -\sqrt{2},$$

Lagrange Multiplier

- New Function

$$\Lambda(x,y,\lambda) = f(x,y) + \lambda(g(x,y) - c) = x + y + \lambda(x^2 + y^2 - 1)$$

Set the derivative $d\Lambda = 0$, which yields the system of equations:

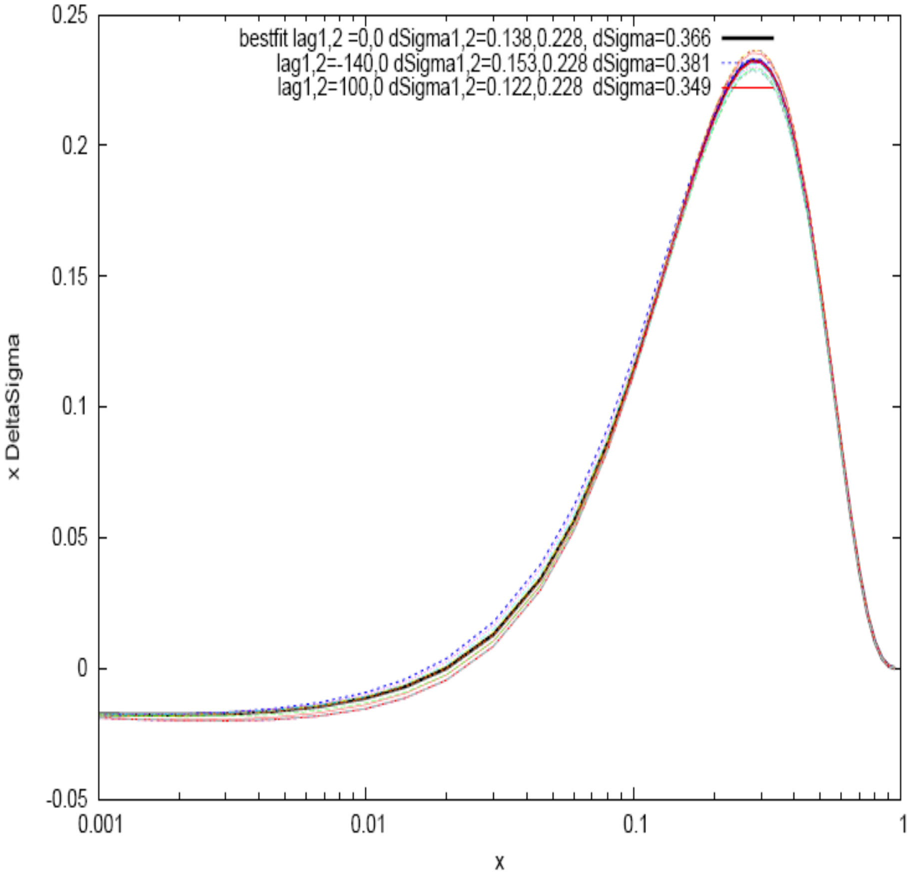
$$\frac{\partial \Lambda}{\partial x} = 1 + 2\lambda x = 0, \quad \text{(i)}$$

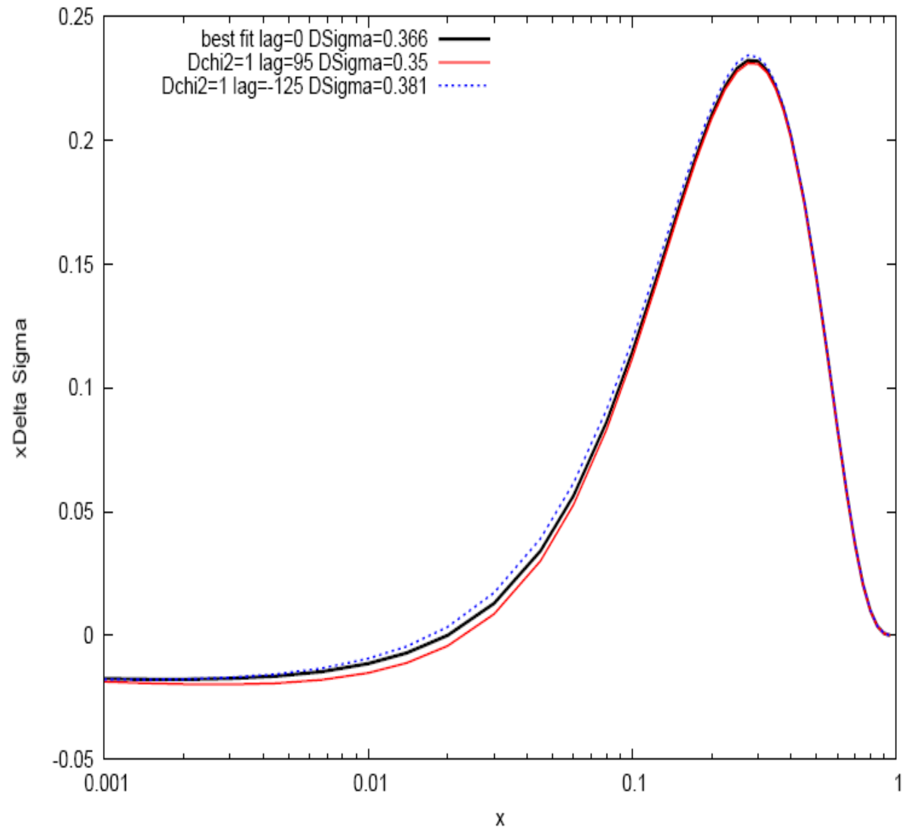
$$\frac{\partial \Lambda}{\partial y} = 1 + 2\lambda y = 0, \quad \text{(ii)}$$

$$\frac{\partial \Lambda}{\partial \lambda} = x^2 + y^2 - 1 = 0, \quad \text{(iii)}$$

$$f(\sqrt{2}/2, \sqrt{2}/2) = \sqrt{2} \text{ and } f(-\sqrt{2}/2, -\sqrt{2}/2) = -\sqrt{2}, \qquad \lambda = -\frac{1}{\sqrt{2}}$$

Polarized quarks x distribution and uncertainties at $\Delta\chi^2 = 1$ from constraints on $\Delta\Sigma$'s (x range [0.001, 0.2] and [0.2, 1])





Polarized quarks x distribution and uncertainties at $\Delta\chi^2 = 9$ from constraints on $\Delta\Sigma$'s (x range [0.001, 0.2] and [0.2, 1])